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Cámpus de Ilha Solteira

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UM MODELO BI-NÍVEL PARA O INVESTIMENTO MULTI-ETAPA EM CAPACIDADE DE TRANSMISSÃO E GERAÇÃO DE ENERGIA ELÉTRICA EM MERCADOS COMPETITIVOS

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Especialidade: Automação.

Prof. Dr. Marcos Julio Rider Flores
Orientador

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CERTIFICADO DE APROVAÇÃO

TÍTULO: Un modelo binivel para la inversión multietapa en capacidad de transmisión y generación de energía eléctrica en mercados competitivos

AUTOR: JORGE HANS ALAYO GAMARRA
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To my mother Maruja, for all she gave me.

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- God, for giving me the force and health;
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- My wife Miriam, for her love;
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The future has many names:

For the weak, it means the unattainable.

For the fearful, it means the unknown.

For the courageous, it means opportunuty. Victor Hugo

RESUMO

Esta dissertação de mestrado discute sobre o investimento em capacidade de transmissão em mercados elétricos competitivos e sua relação com o investimento em capacidade de geração. O principal problema que surge ao descentralizar os investimentos são as externalidades, devido à dependência entre os custos de oportunidade da capacidade transmissão e geração. As externalidades são distorções no mercado que se apresentam quando as decisões de um agente afetam o bem-estar de outro agente, porém não vice-versa. Em presença de externalidades não é válido afirmar que a solução descentralizada seja igual ao resultado centralizado (ótimo de Pareto). Para solucionar este problema deve-se implementar um processo descentralizado através de esquemas regulatórios. Logo, é proposto um modelo binível que procura o ótimo de Pareto e serve como referência para implementação de esquemas regulatórios que permitam avaliar os investimentos em capacidade de transmissão e geração em um mercado competitivo. O modelo binível proposto é transformado em um problema de programação linear inteira mista, usando a teoria de dualidade e técnicas de linearização. O modelo proposto foi implementado em AMPL e solucionado usando o solver comercial CPLEX. Finalmente, são apresentados os resultados obtidos para dois sistemas testes e um sistema real.

Palavras-chave: Planejamento da expansão de sistema de transmissão. Modelos de programação binível. Mercados elétricos. Externalidades.

ABSTRACT

This dissertation discusses about the investing in transmission capacity in competitive electricity markets and its relation with the investing in generation capacity. Since opportunity costs of transmission and generation capacity are dependent, externalities arise when the investments decisions are decentralized. Externalities are present whenever a decision of a certain agent affects another agent's welfare but not vice versa. In presence of externalities, the decentralized outcome does not lead to a Pareto optimal solution. In order to overcome this problem, the Pareto optimal solution should be found, and set in the market by means of regulation. Moreover, a bi-level multistage model is proposed, which finds the Pareto optimal solution. This solution can be used as reference for the implementation of regulatory mechanisms that asses the investments in transmission and generation capacity. The proposed bi-level model is transformed into a mixed integer linear problem using duality theory and linearization techniques. Finally, the proposed model is implemented in AMPL and solved using CPLEX; results for several study cases are presented.

Keywords: Transmission planning. Bi-level models. Electricity markets. Externalities.

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LIST OF ABBREVIATIONS

CD	Cost of debt.
CP	Cost of generation project.
CS	Consumer surplus.
FC	Fixed cost.
I	Income.
IC	Investment cost.
K	Capital.
KKT	Karush Kuhn Tucker.
O & M	Operation and manteinnance cost .
OPF	Optimal Power Flow.
PS	Producer surplus.
PV	Present Value.
TPP	Transmission planning problem.
W	Welfare.
VC	Variable cost.

LIST OF SYMBOLS

Index:

a	Index for the set of consumers.
i	Index for the set of nodes.
ij	Index for the branch between nodes $i-j$ for the set of branches.
k	Index for the set of generators.
o	Index for the set of demand levels.
q	Index for the set of candidate transmission lines belonging to a branch.
s	Index for the set of generation technologies.
t	Index for the set of stages.
u	Index for the set of candidate generation units belong to a technology.
v	Index for the set of generation firms.

Sets:

Y	Production set for the firm.
Ω^1	Set of binary variables which represent the investment in generation units for a specific technology.
Ω^2	Set of binary variables which represent the investment in transmission circuits in a specific branch.
Ω^D	Set of demand levels.
Ω^G	Set of existing and future generators.
Ω_i^G	Set of existing generators connected at node i.
Ω^{G0}	Set of existing generators with fixed capacity.
Ω_i^{G0}	Subset of generators from Ω^{G0} connected at node i.
Ω^{G1}	Set of generators whose initial capacity can be increased continuously.
Ω_i^{G1}	Subset of generators from Ω^{G1} connected at node i.
Ω^{L0}	Set of branches with existing transmission circuits.
Ω_i^{L0}	Subset of branches from Ω^{L0} connected at node i.

$\Omega^{L'}$	Set of candidate branches.
$\Omega_i^{L'}$	Subset of branches from $\Omega^{L'}$ connected at node i.
Ω^{L}	Set of existing and candidate branches $\Omega^{L'} \cup \Omega^{L0}$.
Ω_i^{L}	Subset of branches from Ω^{L} connected at node i.
Ω^{N}	Set of nodes.
Ω^{C}	Set of consumers.
Ω_i^{C}	Set of consumers connected at node i.
Ω^{S}	Set of candidate technologies with fixed capacity generation units.
Ω^{S2}	Set of candidate technologies whose initial capacity can be increased continuously.
Ω_i^{S}	Subset of generation units from Ω^{S} connected at node i.
Ω^{T}	Set of stages.
Ω^{V}	Set of generation firms.
Ω_v^{G0}	Subset of generators from Ω^{G0} belonging to firm v.
Ω_v^{G1}	Subset of generators from Ω^{G1} belonging to firm v.

Functions:

$C(x)$	Cost function.
$C(\overline{f})$	Transmission capacity investment cost.
$C(g)$	Operation cost.
$C(\overline{g})$	Generation capacity investment cost.
x_i^d	Demand function of good i.
x_i^o	Supply function of good i.
$U(x)$	Utility function for the consumer.
W	Social welfare function.
π	Profit function for the firm.

Variables:

d_a	Demand for consumer a.
f_{ij}	Active power flow in branch ij.
$f_{ij,t}$	Active power flow in branch ij, at stage t.
$f_{ij,o}$	Active power flow in branch ij, at demand level o.

$f_{ij,o,t}$	Active power flow in branch ij, for demand level o, at stage t.
$f_{q,ij,o,t}$	Active power flow at transmission circuit q, at branch ij, at demand level o, at stage t.
$f_{ij}^{max,0}$	Initial capacity for line ij, whose capacity can be increased continuously.
f_{ij}^{max}	Final capacity for line ij, whose capacity can be increased continuously.
g_k	Active power produced by generator k.
$g_{k,o}$	Active power produced by generator k, at demand level o.
$g_{k,o,t}$	Active power produced by generator k, at demand level o, at stage t.
$g_{s,i,o}$	Active power produced by technology s, at node i, at demand level o.
$g_{u,s,i,o,t}$	Active power produced by generator u, with technology s, at node i, at demand level o, at stage t.
$g_k^{max,0}$	Initial capacity of generator k, whose capacity can be increased continuously.
g_k^{max}	Final capacity of generator k, whose capacity can be increased continuously.
$g_{s,i}^{max}$	Final capacity of generator s, at node i, whose capacity can be increased continuously.
n_{ij}	Number of new lines in branch ij.
p	Price vector.
p_i	Marginal price at node i.
$p_{o,t}$	Marginal price at demand level o, at stage t.
r_a	Load curtailment for consumer a.
$r_{i,o}$	Load curtailment at node i, at demand level o.
x	Vector of good quantities.
y	Production vector of the firm.
$w_{u,s,i,t}$	Binary variable which is 1 if unit u with technology s is built, 0 otherwise.
z_{ij}	Binary variable which is 1 if circuit in branch ij is built, 0 otherwise.
$z_{q,ij,t}$	Binary variable which is 1 if the circuit q in branch ij is built, 0 otherwise.
λ^*	Lagrange multiplier indexed to restriction $*$.
θ_i	Angle at node i.
$\theta_{i,o}$	Angle at node i, at demand level o.
$\theta_{i,t}$	Angle at node i, at stage t.
$\theta_{i,o,t}$	Angle at node i, at demand level o, at stage t.

Parameters:

b_{ij}	Susceptance of lines in branch ij.
c_{ij}	Investment cost of lines in branch ij.
$d_{i,o}$	Demand at node i, at demand level o.
$d_{i,t}$	Demand at node i, at stage t.
$d_{i,o,t}$	Demand at node i, at demand level o, at stage t.
$\overline{d_a}$	Maximum demand for consumer a.
$\overline{f}_{ij}$	Fixed capacity for lines in branch ij.
$\overline{g}_k$	Fixed capacity of generator k.
$\overline{L}$	Budget.
l_{ij}	distance of lines in branch ij.
n^0_{ij}	Number of existing circuits in branch ij.
$\overline{n}_{ij}$	Maximum number of lines which can be built in branch ij.
α^0, α^1	Parameters for linear demand function.
β_t	Interest rate for stage t.
γ_{ij}	Charge for transmit power from node i to node j.
δ_a	Reserve price for consumer a.
ε_s	Investment cost for generation unit with technology s.
η_{ij}	Annualized investment cost of line ij in $\$/(MW \cdot km \cdot \text{year})$.
θ_{slack}	Reference angle for the system.
ξ_k	Variable cost diminution for increasing initial generation capacity.
$\overline{\pi}$	Opportunity cost for the investor.
ρ_a	Cost of load curtailment for consumer a.
σ_k	Variable cost for generator k.
σ_s	Variable cost for technology s.
$\sigma_{s,i}$	Variable cost for technology s at node i.
τ_o	number of hours for the demand level o.
ϕ	Margin reserve factor.
$\varphi_{ij,i}$	Power distribution factor for line ij with respect to injection at node i.
ω	cost of labor.

CONTENTS

1 INTRODUCTION

The transmission planning problem (TPP) consist in finding the new transmission lines which should be built in order to have an adequate operation for the future. Traditionally, the TPP is formulated as the minimization of the total investment cost for a specific point in the future, subject to the network equations and the transmission lines limits.

In mathematical terms, the TPP corresponds to a mixed integer nonlinear programming problem and for the moment there is no algorithm that guarantees finding the global optimal solution. In consequence, a gradual progress has been made in respect to the modeling of the problem and solution techniques. The more complex is the model the more difficult to solve is. Thus, the main approach to TPP is to use the DC representation of the network, which is given by the two Kirchhoff's laws equations.

The first application of optimization techniques to TPP begins in 1970 with the article of Garver (GARVER, 1970). Garver only uses the Kirchhoff's current law, and presents a constructive heuristic algorithm for finding solutions. Later articles in this regard proposed very similar algorithms based on the idea of Garver; algorithms such the minimum effort (MONTICELLI et al., 1982) and the Villasana - Garver - Salon (VILLASANA; GARVER; SALON, 1985) were very popular. Constructive Heuristic algorithms can find solutions through a sequence of linear programming problems; a candidate is selected at each step according to a sensibility index. These algorithms are simple, fast but they can lead to poor quality solutions.

Moreover, classic optimization techniques were applied to the TPP, including Benders decomposition (ROMERO; MONTICELLI, 1994b) an the Branch and Bound algorithm (ROMERO; MONTICELLI, 1994a; HAFFNER et al., 2000). These algorithms find the global optimal solution through a systematized search in the solution space. This is possible only with some simplifications and a lot of computational effort, so these algorithms usually present convergence troubles for large systems.

In the middle of the nineties, metaheuristic algorithm were applied to the TPP, including Simulated Annealing (ROMERO; GALLEGO; MONTICELLI, 1996), genetic algorithms (ROMERO; RIDER; SILVA, 2007; GALLEGO; MONTICELLI; ROMERO, 1998), Tabu search (GALLEGO; ROMERO; MONTICELLI, 2000; SILVA et al., 2001), GRASP (FARIA H. et al., 2005), Ant Colony, etc. These algorithms make an intelligent search through the solution space; however, they do not guarantee finding the global optimal solution. Nonetheless, these algorithms are simple, fast and can find solutions of good quality.

Finally, the disjunctive linear model was proposed in (BAHIENSE et al., 2001). In simple words, the original problem is represented by a mixed integer linear model by using the Fortuny-Amat representation (FORTUNY-AMAT; MCCARL, 1981). In consequence, the global optimal solution can be found in the equivalent system. Nowadays, this model is the general reference for further research (VINASCO; RIDER; ROMERO, 2011; RAHMANI et al., 2013).

The referred models use the DC model of the network since it's highly complex of obtain solutions, even for such simplification. However, with the advances in the operation research, it has been claimed that AC representation of the network is possible. The article of Rider (RIDER; GARCIA; ROMERO, 2007) has the first AC formulation of the transmission planning problem. Since then, many others have researched in this regard (ZHANG et al., 2012; JABR, 2013; TAYLOR; HOVER, 2013). The AC model is the natural extension of the problem and it is a research topic in development. This dissertation always refers to the DC model unless specification of the opposite.

Furthermore, the classic TPP was formulated as a static problem, in other words, transmission reinforces should be found for a certain point in the future. The real problem should represent the dynamics of the generation and demand for the planning horizon. The dynamic or multistage formulation of the problem finds the optimal solution for a set of stages subject to inter-temporal restrictions: a line built at stage t must be available for the stage $t+1$. In this regard, the article of Escobar (ESCOBAR; GALLEGO; ROMERO, 2004) proposes the nonlinear formulation of the problem. Then, Vinasco (VINASCO; RIDER; ROMERO, 2011) presents the mixed integer linear formulation of the problem.

Traditional minimization of total investment cost is valid only for vertical integrated market, in which a monopoly is in charge of the generation, transmission and distribution activities. The main problem of a monopoly is that it has no incentive for being efficient and can create unnecessary investments. Economic theory claims that a competitive market would lead to a more efficient outcome (MAS-COLLEL; WHISTON; GREEN, 1995). For this reason, at the end of the seventies competition was introduced in several public utilities including electricity (KIRSCHEN; STRBAC, 2004).

With the introduction of competition in the electricity market, traditional approach in not valid. In a competitive market, there is no central planner who decide investments in generation and transmission capacity. Investments are made in function of the possible profit that agents can made in the market. In contrast to perfect competition, each agent has its own objective function and the equilibrium is the simultaneous solution of all optimization problems, which is known as market equilibrium (TIROLE, 1988).

In a competitive framework, the transmission and generation investing problem can be represented by a multilevel optimization problem. A multilevel problem represents the Nash equilibrium of a game in which a certain player makes the first move. In specialized literature, several multilevel models have been proposed in order to represent the investments in transmission and generation capacity in competitive markets, for further details see (GARCES et al., 2009; JENABI; FATEMI; SMEERS, 2013; POZO; SAUMA; CONTRERAS, 2013).

In this context, investment models for transmission capacity implicitly take the premise that opportunity cot of transmission capacity depends only in the difference of marginal prices between the terminal (HOGAN, 1999; HOGAN; ROSELLON; VOGELSANG, 2010). For a static game, this is correct; however, for a multistage model this proposition is not correct.

This dissertation recognizes that opportunity cost of transmission capacity not only depends on the marginal costs but also on the possible generation investments. So, when decentralizing investment decisions implies existing of externalities between the generation and transmission investments, those externalities can make that decentralized result can reach a Pareto optimal solution, market failures and a bad use of the economy resources.

The problem of externalities can be overcome through regulatory mechanism. In this sense, it is necessary to find the the Pareto optimal solution and set in the decentralized market; examples of regulatory mechanisms are taxes, price regulation, normativeness, etc. Finally, a multistage bi-level model is proposed, which finds the Pareto optimal solution. This solution can be used as a reference for the implementation of regulatory mechanism. The proposed model is implemented and solved with CPLEX and results for several study cases are shown.

1.1 OBJECTIVES

The general objectives of this dissertation are the following:

- To review the transmission and generation planning problem in a competitive market.
- To develop a multistage bi-level mode for investing in transmission and generation capacity for competitive electricity markets.

The specific objectives of this dissertation are the following:

- To present the microeconomic concepts for electricity markets.
- To review the externalities and how they can make that a market cannot reach Pareto optimal solution.
- To present multilevel optimization as a standard for modeling sequential decisions and how can be applied to the investments in transmission and generation capacity.

- To present the opportunity cost dependence of the investments in transmission and generation capacity.
- To present a discussion showing how externalities surge when decentralizing investments in transmission and generation capacity.
- To formulate a model that finds Pareto optimal solution as a reference solution.
- To present the results of the proposed model and show the developed discussion.

1.2 CONTRIBUTIONS

This dissertation has important contributions. From a theoretical standpoint, this dissertation explains how externalities arise when the investment decisions are decentralized; thus, market outcome does not lead to a Pareto optimal solution. Implications of the externalities can change the way transmission planning is perceived, since a decentralized market is not completely efficient in comparison to a centralized market.

From a practical standpoint, this dissertation presents a multistage bi-level model which finds the optimal solution considering the electricity as the only good in the market (known as partial equilibrium, in microeconomics). The proposed model can be used to asses investments in transmission and generation capacity as an additional tool in the decision process.

1.3 ORGANIZATION OF THE WORK

This dissertation is divided into five chapters, where the first chapter is the introduction. The remaining chapters have the following structure:

- Chapter 2 presents the theoretical framework of the dissertation: fundamental of microeconomics, electricity markets, investing in transmission and generation capacity, multilevel optimization and review of existing multilevel models for investing in transmission and generation capacity.
- Chapter 3 discusses about transmission investments and its dependence with investments in generation capacity, the problem of externalities is pointed out and a bi-level model is proposed from the discussion bases.
- Chapter 4 presents the results for several study cases.
- Finally, Chapter 5 presents the conclusions of the dissertation.

2 THEORETICAL FRAMEWORK

This chapter is organized as follows. First, section 2.1 introduces the key microeconomic concepts in order to understand the market performance: how prices are determined, what is the outcome of perfect competition and how perfect competition leads to a Pareto optimal solution. Moreover, market failures are discussed, specially the externalities and how market outcome does not lead to a Pareto optimal solution because of the externalities. Section 2.2 presents the fundamentals of the electricity market operation. Section 2.3 discusses the nature of investing in transmission and generation capacity; a basic mathematical formulation is presented. Section 2.4 introduces multilevel optimization as a useful tool for modeling sequential decisions in which one player makes the first movement. In this sense, investments in capacity are made before the market operation, so the investment models can be expressed as a multilevel optimization problem. Finally, section 2.5 presents the state-of-the-art models; their main advantages and disadvantages are discussed.

2.1 FUNDAMENTALS OF MICROECONOMICS

Microeconomics is the branch of economics that studies the behavior of economic agents and its interrelation in the market; this section provides a basic introduction to the key microeconomic concepts which will be used later.

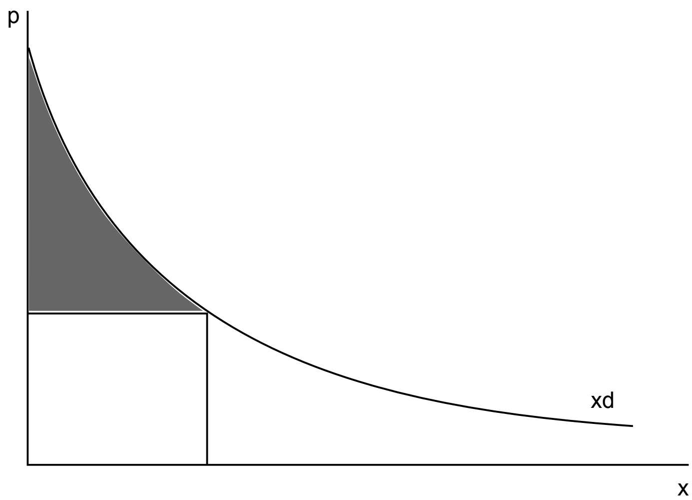
2.1.1 Consumer theory

The consumer is the basic agent of the economy; it is modeled as an optimizing agent with preferences over goods. Preferences are represented by a convex function $U(x)$ called utility function; where x is a vector of quantities of all consumption goods. Prices are represented by the price vector p. Also, the consumer has a limited budget $\overline{L}$, then that problem of the consumer consists in maximizing its utility $U(x)$ subject to its consumption possibilities set (MAS-COLLEL; WHISTON; GREEN, 1995):

$$
\begin{aligned}
&\text{Maximize}_{\,x} \quad U(x) \\
&\text{Subject to:} \\
&\qquad\qquad p \cdot x \leq \overline{L} \\
&\qquad\qquad x \geq 0
\end{aligned}
\tag{1}
$$

The demand function $x_i^d = x_i(p, \overline{L})$ for each good is derived from the solution of the consumer problem. This function depends on the prices p and the budget $\overline{L}$. The plot of the demand function versus its price is known as the demand curve. Figure 1 shows a demand curve for a certain good.

Figure 1 - Demand curve.



Source: Kirschen e Strbac (2004).

According to Figure 1, when the price is high, the consumer would decide to buy less units. If the price decreases, consumer would buy more units until the price equals its valuation. Then, consumer would be disposed to pay more money for the first consumption units and less for the next units, however in a market the consumer always pays the same price for all the units. This difference between consumer valuation and the market price represents a profit, which decrease until the valuation reaches the market price. The total difference between valuation and price market is called net consumer surplus (CS) and its represented in Figure 1 by the shaded area. Later, it will be shown that through a market a consumer can get the maximum consumer net surplus.

2.1.2 Producer theory

Producers, or firms, are economic agents which are able to transform goods. Each firm is determined by a function of transformation of goods called technology The objective of a firm is to maximize its profit subject to its technological restriction.

A firm has a production vector y whose components are the inputs and output for the production of goods; the negative sign of the components of y is to distinguish between an input and an output. Price vector p represents price for the input and outputs. The problem of the firm is to maximize its profit π subject to the set of production possibilities Y (MAS-COLLEL; WHISTON; GREEN, 1995):

$$\begin{aligned} &\text{Maximize}_{\,y} \quad p \cdot y \\ &\text{Subject to:} \\ &\qquad\qquad\quad y \in Y \end{aligned} \tag{2}$$

The previous problem can be expressed as the maximization of the difference between income and cost function $C(x)$. For this moment, consider that the firm takes the price p as given, which id true for perfect competition and will be discussed in the next section. Also, consider the firm only produces one good x, optimal production is given by the following problem:

$$\text{Maximize } \pi(x) = p \cdot x - C(x) \tag{3}$$

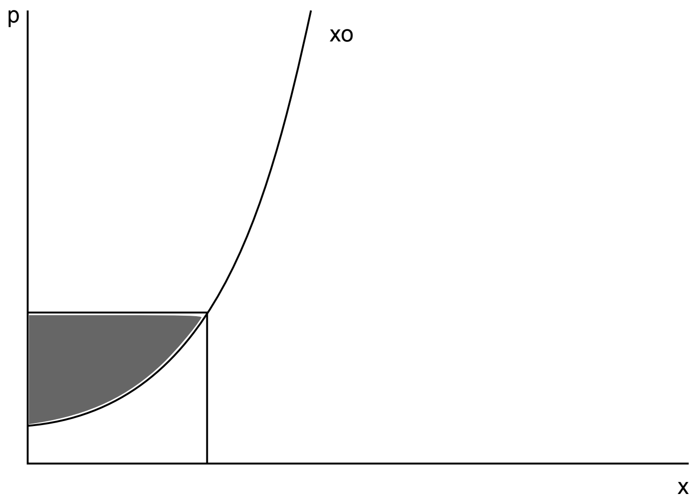
$$\frac{d(p \cdot x)}{dx} = \frac{dC(x)}{dx} \tag{4}$$

$$p(x) = \frac{dC(x)}{dx} \tag{5}$$

The economic cost of producing an additional unit is called the*marginal cost* and it is given by the derivative of the cost function respect the produced quantity. This cost not only includes the accounting costs, but also the opportunity cost. The opportunity cost is the cost of the following best alternative; in other words, it is what an agent refuses when it takes a decision. According to last equation, the firm maximizes its profit when its marginal cost equals the market price. This basic principle says that a price should reflect the economic cost of producing an additional unit of the good, in perfect competition (ALBOUE, 1983).

The inverse function $x_i^o = p^{-1}(x)$ is called the supply function, and is represented in the Figure 2. For each market price, the firm produce quantity x^* which maximize its profit.

Figure 2 - Supply curve.



Source: Kirschen e Strbac (2004).

According to Figure 2, the marginal cost is an increasing function. Then, the first produced units are cheaper than the next units, this cost increases as production does. The difference between the market price and the marginal cost represents the profit of the firm and its known as producer surplus (PS). Figure 2 represents the PS by the shaded area; in the next section it is shown that through a market the firm can get the maximum surplus.

2.1.3 Competitive equilibrium of the market

After modeling agent behavior, it is important to model their interaction in the market. In the previous section, market price was assumed to be given; if this is true, the market is of perfect competition, because no part can modify the price by itself. In a perfectly competitive market, the joint action of consumers and firms is what determines the price of a good. A market of perfect competition has the following characteristics:

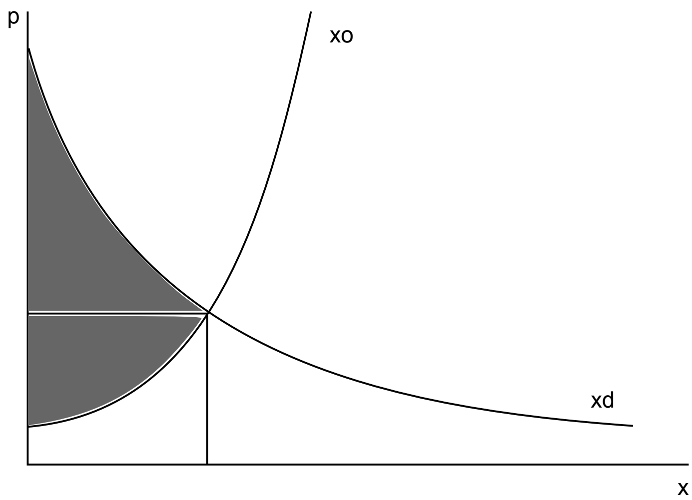
- There are a large number of firms and consumers.
- Products are homogeneous, in other words, the output of a firm is exactly the same of the output of another firm for the same good.
- There is perfect information among all consumers and firms.
- The firms take the price as given, so no individual action can affect the price.
- There is perfect mobility for entry and exit of firms.

Then, the market equilibrium is reached when supply is equal to demand for all goods.

$$x_i^d = x_i^o \tag{6}$$

Figure 3 presents the market equilibrium for a certain good, equilibrium prices are obtained intercepting the supply and demand functions. The shaded area represents the sum of the net consumer surplus plus the net producer surplus, which is called social welfare. The social welfare is an index of the welfare of the overall economy.

Figure 3 - Market equilibrium - Pareto optimal solution.



Source: Kirschen e Strbac (2004).

According to Figure 3, market equilibrium reaches the maximum social welfare. This important property is known as the first welfare theorem and says that every market equilibrium is a Pareto optimal solution so maximum social welfare is reached (MAS-COLLEL; WHISTON; GREEN, 1995).

The First Welfare Theorem is very important because it implies that the global optimal solution can be implemented through a market in a natural fashion from the own agent's behavior; which is the main argue for introducing competence in the electricity sector.

Finally, the first welfare theorem is based on very restrictive premises, sadly this is very ideal. however it can be use as a reference and with the help of a regulator, the market outcome can be near to the global optimal solution (ALBOUE, 1983).

2.1.4 Market failures: externalities

The market equilibrium discussed in the previous section is referential, because there are many premises that do not hold in practice. In this sense, the presence of market failures would not lead to a Pareto optimal solution; one of these failures are the *externalities* (DAMMERT; GARCIA; MOLLINELLI, 2010).

Externalities are present when the utility of one individual includes decision variables that are made by another individual without taking in count the welfare of the first individual. The second condition is that the individual who make the decisions that affect other individuals does not receive any pay for it (BAUMOL; OATES, 1988).

It can be demonstrated that in presence of externalities, the market outcome does leads to Pareto optimal solution, for further details see (BAUMOL; OATES, 1988). The following example shows this proposition. Consider two firms, firm A produces steel and firm M produces apples. The production level of steel affects negatively the apple production, but not vice versa. Let x_a and and x_m be steel production and apple production respectively, production functions are given by the following expressions:

$$x_a = f(L_a) \tag{7}$$

$$x_m = g(L_m) + h(x_a) \tag{8}$$

$$\frac{\partial h}{\partial x_a} \leq 0 \tag{9}$$

Where L_a is the labor in producing steel and L_m is the labor in producing apples. Furthermore, total labor is restricted to a number of hours a day $L_a + L_m = \overline{L}$. The central planner maximizes the social welfare given by the profit of both firms. If p_a and p_m are the prices for steel and apples respectively. Then, central planner solves the following problem:

$$\begin{aligned}
\underset{L_a, L_m}{\text{Maximize}} \quad & p_a \cdot x_a + p_m \cdot x_m \\
\text{Subject to:} \quad & \\
& L_a + L_m = \overline{L} \\
& x_a = f(L_a) \\
& x_m = g(L_m) + h(x_a)
\end{aligned} \tag{10}$$

Replacing the restrictions in the objective function leads to the following Lagrangian:

$$Ł : p_a \cdot f(L_a) + p_m \cdot (g(L_m) + h(x_a)) + \lambda(\bar{L} - L_a - L_m) \tag{11}$$

First order conditions are the following:

$$\frac{\partial Ł}{\partial L_a} = p_a \cdot \frac{\partial f}{\partial L_a} + p_m \cdot \frac{\partial h}{\partial x_a} \cdot \frac{\partial f}{\partial L_a} - \lambda = 0 \tag{12}$$

$$\frac{\partial Ł}{\partial L_m} = p_m \cdot \frac{\partial g}{\partial L_m} - \lambda = 0 \tag{13}$$

In the other hand, in a decentralized market each firm maximizes its profit. Let ω be the cost of labor, firm A maximizes its profit:

$$\pi_a = p_a \cdot x_a - \omega \cdot L_a \tag{14}$$

$$\frac{\partial \pi_a}{\partial L_a} = 0 \tag{15}$$

$$\implies p_a \cdot \frac{\partial f}{\partial L_a} - \omega = 0 \tag{16}$$

In a similar way, firm M maximizes its profit:

$$\pi_m = p_m \cdot x_m - \omega \cdot L_m \tag{17}$$

$$\frac{\partial \Pi_m}{\partial L_m} = 0 \tag{18}$$

$$\implies p_m \cdot \frac{\partial g}{\partial L_m} - \omega = 0 \tag{19}$$

Comparing the results for the central planner (*) and decentralized market (d), Lagrange multiplier of the labor restriction is equal to the cost of labor ($\lambda = \omega$). The production of apples is the same in both cases; nonetheless, the production of steel is different. The following inequality stands for steel production:

$$\implies p_m \cdot \frac{\partial h}{\partial x_a} \cdot \frac{\partial f^*}{\partial L_a} \leq 0 \tag{20}$$

Replacing the condition $\lambda = \omega$ in the inequality:

$$p_a \cdot \frac{\partial f^*}{\partial L_a} + p_m \cdot \frac{\partial h}{\partial x_a} \cdot \frac{\partial f^*}{\partial L_a} = p_a \cdot \frac{\partial f^d}{\partial L_a} \tag{21}$$

$$\Longrightarrow \frac{\partial f^*}{\partial L_a} \geq \frac{\partial f^d}{\partial L_a} \tag{22}$$

$$\Longrightarrow x_a^* \leq x_a^d \tag{23}$$

This result says that the production of steel in a decentralized market x_a^d is higher than the Pareto optimal solution x_a^*. Thus, welfare for a decentralized market is lower since with a reduction of steel production leads to a higher social welfare.

2.2 FUNDAMENTALS OF ELECTRICITY MARKETS

Electricity is a good with very particular characteristics that are not observed in other markets; in this section the fundamentals of electricity markets and its main characteristics are introduced.

2.2.1 Organization of the electricity market

A modern electricity market is organized according to the generation, transmission and distribution activities. The following functions can be found in a modern market (STOFT, 2002):

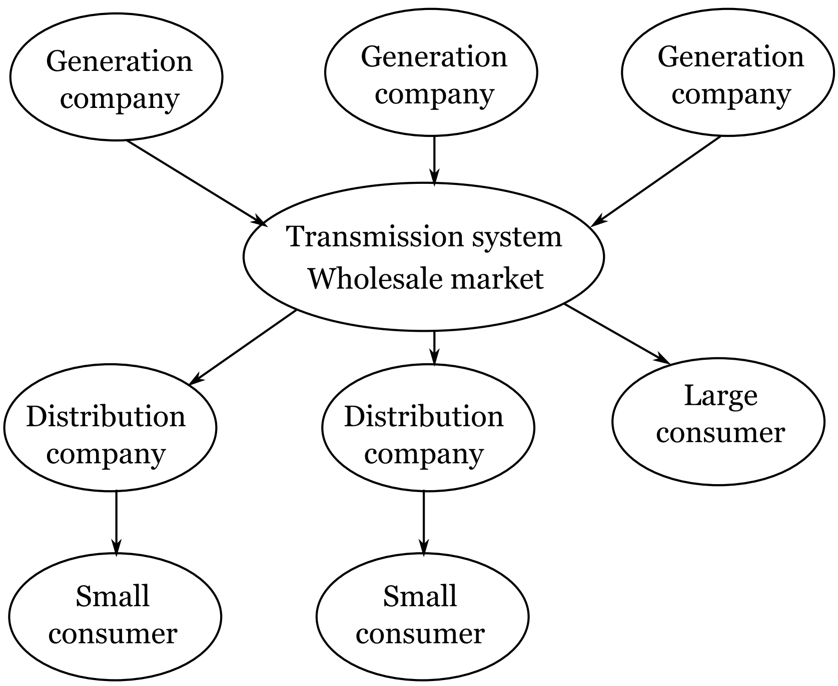
- *Generation companies* that produce and sell electricity. They can own one generation unit or a portfolio of generation units with different technologies.
- *Distribution companies* which own and operate the distribution networks. They have the monopoly for the sale of electricity to all the consumers connected to their network.
- *The Market Operator* who matches the bid and offers and set the financial equilibrium between supply and demand. Cares about the commercial relation among consumers and producers.
- *The Independent System Operator* (ISO) determine the equilibrium of supply and demand in real time, maintains the security in the electric system in a way that does not favor any party.
- *Transmission company* which own the transmission network: lines, transformers and compensation equipments. Usually there is only one company because the transmission

activity is a natural monopoly; however, competition can exist.

- *The regulator* is the governmental body for ensuring fair and efficient operation of the electric sector. It determines and approve the market rules and set the prices for the monopolies in the sector.
- *Small consumers*, usually are the residential consumers, buy electricity to distribution companies that own the monopoly over their geographic area. The prices they pay are set by the *regulator*.
- *Large consumers*, usually are industrial consumers who buy electricity directly from the market. These consumers can be connected directly to the transmission system or the distribution system.

The agents usually interact according to the structure of a wholesale market shown in Figure 4. In this structure, no central organization is in charge of the sale of electricity; distribution companies and large consumers buy electricity directly from the generation companies.

Figure 4 - Organization of the electricity market: wholesale market.



Source: Stoft (2002).

2.2.2 Market operation

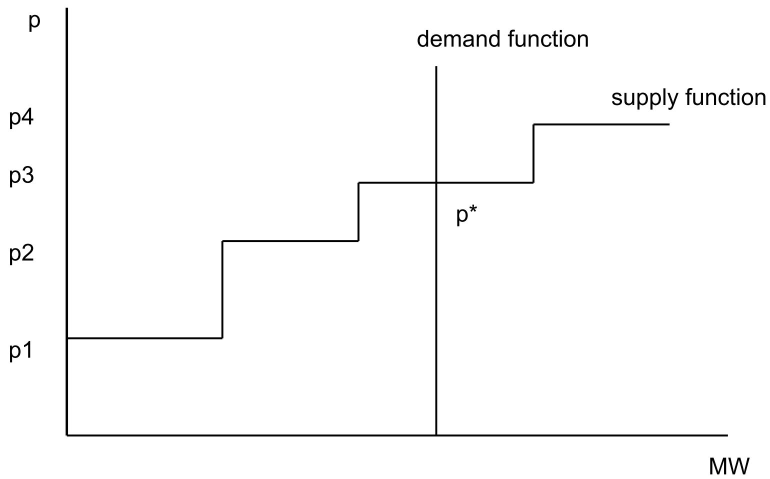
The real time market operation is operated as a wholesale market, also known as the *spot market*. A spot market is the most traditional form of a market; producers sell their product and buyers pay on the spot the real time price.

The spot market must be managed in a centralized manner, since it is very difficult to reach a market equilibrium in real time through interaction of generation companies and consumers. Thus, spot market operates as a *pool*; a pool basically operates in the following manner (HUNT, 2002):

- Generation companies submit bids to supply a certain amount of electricity at an certain price for a certain period. These bids are ranked in order of increasing price and from this rank the supply curve is determined.
- A similar procedure can be applied for obtaining the demand curve. However, a forecast of the demand can be used instead, since electricity demand can be considered as an inelastic vertical line.
- Intersection of supply and demand curves represents the market equilibrium at a equilibrium price, which represent the marginal cost of the system. The marginal cost reflects the additional cost of providing one more MWh.
- The operator pays the marginal cost to all the generation companies for each MWh they produce, independently of the price they bid.

The spot market equilibrium is represented in Figure 5.

Figure 5 - *Spot* market equilibrium.



Source: Hunt (2002).

The previous procedure is valid only when generation and demand are located on the same node. However, almost never, loads are located at generation nodes. Thus, a transmission system is needed to transmit the electricity. Since the transmission system is composed by a set of lines, the market operation is constrained by its capacity limits. Then, the market operator must represent the network equations in its problem.

For this purpose, the network is represented by a set of nodes Ω^N and a set of existing branches Ω^{L0}. Each node i has an angle θ_i. Also, each line $ij \in \Omega^{L0}$ has a susceptance b_{ij}, a transmission capacity $\overline{f_{ij}}$, and a power flow f_{ij}. Ω_i^{L0} represents the set of lines connected to node i. The supply side is represented by a set of existing generators Ω^{G0}, where Ω_i^{G0} is the set of generators connected to node i. Each generator has a variable cost σ_k, a fixed capacity $\overline{g_k}$ and a production g_k. The market operator maximizes the social welfare subject to the network equations and the capacity limits. Let W_i be the social welfare at node i, then the market operator solves the following optimization problem (KIRSCHEN; STRBAC, 2004):

$$
\begin{aligned}
\underset{g_k, f_{ij}, \theta_i}{\text{Maximize}} \quad & \sum_{i \in \Omega^N} W_i \\
\text{Subject to:} \quad & \\
& \sum_{ij \in \Omega_i^{L0}} f_{ij} - \sum_{ji \in \Omega_i^{L0}} f_{ji} + \sum_{k \in \Omega_i^{G0}} g_k = d_i \quad && \forall i \in \Omega^N \\
& b_{ij} \cdot (\theta_i - \theta_j) = f_{ij} && \forall ij \in \Omega^{L0} \\
& |f_{ij}| \leq \overline{f_{ij}} && \forall ij \in \Omega^{L0} \\
& g_k \leq \overline{g_k} && \forall k \in \Omega^{G0} \\
& g_k \geq 0 && \forall k \in \Omega^{G0}
\end{aligned} \tag{24}
$$

The social welfare is given by he area between the supply and demand curve. If demand is inelastic (constant with variations of price), the social welfare is given only by the supply curve. Thus, the maximization problem reduces to the following minimization of total cost (KIRSCHEN; STRBAC, 2004):

$$
\begin{aligned}
&\underset{g_k,\, f_{ij},\, \theta_i}{\text{Minimize}} \quad \sum_{k \in \Omega^{G0}} \sigma_k \cdot g_k \\
&\text{Subject to:} \\
&\qquad \sum_{ij \in \Omega_i^{L0}} f_{ij} - \sum_{ji \in \Omega_i^{L0}} f_{ji} + \sum_{k \in \Omega_i^{G0}} g_k = d_i \qquad \forall i \in \Omega^N \\
&\qquad b_{ij} \cdot (\theta_i - \theta_j) = f_{ij} \qquad \forall ij \in \Omega^{L0} \\
&\qquad |f_{ij}| \leq \overline{f}_{ij} \qquad \forall ij \in \Omega^{L0} \\
&\qquad g_k \leq \overline{g}_k \qquad \forall k \in \Omega^{G0} \\
&\qquad g_k \geq 0 \qquad \forall k \in \Omega^{G0}
\end{aligned}
\tag{25}
$$

The problem is compatible with the optimal power flow problem OPF (WOOD; WOLLENBERG, 1992). The market equilibrium is given by the joint solution of the market operator problem and the profit maximization problems for each firm. Finally, the market operator allocates the production and sets the nodal prices equal to the nodal marginal cost.

2.3 INVESTMENTS IN THE ELECTRICITY MARKET

In the long run, transmission and generation capacity need to be increased in order to meet the demand. This section introduces the key concepts of investing in transmission and generation capacity, and presents the basic modeling of both problems.

2.3.1 Investing in generation capacity

For the investor, investing in generation capacity depends on the possible revenues he could make in the market during the lifetime of the project. Note that decision investments not only depends on electricity prices, but also on the opportunity costs of the investors. Depending on their opportunity costs, investors would decide to invest in other industries instead.

Thus, investors forecast electricity prices for the lifetime of the project and make some financial calculus in order to decide if they invest. This approach is compatible with the theory of producers and both lead to the same result.

Involved costs must be analyzed for the financial evaluation of a generation project. Basically, generation costs are divided into two groups: fixed costs FC and variable costs VC. The fixed costs consist in the investment cost of the plant IC plus the operation and maintenance cost $O\&M$.

$$FC = IC + O\&M \tag{26}$$

The investment cost is not necessarily equal to the cost of the project CP. Since the involved cost are high and investors usually has a own capital $K \leq CP$, part of the cost of the project is financed with debt D at an interest rate β which depends on the project lifetime and the risk aversion of the lender.

$$CP = K + D \tag{27}$$

Then, inside the annual fixed costs the investor should consider the annual payment of the debt CD_t at an interest rate β. In order to compute CD_t , we consider a project lifetime of T years; the present value of the cash flows must be equal to the total debts.

$$D = \frac{CD_t}{(1+\beta)} + \frac{CD_t}{(1+\beta)^2} + \cdots + \frac{CD_t}{(1+\beta)^T} \tag{28}$$

Making some simplifications We have the following expression:

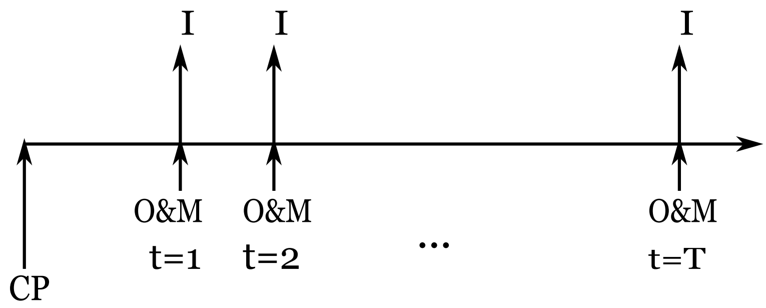
$$CD_t = \frac{\beta \cdot D}{1 + \frac{1}{(1+\beta)^T}} \tag{29}$$

In the other hand, variable costs VC are given by the consumption of fuel associated to a certain technology. In consequence, total cost faced by the investor is given by the following expression:

$$C_t = K_t + CD_t + O\&M_t + VC_t \tag{30}$$

Cash flow during the project lifetime is presented in Figure 6. In the first period the investment is made. In the following periods, the investor has an income for the sell of electricity and faces the variable cost plus the cost of operation and maintenance. Also, the payment of the debt is considered during the project lifetime.

Figure 6 - Cash flow for the investor in generation capacity.



Source: the author.

The revenue of the investor is given by the income minus the cost. This income depends on the electricity prices and also a capacity payment would exist depending on the regulation of the country. Investor maximizes his profits given by the present value of his income $PV(I)$ minus the present value of his costs $VP(C)$ subject to opportunity cost $\bar{\Pi}$ and available capital $\bar{K}$.

$$\begin{aligned} \text{Maximize } \pi = \quad & PV(I) - PV(C) \\ \text{Subject to:} \quad & \\ & \pi \geq \bar{\pi} \\ & K \leq \bar{K} \end{aligned} \tag{31}$$

The solution space of the problem is given for all projects whose profits are higher than the investor's opportunity cost. Thus, the investor would choose a project that has the highest profit. Note that opportunity costs must no be considered in the costs, since they are explicitly included in the restrictions of the problem.

It is possible to reduce the model using an adequate interest rate. For an given cash flow, there is an associated interest rate for which its present value is equal to zero. For a investor, the opportunity cost can be represented by a minimum acceptable rate of return. Thus, if the cash flow of the selected project is zero, the investor recovers his opportunity cost at least. In consequence, $\bar{\pi} = 0$ and the restriction is not necessary anymore.

2.3.2 Investing in transmission capacity

The transmission business is considered a natural monopoly because of its high fixed costs and low variable costs. This leads to only one company in the market structure; so, the transmission business must be regulated. There are two used approaches in order to incentive investments in transmission capacity: an approach based on cost and approach based on value.

In the approach based on cost, the transmission company receives enough income to recover his investment cost plus an attractive rate of return. The transmission company prepares a transmission plan, the regulator reviews the plan and decides which reinforces should be built. Finally, the transmission company recovers its investment through a charge in the tariff of consumers. Under this approach the regulator only has to create mechanisms that incentive efficient investments.

The other approach is based on value. The transmission company invests in those transmission lines that produce revenues through the market . The main idea for any transportation business is to bring a product from a certain location to a more expensive location. Then the transportation is efficient if the transportation cost does not exceed the price difference of the locations. The same idea can be used to value a transmission line and the transmission company can recover its investment through a pay for the use of the network.

For both approaches, it has been argued that the opportunity cost of the transmission capacity depends on the price difference of the line terminals. If the product of a transmission company is transmission capacity, then a transmission line should be built only when its marginal cost equals to its opportunity cost, which is given by the price difference of the line terminals (HOGAN, 1999).

Transmission line ij has a cost function C with a fixed cost FC that does not depends on line capacity $\overline{f}_{ij}$ and a variable cost VC which depends on the line capacity $\overline{f}_{ij}$, the line longitude l_{ij} and the annualized cost per unit of km and capacity η_{ij}. The cost η_{ij} is obtained from the following expression:

$$\eta_{ij} = \beta \cdot c_{ij}/l_{ij} \cdot \overline{f}_{ij} \cdot \left(1 + \frac{1}{(1+\beta)^T}\right) \tag{32}$$

If we consider that transmission capacity f_{ij}^{max} can be increased continuously, we have the following marginal cost of transmission capacity:

$$C(f_{ij}^{max}) = FC + VC(f_{ij}^{max}) \tag{33}$$

$$VC(f_{ij}^{max}) = \eta_{ij} \cdot l_{ij} \cdot f_{ij}^{max} \tag{34}$$

$$\frac{dC_T}{d f_{ij}^{max}} = \eta_{ij} \cdot l_{ij} \tag{35}$$

The previous marginal cost assumes that transmission capacity can be increased continuously. If this is accepted, optimal capacity for each line ca be found by equation 35. Furthermore, the planner needs to value the performance of the overall transmission system. Thus, the planner uses the concept of a reference network. Topologically, a reference network considers the existing network and the candidate lines, but with variable line capacities. The planner objective is to find optimal capacity for each line minimizing total cost. (KIRSCHEN; STRBAC, 2004).

Let Ω^D be the set of demand levels with a τ_o number of hours associated at each level; let Ω^G be the set of generators with marginal cost σ_k and production $g_{k,o}$; let Ω^N be the set of nodes and $Omega^L$ the set of lines; each line has a power flow f_{ij} for $ij \in \Omega^L$ and the constant factors $\varphi_{ij,i}$ represent the power distribution factors. The central planner solves the following problem:

$$
\begin{aligned}
\underset{g_{k,o},\, f_{ij}^{max}}{\text{Minimize}} \quad & \sum_{o\in\Omega^D}\sum_{k\in\Omega^G} \tau_o \cdot \sigma_k \cdot g_{k,o} + \sum_{ij\in\Omega^L} \eta_{ij} \cdot l_{ij} \cdot f_{ij}^{max} \\
\text{Subject to:} \quad & \\
& \sum_{ij\in\Omega_i^L} f_{ij,o} - \sum_{ji\in\Omega_i^L} f_{ij,o} + \sum_{k\in\Omega_i^G} g_{k,o} = d_{i,o} \qquad \forall i \in \Omega^N, \forall o \in \Omega^D \\
& f_{ij,o} = \sum_{i\in\Omega^N} \varphi_{ij,i} \cdot (\sum_{k\in\Omega_i^G} g_{k,o} - d_{i,o}) \qquad \forall ij \in \Omega^L, \forall o \in \Omega^D \\
& |f_{ij,o}| < f_{ij}^{max} \qquad \forall ij \in \Omega^L, \forall o \in \Omega^D
\end{aligned}
\tag{36}
$$

Whenever the objective function considers operation plus investment costs, implicitly it is recognized that the opportunity cost of a transmission line only depends on the price difference of the line terminals. A detailed discussion on these issues is developed in Chapter 3.

2.4 MULTILEVEL OPTIMIZATION AND TRANSMISSION PLANNING

2.4.1 Formulation and solution of multilevel optimization problems

Multilevel models are optimization problems that has a subset of variables restricted to be an optimal solution of other optimization problems which are parameterized on the remaining variables. A bi-level problem can be represented by the following formulation:

First level:

$$
\begin{aligned}
&\text{Minimize}_{x,y} \quad F(x,y) \\
&\text{Subject to:} \\
&\qquad\qquad g(x,y) \leq 0
\end{aligned}
\tag{37}
$$

Where y must be the solution of the lower level problem given the optimal solution of x:

Second level:

$$
\begin{aligned}
&\text{Minimize}_{y} \quad f(x,y) \\
&\text{Subject to:} \\
&\qquad\qquad h(x,y) \leq 0
\end{aligned}
\tag{38}
$$

Multilevel problems are very difficult to solve; the classic procedure is to transform the problem into just one level problem. Consider the following second level problem:

$$
\begin{aligned}
&\text{Minimize} \quad c^t \cdot x \\
&\text{Subject to:} \\
&\qquad Ax = b : y \\
&\qquad Dx \leq e : w \\
&\qquad x_i \text{ unrestricted} \quad \forall x_i \in \Omega_1 \\
&\qquad x_i \geq 0 \qquad\qquad \forall x_i \in \Omega_2
\end{aligned}
$$

Where y and w are the dual variables of the restrictions. The dual problem is given for the following:

$$
\begin{aligned}
&\text{Maximize} \quad b^t \cdot y + e^t \cdot w \\
&\text{Subject to:} \\
&\qquad A^t y + D^t w = c \\
&\qquad w \leq 0 \\
&\qquad y \text{ unrestricted}
\end{aligned}
\tag{39}
$$

According to the Karush-Kuhn-Tucker conditions, the following conditions applies for the optimal solution:

$$
A \cdot x^* = b,\ D \cdot x^* \leq e,\ x \text{ unrestricted} \tag{40}
$$

$$A^t \cdot y^* + D^t \cdot w^* = c,\ w \leq 0, y \text{ unrestricted} \tag{41}$$

$$(e - D \cdot x^*)^t \cdot w^* = 0 \tag{42}$$

Rearranging the third condition:

$$e^t w^* = (x^*)^t D^t w^* \tag{43}$$

Multiplying by $(x^*)^t$:

$$b^t \cdot y^* + e^t w^* = c^t \cdot x^* \tag{44}$$

Equation (44) is know as the strong dual condition. Thus, the second level problem can be represented by the following system:

$$\left.\begin{aligned} c^t x - b^t y - e^t w = 0 \\ A \cdot x = b \\ D \cdot x \leq e \\ x \text{ unrestricted} \\ A^t y + D^t w = c \\ w \leq 0 \\ y \text{ unrestricted} \end{aligned}\right\} \tag{45}$$

The previous system can be included in the first level problem so that the bi-level model is formulated as one level problem. Since Karush-Kuhn-Tucker conditions produce nonlinear terms; the one level problem usually is linearized using the Fortuny - Amat representation(FORTUNY-AMAT; MCCARL, 1981), for further details see Section 3.2.2.

In game theory, a bi-level model represents a sequential game of two players. In this sequential game, a certain player makes the first move; in contrast to the classic static game, where both players move at the same time. This model was first proposed by Stackelber (TIROLE, 1988). The classic example consists in a game of two firms deciding their capacity and then they compete using the produced quantity as their decision variable.

Let k_1 be the capacity of the first firm and k_2 be the capacity of the second firm, demand function is given by $p = 1 - k_1 - k_2$. Then, profit for both firms are given by the following functions:

$$\pi_1(k_1, k_2) = k_1 \cdot (1 - k_1 - k_2) \tag{46}$$

$$\pi_2(k_1, k_2) = k_2 \cdot (1 - k_1 - k_2) \tag{47}$$

If both firms decide their capacity at the same time, both firms maximize their profit taking as given their rivals decisions:

$$\frac{\partial \pi_1}{\partial k_1} = \frac{\partial \pi_2}{\partial k_2} = 0 \tag{48}$$

$$k_1 = \frac{1 - k_2}{2}$$

$$k_2 = \frac{1 - k_1}{2}$$

Solving the system of equations leads to the following result:

$$k_1 = k_2 = 1/3$$

$$\pi_1 = \pi_2 = 1/9$$

In the Stackelberg model, the first firm maximizes its profit taking in count decision of the second firm. Thus, the first firm replaces optimality condition of the second firm in its profit function:

$$\pi_1(k_1, k_2) = k_1 \cdot (1 - k_1 - \frac{1 - k_1}{2}) \tag{49}$$

$$\pi_1(k_1, k_2) = k_1 \cdot (\frac{1 - k_1}{2}) \tag{50}$$

Then, the first firm maximizes its profit:

$$\frac{\partial \Pi_1}{\partial k_1} = 0 \tag{51}$$

$$k_1 = 1/2,\ k_2 = 1/4$$

$$\pi_1 = 1/8, \quad \pi_2 = 1/16$$

The player who moves first has a better profit in comparison to the simultaneous move game. In general, this result holds for any problem.

2.4.2 Transmission planning and multilevel optimization

The problem of investing in transmission and generation capacity can be formulated as a multilevel optimization problem. Previously, the market operation problem and the investing problem were presented as two separate problems. Actually, these two problems are related; investing in transmission and generation capacity depends on future market prices and the market operation depends on the new transmission and generation capacity.

In order to formulate the multilevel problem, investing problem should be represented at the first level since investments in new capacity must anticipate the market operation. Moreover, generation companies anticipate the transmission decisions. In (SAUMA; OREN, 2006), it is claimed that the transmission company can anticipate the generation investments in a proactive manner; but in practice, the new generation units are the main drivers for transmission investments, for further details see Chapter 3.

Consider the price vector p, the production vector g , $\overline{f}$ is the new transmission capacity and $\overline{g}$ is the new generation capacity. The basic multilevel formulation of the problem takes the following form:

First level:

$$\begin{aligned} &\text{Minimize}_{\overline{f}} \quad \text{Investment cost}(p,g) \\ &\text{Subject to:} \\ &\qquad\qquad\quad \text{Investment restrictions} \end{aligned} \tag{52}$$

Second level:

$$\begin{aligned} &\text{Maximize}_{p,g} \quad \text{Social welfare}(\overline{f},\overline{g}) \\ &\text{Subject to:} \\ &\qquad\qquad\quad \text{Equipment limits} \\ &\qquad\qquad\quad \text{Network equations} \end{aligned} \tag{53}$$

This basic model has two levels: the first level has the investment problem while the second level has the market operation problem. This basic structure is used in several existing models as shown in the next section.

2.5 STATE-OF-THE ART MODELS

2.5.1 Garces, Conejo, Bertrand and Romero Model

Garces, Conejo, Bertrand and Romero (GARCES et al., 2009) propose a bi-level model for investing in transmission capacity. The first level represents the investment problem where decisions are made by a central planner who maximize the social welfare. The second level represents the market operator problem who minimizes the operation cost of the spot market.

The network is modeled by a set of nodes Ω^N and a set of branches Ω^L. Each node i has an angle θ_i. Each branch ij has a susceptance b_{ij}, a fixed capacity $\overline{f_{ij}}$ and a power flow f_{ij}. The investment cost of a circuit in branch ij is given by c_{ij}, total cost is multiplication of each cost multiplied by a the binary variable z_{ij} which represents investment decisions.

The demand is represented by a set of consumers Ω^C with a demand d_a and a reserve price δ_a. Also, there could be a level of load curtailment r_a with cost ρ_a for each consumer. The supply is represented by a set of generators Ω^G, with variable cost σ_k, fixed capacity $\overline{g_k}$ and production g_k.

The objective function of the first level problem is the social welfare minus the total investment cost in transmission capacity. Restrictions are the following in order of appearance: (i) budget restriction $\overline{L}$, (ii) $z = 1$ for existing lines and (iii) decision variables must be binary.

The objective function of the second level problem is the social welfare for the considered period (τ hours). Restrictions are the following in order of appearance: (i) Kirchhoff's current law, (ii) Kirchhoff's voltage law, (iii) capacity limits of the lines, (iv) angle limits, (v) generation capacity limits, (vi) consumer demand limits, (vii) load curtailment limit for each consumer, (viii) slack angle for the network and (ix) non negativity of the variables. The mathematical formulation of the model is the following:

First level:

$$\underset{z_{ij}}{\text{Maximize}} \quad \tau\left[\sum_{a\in\Omega^C}\delta_a\cdot d_a-\sum_{k\in\Omega^G}\sigma_k\cdot g_k-\sum_{a\in\Omega^C}\rho_a\cdot r_a\right]-\sum_{ij\in\Omega^{L'}}c_{ij}\cdot z_{ij}$$

Subject to:

$$\begin{aligned}
&\sum_{ij\in\Omega^L}c_{ij}\cdot z_{ij}\leq\overline{L} && \\
&z_{ij}=1 && \forall ij\in\Omega^{L0}\\
&z_{ij}\in\{0,1\} && \forall ij\in\Omega^L
\end{aligned} \tag{54}$$

Second level:

$$\underset{g_k,f_{ij},\theta_i,d_a,r_a}{\text{Maximize}} \quad \sum_{a\in\Omega^C}\delta_a\cdot d_a-\sum_{k\in\Omega^G}\sigma_k\cdot g_k-\sum_{a\in\Omega^C}\rho_a\cdot r_a$$

Subject to:

$$\begin{aligned}
&\sum_{k\in\Omega_i^G}g_k+\sum_{ij\in\Omega_i^L}f_{ij}-\sum_{ji\in\Omega_i^L}f_{ij}+\sum_{a\in\Omega_i^C}r_a=\sum_{a\in\Omega_i^C}d_a && \forall i\in\Omega^N\\
&f_{ij}=b_{ij}\cdot(\theta_i-\theta_j)\cdot z_{ij} && \forall ij\in\Omega^L\\
&|f_{ij}|\leq\overline{f_{ij}} && \forall ij\in\Omega^L\\
&|\theta_i|\leq\pi && \forall i\in\Omega^N\\
&g_k\leq\overline{g_k} && \forall k\in\Omega^G\\
&d_a\leq\overline{d_a} && \forall a\in\Omega^C\\
&r_a\leq\overline{d_a} && \forall a\in\Omega^C\\
&\theta_{slack}=0 && \\
&g_k,r_a,d_a\geq 0 &&
\end{aligned} \tag{55}$$

2.5.2 Jenabi, Ghomi and Smeers Model

Jenabi, Ghomi and Smeers (JENABI; FATEMI; SMEERS, 2013) propose two bi-level models for investing in transmission and generation capacity. For both models, the network is model by a set of nodes Ω^N with angle θ_i and a set of branches Ω^L. The set Ω^L is the union of the set of existing branches Ω^{L0} and the set of candidate branches $\Omega^{L'}$. Each branch has a susceptance b_{ij}, a fixed capacity $\overline{f_{ij}}$ and a power flow $f_{ij,o}$ at demand level o. The total investment cost in transmission capacity is given by the cost of each line c_{ij} multiplied by the investment decision variable z_{ij}.

The proposed models consider a linear function demand at each node:

$$p_i = \alpha_i^0 + \alpha_i^1 \cdot d_i \tag{56}$$

Where α^0 and α^1 are constant parameters. Also, a set of demand levels Ω^D with a duration of τ_o hours for each level is considered. The consumer surplus $CS_{i,o}$ at node i at the demand level o is given by the following expression:

$$CS_{i,o} = \int_0^{d_{i,o}} (\alpha_i^0 + \alpha_i^1 \cdot h) dh \tag{57}$$

$$CS_{i,o} = \alpha_i^0 \cdot d_{i,o} + \frac{\alpha_i^1}{2} \cdot d_{i,o}^2 \tag{58}$$

$$CS_{i,o} = \alpha_i^0 \cdot d_{i,o} + \frac{\alpha_i^1}{2} \cdot (d_{i,o})^2 \tag{59}$$

Where $d_{i,o}$ is the demand at node i at demand level o. So, the total consumer surplus is the sum of consumer surplus for all nodes in the network, total consumer surplus is given by the following expression:

$$CS = \sum_{o \in \Omega^D} CS_o = \sum_{o \in \Omega^D} \sum_{i \in \Omega^N} \left[\alpha_i^0 \cdot d_{i,o} + \frac{\alpha_i^1}{2} \cdot (d_{i,o})^2 \right] \tag{60}$$

The supply is represented by set of existing generators with fixed capacity Ω^{G0} and a set of candidate technologies Ω^{S2} whose capacity $g_{s,i}^{max}$ can be increased continuously. Each technology has an investment cost ε_s and production $g_{s,i,o}$ at node i for demand level o. Furthermore, each existing generator has fixed capacity $\bar{g}_k$, marginal cost σ_k and production $g_{k,o}$ at demand level o.

The first model consider a transmission company which maximize the social welfare subject to the operation of the spot market. The first level has an objective function that represent the social welfare; restrictions are the following in order of appearance: (i) $z = 1$ for existing lines and (ii) investment decision variables must be binary. The second level represents the market operator problem which maximizes the social welfare minus the total investment cost; restrictions are the following in order of appearance: (i) Kirchhoff's current law, (ii) Kirchhoff's voltage law, (iii) line capacity limits, (iv) slack angle in the network, (v) generation capacity limits for candidate technologies and (vi) generation capacity limits for existing generators. The mathematical formulation is the following:

First level:

$$\begin{aligned}
\underset{z_{ij},\, g_{s,i}^{max}}{\text{Maximize}} \quad & CS - \sum_{s\in\Omega^{S2}} \sum_{i\in\Omega^{N}} \varepsilon_s \cdot g_{s,i}^{max} - \sum_{ij\in\Omega^{L'}} c_{ij} \cdot z_{ij} \\
& - \sum_{o\in\Omega^{D}} \sum_{i\in\Omega^{N}} \sum_{s\in\Omega^{S2}} \tau_o \cdot \sigma_s \cdot g_{s,i,o} - \sum_{o\in\Omega^{D}} \sum_{k\in\Omega^{G0}} \tau_o \cdot \sigma_k \cdot g_{k,o}
\end{aligned} \tag{61}$$

Subject to:

$$\begin{aligned}
& z_{ij} = 1 && \forall ij \in \Omega^{L0} \\
& z_{ij} \in \{0,1\} && \forall ij \in \Omega^{L}
\end{aligned}$$

Second level:

$$\begin{aligned}
\underset{g_{s,i,o},\, g_{k,o},\, f_{ij,o},\, \theta_{i,o},\, d_{i,o}}{\text{Maximize}} \quad & EC - \sum_{i\in\Omega^{N}} \sum_{s\in\Omega^{S2}} \varepsilon_{i,s} \cdot g_{s,i}^{max} - \sum_{ij\in\Omega^{L'}} c_{ij} \cdot z_{ij} \\
& - \sum_{o\in\Omega^{D}} \sum_{i\in\Omega^{N}} \sum_{s\in\Omega^{S2}} \tau_o \cdot \sigma_s \cdot g_{s,i,o} \\
& - \sum_{o\in\Omega^{D}} \sum_{k\in\Omega^{G0}} \tau_o \cdot \sigma_k \cdot g_{k,o}
\end{aligned}$$

Subject to:

$$\begin{aligned}
& \sum_{ij\in\Omega_i^{L}} f_{ij,o} - \sum_{ji\in\Omega_i^{L}} f_{ji,o} + \sum_{s\in\Omega_i^{S2}} g_{i,s,o} \\
& + \sum_{k\in\Omega_i^{G0}} g_{k,o} = d_{i,o} && \forall i \in \Omega^{N}, \forall o \in \Omega^{D} \\
& f_{ij,o} = b_{ij} \cdot (\theta_{i,o} - \theta_{j,o}) \cdot z_{ij} && \forall ij \in \Omega^{L}, \forall o \in \Omega^{D} \\
& |f_{ij,o}| \le z_{ij} \cdot \overline{f}_{ij} && \forall ij \in \Omega^{L}, \forall o \in \Omega^{D} \\
& \theta_{slack,o} = 0 && \forall o \in \Omega^{D} \\
& g_{s,i,o} \le g_{s,i}^{max} && \forall i \in \Omega^{N}, \forall s \in \Omega^{S2}, \forall o \in \Omega^{D} \\
& g_{k,o} \le \bar{g}_k && \forall k \in \Omega^{G0}, \forall o \in \Omega^{D}
\end{aligned} \tag{62}$$

For the second model, the first level represents the investment problem of the transmission company which maximizes its profit; a pay γ_{ij} to transmit power from node i to node j is considered. The objective function of the first level represents the sum of all payments minus the investment cost. The second level problem has the same formulation of the previous model; the mathematical formulation is the following:

First level:

$$
\begin{aligned}
&\underset{z_{ij}}{\text{Maximize}} \quad \sum_{o\in\Omega^D}\sum_{ij\in\Omega^{L'}} \tau_o\cdot\gamma_{ij}\cdot|f_{ij,o}| - \sum_{ij\in\Omega^{L'}} c_{ij}\cdot z_{ij}\\
&\text{Subject to:}\\
&z_{ij} = 1 \qquad\qquad \forall ij\in\Omega^{L0}\\
&z_{ij}\in\{0,1\} \qquad\qquad ij\in\Omega^{L}
\end{aligned}
\tag{63}
$$

Second level:

$$
\begin{aligned}
&\underset{g_{s,i,o},g_{k,o},f_{ij,o},\theta_{i,o},d_{i,o}}{\text{Maximize}} \quad CS - \sum_{i\in\Omega^N}\sum_{s\in\Omega^{S2}} \varepsilon_{i,s}\cdot g_{s,i}^{max} - \sum_{ij\in\Omega^{L'}} c_{ij}\cdot z_{ij}\\
&\qquad - \sum_{o\in\Omega^D}\sum_{i\in\Omega^N}\sum_{s\in\Omega^{S2}} \tau_o\cdot\sigma_s\cdot g_{s,i,o}\\
&\qquad - \sum_{o\in\Omega^D}\sum_{k\in\Omega^{G0}} \tau_o\cdot\sigma_k\cdot g_{k,o}\\
&\text{Subject to:}\\
&\sum_{ij\in\Omega_i^L} f_{ij,o} - \sum_{ji\in\Omega_i^L} f_{ji,o} + \sum_{s\in\Omega_i^{S2}} g_{i,s,o}\\
&+ \sum_{k\in\Omega_i^{G0}} g_{k,o} = d_{i,o} \qquad \forall i\in\Omega^N,\forall o\in\Omega^D\\
&f_{ij,o} = b_{ij}\cdot(\theta_{i,o}-\theta_{j,o})\cdot z_{ij} \qquad \forall ij\in\Omega^L,\forall o\in\Omega^D\\
&|f_{ij,o}|\le z_{ij}\cdot\overline{f}_{ij} \qquad \forall ij\in\Omega^L,\forall o\in\Omega^D\\
&\theta_{slack,o} = 0 \qquad \forall o\in\Omega^D\\
&g_{s,i,o}\le g_{s,i}^{max} \qquad \forall i\in\Omega^N,\forall s\in\Omega^{S2},\forall o\in\Omega^D\\
&g_{k,o}\le\bar{g}_k \qquad \forall k\in\Omega^{G0},\forall o\in\Omega^D
\end{aligned}
\tag{64}
$$

2.5.3 Pozo, Sauma and Contreras Model

Pozo, Sauma and Contreras (POZO; SAUMA; CONTRERAS, 2013) propose a three level model for investing in transmission and generation capacity. The network is represented by a set of nodes Ω^N and a set of branches Ω^L. Each node i has an angle θ_i. Also, each branch ij has a susceptance b_{ij}, a initial capacity $f_{ij}^{max,0}$ which can be increased continuously without affecting the line reactance, a final capacity f_{ij}^{max} with an annualized cost η_{ij} and a power flow f_{ij}. Also, there is a set of demand levels Ω^D and each level has a duration of τ_o hours and a demand $d_{i,o}$ associated to node i.

The supply is represented by a set of generators Ω^{G0} with fixed capacity $\bar{g}_k$ and a set of generators Ω^{G1} whose initial capacity $g_k^{max,0}$ can be increased continuously, the final capacity is given by g_k^{max} with a investment cost ε_k. The sets $Omega^{G0}$ and Ω^{G1} are indexed to the set of nodes Ω^N so that the generator k is connected to node i and $k = i$. The model considers that marginal cost decrease linearly with the installed capacity and is given by $\sigma_k + \xi_k \Delta g_k^{max}$. Finally, each generator has a production $g_{k,o}$.

The first level problem represents the investment problem in transmission capacity; the objective function represent the investment cost in transmission and generation capacity plus operation costs of the spot market:

First level:

$$\begin{aligned} \underset{f_{ij}^{max}}{\text{Minimize}} \quad & \sum_{o\in\Omega^D} \tau_o \left\{ \sum_{k\in\Omega^{G1}} [p_{i,o} - (\sigma_k - \xi_k \cdot \Delta g_k^{max}] g_{k,o} + \sum_{k\in\Omega^{G0}} [p_{i,o} - \sigma_k] g_{k,o} \right\} \\ & - \sum_{i\in\Omega^{G1}} \varepsilon_k \cdot \Delta g_k^{max} - \sum_{ij\in\Omega^L} \eta_{ij} (f_{ij}^{max} - f_{ij}^{max,0}) \end{aligned} \tag{65}$$

The second level represents the investment problem in generation capacity; in this level each firm $v \in \Omega^V$ choose their capacity:

Second level:

$$\underset{g_k^{max}}{\text{Maximize}} \quad \left. \begin{aligned} & \sum_{o\in\Omega^D} \tau_o \left\{ \sum_{k\in\Omega_v^{G1}} [p_{i,o} - (\sigma_k - \xi_k \Delta g_k^{max})] g_{k,o} + \sum_{k\in\Omega_v^{G2}} [p_{i,t} - \sigma_k] g_{k,o} \right\} \\ & - \sum_{k\in\Omega_v^{G0}} \varepsilon_k \cdot \Delta g_k^{max} \end{aligned} \right\} \forall v \in \Omega^V \tag{66}$$

Finally, the third level represents the market operator problem; in this level the market operator decide the prices and the production of each generator. Restrictions are the following in order of appearance: (i) capacity limits for existing generators, (ii) capacity limits for candidate generators, (iii) Kirchhoff's voltage law using constant power distribution factors, (iv) capacity limits for lines, (v) Kirchhoff current law and (vi) non negativity of production.

Third level:

$$\underset{g_{k,o},\,f_{ij,o},\,r_{i,o}}{\text{Minimize}} \quad \sum_{o\in\Omega^D}\tau_o\left\{\sum_{k\in\Omega^{G1}}[\sigma_k-\xi_k\cdot\Delta g_k^{max}]g_{k,o}+\sum_{k\in\Omega^{G2}}\sigma_k\cdot g_{k,o}\right\}$$

Subject to:

$$\begin{aligned}
&g_{k,o}\le\bar{g}_k && \forall k\in\Omega^{G0},\forall o\in\Omega^D\\
&g_{k,o}\le g_k^{max,0} && \forall k\in\Omega^{G1},\forall o\in\Omega^D\\
&f_{ij,o}=\sum_{i\in N}\varphi_{ij,i}\cdot(g_{k,o}-d_{i,o}) && \forall ij\in\Omega^L,\forall o\in\Omega^D\\
&|f_{ij,o}|\le\bar{f}_{ij} && \forall ij\in\Omega^L\\
&g_{k,o}+r_{i,o}=d_{i,o} && \forall i\in\Omega^N,o\in\Omega^D\\
&g_{k,o}\ge 0 && \forall k\in\Omega^{G1}\cup\Omega^{G0},\forall o\in\Omega^D
\end{aligned} \tag{67}$$

Each firm must choose their bid prices according to the following problem; restrictions are the following in order of appearance: (i) capacity limits for existing generators, (ii) capacity limits for candidate generators and (iii) non negativity of production.

$$\left.\begin{aligned}
&\underset{g_{k,o}}{\text{Maximize}} \quad \sum_{o\in\Omega^D}\sum_{k\in\Omega_v^{G1}}\tau_o[p_{i,o}-(\sigma_k-\xi_k\cdot\Delta g_k^{max})]g_{k,o}\\
&\qquad\qquad +\sum_{o\in\Omega^D}\sum_{k\in\Omega_v^{G0}}\tau_o[p_{i,o}-\sigma_k]g_{k,o}\\
&\text{Subject to:}\\
&g_{k,o}\le\bar{g}_k \qquad \forall k\in\Omega_V^{G0},\forall o\in\Omega^D\\
&g_{k,o}\le g_k^{max,0} \qquad \forall k\in\Omega_v^{G1},\forall o\in\Omega^D\\
&g_{k,o}\ge 0 \qquad \forall k\in\Omega_v^{G1}\cup\Omega_v^{G},\forall o\in\Omega^D
\end{aligned}\right\}\forall v\in\Omega^V \tag{68}$$

2.5.4 Fan and Cheng Model

Fan and Cheng (FAN; CHENG; YAO, 2009) propose a multistage bi-level model for investing in transmission capacity. The model considers a set of stages Ω^T. The network is represented by a set of nodes Ω^N and a set of branches Ω^L. Each node has angle $\theta_{i,t}$ at stage t.The set Ω^L is the union of the existing transmission lines Ω^{L0} and the set of candidate lines $\Omega^{L'}$. Each line ij has a susceptance b_{ij}, a fixed capacity $\bar{f}_{ij}$, and a power flow $f_{ij,t}$ at stage t. Variables $n_{ij,t}$ represent the investment decisions. The supply is represented by a set of generators Ω^G with a production $g_{k,t}$ at stage t.

In order to define the objective function for the first level, the profit function $\pi_t\left(\overline{f}\right)$ is used. These profits are brought to present value using a set or interest rates β_t. The social welfare is given by $W(d_{i,t}, g_{k,t})$. Restrictions are the following in order of appearance: (i) lines built at stage t^* are available for stage t^*+1 and (ii) investment decision variables must be integer.

The second level represents the market operator problem which maximizes the social welfare; restrictions are the following in order of appearance: (i)Kirchhoff voltage law for existing lines, (ii) Kirchhoff's voltage law for candidate lines, (iii) Kirchhoff's current law, (iv) capacity limits for existing lines, (v) capacity limits for candidate lines, (vi) reference angle for the network and (vii) capacity limits for generators. The mathematical formulation of the following :

First level:

$$\begin{aligned}
&\underset{n_{ij,t}}{\text{Maximize}} \quad \sum_{t\in\Omega T} \frac{1}{(1+\beta_t)^t}\cdot\pi_t\left(\overline{f}_{ij,t}\right)\\
&\text{Subject to:}\\
&\qquad n_{ij,t} \geq n_{ij,t+1} \qquad \forall ij \in \Omega^L\\
&\qquad n_{ij}\ \text{integer} \qquad \forall ij \in \Omega^L
\end{aligned} \tag{69}$$

Second level:

$$\left.\begin{aligned}
&\underset{g_{k,t},\, f_{ij,t},\, \theta_{i,t}}{\text{Maximize}} \quad \sum_{t\in\Omega T} W(d_{i,t}, g_{k,t})\\
&\text{Subject to:}\\
&\quad f_{ij,t} = (\theta_{i,t}-\theta_{j,t})\cdot b_{ij} && \forall ij \in \Omega^L\\
&\quad f_{ij,t} = n_{ij,t}\cdot(\theta_{i,t}-\theta_{j,t})\cdot b_{ij} && \forall ij \in \Omega^L\\
&\quad \sum_{ij\in\Omega_i^L} f_{ij,t} - \sum_{ji\in\Omega_i^L} f_{ji,t} + \sum_{k\in\Omega_i^{G0}} g_{k,t} = d_{i,t} && \forall i \in \Omega^N\\
&\quad |f_{ij,t}| \leq \overline{f}_{ij} && \forall ij \in \Omega^{L0}\\
&\quad |f_{ij,t}| \leq n_{ij,t}\cdot\overline{f}_{ij} && \forall ij \in \Omega^{L'}\\
&\quad \theta_{slack,t} = 0\\
&\quad 0 \leq g_{k,t} \leq \overline{g}_k && \forall k \in \Omega^G
\end{aligned}\right\} \forall t \in \Omega^T \tag{70}$$

2.5.5 Centeno, Wogrin, Lopez-Peña and Vasquez Model

Centeno, Wogrin, Lopez Peña and Vazquez (CENTENO et al., 2011) propose a multistage bi-level model for investing in generation capacity. The model only considers investment decisions for one company, taking in count that the remaining firms decision's are given. The company faces a trade -off between investing more and selling more with lower prices, or investing less and selling less with higher prices.

The model considers that generation and load are located on the same node. The supply is represented by a set of existing generators Ω^G with fixed capacity $\bar{g_k}$ and a set of technologies Ω^{S2} whose capacity $g_{s,t}^{max}$ can be increased continuously. Each generator has a production $g_{k,o,t}$ at demand level $o \in \Omega^D$, at stage $t \in \Omega^T$. Each demand level $o \in \Omega^D$ has a number of hours τ_o and a demand $d_{o,t}$ at stage t. The cost of new capacity of technology s is ε_s with marginal cost σ_s. The market prices are given by the dual variables $p_{o,t}$ from the power balance equation.

In the first level the company maximizes its profit. In the second level the market operator determines prices and production for the time horizon.
Objective function for the first level is given by the firm profit during the time horizon, the cash flows are brought to present value using a set of interest rates β_t. The restriction for the first level is that capacity at stage t^* must be available at stage t^*+1. The second level represents the spot market equilibrium, the objective function represents the social welfare; restrictions are the following in order of appearance: (i) capacity limits for existing generators, (ii) capacity limits for new generators and (iii) power balance equation. The mathematical formulation is the following:

First level:

$$\begin{aligned}
\underset{g_{s,t}^{max}}{\text{Maximize}} \quad & \sum_{t\in\Omega^T}\sum_{o\in\Omega^D}\sum_{s\in\Omega^{S2}} \frac{\tau_o}{(1+\beta_t)^t}\cdot(p_{o,t}-\sigma_s)\cdot g_{s,o,t} \\
& -\sum_{t\in T}\sum_{s\in S2}\frac{1}{(1+\beta_t)^t}\cdot\varepsilon_s\cdot(g_{s,t+1}^{max}-g_{s,t}^{max}) \qquad (71)
\end{aligned}$$

Subject to:

$$g_{s,t}^{max} \leq g_{s,t+1}^{max} \qquad \forall s\in\Omega^{S2}, \forall t\in\Omega^T | t<T$$

Second level:

$$\left.\begin{aligned}
\underset{g_{k,o,t}}{\text{Maximize}} \quad & W(d_{o,t}, g_{k,o,t}, g_{k,o,t}) \\
\text{Subject to:} \quad & \\
& g_{k,o,t} \leq \bar{g_k} && \forall k\in\Omega^G \\
& g_{s,o,t} \leq g_{s,t}^{max} && \forall s\in\Omega^{S2} \\
& \sum_{k\in\Omega^{G0}} g_{k,o,t} + \sum_{s\in S2} g_{s,o,t} = d_{o,t} \quad : p_{o,t} &&
\end{aligned}\right\} \forall o\in\Omega^D, \forall t\in\Omega^T \qquad (72)$$

2.5.6 Discussion about the state-of-the-art multilevel models

The state-of-the art models presented in the previous section have important contributions to the TPP. A common characteristic is that all models use the DC representation of the network, since the AC models would be very complex to solve. The objective functions for the market operator problem seems to differ from one model to another. However, the minimization of total costs is equivalent to social welfare maximization for an inelastic demand; thus, the objective functions are the same in all cases.

The main difference between the state-of-the-art models is that some of them (Garcés and Fan) minimizes the total investment cost plus operation cost taking as given generation investments. When only investment plus operation costs are considered, implicitly the models consider that opportunity cost of transmission capacity depends only on the marginal cost of line terminals.

Only, the first model proposed by Jenabi as well the model of Pozo consider investing in generation; however, the models are static and they cannot see the dynamic of investments. In practice, opportunity costs of transmission and generation capacity can be observed during a time horizon and not in a static point of view.

Furthermore, the model of Pozo considers constant power distribution factors, but when increasing capacity those distribution factors do not remain constant. The models of Jenabi, Pozo and Centeno consider that line capacity can be increased continuously which is not a real assumption.

The proposed model in this dissertation make some contributions to the state-of-the-art models. The details of the proposed model are discussed in the next chapter.

3 PROPOSAL OF A MULTISTAGE BI-LEVEL MODEL FOR INVESTING IN TRANSMISSION AND GENERATION CAPACITY

Opportunity cost of transmission capacity is closely dependent on the generation capacity investments. In consequence, externalities arise when investment decisions in transmission and generation capacity are decentralized. This chapter discuses these issues and leads to the conclusion that transmission and generation investment should clear in the same market. Thus, a multistage bi-level model is proposed in order to find the Pareto optimal solution, which can be used for implementation of regulatory mechanisms.

3.1 COMPLEMENTARITY IN TRANSMISSION AND GENERATION CAPACITY

This section explains how opportunity cost of transmission capacity depends on the generation capacity investments. Furthermore, it is shown how externalities arise when investment decisions are decentralized.

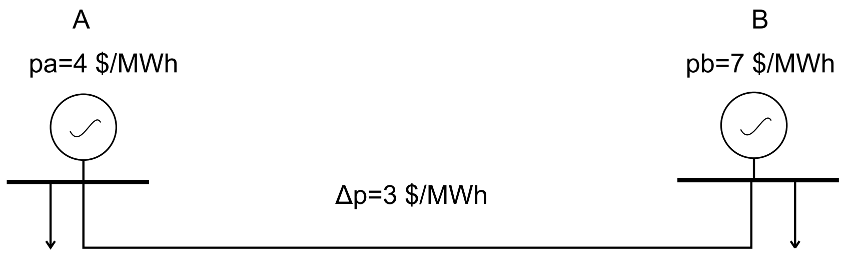
3.1.1 The opportunity cost of the transmission capacity

Section 2.3 considered that opportunity cost of transmission capacity depends only on the marginal costs of line terminals (HOGAN, 1999). Thus, if electricity demand is inelastic, minimization of total investment cost in transmission capacity plus the total operation costs leads to the Pareto optimal solution. This result is correct for a static approach, where there is no need to increase generation capacity. However, with demand growth, new generation capacity must be built and this can also change marginal prices. Then, opportunity cost of transmission capacity must take in count investments in generation capacity.

Furthermore, it is not correct to consider that transmission capacity has an variable cost. Investments in transmission capacity has fixed capacity and are a sunk cost. After a transmission line starts its operation, the additional cost to transmit a MW from one node to another is zero. In conclusion, it is not correct to consider the annualized investment cost as the variable cost since this cost does not exist.

To put it simple, consider the following example of the two bus system showed in Figure 7. If electricity is produced at node A at price $p_A = 4\$/MWh$ and it has to be transported to node B with price $p_B = 7\$/MWh$, the transmission businesses is efficient only if the transportation cost does not exceed the price difference between nodes ($\Delta p = 3\$/MWh$). In this static approach, it is assumed that the line is already built and also a transportation cost is considered.

Figure 7 - Results for the two bus system - base case.



Source: the author.

Now consider that the transmission line is not built yet. Then, transmission capacity must be built until it equals it opportunity cost. In the short run, the opportunity cost of transmission capacity is given by the possibility of buying electricity from the remote node. In the long run, the opportunity cost is also given by the possibility of building local generation with equal or less marginal price. Then, the investment is efficient only if investment cost plus operation cost does not exceed investment cost of local generation plus new operation cost.

From the previous example, consider that transmission line is not built and there are two options: building a transmission line of capacity $\overline{f} = 1000$ MW (see Figure 7) whose investment cost is $C(\overline{f}) = 1 \cdot 10^6$ \$. or building local generation at node B with capacity $\overline{g} = 1000$ MW whose investment cost is $C(\overline{g}) = 200 \cdot 10^6$ \$ with marginal price $\sigma = 2\$/MWh$. Node B needs 1000 MW for 15 years in order to supply its demand. Then , total costs are compared for both options:

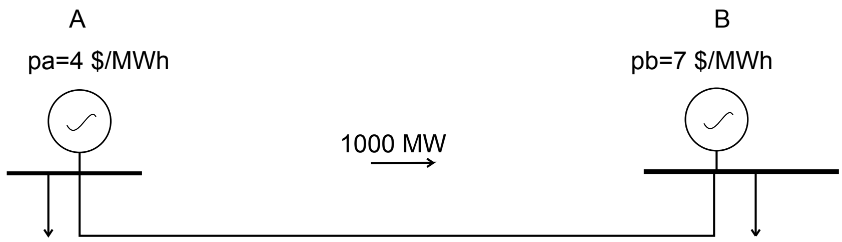
$$C(\overline{f}) + C(g) \leq C(\overline{g}) + C(g') \tag{73}$$

$$\$1 \cdot 10^6 + \sum_{j=1}^{15} \frac{8760h \cdot 1000MW \cdot 4\$/MWh}{(1+0.1)^j} \leq \$200 \cdot 10^6 + \sum_{j=1}^{15} \frac{8760h \cdot 1000MW \cdot 2\$/MWh}{(1+0.1)^j}$$

$$\$2,68 \cdot 10^8 \leq \$3,33 \cdot 10^8$$

In this first case, building the transmission line is better than building local generation, solution is showed in Figure 8.

Figure 8 - Results for the two bus system - case 1.



Source: the author.

Now consider a second case in which investment cost for local generation at node B decreases to $ $100 \cdot 10^6$ (see Figure 9). Total costs are compared for both options.

$$C(\overline{f}) + C(g) \geq C(\overline{g}) + C(g') \tag{74}$$

$$\$1 \cdot 10^6 + \sum_{j=1}^{15} \frac{8760h \cdot 1000MW \cdot 4\$/MWh}{(1+0.1)^j} \geq \$100 \cdot 10^6 + \sum_{j=1}^{15} \frac{8760h \cdot 1000MW \cdot 2\$/MWh}{(1+0.1)^j}$$

$$\$2,68 \cdot 10^8 \geq 2,33 \cdot 10^8$$

Figure 9 - Results for the two bus system - case 2.

Source: the author.

In this second case, building local generation is better than building the transmission line. These results shows that opportunity costs depends on the investments on generation; thus, investments must be cleared in the same market.

3.1.2 Externalities in the investments in transmission and generation capacity

Externalities are present when one agent decisions affect the welfare of another agent but not vice-versa. In a decentralized framework investments follows this temporal setting:

- $t = 1$ Generation companies decide their investments in function of available resources and future prices in the spot market.
- $t = 2$ The transmission company decides the new transmission capacity for the generation built at $t = 1$.
- $t = 3$ Given generation and transmission capacity built at $t = 1$ and $t = 2$, market operator maximizes the social welfare and determine market prices.

Since generation companies decide first, they maximize their profit without considering the transmission capacity cost. This introduce negative externalities to the the transmission capacity investment problem and can lead to a non-optimal solution.

The mathematical formulation of the problem consists of three levels. At first level, generation companies choose their generation capacity in order to maximize their profits subject to its opportunity cost $\bar{\pi}_k$. Profit is the income $p \cdot g_k$ minus the investment cost $C(\bar{g}_k)$ and the operation cost $C_k(g_k)$. The mathematical formulation of the first level is the following:

First level:

$$\left.\begin{array}{ll} \underset{\bar{g}}{\text{Maximize}} & p \cdot g_k - C_k(\bar{g}_k) - C_k(g_k) \\ \text{Subject to:} & \\ & \pi_k \geq \bar{\pi}_k \end{array}\right\} \forall v \in \Omega^V \tag{75}$$

Furthermore, the transmission company chooses the new transmission capacity $\bar{f}$ so that the company minimizes total investment cost $C(\bar{f})$ plus operation cost $C(g)$ subject to network equations. The transmission company takes as given the generation $\bar{g}$ built at the first level:

Second level:

$$\begin{array}{ll} \underset{\bar{f}}{\text{Minimize}} & C'(\bar{f}) + C(g) \\ \text{Subject to:} & \\ & \text{Restrictions}(\bar{g}, \bar{f}, g) \end{array} \tag{76}$$

Finally, at the third level, the market operator minimizes the total operation cost $C(g)$.

Third level:

$$
\begin{array}{cl}
\underset{\bar{g}}{\text{Minimize}} & C(g) \\
\text{Subject to:} & \\
 & \text{Restrictions}(\bar{g}, \bar{f}, g)
\end{array}
\tag{77}
$$

Generation companies do not take in count the transmission costs. In the other hand, Pareto optimal solution is given by the following two level problem :

First level:

$$
\begin{array}{cl}
\underset{\bar{f},\bar{g}}{\text{Minimize}} & C(\bar{g}) + C'(\bar{f}) + C(g) \\
\text{Subject to:} & \\
 & \text{Restrictions}(\bar{g}, g)
\end{array}
\tag{78}
$$

Second level:

$$
\begin{array}{cl}
\underset{\bar{g}}{\text{Minimize}} & C(g) \\
\text{Subject to:} & \\
 & \text{Restrictions}(\bar{g}, \bar{f}, g)
\end{array}
\tag{79}
$$

It can be seen that optimality conditions for both problems are different. Thus, the decentralized outcome does not lead to a Pareto optimal solution. Since Pareto optimal solution gives a solution of maximum social welfare, a decentralized outcome would not lead to the maximum social welfare.

In order to overcome the externalities, regulatory mechanisms can be proposed so that generation companies take in count the transmission costs on their objective functions (BAUMOL; OATES, 1988); this can be accomplished through taxes, normativeness, etc. The proposition of such regulatory mechanism is out of the purposes of the present dissertation.

3.2 THE PROPOSED MODEL

The proposed model finds the Pareto optimal solution which can be used as a reference for implementation of regulatory mechanisms. Without loss of generality, some aspects are left apart such reliability and security of the system; these aspects can be added posteriorly, for further details see (GARCES; ROMERO; LOPEZ-LEZAMA, 2010).

3.2.1 Mathematical formulation of the model

The proposed model considers the DC model of the network, which is represented by a set of nodes Ω^N ans a set of branches Ω^L. Each branch ij connects nodes i and j. Each branch ij has a number of existing circuits n_{ij}^0, with suceptance b_{ij}, and fixed capacity $\overline{f_{ij}}$. Moreover, a set of stages Ω^T is considered so that transmission an generation capacity is fixed during each stage. Each stage has a subset of demand levels Ω^D of duration τ_o hours with a demand $d_{i,o,t}$ at node i, at demand level o and at stage t.

Each node has an angle $\theta_{i,o,t}$ associated to demand level o at stage t. A reference angle *slack* is considered so that $\theta_{\text{slack},o,t} = 0$. Power flows though existing lines are represented by variables $f_{ij,o,t}$. Candidate lines are represented by a set of circuits $\Omega^2, q \in \Omega^2 = 1, 2 \cdots \overline{q}$ available at each branch ij, with investment cost c_{ij}. Binary variable $z_{q,ij,t}$ represents investment decision for circuits q, in branch ij at stage t. Power flows in candidate circuits are given by variables $f_{q,ij,o,t}$ at demand level o and stage t.

The supply is given by a set of existing generators Ω^{G0}. Each generator k has a marginal cost σ_k and a fixed capacity $\overline{g_k}$. At each demand level, existing generator has a production $g_{k,o,t}$. Candidate generators at node i are represented by a set of technologies Ω^S, each technology has a set of candidate units of similar characteristics $\Omega^1, u \in \Omega^1 = 1, 2 \cdots \overline{u}$. Binary variables $w_{u,s,i,t}$ represent investment decisions for unit u, for technology s, at node i, at stage t. The investment cost for technology s at node i is given by $\varepsilon_{s,i}$ with marginal cost $\sigma_{s,i}$. Each unit has fixed capacity $\overline{g_{s,i}}$ and a production $g_{u,s,i,o,t}$ at demand level o. It is considered that generation units lifetime is the same for all technologies and it is longer than the horizon time in analysis.

The first level problem is given by the investment problem in transmission and generation capacity. Objective function is given by the present value of the total investment cost plus the operation cost. Restrictions are the following in order of appearance: (i) margin reserve of generation capacity , (ii) maximum number of generation units at a certain node, (iii) inter-temporal constraint for generation units: units built at stage t^* are available at stage t^*+1, (iv) construction order for generation units, (v) maximum number of circuits at a certain branch, (vi) inter-temporal restriction for transmission circuits: circuits built at t^* are available at stage t^*+1, (vii) construction order of transmission circuits, (viii) investment decision variables in transmission circuits must be binary and (ix) investment decision variables din generation units must be binary.

The second level is given by the market operator problem for each demand level at each stage. Objective function represents total operation cost. restrictions are the following in order of appearance: (i) Kirchhoff's current law, (ii) Kirchhoff's voltage law for existing lines, (iii) capacity limits of existing lines, (iv) Kirchhoff's voltage law for candidate lines, (v) capacity limits for candidate lines, (vi) capacity limits for existing generators and (vii) capacity limits for candidate generators. The mathematical formulation is the following:

First level

$$\begin{aligned}
\underset{z_{q,ij,t},\, w_{u,s,i,t}}{\text{Minimize}} \quad & \sum_{ij\in\Omega^L}\sum_{q\in\Omega^2}\frac{1}{(1+\beta_1)}\cdot c_{ij}\cdot z_{q,ij,1}+\\
& \sum_{t\in\Omega^T|t>1}\sum_{ij\in\Omega^L}\sum_{r\in\Omega^2}\frac{1}{(1+\beta_t)^t}\cdot c_{ij}\cdot(z_{q,ij,t}-z_{q,ij,t-1})+\\
& \sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\frac{1}{(1+\beta_1)}\cdot\varepsilon_{s,i}\cdot\overline{g}_{s,i}\cdot w_{u,s,i,1}+\\
& \sum_{t\in\Omega^T|t>1}\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\frac{1}{(1+\beta_t)^t}\cdot\varepsilon_{s,i}\cdot\overline{g}_{s,i}\cdot(w_{u,s,i,t}-w_{u,s,i,t-1})+\\
& \sum_{t\in\Omega^T}\sum_{o\in\Omega^D}\sum_{k\in\Omega^{G0}}\frac{1}{(1+\beta_t)^t}\cdot\tau_o\cdot\sigma_k\cdot g_{k,o,t}+\\
& \sum_{t\in\Omega^T}\sum_{o\in\Omega^D}\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\frac{1}{(1+\beta_t)^t}\cdot\tau_o\cdot\sigma_{s,i}\cdot g_{u,s,i,o,t}
\end{aligned}$$

Subject to:

$$\begin{aligned}
& \sum_{k\in\Omega^{G0}}\overline{g}_k+\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\overline{g}_{s,i}\cdot w_{u,s,i,t}\geq\phi\cdot\sum_{i\in\Omega^N}d_{i,o,t} && \forall o\in\Omega^D, t\in\Omega^T\\
& \sum_{u\in\Omega^S}w_{u,s,i,T}\leq\overline{w}_{s,i} && \forall s\in\Omega^S, i\in\Omega^N\\
& w_{u,s,i,t-1}\leq w_{u,s,i,t} && \forall u\in\Omega^S, s\in\Omega^S, i\in\Omega^N, t>1\in\Omega^T\\
& w_{u,s,i,t}\leq w_{u-1,s,i,t} && \forall u>1\in\Omega^S, s\in\Omega^S, i\in\Omega^N, t\in\Omega^T\\
& \sum_{q\in\Omega^2}z_{q,ij,T}\leq\overline{n}_{ij} && \forall ij\in\Omega^L\\
& z_{q,ij,t-1}\leq z_{q,ij,t} && \forall q\in\Omega^2, ij\in\Omega^L, t>1\in\Omega^T\\
& z_{q,ij,t}\leq z_{q-1,ij,t} && \forall q>1\in\Omega^2, ij\in\Omega^L, t\in\Omega^T\\
& z_{q,ij,t}\in\{0,1\} && \forall q\in\Omega^2, ij\in\Omega^L, t\in\Omega^T\\
& w_{u,s,i,t}\in\{0,1\} && \forall u\in\Omega^S, s\in\Omega^S, i\in\Omega^N, t\in\Omega^T
\end{aligned} \tag{80}$$

The two first lines of the objective function correspond to the investment cost in transmission capacity, the next two lines correspond to the investment cost in generation capacity and the last two lines correspond to the total operating cost of the spot market.

$$
\left.
\begin{array}{lll}
\underline{\text{Second level}} & & \\
\underset{g_k, g_s, f_{ij}, \theta}{\text{Minimize}} & \sum_{k\in\Omega^{G0}} \sigma_k \cdot g_{k,o,t} + \sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1} \sigma_{s,i}\cdot g_{u,s,i,o,t} & \\
\text{Subject to:} & & \\
& \sum_{ij\in\Omega_i^{L'}}\sum_{q\in\Omega^2} f_{q,ij,o,t} - \sum_{ji\in\Omega_i^{L'}}\sum_{q\in\Omega^2} f_{q,ji,o,t} & \\
& + \sum_{ij\in\Omega_i^{L0}} f_{ij,o,t} - \sum_{ji\in\Omega_i^{L0}} f_{ij,o,t} & \\
& + \sum_{k\in\Omega_i^{G0}} g_{k,o,t} + \sum_{s\in\Omega_i^S}\sum_{u\in\Omega^1} g_{u,s,i,o,t} = d_{i,o,t} & \forall i\in\Omega^N \\
& f_{ij,o,t} - b_{ij}\cdot n_{ij}^0 \cdot (\theta_{i,o,t} - \theta_{j,o,t}) = 0 & \forall ij\in\Omega^L \\
& |f_{ij,o,t}| \le \overline{f}_{ij}\cdot n_{ij}^0 & \forall ij\in\Omega^L \\
& f_{q,ij,o,t} - b_{ij}\cdot z_{q,ij,t}\cdot(\theta_{i,o,t}-\theta_{j,o,t}) = 0 & \forall q\in\Omega^2, ij\in\Omega^L \\
& |f_{q,ij,o,t}| \le \overline{f}_{ij} & \forall q\in\Omega^2, ij\in\Omega^L \\
& 0 \le g_{k,o,t} \le \overline{g}_k & \forall k\in\Omega^{G0} \\
& 0 \le g_{u,s,i,o,t} \le \overline{g}_{s,i}\cdot w_{u,s,i,t} & \forall u\in\Omega^1, s\in\Omega^S, i\in\Omega^N \\
& \theta_{\text{slack},p,t} = 0 &
\end{array}
\right\}
\begin{array}{c}\forall o\in\Omega^D,\\ t\in\Omega^T\end{array}
\tag{81}
$$

The second level is linear in its variables: g, f_{ij},and θ; this because market operator take as given first level variables $z_{q,ij,t}$ and $w_{u,s,i,t}$. Thus, its dual problem can be formulated as shown in section 2.4.

3.2.2 Single-level formulation of the bi-level model

According to section 2.4, a bi-level problem can be transformed in one level problem replacing the dual problem of the second level in the first level. In (82), dual variables are presented next to each restriction: λ^1, λ^2, λ^3, λ^4, λ^6, λ^7, λ^8, λ^9, and λ^{10}. Dual variable λ^1 has an economic interpretation, since Lagrange multipliers of the power balance equation represents additional cost of producing one more MW for a certain node, dual variables $\lambda^1 = p_i$ represent the nodal marginal cost.

$$\left.\begin{array}{ll}
\sum_{ij\in\Omega_i^{L'}}\sum_{q\in\Omega^2} f_{q,ij,o,t} - \sum_{ji\in\Omega_i^{L'}}\sum_{q\in\Omega^2} f_{q,ji,o,t} + \sum_{ij\in\Omega_i^{L0}} f_{ij,o,t} & \\
-\sum_{ji\in\Omega_i^{L0}} f_{ji,o,t} + \sum_{k\in\Omega_i^{G0}} g_{k,o,t} + \sum_{s\in\Omega_i^S}\sum_{u\in\Omega^1} g_{u,s,i,o,t} = d_{i,o,t} \quad : \lambda^1_{i,o,t} & \forall i\in\Omega^N \\
f_{ij,o,t} - b_{ij}\cdot n^0_{ij}\cdot(\theta_{i,p,t}-\theta_{j,p,t}) = 0 \quad : \lambda^2_{ij,o,t} & \forall ij\in\Omega^L \\
-f_{ij,o,t} \geq -\overline{f}_{ij}\cdot n^0_{ij} \quad : \lambda^3_{ij,o,t} & \forall ij\in\Omega^L \\
f_{ij,o,t} \geq -\overline{f}_{ij}\cdot n^0_{ij} \quad : \lambda^4_{ij,o,t} & \forall ij\in\Omega^L \\
f_{q,ij,o,t} - b_{ij}\cdot z_{q,ij,p,t}\cdot(\theta_{i,o,t}-\theta_{j,o,t}) = 0 \quad : \lambda^5_{q,ij,o,t} & \forall q\in\Omega^2, ij\in\Omega^L \\
-f_{q,ij,o,t} \geq -\overline{f}_{ij} \quad : \lambda^6_{q,ij,o,t} & \forall q\in\Omega^2, ij\in\Omega^L \\
f_{q,ij,o,t} \geq -\overline{f}_{ij} \quad : \lambda^7_{q,ij,o,t} & \forall q\in\Omega^2, ij\in\Omega^L \\
-g_{k,o,t} \geq -\overline{g}_k \quad : \lambda^8_{k,o,t} & \forall k\in\Omega^{G0} \\
-g_{u,s,i,o,t} \geq -\overline{g}_{s,i}\cdot w_{u,s,i,t} \quad : \lambda^9_{u,s,i,o,t} & \forall u\in\Omega^1, s\in\Omega^S, i\in\Omega^N \\
\theta_{\text{slack},o,t} = 0 \quad : \lambda^{10}_{o,t} & \\
g_{k,o,t}, g_{u,s,i,o,t} \geq 0 &
\end{array}\right\} \begin{array}{c}\forall o\in\Omega^D,\\ t\in\Omega^T\end{array} \tag{82}$$

The strong dual condition states that objective of primal problem equals the dual problem objective at the optimal solution:

$$\begin{aligned}
0 \; = \; & \sum_{i\in\Omega^N} d_{i,o,t}\cdot\lambda^1_{i,o,t} \\
& - \sum_{ij\in\Omega^L} n^0_{ij}\cdot\overline{f}_{ij}\cdot(\lambda^3_{ij,o,t}+\lambda^4_{ij,o,t}) \\
& - \sum_{ij\in\Omega^L}\sum_{q\in\Omega^2}\overline{f}_{ij}\cdot(\lambda^6_{q,ij,o,t}+\lambda^7_{q,ij,o,t}) \\
& - \sum_{k\in\Omega^{G0}}\overline{g}_k\cdot\lambda^8_{k,o,t} - \sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\overline{g}_{s,i}\cdot\lambda^9_{u,s,i,o,t}\cdot w_{u,s,i,o,t} \\
& - \sum_{k\in\Omega^{G0}}\sigma_k\cdot g_{k,o,t} - \sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\sigma_{s,i}\cdot g_{u,s,i,o,t}
\end{aligned} \tag{83}$$

Finally the dual problem is given by the following problem:

$$
\left.
\begin{array}{lll}
\underset{\lambda^{1\ldots 10}}{\text{Maximize}} & \displaystyle\sum_{i\in\Omega^N} d_{i,o,t}\cdot\lambda^1_{i,o,t} - \sum_{ij\in\Omega^L} n^0_{ij}\cdot\overline{f}_{ij}\cdot(\lambda^3_{ij,o,t}+\lambda^4_{ij,o,t}) & \\
& \displaystyle -\sum_{ij\in\Omega^L}\sum_{r\in\Omega^2}\overline{f}_{ij}\cdot(\lambda^6_{q,ij,o,t}+\lambda^7_{q,ij,o,t}) & \\
& \displaystyle -\sum_{k\in\Omega^{G0}}\overline{g}^0_k\cdot\lambda^8_{k,o,t} - \sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\overline{g}_{s,i}\cdot\lambda^9_{u,s,i,o,t}\cdot w_{u,s,i,o,t} & \\
\text{Subject to:} & & \\
& \lambda^1_{j,o,t}-\lambda^1_{i,o,t}+\lambda^2_{ij,o,t}-\lambda^3_{ij,o,t}+\lambda^4_{ij,o,t}=0 & \forall ij\in\Omega^L \\
& \lambda^1_{j,o,t}-\lambda^1_{i,o,t}+\lambda^5_{q,ij,o,t}-\lambda^6_{q,ij,o,t}+\lambda^7_{q,ij,o,t}=0 & \forall r\in\Omega^2, ij\in\Omega^L \\
& \lambda^1_{k,o,t}-\lambda^8_{k,o,t}\le\sigma_k & \forall k\in\Omega^{G0} \\
& \lambda^1_{k,o,t}-\lambda^9_{u,s,i,o,t}\le\sigma_{s,i} & \forall u\in\Omega^1, s\in\Omega^S, i\in\Omega^N \\
& \displaystyle -\sum_{ij\in\Omega^L} b_{ij}\cdot n^0_{ij}\cdot\lambda^2_{ij,o,t} & \\
& \displaystyle -\sum_{ij\in\Omega^L}\sum_{q\in\Omega^2} b_{ij}\cdot z_{q,ij,t}\cdot\lambda^5_{q,ij,o,t}=0 & \forall i\in\Omega^N, i\neq\text{slack} \\
& \displaystyle -\sum_{ij\in\Omega^L} b_{ij}\cdot n^0_{ij}\cdot\lambda^2_{ij,o,t} & \\
& \displaystyle -\sum_{ij\in\Omega^L}\sum_{q\in\Omega^2} b_{ij}\cdot z_{q,ij,t}\cdot\lambda^5_{q,ij,o,t}+\lambda^{10}_{o,t}=0 & i=\text{slack} \\
& \lambda^3_{ij,o,t}\ge 0 & \forall ij\in\Omega^L \\
& \lambda^4_{ij,o,t}\ge 0 & \forall ij\in\Omega^L \\
& \lambda^6_{q,ij,o,t}\ge 0 & \forall r\in\Omega^2, ij\in\Omega^L \\
& \lambda^7_{q,ij,o,t}\ge 0 & \forall r\in\Omega^2, ij\in\Omega^L \\
& \lambda^8_{k,o,t}\ge 0 & \forall k\in\Omega^{G0} \\
& \lambda^9_{u,s,i,o,t}\ge 0 & \forall u\in\Omega^1, s\in\Omega^S, i\in\Omega^N
\end{array}
\right\}
\begin{array}{c}\forall o\in\Omega^D,\\ t\in\Omega^T\end{array}
\tag{84}
$$

The second level problem can be represented by the primal problem restrictions (82), the dual problem restrictions (83) and the strong dual condition (84) of the second level problem.

3.2.3 Mixed integer linear formulation of the model

With the one level representation the problem turns no lineal since some equations has bilinear terms where two decision variables are multiplied. In order to linearize the bilinear terms, the Fortuny - Amat representation is used. Let x be a binary variable and let y a continuous variable, then the following relations applies:

$$x \cdot y \implies \begin{cases} x \cdot y = y - y^* \\ |y - y^*| \leq M \cdot x \\ |y^*| \leq M \cdot (1 - x) \end{cases} \tag{85}$$

Where y^* is an auxiliary continuous variable, M is the upper limit of y. If $x = 0 \implies y - y^* = 0$ and if $x = 1 \implies y - y^* = y$. By the use of the Fortuny - Amat representation the nonlinear problem can be expressed as a mixed integer linear problem. For the equation 86; bilinear terms have multiplications of $z_{r,ij,p,t}$ and $\theta_{i,p,t}$.

$$f_{q,ij,o,t} - b_{ij} \cdot z_{q,ij,o,t} \cdot (\theta_{i,o,t} - \theta_{j,o,t}) = 0 \tag{86}$$

Equation 86 can be replaced by inequalities (87) and (88); if $z_{q,ij,t} = 1$ then $|f_{r,ij,p,t} - b_{ij} \cdot (\theta_{i,p,t} - \theta_{j,p,t})| \leq 0$ which implies that the term is zero. If $z_{q,ij,t} = 0$ then $|f_{r,ij,p,t}| \leq 0 \implies f_{r,ij,p,t} = 0$, where M is a large number in order to not constraint the problem.

$$|f_{q,ij,o,t} - b_{ij} \cdot (\theta_{i,o,t} - \theta_{j,o,t})| \leq M \cdot (1 - z_{q,ij,t}) \tag{87}$$

$$|f_{q,ij,o,t}| \leq \overline{f}_{ij} \cdot z_{q,ij,t} \tag{88}$$

Similarly, for equation (89); bilinear terms have multiplications of variables $z_{q,ij,t}$ and $\lambda^5_{q,ij,o,t}$.

$$-\sum_{ij \in \Omega^L} b_{ij} \cdot n^0_{ij} \cdot \lambda^2_{ij,o,t} - \sum_{ij \in \Omega^L} \sum_{q \in \Omega^2} b_{ij} \cdot z_{q,ij,t} \cdot \lambda^5_{q,ij,o,t} = 0 \tag{89}$$

Equation (89) can be replaced by inequalities (90), (91) and (92). If $z_{q,ij,t} = 0$ then $\lambda^5_{q,ij,o,t} - \lambda^{5*}_{q,ij,o,t} = 0$ and $|\lambda^{5*}_{q,ij,o,t}| \leq M'$. If $z_{q,ij,t} = 1$ then $\lambda^{5*}_{q,ij,o,t} = 0$ and $|\lambda^5_{q,ij,o,t}| \leq M'$, where M' is a large number in order to not constrain the problem.

$$-\sum_{ij \in \Omega^L} b_{ij} \cdot n^0_{ij} \cdot \lambda^2_{ij,o,t} - \sum_{ij \in \Omega^L} \sum_{r \in \Omega^2} b_{ij} \cdot (\lambda^5_{q,ij,o,t} - \lambda^{5*}_{q,ij,o,t}) = 0 \tag{90}$$

$$|\lambda^5_{q,ij,o,t} - \lambda^{5*}_{q,ij,o,t}| \leq M' \cdot z_{q,ij,t} \tag{91}$$

$$|\lambda^{5*}_{q,ij,o,t}| \leq M' \cdot (1 - z_{q,ij,t}) \tag{92}$$

Finally, for the strong dual condition, bilinear terms have multiplication of variables $w_{u,s,i,p,t}$

and $\lambda^9_{u,s,i,p,t}$.

$$
\begin{aligned}
-\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\sigma_{s,i}\cdot g_{u,s,i,o,t} \quad &= \quad \sum_{i\in\Omega^N} d_{i,o,t}\cdot\lambda^1_{i,o,t} \\
-\sum_{k\in\Omega^{G0}}\sigma_k\cdot g_{k,o,t} \qquad & \qquad -\sum_{ij\in\Omega^L} n^0_{ij}\cdot\overline{f}_{ij}\cdot(\lambda^3_{ij,o,t}+\lambda^4_{ij,o,t}) \\
& \qquad -\sum_{ij\in\Omega^L}\sum_{r\in\Omega^2}\overline{f}_{ij}\cdot(\lambda^3_{q,ij,o,t}+\lambda^7_{q,ij,o,t}) \\
& \qquad -\sum_{k\in\Omega^{G0}}\overline{g}_k\cdot\lambda^8_{k,o,t}-\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\overline{g}_{s,i}\cdot\lambda^9_{u,s,i,o,t}\cdot w_{u,s,i,o,t}
\end{aligned}
\tag{93}
$$

Strong dual condition can be replaced by inequalities (94), (95) y (96). If $w_{u,s,i,o,t}=0$ then $\lambda^9_{u,s,i,o,t}-\lambda^{9*}_{u,s,i,o,t}=0$ and $|\lambda^{9*}_{u,s,i,o,t}|\leq M''$. If $w_{u,s,i,p,t}=1$ then $|\lambda^{9*}_{u,s,i,p,t}|=0$ and $|\lambda^9_{u,s,i,o,t}|\leq M''$, where M'' is a large number in order to not constrain the problem.

$$
\begin{aligned}
-\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\sigma_{s,i}\cdot g_{u,s,i,o,t} \quad &= \quad \sum_{i\in\Omega^N} d_{i,o,t}\cdot\lambda^1_{i,o,t}-\sum_{ij\in\Omega^L} n^0_{ij}\cdot\overline{f}_{ij}\cdot(\lambda^3_{ij,o,t}+\lambda^4_{ij,o,t}) \\
-\sum_{k\in\Omega^{G0}}\sigma_k\cdot g_{k,o,t} \qquad & \qquad -\sum_{ij\in\Omega^L}\sum_{r\in\Omega^2}\overline{f}_{ij}\cdot(\lambda^3_{q,ij,o,t}+\lambda^7_{q,ij,o,t})-\sum_{k\in\Omega^{G0}}\overline{g}^0_k\cdot\lambda^8_{k,o,t} \\
& \qquad -\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\overline{g}_{s,i}\cdot(\lambda^9_{u,s,i,o,t}-\lambda^{9*}_{u,s,i,o,t})
\end{aligned}
\tag{94}
$$

$$|\lambda^9_{u,s,i,o,t}-\lambda^{9*}_{u,s,i,o,t}|\leq M''\cdot w_{u,s,i,o,t} \tag{95}$$

$$|\lambda^{9*}_{u,s,i,o,t}|\leq M''\cdot(1-w_{u,s,i,o,t}) \tag{96}$$

Finally, replacing the bilinear terms by their linear representation leads to the following mixed integer linear problem:

Minimize

$$
\begin{aligned}
\min_{\substack{z,w\\ g,f,\theta\\ \lambda^{1\ldots 10}}} \quad & \sum_{ij\in\Omega^L}\sum_{q\in\Omega^2}\frac{1}{(1+\beta_1)}\cdot c_{ij}\cdot z_{q,ij,1}+\\
& \sum_{t\in\Omega^T|t>1}\sum_{ij\in\Omega^L}\sum_{q\in\Omega^2}\frac{1}{(1+\beta_t)^t}\cdot c_{ij}\cdot(z_{q,ij,t}-z_{q,ij,t-1})+\\
& \sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\frac{1}{(1+\beta_1)}\cdot\varepsilon_{s,i}\cdot\overline{g}_{s,i}\cdot w_{u,s,i,1}+\\
& \sum_{t\in\Omega^T|t>1}\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\frac{1}{(1+\beta_t)^t}\cdot\varepsilon_{s,i}\cdot\overline{g}_{s,i}\cdot(w_{u,s,i,t}-w_{u,s,i,t-1})+\\
& \sum_{t\in\Omega^T}\sum_{o\in\Omega^D}\sum_{k\in\Omega^{G0}}\frac{1}{(1+\beta_t)^t}\cdot\tau_o\cdot\sigma_k\cdot g_{k,o,t}+\\
& \sum_{t\in\Omega^T}\sum_{o\in\Omega^D}\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\frac{1}{(1+\beta_t)^t}\cdot\tau_o\cdot\sigma_{s,i}\cdot g_{u,s,i,o,t}
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
& \sum_{k\in\Omega^{G0}}\overline{g}_k+\sum_{i\in\Omega^N}\sum_{s\in\Omega^S}\sum_{u\in\Omega^1}\overline{g}_{s,i}\cdot w_{u,s,i,t}\geq\phi\cdot\sum_{i\in\Omega^N}d_{i,o,t} && \forall o\in\Omega^D,t\in\Omega^T\\
& \sum_{u\in\Omega^1}w_{u,s,i,T}\leq\overline{w}_{s,i} && \forall s\in\Omega^S,i\in\Omega^N\\
& w_{u,s,i,t-1}\leq w_{u,s,i,t} && \forall u\in\Omega^1,s\in\Omega^S,i\in\Omega^N,t>\in\Omega^T\\
& w_{u,s,i,t}\leq w_{u-1,s,i,t} && \forall u>1\in\Omega^1,s\in\Omega^S,i\in\Omega^N,t\in\Omega^T\\
& \sum_{q\in\Omega^2}z_{q,ij,T}\leq\overline{n}_{ij} && \forall ij\in\Omega^L\\
& z_{q,ij,t-1}\leq z_{q,ij,t} && \forall q\in\Omega^2,ij\in\Omega^L,t>1\in\Omega^T\\
& z_{q,ij,t}\leq z_{q-1,ij,t} && \forall q>1\in\Omega^2,ij\in\Omega^L,t\in\Omega^T\\
& z_{q,ij,t}\in\{0,1\} && \forall q\in\Omega^2,ij\in\Omega^L,t\in\Omega^T\\
& w_{u,s,i,t}\in\{0,1\} && \forall u\in\Omega^1,s\in\Omega^S,i\in\Omega^N,t\in\Omega^T\\
& \sum_{ij\in\Omega_i^{L'}}\sum_{q\in\Omega^2}f_{q,ij,o,t}-\sum_{ji\in\Omega_i^{L'}}\sum_{q\in\Omega^2}f_{q,ji,o,t}+\sum_{ij\in\Omega_i^{L0}}f_{ij,o,t}\\
& \sum_{ji\in\Omega_i^{L0}}f_{ji,o,t}+\sum_{k\in\Omega_i^{G0}}g_{k,o,t}+\sum_{s\in\Omega_i^S}\sum_{u\in\Omega^1}g_{u,s,i,o,t}=d_{i,o,t} && \forall i\in\Omega^N,o\in\Omega^D,t\in\Omega^T\\
& f_{ij,o,t}-b_{ij}\cdot n_{ij}^0\cdot(\theta_{i,o,t}-\theta_{j,o,t})=0 && \forall ij\in\Omega^L,o\in\Omega^D,t\in\Omega^T\\
& |f_{ij,o,t}|\leq\overline{f}_{ij}\cdot n_{ij}^0 && \forall ij\in\Omega^L,o\in\Omega^D,t\in\Omega^T\\
& |f_{q,ij,o,t}-b_{ij}\cdot(\theta_{i,o,t}-\theta_{j,o,t})|\leq M\cdot(1-z_{q,ij,t}) && \forall q\in\Omega^2,ij\in\Omega^L,o\in\Omega^D,t\in\Omega^T\\
& |f_{q,ij,o,t}|\leq\overline{f}_{ij}\cdot z_{q,ij,t} && \forall q\in\Omega^2,ij\in\Omega^L,o\in\Omega^D,t\in\Omega^T\\
& 0\leq g_{k,o,t}\leq\overline{g}_k && \forall k\in\Omega^{G0},\forall o\in\Omega^D,\forall t\in\Omega^T\\
& 0\leq g_{u,s,i,o,t}\leq\overline{g}_{s,i}\cdot w_{u,s,i,t} && \forall u\in\Omega^1,s\in\Omega^S,i\in\Omega^N,o\in\Omega^D,t\in\Omega^T\\
& \theta_{\text{slack},o,t}=0 && \forall o\in\Omega^D,t\in\Omega^T
\end{aligned}
\tag{97}
$$

$$
\begin{aligned}
&\lambda^1_{j,o,t} - \lambda^1_{i,o,t} + \lambda^2_{ij,o,t} - \lambda^3_{ij,o,t} + \lambda^4_{ij,o,t} = 0 && \forall ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^1_{j,o,t} - \lambda^1_{i,o,t} + \lambda^5_{r,ij,p,t} - \lambda^6_{q,ij,o,t} + \lambda^7_{q,ij,o,t} = 0 && \forall r \in \Omega^2, ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^1_{k,o,t} - \lambda^8_{k,o,t} \leq \sigma_k && \forall k \in \Omega^{G0}, o \in \Omega^D, t \in \Omega^T \\
&\lambda^1_{i,o,t} - \lambda^9_{u,s,i,o,t} \leq \sigma_{s,i} && \forall u \in \Omega^1, s \in \Omega^S, i \in \Omega^N, o \in \Omega^D, t \in \Omega^T \\
&- \sum_{ij \in \Omega^L} b_{ij} \cdot n^0_{ij} \cdot \lambda^2_{ij,o,t} && \\
&- \sum_{ij \in \Omega^L} \sum_{r \in \Omega^2} b_{ij} \cdot (\lambda^5_{q,ij,o,t} - \lambda^{5*}_{r,ij,o,t}) = 0 && \forall i \in \Omega^N | i \neq \text{slack}, o \in \Omega^D, t \in \Omega^T \\
&- \sum_{ij \in \Omega^L} b_{ij} \cdot n^0_{ij} \cdot \lambda^2_{ij,o,t} && \\
&- \sum_{ij \in \Omega^L} \sum_{r \in \Omega^2} b_{ij} \cdot (\lambda^5_{q,ij,o,t} - \lambda^{5*}_{r,ij,o,t}) + \lambda^{10}_{o,t} = 0 && i = \text{slack}, \forall o \in \Omega^D, t \in \Omega^T \\
&|\lambda^5_{q,ij,o,t} - \lambda^{5*}_{q,ij,o,t}| \leq M' \cdot z_{q,ij,t} && \forall q \in \Omega^2, ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&|\lambda^{5*}_{q,ij,o,t}| \leq M' \cdot (1 - z_{q,ij,t}) && \forall q \in \Omega^2, ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^3_{ij,o,t} \geq 0 && \forall ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^4_{ij,o,t} \geq 0 && \forall ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^6_{q,ij,o,t} \geq 0 && \forall r \in \Omega^2, ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^7_{q,ij,o,t} \geq 0 && \forall r \in \Omega^2, ij \in \Omega^L, o \in \Omega^D, t \in \Omega^T \\
&\lambda^8_{k,o,t} \geq 0 && \forall k \in \Omega^{G0}, o \in \Omega^D, t \in \Omega^T \\
&\lambda^9_{u,s,i,o,t} \geq 0 && \forall u \in \Omega^1, s \in \Omega^S, i \in \Omega^N, o \in \Omega^D, t \in \Omega^T \\
&\sum_{i \in \Omega^N} d_{i,o,t} \cdot \lambda^1_{i,o,t} - \sum_{ij \in \Omega^L} n^0_{ij} \cdot \overline{f}_{ij} \cdot (\lambda^3_{ij,o,t} + \lambda^4_{ij,o,t}) && \\
&- \sum_{ij \in \Omega^L} \sum_{q \in \Omega^2} \overline{f}_{ij} \cdot (\lambda^6_{q,ij,o,t} + \lambda^7_{q,ij,o,t}) && \\
&- \sum_{k \in \Omega^{G0}} \overline{g}^0_k \cdot \lambda^8_{k,o,t} - \sum_{i \in \Omega^N} \sum_{s \in \Omega^S} \sum_{u \in \Omega^1} \overline{g}_{s,i} \cdot (\lambda^9_{u,s,i,o,t} - \lambda^{9*}_{u,s,i,o,t}) && \\
&- \sum_{k \in \Omega^{G0}} \sigma_k \cdot g^0_{k,o,t} - \sum_{i \in \Omega^N} \sum_{s \in \Omega^S} \sum_{u \in \Omega^1} \sigma_{s,i} \cdot g'_{u,s,i,o,t} = 0 && \forall o \in \Omega^D, t \in \Omega^T \\
&|\lambda^9_{u,s,i,o,t} - \lambda^{9*}_{u,s,i,o,t}| \leq M'' \cdot w_{u,s,i,o,t} && \forall u \in \Omega^1, s \in \Omega^S, i \in \Omega^N, o \in \Omega^D, t \in \Omega^T \\
&|\lambda^{9*}_{u,s,i,o,t}| \leq M'' \cdot (1 - w_{u,s,i,o,t}) && \forall u \in \Omega^1, s \in \Omega^S, i \in \Omega^N, o \in \Omega^D, t \in \Omega^T
\end{aligned}
$$

Since the problem is linear, it can be solve by a linear programming solver such CPLEX as shown in the next chapter.

4 TEST AND RESULTS

In this chapter, test and results for some systems are presented: the Garver system, the IEEE 24 buses system and the Peruvian system. For each system, several study cases are presented which are divided in two groups. The first group is intended to verify the results of the proposed model with the standard results known in the literature. The second group is intended to show discussion of the previous chapter: how opportunity cost of investment in transmission and generation capacity are dependent (Garver system), and how externalities arise when the investment decisions are decentralized (IEEE 24 buses system). Finally, the Peruvian system study cases are intended to show how the proposed model can be applied to a real system.

4.1 THE GARVER TEST SYSTEM

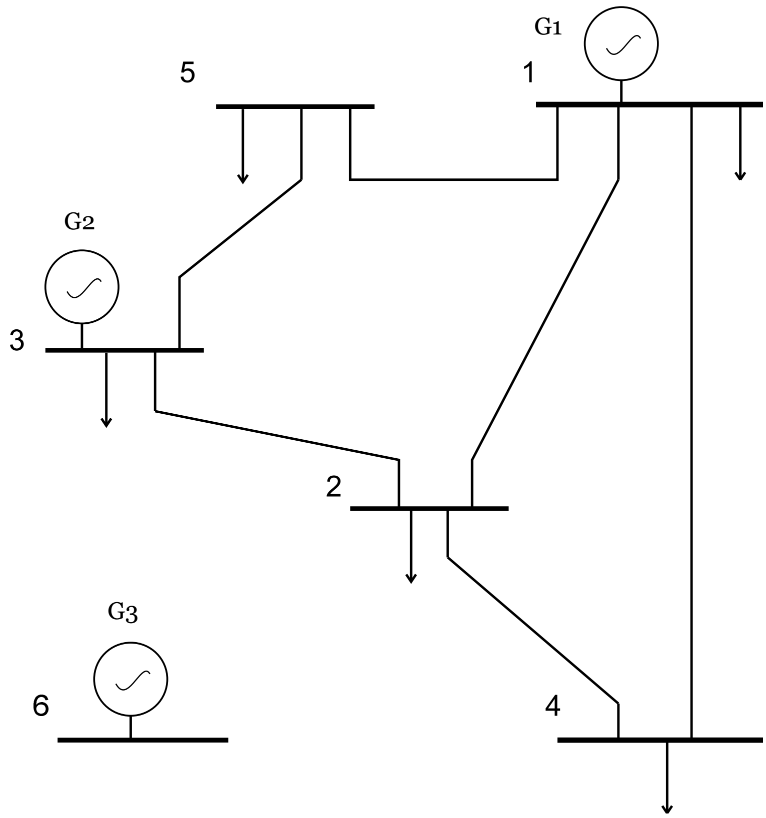
This system was proposed by L. Garver (GARVER, 1970). The system has 6 nodes, 3 existing generators and 15 branches. The data for the Garver test system is shown in Appendix A1; Figure 10 shows the Garver system topology:

There were considered the following study cases:

- Case 1: only consider one stage, existing capacity is enough to supply the demand. Marginal costs of existing generators are zero, in consequence operation cost is not relevant in the objective function.
- Case 2: similar to the previous study case with the only difference that existing generation is greater than demand, then is possible generation reprogramming.
- Case 3: similar to the previous study case with the only difference that the marginal cost of generators G1 and G2 increase to 150 \$/MWh so that operation cost is relevant in the objective function.
- Case 4: consider two stages; demand grows for the second stage. The marginal cost of existing generators are equal to zero.
- Case 5: similar to the previous case with the difference that the marginal cost for diesel generators π_{diesel} decreases from 74.4 \$/MWh to 1 \$/MWh.
- Case 6: similar to case 4 with the difference that existing generators have a marginal cost of 75 \$/MWh.

- Case 7: similar to the previous case, but investment costs for diesel units decreases to $\varepsilon_{\text{diesel}} = 10^5 \$/MW$.

Figure 10 - Garver system.



Source: the author.

The first two cases are intended to verify the results of the proposed model with standard results for the Garver system without generation reprogramming and with generation reprogramming respectively (ROMERO et al., 2002). The third case is intended to show how marginal costs can affect investment decisions. The next four study cases are intended to show how opportunity costs of transmission and generation capacity are dependent.

For all cases, only one demand level was considered with $\tau = 8760$ hours. The margin of reserve used was $\phi = 1$. The same generation candidates were considered for all nodes, the investment and operation cost were adapted from (DAMMERT; GARCIA; MOLLINELLI, 2010). An interest rate of 5% was used and the parameters $M = \pi/2$, $M' = 1000000$ and $M'' = 1000000$ were obtained so that they do not constrain the problem as explained in the previous chapter.

Table 1 presents the new transmission lines for the three first study cases, Table 2 presents the production in MW for existing generation and Table 3 presents the summary of results. The results coincide with the optimal global solution (ROMERO et al., 2002).

Also, Figures 11 and 12 presents DC power flow solutions for cases 1 and cases 2 and 3 respectively. Power flow simulations were done using the educational version of the program Powerworld Simulator (OVERBYE et al., 1995).

Table 1 - New transmission lines - Garver system, cases 1,2 and 3

		Case 1	Case 2	Case 3
i	j	n_{ij}	n_{ij}	n_{ij}
1	5	0	0	1
2	6	4	0	4
3	5	1	1	0
4	6	2	3	2

Source: the author

Table 2 - Production for existing generation in MW - Garver system, cases 1, 2 and 3

Unit	Case 1	Case 2	Case 3
G1	50	150	126.66
G2	165	312.1	33.34
G3	545	297.9	600

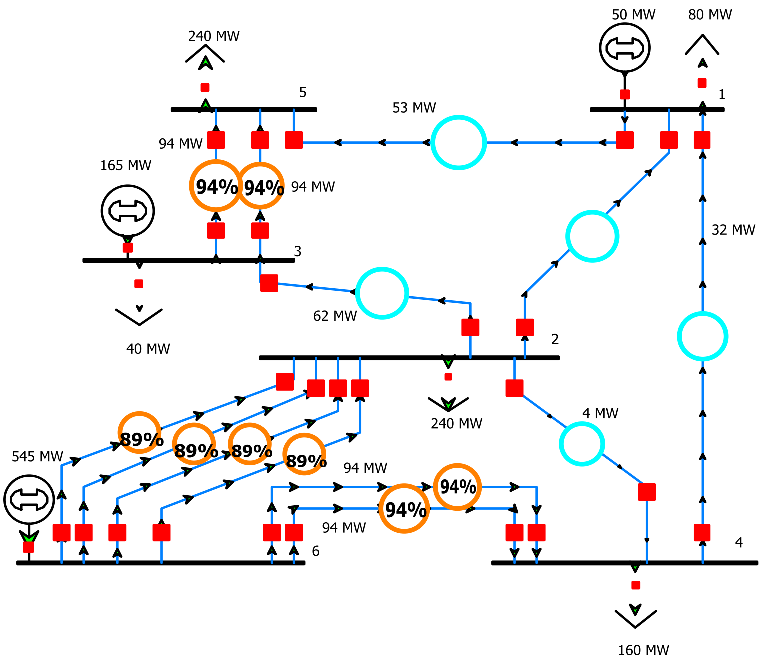
Source: the author

Table 3 - Sumary of results - Garver system, cases 1, 2 and 3

	Case 1	Case 2	Case 3
Investment in transmission	$ $200 \cdot 10^5$	$ $110 \cdot 10^5$	$200 \cdot 10^5$
Investment in generation	$0	$ 0	$ 0
Operating cost	$0	$ 0	$ $607,19 \cdot 10^5$

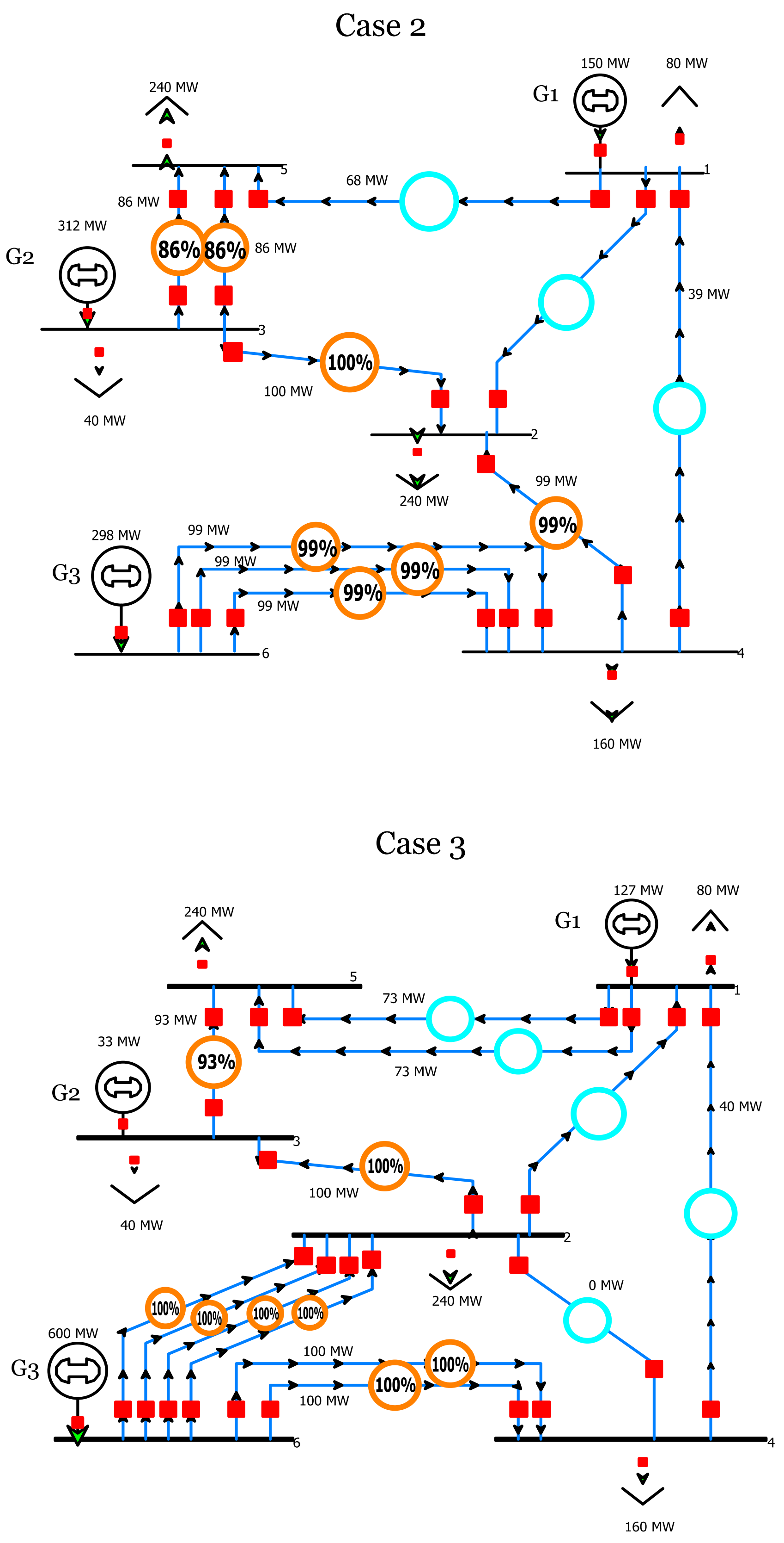
Source: The author

Figure 11 - Results for the Garver system - case 1.



Source: the author.

Figure 12 - Results for Garver system cases 2 and 3.



Source: the author.

It is interesting to compare results for cases 2 and 3. The main difference between both cases is that the marginal cost for existing generation is greater in case 3. In case 2, operation cost is zero, the generation at the isolated node is limited in order to reduce the investment cost. In contrast, in case 3 the isolated node has its maximum generation output because the investment cost is less than the savings in the operating cost (its opportunity cost).

The additional investment cost would be $\Delta CI = 200 \cdot 10^5 - 110 \cdot 10^5 = \$90 \cdot 10^5$, the savings would be $\Delta CO = 8760h \cdot 15\$/MWh \cdot (150 + 312.1 - 126.66 - 33.34)MW = \$3969 \cdot 10^5$, then the additional investment cost is less than the savings in the operation cost.

In a static approach, opportunity cost of transmission capacity only depends on the marginal cost at nodes. In consequence, this proposition is correct when one stage is considered.

The following four study cases have two stages with load growth. Table 4 presents new transmission lines for each study case, Table 4 presents the production for existing generation and Table presents the summary of results for the four study cases.

In contrast to the previous study cases, the model obtains new generation capacity; diesel units were selected in all cases. Figures 13, 14, 15 and 16 present the DC power flow solutions for cases 4, 5, 6 and 7 respectively, new generation units are colored in orange. Power flow simulations were done using the educational version of the program Powerworld Simulator (OVERBYE et al., 1995).

Table 4 - New transmission lines - Garver system, cases 4, 5, 6 and 7

		Case 4		Case 5		Case 6		Case 7	
i	j	$n_{ij,t=1}$	$n_{ij,t=2}$	$n_{ij,t=1}$	$n_{ij,t=2}$	$n_{ij,t=1}$	$n_{ij,t=2}$	$n_{ij,t=1}$	$n_{ij,t=2}$
2	6	0	4	3	1	1	3	1	3
3	5	1	1	2	0	2	0	1	1
4	6	3	0	0	2	2	0	1	1

Source: the author

Table 5 - Production for existing generation in MW - Garver system, cases 4, 5, 6 and 7

Unit	Case 4		Case 5		Case 6		Case 7	
	$t=1$	$t=2$	$t=1$	$t=2$	$t=1$	$t=2$	$t=1$	$t=2$
G1	150	150	137.5	150	150	150	150	150
G2	312.12	360	322.5	360	350	360	280	360
G3	297.88	600	300	590	260	590	170	590

Source: the author

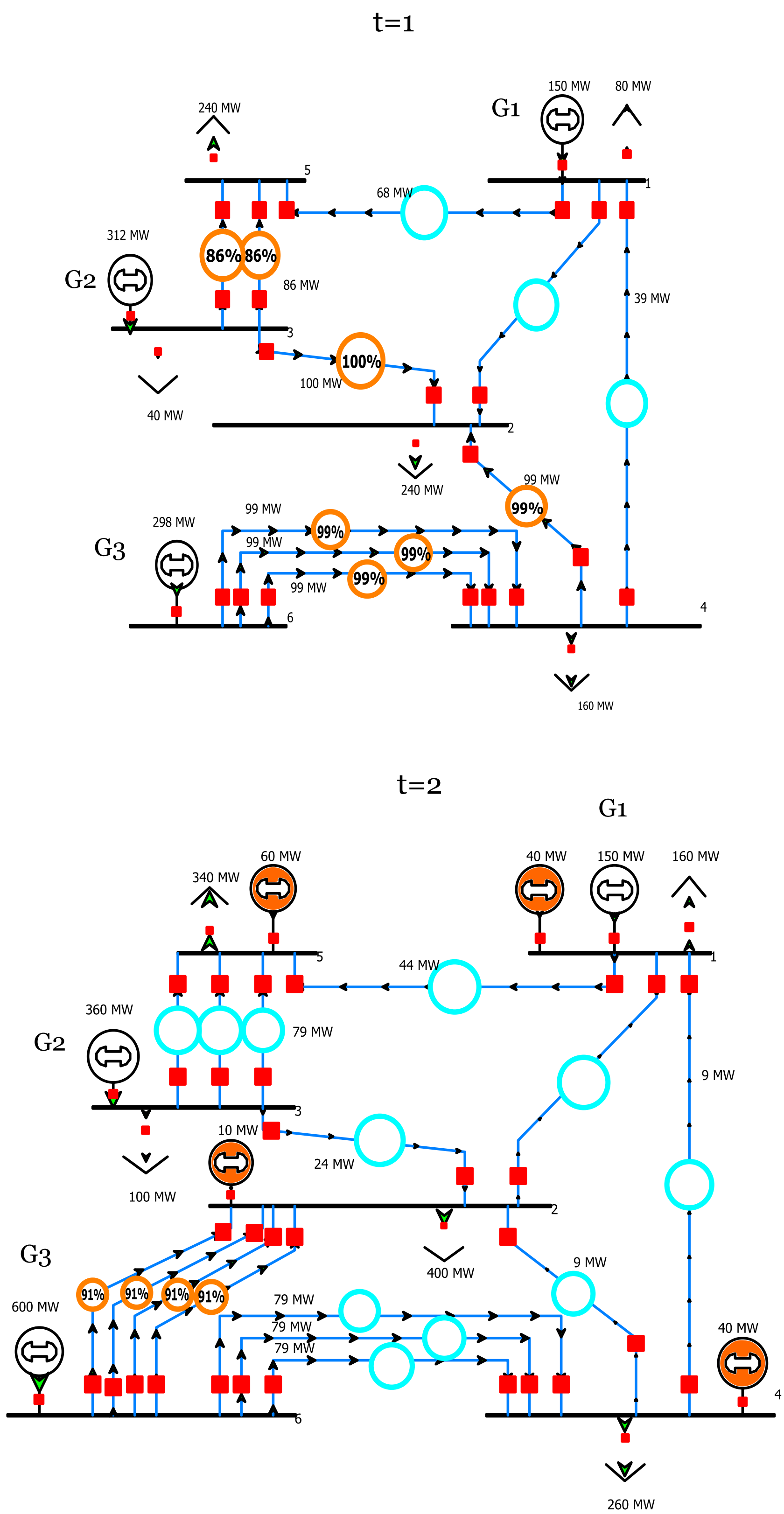
Table 6 - Summary of results - Garver system, cases 4, 5, 6 and 7

	Case 4		Case 5	
	$t=1$	$t=2$	$t=1$	$t=2$
Investment in transmission	$\$ 100 \cdot 10^5$	$\$ 140 \cdot 10^5$	$\$ 130 \cdot 10^5$	$\$ 90 \cdot 10^5$
Investment in generation	$ 0	$\$ 56 \cdot 10^6$	$0	$\$ 56 \cdot 10^6$
Operating cost	$ 0	$\$ 997 \cdot 10^5$	$0	$\$ 14 \cdot 10^5$

	Case 6		Case 7	
	$t=1$	$t=2$	$t=1$	$t=2$
Investment in transmission	$\$ 140 \cdot 10^5$	$\$ 90 \cdot 10^5$	$\$ 90 \cdot 10^5$	$\$ 140 \cdot 10^5$
Investment in generation	$ 0	$\$ 56 \cdot 10^6$	$\$ 56 \cdot 10^6$	$ 0
Operating cost	$\$ 4993 \cdot 10^5$	$\$ 8251 \cdot 10^5$	$\$ 4984 \cdot 10^5$	$\$ 8251 \cdot 10^5$

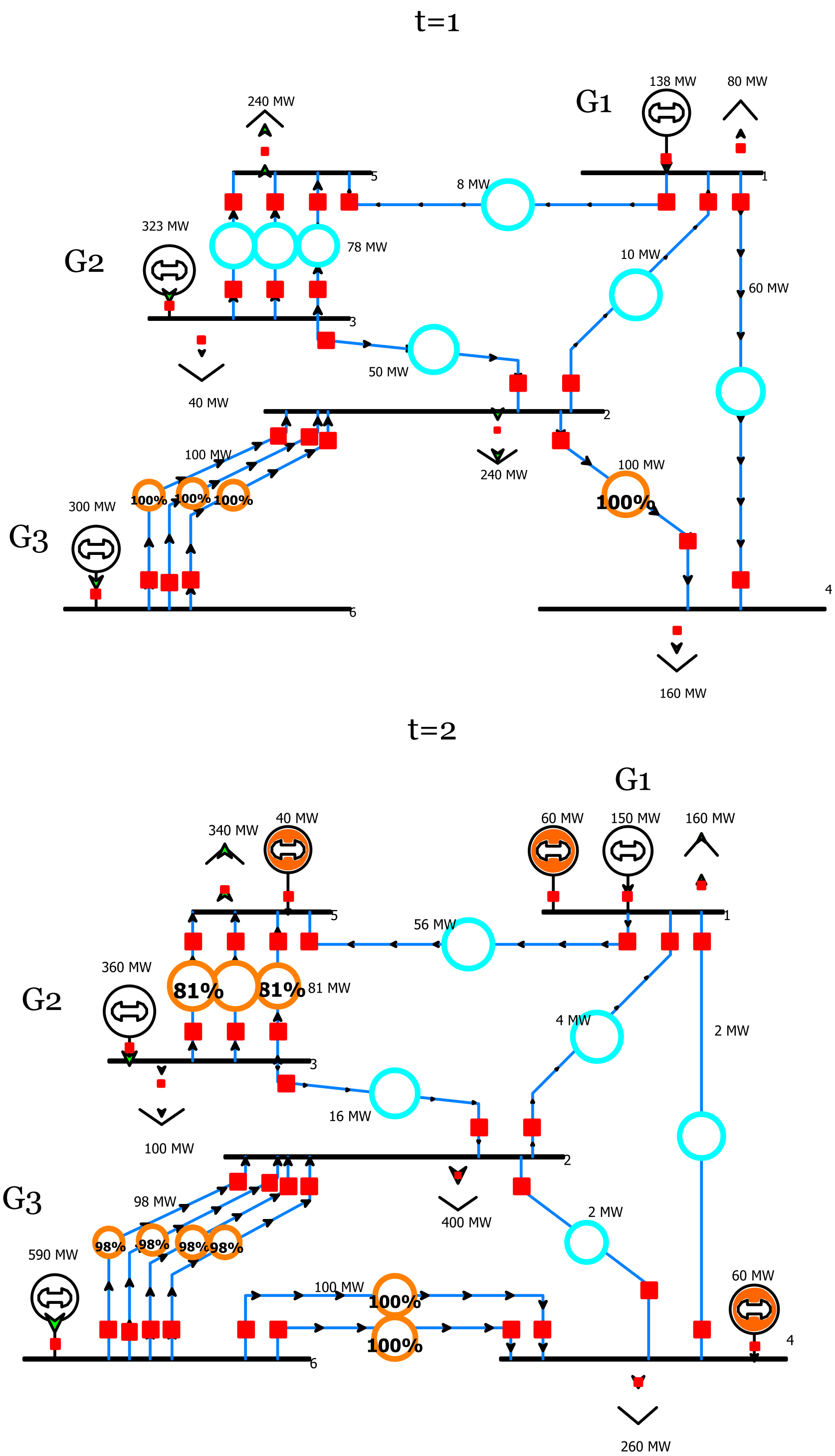
Source: the author

Figure 13 - Results for the Garver system - case 4



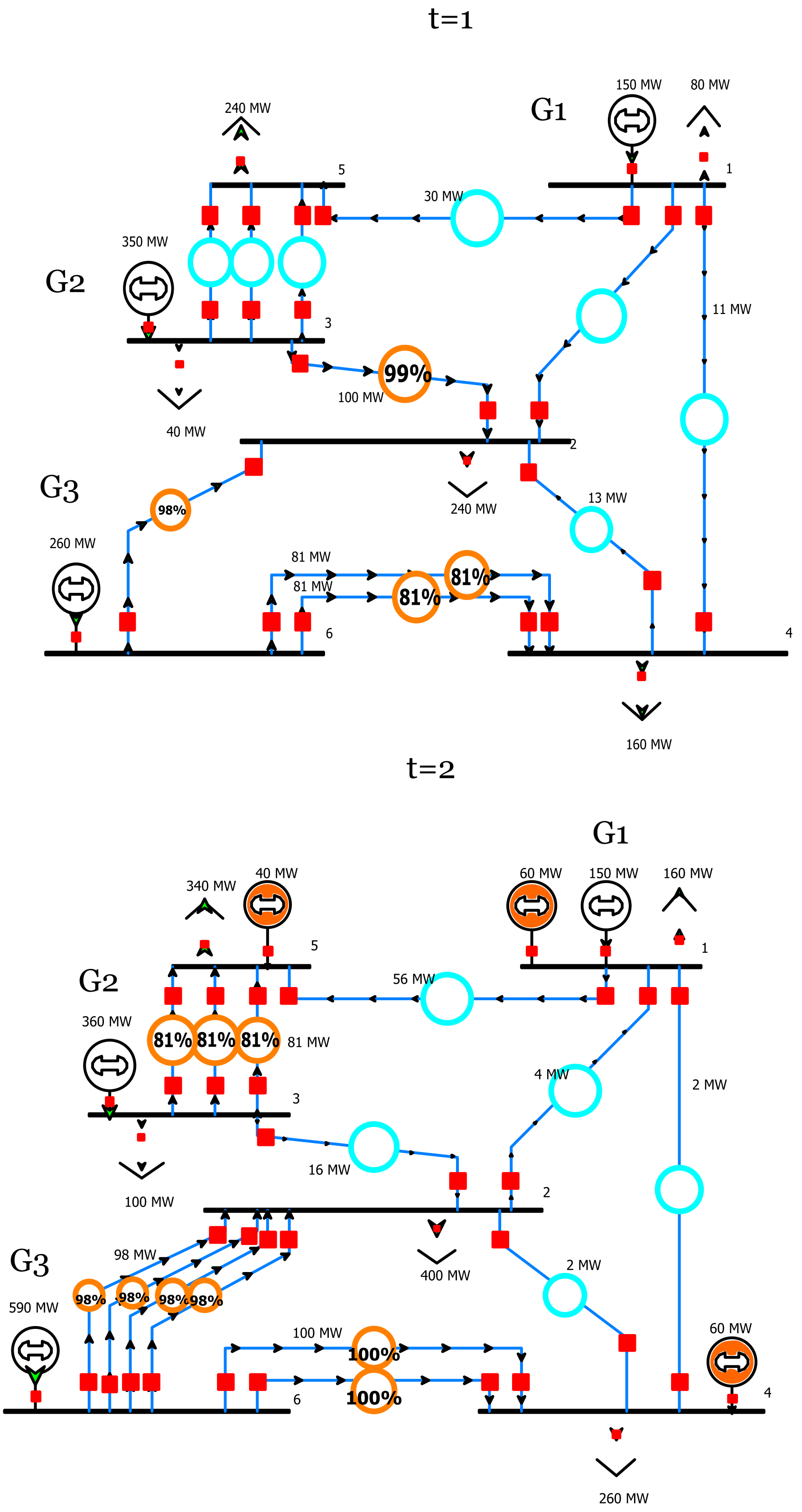
Source: the author.

Figure 14 - Results for the Garver system - case 5.



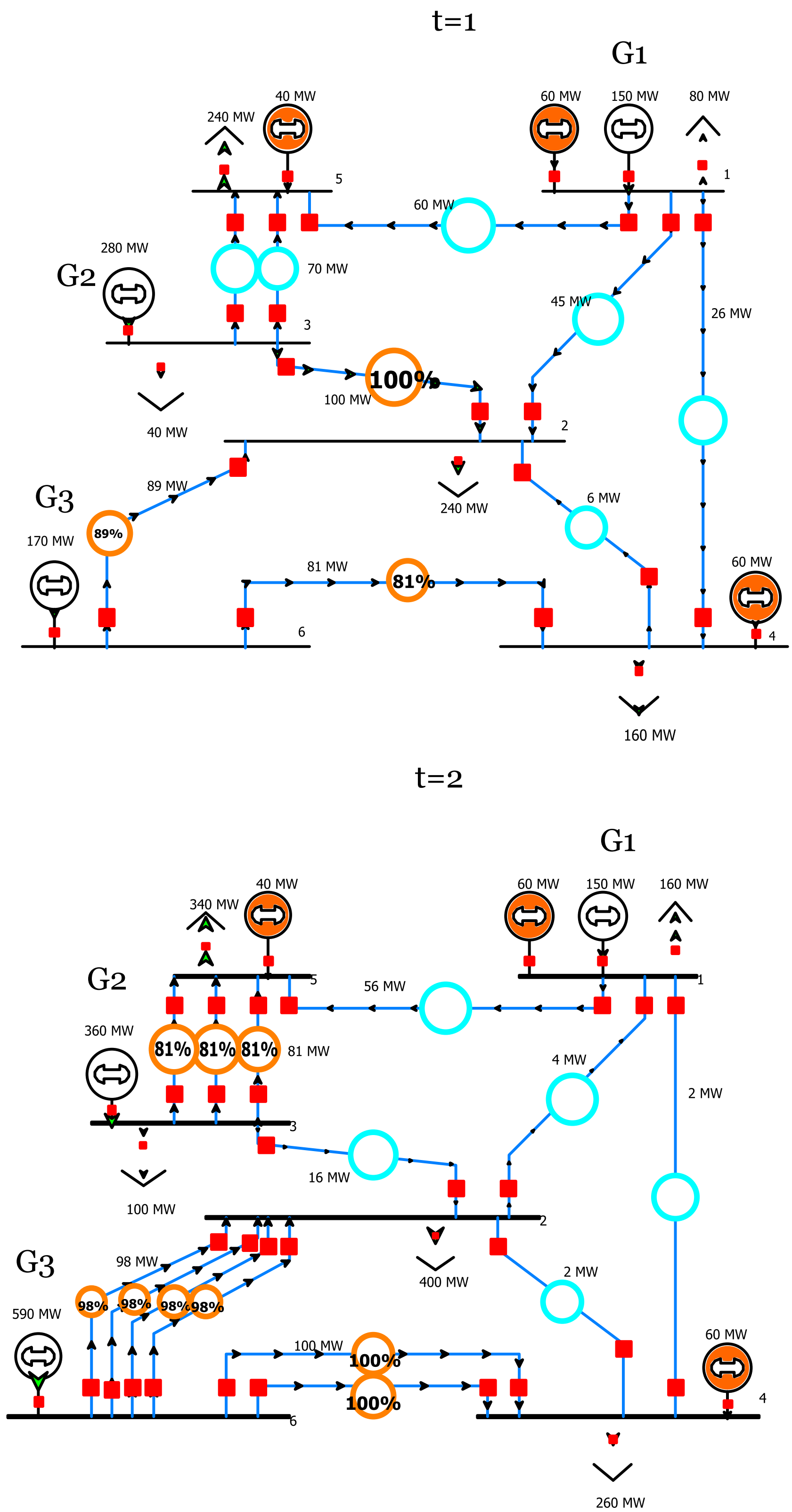
Source: the author.

Figure 15 - Results for the Garver system - case 6.



Source: the author.

Figure 16 - Results for the Garver system - case 7.



Source: the author.

For the last four study cases, generation units are located at the demand nodes instead of the isolated node.

Let's compare cases 4 and 5, the main difference is that the marginal costs for diesel units decreases in the case 5. Results are almost the same with the only exception of the stage $t = 2$ of case 4, which has an additional line n_{46} (See Figures 13 and 14). Generation at isolated node is cheaper for the case 4 (0 $/MWh), then generation output at isolated node is at its maximum and construction of additional line n_{46} is justified. In case 5, isolated node only produces 590 MW and the additional line n_{4-6} is not built; this saving is greater than the saving when producing electricity with diesel units. Since the opportunity cost of the transmission capacity depends in part on the marginal costs, if the marginal costs difference decrease then the opportunity cost of the transmission line decreases. In consequence, for case 5, investment cost of line n_{46} is greater than its opportunity cost.

Notice that marginal cost difference affects opportunity costs even in a dynamic approach. However, even with the same investment cost in both cases, location of the generation units are different; this because transmission capacity opportunity cost does not depends only on the marginal costs but also on the location of the generation units.

In the other hand, cases 6 and 7 are almost the same except for the investment cost of diesel generation units. Those examples are important because if opportunity costs only depends on marginal costs, there should be no differences in both solutions. Nonetheless, results show that this assumption is not correct, for stage $t = 1$ of case 7, transmission capacity is replaced with generation capacity (See Figures 15 and 16). Case 6 has two more additional lines: n_{35} and n_{46}; the investments cost of these lines are greater than its opportunity costs, which is not only given by marginal costs but also by the possibility of increase local generation capacity.

Additionally, notice that both study cases have the same results at stage $t = 2$. In spite of at the horizon the results are the same, the possibility of building local generation could make anticipate the investments and make important savings in the operating cost. In general, this is very frequent in real systems; thus, transmission planning must take in count generation investments. All the examples show that opportunity cost of transmission and generation capacity are dependent. Next section shows the main implications of this result when the investment decisions are decentralized.

4.2 THE IEEE 24 BUSES TEST SYSTEM

The IEEE 24 buses system was initially proposed for reliability purposes (SUBCOMMITTEE, 1979); but later it was used as a standard test system for the transmission planning problem. The system has 24 nodes; 10 existing generators and 41 branches. Data for the system is shown in Appendix A2. Figure 17 shows the IEEE 24 buses system topology. The following study cases were considered:

- Case 1: only considers one stage; existing generation capacity is greater than demand.
- Case 2: considers two stages; for the second stage demand grows 30% in respect to the first stage.
- Case 3: similar to the previous case, but for a decentralized framework.

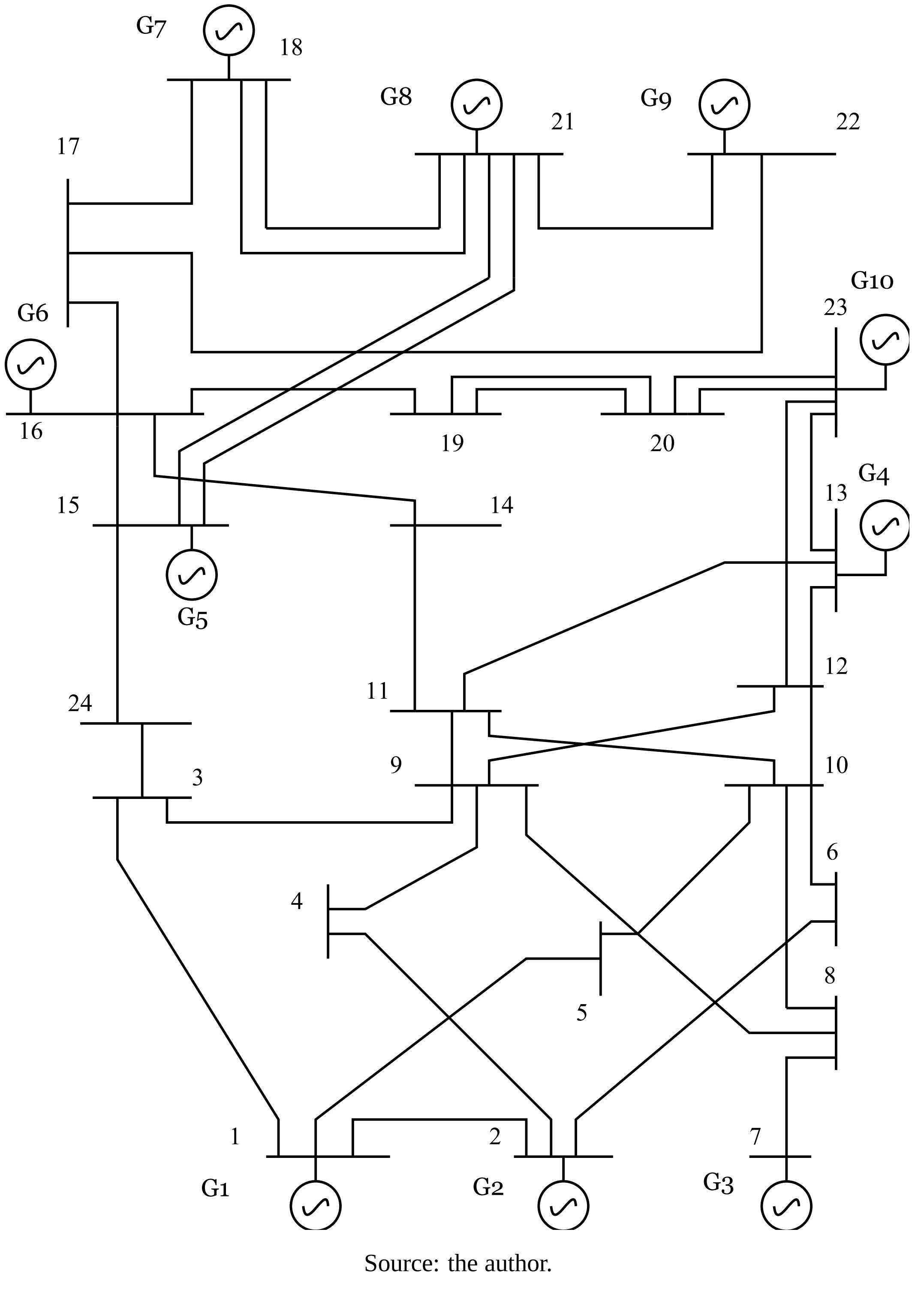
Case 1 is intended to verify results of the proposed model with standard results for the IEEE 24 buses system with generation reprogramming (ROMERO et al., 2002), while cases 2 and 3 are intended to show the difference between a centralized versus a decentralized outcome.

Only one demand level was considered with $\tau = 8760$ hours. Three generation projects were considered, all the projects have the same investment and operation cost, but can be built at different nodes. The reserve margin used was $\phi = 1$. Finally, an interest rate of 5% was used in order to bring costs to present value. Parameters $M = \pi/2$, $M' = 1000000$ and $M'' = 1000000$, were obtained so that they do not constrain the problem as explained in the previous chapter.

Table 7 presents new transmission lines for all study cases, Table 8 presents production for existing generation for all cases and Table 9 shows the summary of results for all cases.

Figures 18, 19, 20, 21 and 22 show the DC power flow solutions for cases 1, 2 ($t = 1$), 2 ($t = 2$), 3 ($t = 1$) and 3 ($t = 2$) respectively. Case 1 does not has new generation units, results coincide with the optimal solution (ROMERO et al., 2002). The two first cases have the same new transmission lines for the first stage.

Figure 17 - IEEE 24 buses system.



Source: the author.

Table 7 - New transmission lines - IEEE 24 buses system, cases 1 and 2

		Case 1	Case 2		Case 3	
i	j	$n_{ij,t=1}$	$n_{ij,t=1}$	$n_{ij,t=2}$	$n_{ij,t=1}$	$n_{ij,t=2}$
1	3	0	0	1	0	1
2	6	0	0	1	1	0
3	24	0	0	1	0	1
4	9	0	0	1	0	1
5	10	0	0	1	1	0
6	10	1	1	0	0	1
7	8	1	1	0	1	0
8	9	0	0	0	1	0
8	10	0	0	1	0	1
9	11	0	0	1	1	0
9	12	0	0	1	1	0
10	12	1	1	0	0	0
11	13	1	1	0	1	0
12	23	0	0	1	0	0
14	16	1	1	0	1	0
15	21	0	0	0	1	1
15	24	0	0	1	0	1
16	17	0	0	1	0	1
20	23	0	0	0	1	1

Source: the author.

Table 8 - Production for existing generation in MW - IEEE 24 buses system, cases 1, 2 and 3

	Case 1	Case 2		Case 3	
Unit	$t = 1$	$t = 1$	$t = 2$	$t = 1$	$t = 2$
G1	571.48	546.56	576	576	576
G2	576	576	576	576	576
G3	717.42	725	796.87	725	837.5
G4	1773	1725.16	1773	1564.73	1773
G5	233.93	267.68	645	645	645
G6	465	465	465	28.54	448.83
G7	442.7	396.79	1094.25	354.73	1200
G8	1156.3	1200	1200	1200	1200
G9	900	900	900	900	900
G10	1714.17	1747.8	1980	1980	1979.9

Source: the author

Table 9 - Summary of results - IEEE 24 buses system, cases 1, 2 and 3

	Case 1	Case 2		Case 3	
	$t = 1$	$t = 1$	$t = 2$	$t = 1$	$t = 2$
Investment in transmission	$\$ 202 \cdot 10^6$	$\$ 202 \cdot 10^6$	$\$ 590 \cdot 10^6$	$\$ 450 \cdot 10^6$	$\$ 397 \cdot 10^6$
Investment in generation	$\$ 0$	$\$ 0$	$\$ 120 \cdot 10^{10}$	$\$ 0$	$\$ 120 \cdot 10^{10}$
Operating cost	$\$ 748,9 \cdot 10^5$	$\$ 748,9 \cdot 10^5$	$\$ 973,6 \cdot 10^5$	$\$ 748,9 \cdot 10^5$	$\$ 973,6 \cdot 10^5$

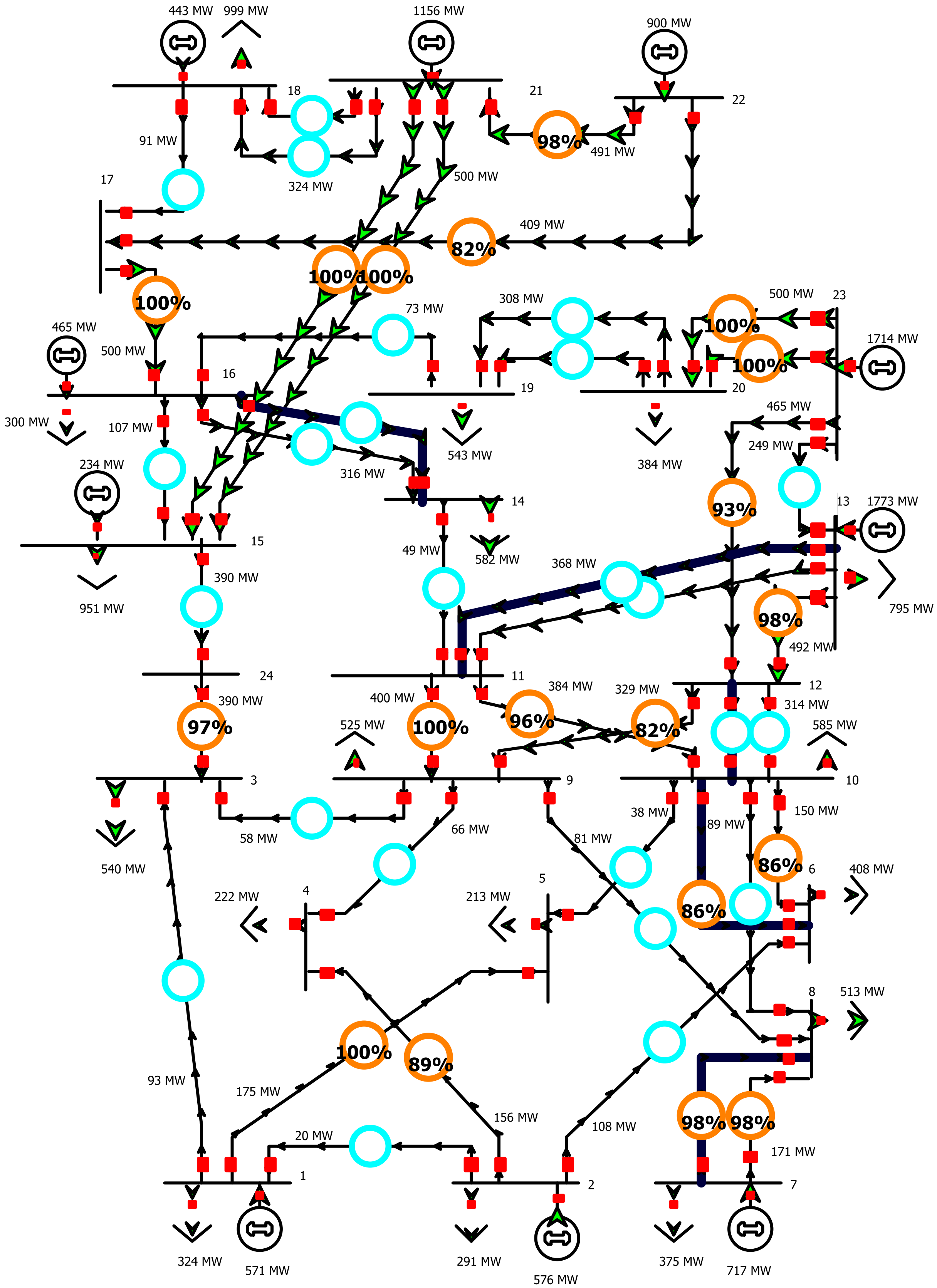
Source: the author.

In order to compare the second study case with a decentralized outcome, it should be added a third level to the proposed model in which generation companies decide their investments. Since this is out of the purposes of the dissertation, let's use the symmetry of the problem. Notice that the marginal cost for existing and candidate units is the same (See Table 19 and 20 from Appendix A2). Also, investments cost for all generation projects are the same independently of the node. Then, if profits for investors are large enough to exceed its opportunity cost, an investor could build a generation unit at any node until meeting the demand. Three generation units are needed to meet the demand at stage $t = 2$; location of the units are not relevant for the investors so any location can be a decentralized outcome.

Let's choose a certain decentralized outcome: building two generation units at node 10 and one generation unit at node 15 (See Figure 22). If generation investments are taken as given, the proposed model can be used choosing proper values of $\bar{w}$.

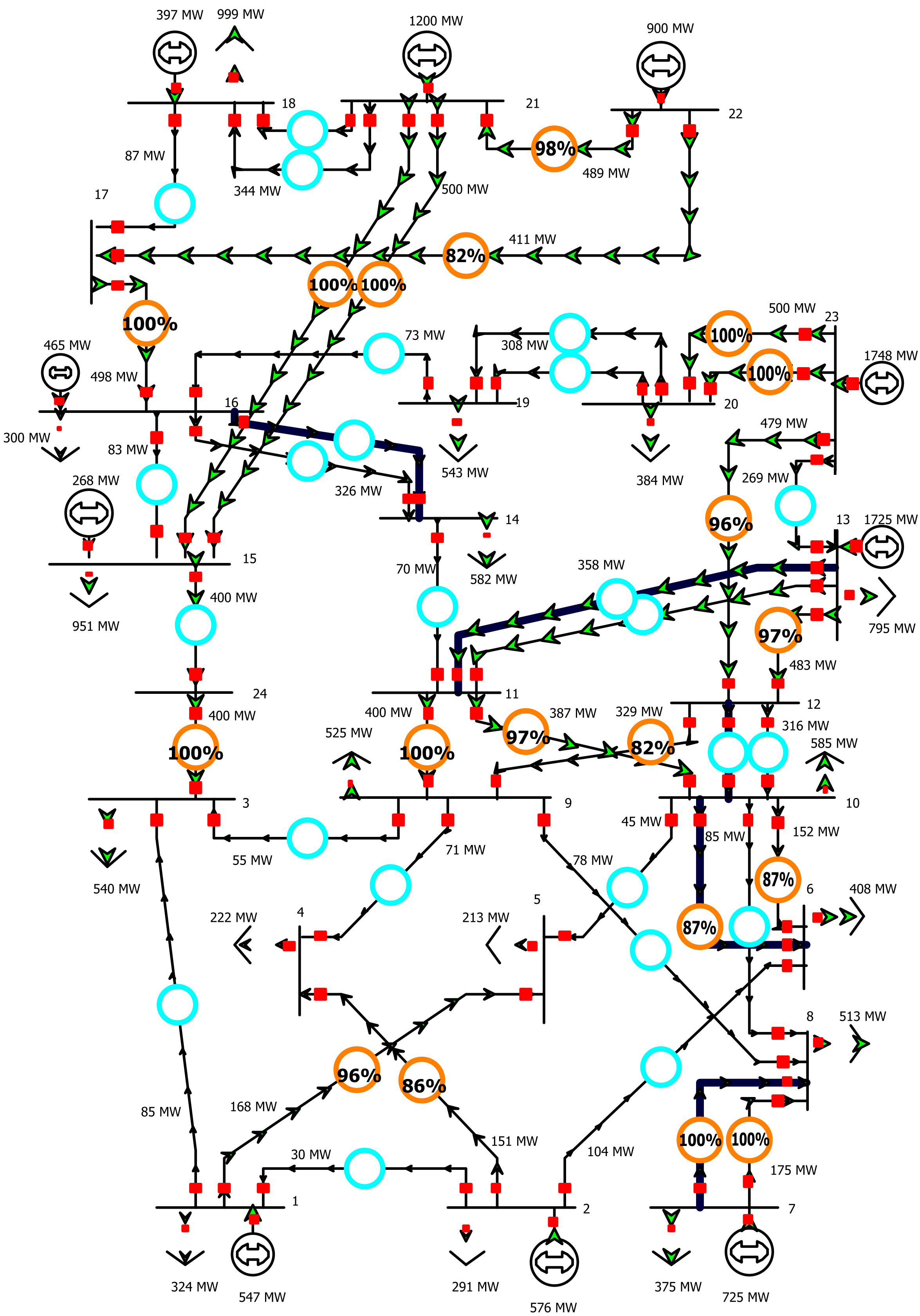
The difference in total cost (generation, transmission and operation cost) between the centralized and the decentralized outcome is $\$55 \cdot 10^6$. This shows that under a decentralized framework, optimal solutions for each agent does not lead to a Pareto optimal solution. In consequence, market outcome does not reach the maximum social welfare in a decentralized electricity market.

Figure 18 - Results for the IEEE 24 buses system, case 1.



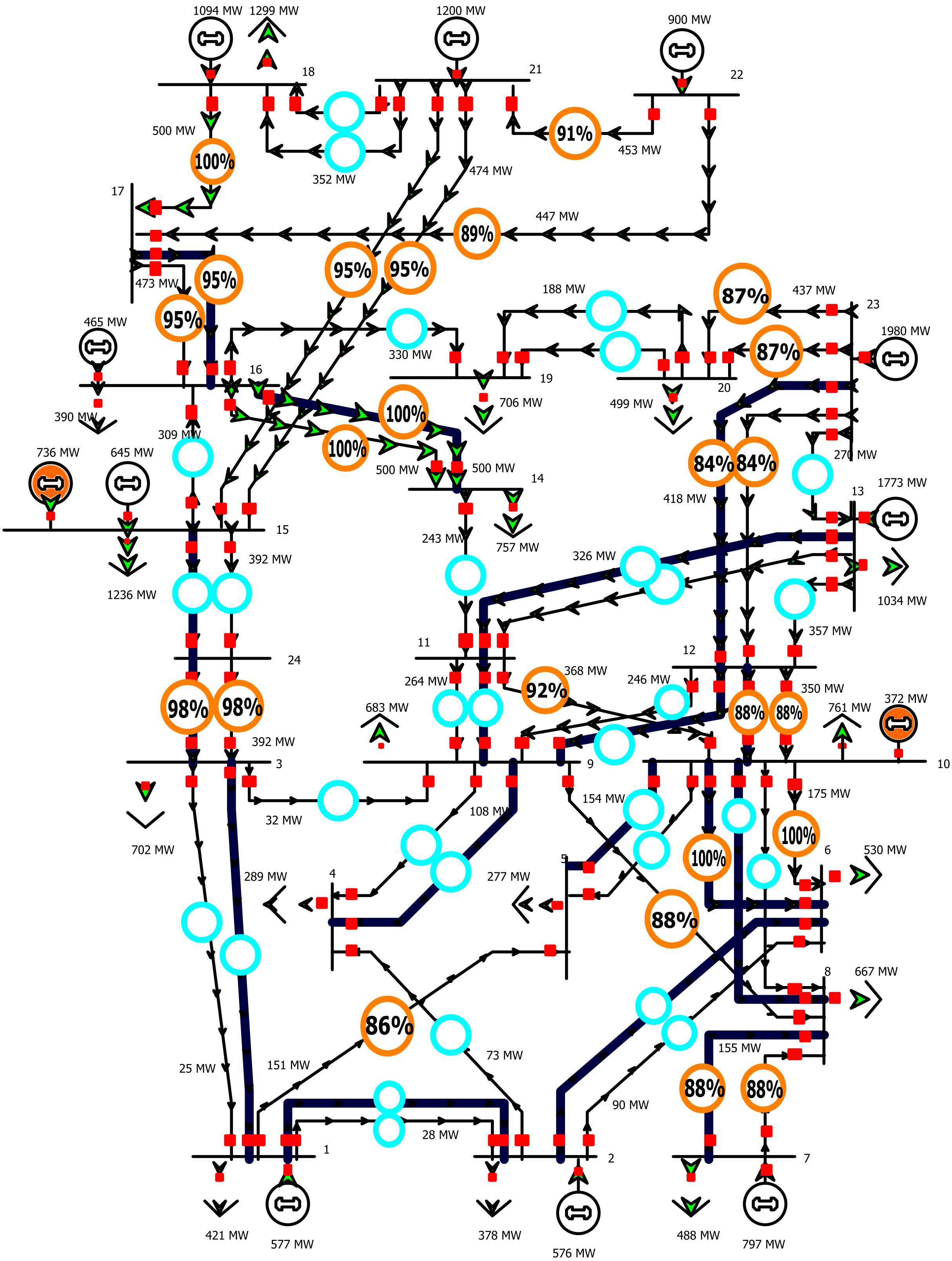
Source: the author.

Figure 19 - Resultados del sistema IEEE 24 barras, caso 2, $t = 1$.



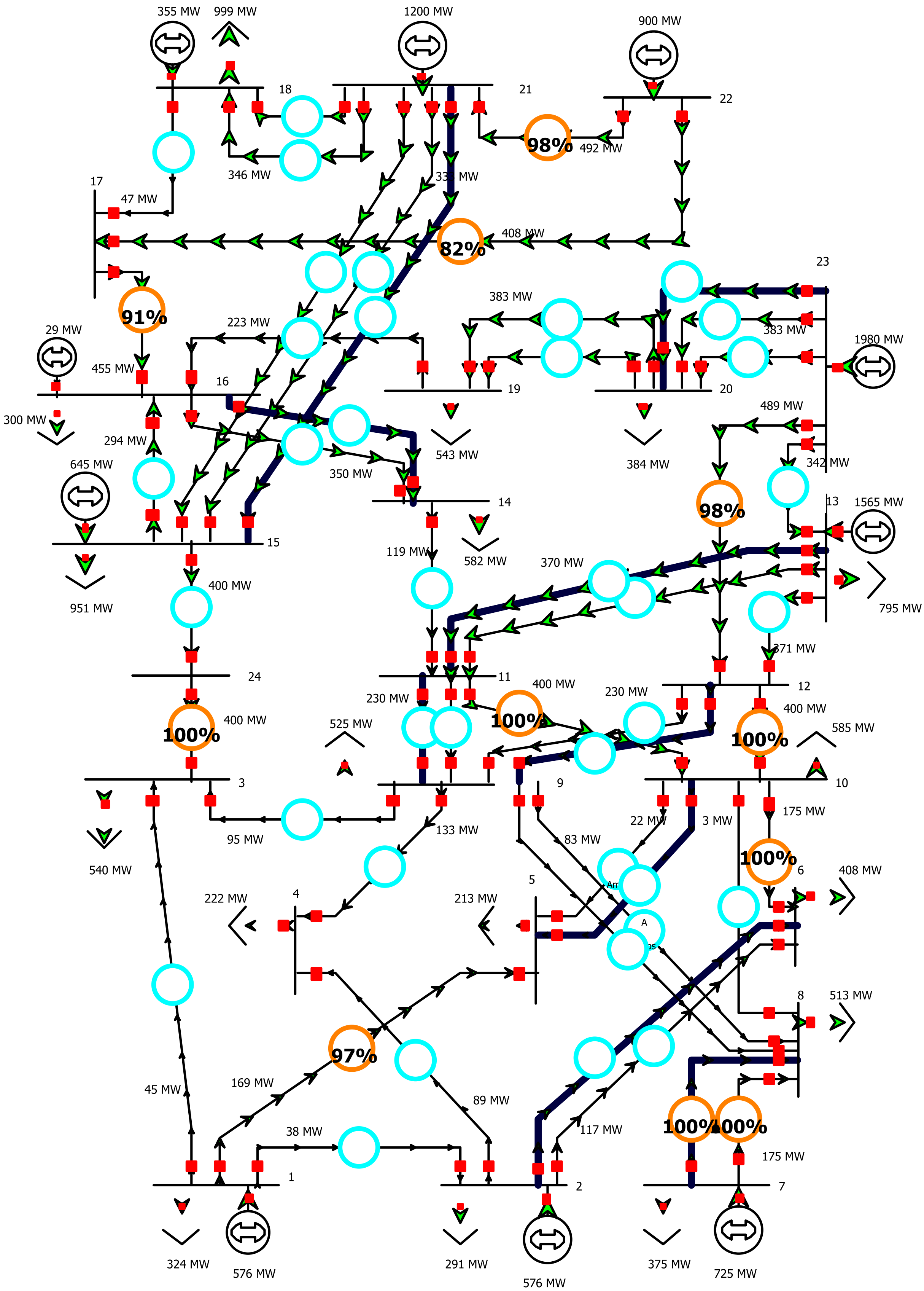
Source: the author.

Figure 20 - Results for the IEEE 24 buses system, case 2, $t = 2$.



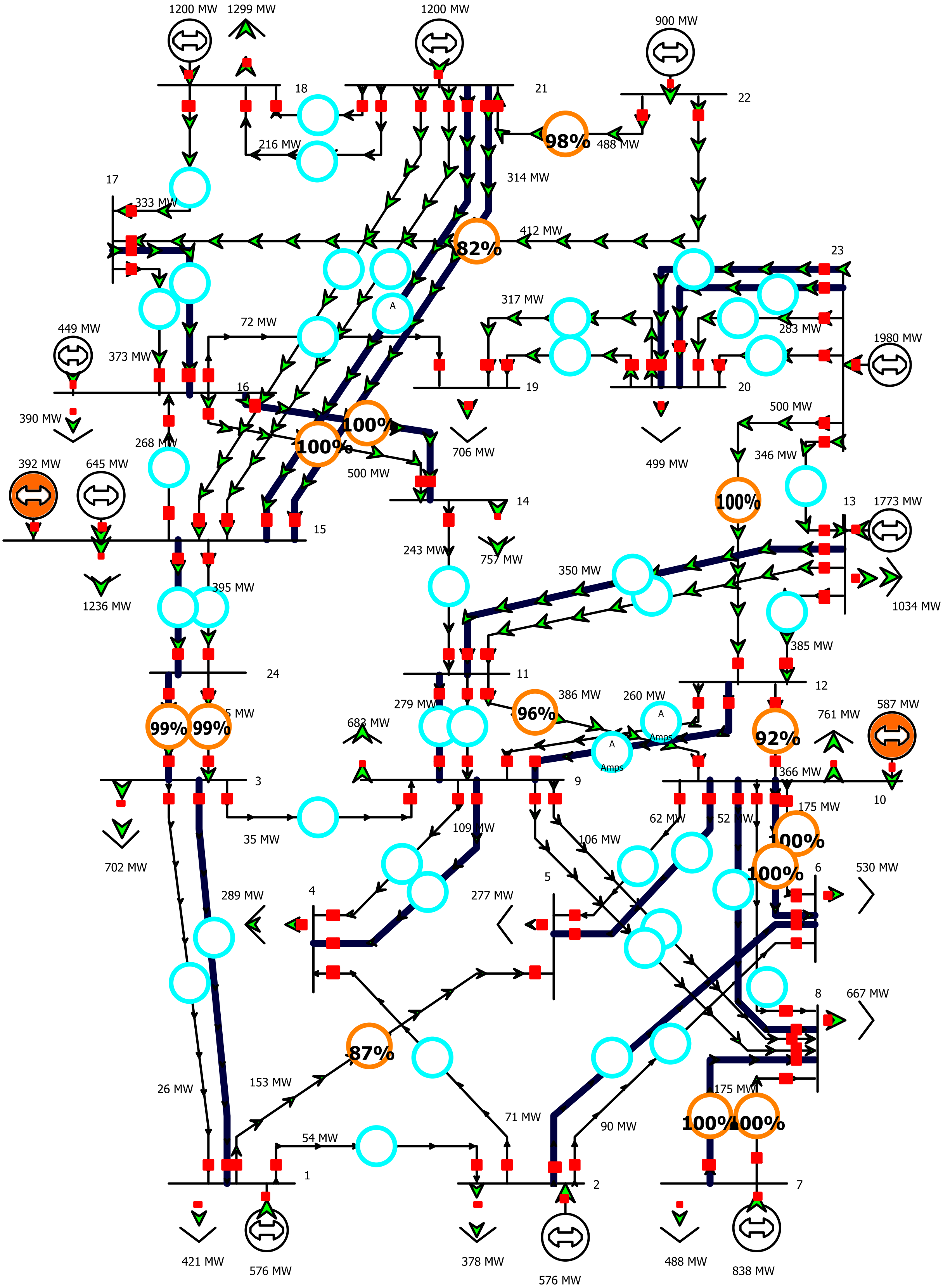
Source: the author.

Figure 21 - Results for the IEEE 24 buses system, case 3, $t = 1$.



Source: the author.

Figure 22 - Results for the IEEE 24 buses system, case 3, $t = 2$.



Source: the author.

4.3 THE PERUVIAN SYSTEM

Data for the Peruvian system was adapted from the data of the Peruvian independent system operator (COES). The system has 131 nodes, 92 existing generators and 202 branches; data for the Peruvian system is shown in Appendix A3.

This system is intended to show how the proposed model can be used with a real system with large scale (131 nodes). Because of the size and the characteristics of the Peruvian system only the main characteristics of the results are commented. Figure 23 presents the Peruvian system topology.

The following study cases were considered:

- Case 1: it is a simplified Peruvian network for years 2019 and 2023.
- Case 2: it is a sensibility of the previous case considering delaying of the generation projects CC El Faro and CT Quillabamba.

Only one demand level was considered with $\tau = 8760$ x 3 hours. The margin of reserve considered was $\phi = 1$. Finally, an interest rate of 5% was used to bring cost to present value and simulation parameters $M = \pi/2$, $M' = 1000000$ and $M'' = 1000000$ were obtained in order to not constrain the problem as explained in the previous chapter.

Table 10 presents new transmission lines for the first study case:

Table 10 - New transmission lines - Peruvian system, case 1

i	j	$n_{ij,t=1}$	$n_{ij,t=2}$
31	32	1	0
42	32	1	0
121	94	1	0
121	81	1	0
125	89	1	0

Source: the author

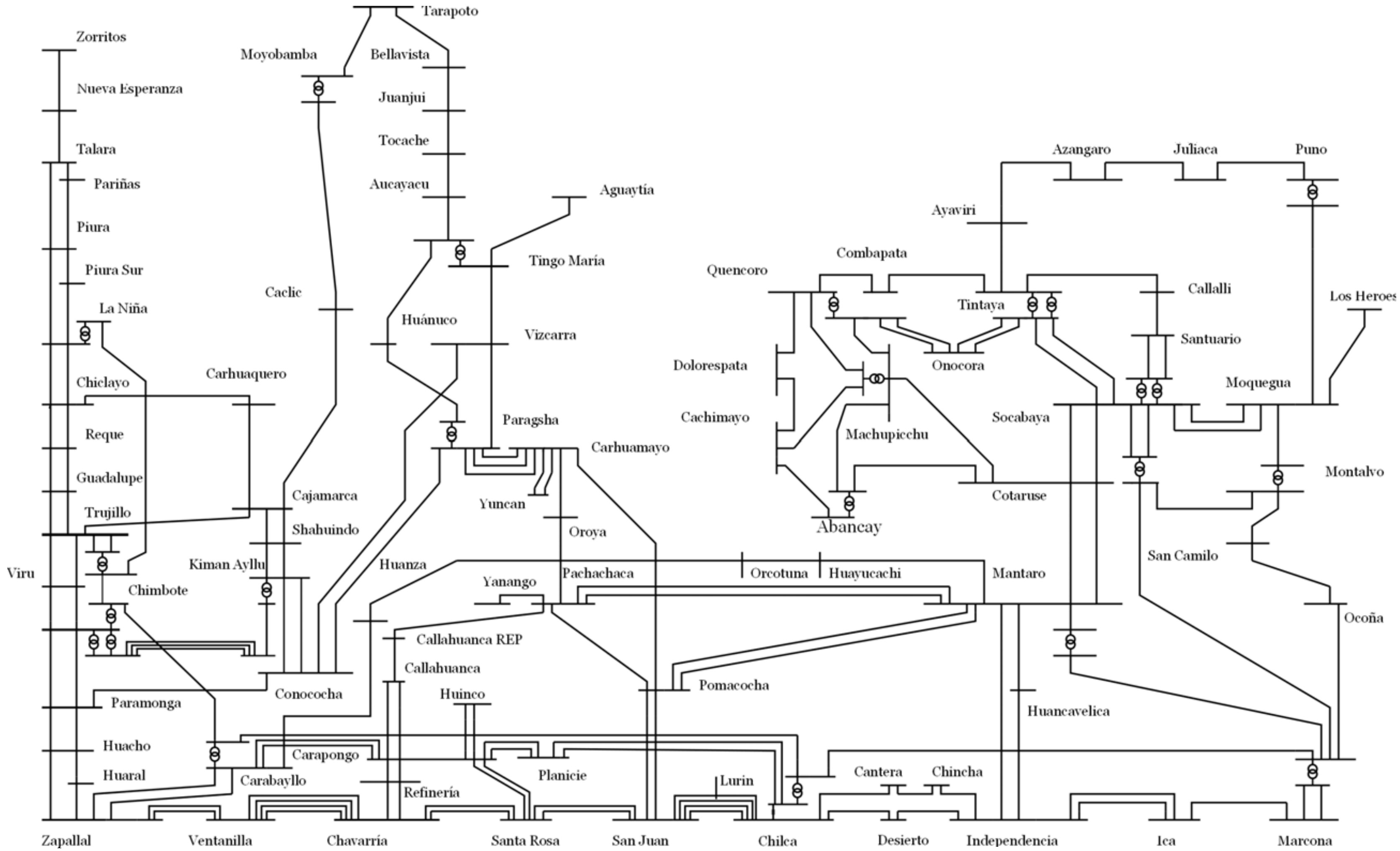


Figure 23 - Peruvian system.

Source: the author.

Table 11 presents production for existing generation for the first study case:

Table 11 - Production for existing generation in MW - Peruvian system, case 1

Unit	$g_{t=1}$	$g_{t=2}$	Unidad	$g_{t=1}$	$g_{t=2}$
Pariac	2	2	Santa Rosa UTI 5	0	50
Yanapampa	2	2	Santa Rosa UTI 6	0	50
Marcara	3	3	Chimay	70	70
Caña Brava	5	5	Marañon	70	70
Las Pizarras	5	5	Rapay	70	70
Macon	5	5	Pisco	0	70.7
Manta	6	6	Aguaytia TG1	80	80
Shima	6	6	Aguaytia TG2	80	80
Curumuy	7	7	Huanza	80	80
Gera	7	7	Quitaracsa	80	80
El carmen	8	8	Santa Teresa	80	80
Esperanza	8	8	Tulumayo	80	80
Mushcapata	8	8	Yaupi	80	80
Tumbes TG1	0	9.17	Machupicchu	87	87
Tumbes TG2	0	9.17	Machupicchu II	90	90
8 de agosto	10	10	Malacas TG4	0	90
Cola	10	10	San Gaban II	90	90
Pias	10	10	Charcani V	100	100
Poechos	10	10	Yuncan	100	100
Santa Cruz	11	11	Cañon del Pato	120	120
Las Cruces	14	14	El Faro CC	120	120
Eolica Marcona	15	15	Matucana	120	120
Eolica Talara	15	15	Santa Rosa TG7	0	121
Gallito Ciego	15	15	Nueva Esperanza	135	135
Renovaandes	15	15	Cheves	140	140
Malacas TG1	0	16	Pucara	140	140
Charcani	18	18	Ilo 2	0	141
Rucuy	18	18	Belo Horizonte	150	150
Vilcanota	18	18	Curibamba	150	150
Cahua	20	20	Huinco	150	150
Pelagatos	20	20	Malacas TGD5	0	177
Runatullo	20	20	Las Flores TG1	192	192
Aipsa	23	23	Santa Rosa TG8	199	199
Independencia	23	23	Quillabamba	56.79	200
Tablazo	29	29	Restitucion	200	200
Callahuanca	30	30	Platanal	210	210
Chancay 2	30	30	ETEN	0	223
Yanango	30	30	Termochilca CC	286	286
Maple	37.5	37.5	Chaglla	360	360
El Angel	40	40	RF Ilo	0	321.31
Eolica Cupisnique	40	40	Ventanilla CC	480	480
RF Pucallpa	0	40	Cerro del Aguila	500	500
Olmos	45	45	Fenix CC	534	534
Tres hermanas	45	45	Mantaro	650	650
Carhuaquero	50	50	Chilca CC	811	811
El Tambo	50	50	Kallpa CC	857	857

Source: the author

In the first study case, new generation capacity is obtained at stage $t = 2$: CH San Gaban I and CH San Gaban III. Both projects are hydro and produce their maximum output for the second stage. In this case is better to reprogram the existing generation instead of building new transmission capacity.

Table 12 presents new transmission lines for the second study case:

Table 12 - New transmission lines - Peruvian system, case 2

i	j	$n_{ij,t=1}$	$n_{ij,t=2}$
31	32	1	0
43	42	1	0
42	32	1	0
95	94	1	0
92	121	1	0
121	94	1	0
42	123	1	0
123	122	1	0
122	32	1	0
121	81	1	0
81	125	1	0
125	89	1	0

Source: the author

Table 13 presents the production for existing generation for the second study case:

Table 13 - Production for existing generation in MW - Peruvian system, case 2

Unit	$g_{t=1}$	$g_{t=2}$	Unit	$g_{t=1}$	$g_{t=2}$
Pariac	2	2	Santa Rosa UTI 5	0	50
Yanapampa	2	2	Santa Rosa UTI 6	0	50
Marcara	3	3	Chimay	70	70
Caña Brava	5	5	Marañon	70	70
Las Pizarras	5	5	Rapay	70	70
Macon	5	5	Pisco	0	70.7
Manta	6	6	Aguaytia TG1	16.79	80
Shima	6	6	Aguaytia TG2	0	80
Curumuy	7	7	Huanza	80	80
Gera	7	7	Quitaracsa	80	80
El carmen	8	8	Santa Teresa	80	80
Esperanza	8	8	Tulumayo	80	80
Mushcapata	8	8	Yaupi	80	80
Tumbes TG1	0	9.17	Machupicchu	87	87
Tumbes TG2	0	9.17	Machupicchu II	90	90
8 de agosto	10	10	Malacas TG4	0	90
Cola	10	10	San Gaban II	90	90
Pias	10	10	Charcani V	100	100
Poechos	10	10	Yuncan	100	100
Santa Cruz	11	11	Cañon del Pato	120	120
Las Cruces	14	14	El Faro CC	0	0
Eolica Marcona	15	15	Matucana	120	120
Eolica Talara	15	15	Santa Rosa TG7	0	121
Gallito Ciego	15	15	Nueva Esperanza	135	135
Renovaandes	15	15	Cheves	140	140
Malacas TG1	0	16	Pucara	140	140
Charcani	18	18	Ilo 2	0	141
Rucuy	18	18	Belo Horizonte	150	150
Vilcanota	18	18	Curibamba	150	150
Cahua	20	20	Huinco	150	150
Pelagatos	20	20	Malacas TGD5	0	177
Runatullo	20	20	Las Flores TG1	192	192
Aipsa	23	23	Santa Rosa TG8	199	199
Independencia	23	23	Quillabamba	0	0
Tablazo	29	29	Restitucion	200	200
Callahuanca	30	30	Platanal	210	210
Chancay 2	30	30	ETEN	0	223
Yanango	30	30	Termochilca CC	286	286
Maple	37.5	37.5	Chaglla	360	360
El Angel	40	40	RF Ilo	0	321.31
Eolica Cupisnique	40	40	Ventanilla CC	480	480
RF Pucallpa	0	40	Cerro del Aguila	500	500
Olmos	45	45	Fenix CC	534	534
Tres hermanas	45	45	Mantaro	650	650
Carhuaquero	50	50	Chilca CC	811	811
El Tambo	50	50	Kallpa CC	857	857

Source: the author

In contrast to the previous study case, the model chooses new generation projects for the first stage: *CH San Gaban III* dispatching 160 MW and a unit of the *Southern Energetic Node* dispatching 500 MW. The new transmission capacity increase with respect to the previous case. The models minimizes joint total cost.If the problem were solved by separate, common sense would indicate that the construction of hydro units is more efficient from the generator perspective. However, this would increase investment cost in transmission capacity so that does not lead to an optimal solution.

4.4 CONCLUSIONS OF THE CHAPTER

It can be seen that there is a closely dependence between investments in transmission and generation capacity. Because of this dependence, externalities arise when investment decisions are decentralized. All the examples make a parallel with the previous chapter and the results support the proposed hypothesis.

5 CONCLUSIONS

An important conclusion of the dissertation is that the transmission and generation investment problem can be successfully modeled as a multilevel optimization problem with the market operator problem in the lower level. Multilevel formulation of the problem can model decision sequence among the market agents; so the investment problem is solved first provided that the market operator maximize the social welfare.

Another important conclusion of the dissertation is that a decentralized market must lead to a Pareto optimal solution in the context of perfect competition. This result is known as the First Welfare Theorem, very well known in economic literature. However, the state-of-the-art models do not give much attention to this theorem. This important theorem allowed to establish a reference model in order to find the Pareto optimal solution, which is found through the solution of the centralized model.

Moreover, it was shown that the opportunity costs of transmission and generation capacity are closely dependent, then Pareto optimal solution is reached only when investments in both capacities are cleared in the same market (in mathematical terms, the investments problems should be modeled in the same level). Since this result, decentralizing the capacity investments leads to a non-Pareto optimal solution. Thus, maximum social welfare is not reached and there will not be an efficient use of resources. Distortions caused by the externalities can be overcome by implementing regulatory mechanisms. Thus, it is concluded that investments in transmission and generation capacity should be under some level of regulation in order to get a Pareto optimal solution.

Finally, a multistage bi-level model was proposed in order to get the Pareto optimal solution for generation and transmission investments. The results were successful and the presented study cases supported the hypothesis of the dissertation.

5.1 FUTURE WORK

During the research, there were observed some future contributions to this dissertation:

- To design proper regulatory mechanisms to overcome the externalities problem.
- To develop a decentralized model and compared the results with the proposed model.
- To propose a stochastic formulation of the problem that consider uncertainty in the de-

mand considering the risk aversion of the decision maker.

- To implement an AC formulation of the model in order to represent more exactly the problem.

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APPENDIX A - TEST SYSTEMS DATA

APPENDIX A.1 - GARVER TEST SYSTEM DATA

Table 14 - Garver system - data for transmission lines

i	j	x_{ij} (pu)	$\overline{f}_{ij}$ (MW)	n_{ij}^0	c_{ij} $(10^5\$)$	$\overline{n}_{ij}$
1	2	0.4	100	1	40	5
1	3	0.38	100	0	38	5
1	4	0.6	80	1	60	5
1	5	0.2	100	1	20	5
1	6	0.68	70	0	68	5
2	3	0.2	100	1	20	5
2	4	0.4	100	1	40	5
2	5	0.31	100	0	31	5
2	6	0.3	100	0	30	5
3	4	0.59	82	0	59	5
3	5	0.2	100	1	20	5
3	6	0.48	100	0	48	5
4	5	0.63	75	0	63	5
4	6	0.3	100	0	30	5
5	6	0.61	78	0	61	5

Source: Garver (1970)

Table 15 - Garver system - data for existing generation

Unit	$\pi^{1,2}$ ($ / MWh)	π^3 ($ / MWh)	nodo i	$\overline{g}_k^1$ MW	$\overline{g}_k^2$ MW	$\overline{g}_k^3$ MW
G1	0	150	1	50	150	150
G2	0	150	3	165	360	360
G3	0	0	6	545	600	600

Source: adapted from Garver (1970)

Table 16 - Garver system - data for investment project in generation units

Node	$\varepsilon_{\text{diesel}}$ (10^6 \$ / MW)	π_{diesel} (\$ / MWh)	$\overline{g}_{\text{diesel}}$ MW	$\overline{w}_{\text{diesel}}$	ε_{gas} (10^6 \$ / MW)	π_{gas} (\$ / MWh)	$\overline{g}_{\text{gas}}$ MW	$\overline{w}_{\text{gas}}$
1	7	74.4	20	3	8	18.2	20	3
2	7	74.4	20	3	8	18.2	20	3
3	7	74.4	20	3	8	18.2	20	3
4	7	74.4	20	3	8	18.2	20	3
5	7	74.4	20	3	8	18.2	20	3
6	7	74.4	20	3	8	18.2	20	3

Source: adapted from Dammert, Garcia e Mollinelli (2010)

Table 17 - Garver system - data for demand levels in MW

Node	$d_{i,t=1}$	$d_{i,t=2}$
1	80	160
2	240	400
3	40	100
4	160	260
5	240	340
6	-	-

Source: adapted from Garver (1970)

APPENDIX A.2 - IEEE 24 BUSES TEST SYSTEM DATA

Table 18 - IEEE 24 buses system - data for transmission lines

i	j	$1/b_{ij}$ (pu)	$\overline{f}_{ij}$ (MW)	n_{ij}^0	c_{ij} $(10^3\$)$	i	j	$1/b_{ij}$ (pu)	$\overline{f}_{ij}$ (MW)	n_{ij}^0	c_{ij} $(10^3\$)$
1	2	0.0139	175	1	3000	13	23	0.0865	500	1	120000
1	3	0.2112	175	1	55000	14	16	0.0389	500	1	54000
1	5	0.0845	175	1	22000	15	16	0.0173	500	1	24000
2	4	0.1267	175	1	33000	15	21	0.049	500	2	68000
2	6	0.192	175	1	50000	15	24	0.0519	500	1	72000
3	9	0.119	175	1	31000	16	17	0.0259	500	1	36000
3	24	0.0839	400	1	50000	16	19	0.0231	500	1	32000
4	9	0.1037	175	1	27000	17	18	0.0144	500	1	20000
5	10	0.0883	175	1	23000	17	22	0.1053	500	1	146000
6	10	0.0605	175	1	16000	18	21	0.0259	500	2	36000
7	8	0.0614	175	1	16000	19	20	0.0396	500	2	55000
8	9	0.1651	175	1	43000	20	23	0.0216	500	2	30000
8	10	0.1651	175	1	43000	21	22	0.0678	500	1	94000
9	11	0.0839	400	1	50000	1	8	0.1344	500	0	35000
9	12	0.0839	400	1	50000	2	8	0.1267	500	0	33000
10	11	0.0839	400	1	50000	6	7	0.192	500	0	50000
10	12	0.0839	400	1	50000	13	14	0.0447	500	0	62000
11	13	0.0476	500	1	66000	14	23	0.062	500	0	86000
11	14	0.0418	500	1	58000	16	23	0.0822	500	0	114000
12	13	0.0476	500	1	66000	19	23	0.0606	500	0	84000
12	23	0.0966	500	1	134000						

Source: Romero et al. (2002)

Table 19 - IEEE 24 buses system - data for existing generation units

Unit	π (\$ / MWh)	node i	$\bar{g}_k$ MW
G1	1	1	576
G2	1	2	576
G3	1	7	900
G4	1	13	1773
G5	1	15	645
G6	1	16	465
G7	1	18	1200
G8	1	21	1200
G9	1	22	900
G10	1	23	1980

Source: adapted from Romero et al. (2002)

Table 20 - IEEE 24 buses system - data for investment projects in generation units

Node	ε (\$/MW)	π (\$ / MWh)	$\bar{g}_k$ MW	$\bar{w}$
10	10^9	1	400	2
15	10^9	1	400	2
18	10^9	1	400	2

Source: adapted from Romero et al. (2002)

Table 21 - IEEE 24 buses system - data for demand levels in MW

Node	$d_{i,t=1}$	$d_{i,t=2}$
1	324	421.2
2	291	378.3
3	540	702
4	222	288.6
5	213	276.9
6	408	530.4
7	375	487.5
8	513	666.9
9	525	682.5
10	585	760.5
11	0	0
12	0	0
13	795	1033.5
14	582	756.6
15	951	1236.3
16	300	390
17	0	0
18	999	1298.7
19	543	705.9
20	384	499.2
21	0	0
22	0	0
23	0	0
24	0	0

Source: adapted from Romero et al. (2002)

APPENDIX A.3 - PERUVIAN SYSTEM DATA

Table 22 - Peruvian system - data for demand levels in MW

Substation	Node	$d_{t=1}$	$d_{t=2}$	Substation	Node	$d_{t=1}$	$d_{t=2}$
Zarumilla 220kV	1	0	0	Mantaro Nueva 220kV	67	0	0
Zorritos 220kV	2	48	54	Cotaruse 220kV	68	324	324
Talara 220kV	3	30	87	Socabaya 220kV	69	26	26
Piura 220kV	4	218	253	Nueva Socabaya 220kV	70	0	0
Pariñas 220kV	5	0	0	Nueva Socabaya 500kV	71	0	0
Piura Sur 220kV	6	31	75	Ocoña 500kV	72	0	0
La Niña 500kV	7	0	0	San Camilo 500kV	73	408	408
La Niña 220kV	8	54	57	Montalvo 500kV	74	0	0
Chiclayo 220kV	9	141	190	Montalvo 220kV	75	0	0
Carhuaquero 220kV	10	20	25	Moquegua 220kV	76	521	514
Reque 220kV	11	6.04	7.41	Cerro Verde 220kV	77	130	130
Guadalupe 220kV	12	109	114	Tajish 220kV	78	0	0
Trujillo 220kV	13	245	307	Antamina 220kV	79	290	302
Trujillo Nueva 220kV	14	0	0	Vizcarra 220kV	80	0	0
Trujillo Nueva 500kV	15	0	0	Tingo Maria 220kV	81	0	0
Viru 220kV	16	0	0	Paragsha 138kV	82	77	73
Chimbote 220kV	17	0	0	Tingo Maria 138kV	83	7	8
Chimbote 500kV	18	0	0	Piedra Blanca 138kV	84	0	0
Chimbote 138kV	19	168	197	Amarilis 138kV	85	35	39
Huallanca 138kV	20	62	74	San Lorenza 138kV	86	0	0
Kiman Ayllu 138kV	21	26	31.25	Aguaytia 220kV	87	55	68
Kiman Ayllu 220kV	22	0	0	Aucayacu 138kV	88	3	3
Shauindo 220kV	23	75	75	Tocache 138kV	89	8	10
Cajamarca 220kV	24	160	397.4	Juanjui 138kV	90	8	9
Conococha 220kV	25	0	0	Bellavista 138kV	91	12	14
Pachapaqui 220kV	26	16	16	Tarapoto 138kV	92	41	49
Paramonga 220kV	27	54	115	Moyobamba 138kV	93	18	23
Huacho 220kV	28	35	53	Moyobamba 220kV	94	87	120
Nueva Huaral 220kV	29	38	53	Caclic 220kV	95	7.6	7.6
Zapallal 220kV	30	72	89	Socabaya 138kV	96	220	230
Carabayllo 220kV	31	262	440	Los Heroes 220kV	97	67	67
Carabayllo 500kV	32	0	0	Santuario 138kV	98	119	119
Ventanilla 220kV	33	207	229	Callalli 138kV	99	21	21
Chavarria 220kV	34	791	849	Tintaya 138kV	100	12	12
Cajamarquilla 220kV	35	80	80	Tintaya 220kV	101	97	97
Carapongo 220kV	36	0	0	Ayaviri 138kV	102	6.5	7
Huinco 220kV	37	0	0	Azangaro 138kV	103	105	115
Santa Rosa 220kV	38	539	570	Azangaro 220kV	104	0	0
Callahuanca 220kV	39	0	0	Juliaca 220kV	105	0	0
Callahuanca REP 220kV	40	0	0	Juliaca 138kV	106	52	53
Pachachaca 220kV	41	0	0	Puno 138kV	107	40	50
Chilca CTM 500kV	42	0	0	Puno 220kV	108	0	0
Chilca CTM 220kV	43	0	0	Combapata 138kV	109	16.4	21
Chilca REP 220kV	44	94	132	Quencoro 138kV	110	18	22
Lurin 220kV	45	142	150	Quencoro 220kV	111	0	0
San Juan 220kV	46	837	963	Onocora 220kV	112	0	0
Pomacocha 220kV	47	161	161	Dolorespata 138kV	113	53	69
Planicie 220kV	48	430	556	Cachimayo 138kV	114	22	26
Huanza 220kV	49	0	0	Machupicchu 138kV	115	9.5	12.4
Orcotuna 220kV	50	23	19	Suriray 138kV	116	0	0
Huayucachi 220kV	51	32	41	Suriray 220kV	117	0	0
Mantaro 220kV	52	14.25	16.5	Abancay 220kV	118	0	0
Oroya 220kV	53	15	24.29	Abancay 138kV	119	28	28
Carhuamayo 220kV	54	31	42	Quellaveco 220kV	120	71	71
Paragsha 220kV	55	24	24	Tarapoto 220kV	121	0	0
Huancavelica 220kV	56	73	74	Carapongo 500kV	122	0	0
Independencia 220kV	57	97	97	Planicie 500kV	123	0	0
Chincha 220kV	58	28	33	Cotaruse 500kV	124	0	0
Desierto 220kV	59	23	15	Tocache 220kV	125	0	0
Cantera 220kV	60	33	40	Yanango 220kV	126	0	0
Ica 220kV	61	106	137	Yanango 500kV	127	0	0
Nazca 220kV	62	24	11	Yuncan 220kV	128	5	6
Marcona 220kV	63	227	255	Cajamarca 500kV	129	0	0
Marcona Nueva 220kV	64	0	0	Yuncan 500kV	130	0	0
Marcona Nueva 500kV	65	0	0	Paramonga 500kV	131	0	0
Mantaro Nueva 500kV	66	0	0				

Source: adapted from the Peruvian system operator

Table 23 - Peruvian system - data for transmission lines
Part I

i	j	$1/b_{ij}$ (pu)	$\overline{f}_{ij}$ (MW)	n_{ij}^0	c_{ij} (10^3\$)	$\overline{n}_{ij}$	i	j	$1/b_{ij}$ (pu)	$\overline{f}_{ij}$ (MW)	n_{ij}^0	c_{ij} (10^3\$)	$\overline{n}_{ij}$
1	2	0.0511	150	1	8.0	0	75	76	0.0035	700	1	0.8	1
2	3	0.1405	150	1	21.2	1	76	69	0.1083	150	2	16.7	1
3	4	0.1405	180	1	16.3	1	69	70	0.0037	346	2	0.6	1
5	4	0.0961	180	1	14.7	1	70	77	0.0060	346	2	0.9	1
3	5	0.0103	180	1	1.6	1	70	71	0.0272	600	1	15.0	1
9	8	0.1132	180	1	17.5	1	71	74	0.0134	700	1	181.6	1
9	8	0.1132	180	1	17.5	1	52	68	0.0817	250	2	46.5	0
8	4	0.1124	180	1	17.4	1	68	69	0.0645	250	2	49.8	0
6	8	0.1030	180	1	16.0	1	52	67	0.0337	600	1	0.5	1
4	6	0.0094	180	1	1.4	1	67	66	0.0272	600	1	15.0	1
8	7	0.0272	600	1	15.0	1	66	65	0.0224	700	1	142.8	1
9	10	0.0840	114	1	13.1	1	65	71	0.0344	700	1	181.6	1
11	9	0.0132	180	1	1.9	1	25	80	0.0538	190	1	8.1	1
11	12	0.0789	180	1	11.3	1	82	55	0.0767	120	1	8.0	1
11	9	0.0132	150	1	1.9	1	82	86	0.1614	75	1	8.3	1
11	12	0.0789	150	1	11.3	1	86	85	0.0806	75	1	4.2	1
12	13	0.1137	180	1	16.3	1	85	84	0.1428	45	1	7.5	1
12	13	0.1137	150	1	16.3	1	84	83	0.0735	45	1	3.8	1
13	24	0.1395	250	1	21.6	1	83	81	0.2910	50	2	1.6	1
13	14	0.0023	345	2	0.6	1	81	78	0.1820	250	1	27.5	1
14	15	0.0224	750	1	15.0	1	78	80	0.0062	250	1	0.9	1
15	18	0.0174	600	1	56.3	1	55	78	0.1248	250	1	19.3	1
16	17	0.0328	150	1	5.2	1	78	80	0.0072	250	1	1.1	1
13	16	0.0998	150	1	15.8	1	80	79	0.0549	228	1	8.2	1
16	17	0.0328	150	1	5.2	1	80	78	0.0085	228	1	1.3	1
13	16	0.0998	150	1	15.8	1	78	79	0.0549	228	1	8.2	1
17	18	0.0224	750	1	15.0	1	81	87	0.0764	190	1	11.6	1
17	19	0.0909	120	1	3.2	1	81	88	0.1203	75	1	11.2	1
17	19	0.0886	120	1	3.2	1	88	89	0.2886	75	1	22.0	1
15	7	0.0385	700	1	124.1	1	89	90	0.3251	75	1	17.2	1
19	20	0.2116	140	3	11.7	1	90	91	0.0655	75	1	11.1	1
20	21	0.0177	120	1	1.0	1	91	92	0.2108	75	1	11.1	1
21	22	0.1204	150	1	9.0	1	92	93	0.2525	75	1	13.3	1
22	23	0.0999	240	2	21.0	1	93	94	0.1200	100	1	7.0	1
23	24	0.0654	240	2	13.7	1	95	94	0.1450	220	1	11.2	1
24	10	0.1040	180	1	15.8	1	95	24	0.1639	220	2	12.7	1
22	25	0.1706	180	2	27.2	1	69	96	0.0629	150	2	9.0	1
17	27	0.2184	180	2	34.8	1	96	98	0.0762	135	2	3.8	1
27	28	0.0575	180	2	8.7	1	98	99	0.2390	110	1	12.4	1
27	26	0.0631	190	1	9.5	1	99	100	0.2422	110	1	12.5	1
26	25	0.0334	190	1	5.1	1	100	102	0.2180	90	1	11.4	1
28	30	0.1104	180	1	16.7	1	102	103	0.1121	90	1	5.8	1
28	29	0.0499	180	1	7.6	1	103	106	0.2061	90	1	10.8	1
29	30	0.0605	180	1	9.2	1	106	107	0.0978	80	1	5.1	1
30	31	0.0063	800	2	1.6	0	107	108	0.1015	120	1	8.0	1
30	33	0.0184	270	1	2.8	0	108	120	0.1009	150	1	15.5	1
30	33	0.0199	270	1	2.8	0	120	76	0.1009	150	1	15.5	1
33	34	0.0109	188	2	1.7	0	76	97	0.1314	228	2	19.6	1
33	34	0.0114	188	2	1.7	0	103	104	0.1015	120	1	8.0	1
34	35	0.0397	340	2	3.3	0	104	105	0.0802	150	1	12.3	1

Source: adapted from the Peruvian system operator

Table 24 - Peruvian system - data for transmission lines
Part II

i	j	$1/b_{ij}$ (pu)	$\overline{f}_{ij}$ (MW)	n^0_{ij}	c_{ij} (10^3\$)	$\overline{n}_{ij}$	i	j	$1/b_{ij}$ (pu)	$\overline{f}_{ij}$ (MW)	n^0_{ij}	c_{ij} (10^3\$)	$\overline{n}_{ij}$
34	38	0.0087	150	2	1.3	0	105	106	0.1015	120	1	8.0	1
38	46	0.0268	150	2	4.1	1	105	108	0.0380	150	1	5.8	1
46	47	0.1136	150	2	17.7	0	100	101	0.0560	125	2	8.0	1
46	44	0.0342	350	2	7.7	0	101	69	0.2148	200	2	32.7	1
46	45	0.0117	350	1	2.5	0	100	109	0.2336	90	1	12.1	1
45	44	0.0230	350	1	5.1	0	109	110	0.2643	90	1	13.8	1
46	44	0.0347	350	1	7.7	0	110	115	0.2711	83	1	13.6	1
44	43	0.0269	600	1	3.0	0	110	113	0.0228	71	1	1.1	1
35	36	0.0055	340	2	0.9	0	115	114	0.2067	93	1	10.8	1
36	39	0.0318	340	2	4.9	0	114	113	0.0355	95	1	1.8	1
44	48	0.0397	350	2	7.9	1	114	119	0.2539	90	1	13.2	1
48	36	0.0096	350	2	1.9	1	115	116	0.0132	250	1	1.3	1
36	31	0.0214	350	2	4.3	1	116	117	0.0556	225	1	16.0	1
18	32	0.0174	600	1	153.4	1	119	118	0.1000	100	1	7.0	1
31	32	0.0272	600	2	15.0	1	117	118	0.0840	454	1	13.3	1
37	36	0.0010	342	2	0.2	1	117	111	0.1247	240	1	24.2	1
36	38	0.0633	342	2	9.6	0	111	110	0.0933	150	1	9.0	1
43	42	0.0272	600	1	15.0	1	111	112	0.0945	240	2	18.3	1
42	32	0.0115	600	1	36.7	1	112	101	0.0692	240	2	13.3	1
39	40	0.0006	380	1	0.1	1	117	68	0.1980	454	1	31.3	1
40	41	0.0733	250	2	11.5	0	68	118	0.1140	454	1	18.0	1
41	47	0.0139	250	1	2.1	1	92	121	0.1000	120	0	8.0	1
41	52	0.2022	150	2	30.7	1	121	94	0.0992	150	0	15.2	1
52	47	0.2025	150	2	30.7	1	123	48	0.0200	600	0	15.0	1
44	60	0.0844	150	1	13.1	1	122	36	0.0200	600	0	15.0	1
44	59	0.1081	150	1	17.0	1	42	123	0.0062	750	0	20.4	1
60	57	0.0832	150	1	13.0	1	123	122	0.0015	750	0	4.9	1
59	58	0.0167	150	1	2.5	1	122	32	0.0033	750	0	11.0	1
58	57	0.0442	150	1	7.0	1	32	66	0.0402	750	0	132.2	1
57	56	0.1979	150	2	28.4	1	66	42	0.0422	750	0	138.7	1
56	52	0.0728	150	2	10.4	1	66	123	0.0391	750	0	128.5	1
52	51	0.0791	150	1	12.0	1	121	81	0.3867	150	0	60.4	1
49	31	0.0812	250	1	12.3	1	68	124	0.0133	750	0	15.0	1
49	50	0.1281	150	1	19.6	1	66	124	0.0152	750	0	122.4	1
51	50	0.0393	150	1	6.0	1	65	124	0.0109	750	0	87.7	1
41	53	0.0227	250	1	3.3	1	81	125	0.1539	180	0	24.0	1
47	54	0.1136	180	1	17.4	1	125	121	0.2318	180	0	36.2	1
53	54	0.0777	150	1	12.0	1	125	89	0.1000	120	0	8.0	1
54	55	0.0443	150	1	6.8	1	126	41	0.0960	285	1	14.1	1
54	55	0.0437	150	2	6.8	1	126	127	0.0160	750	0	15.0	1
55	25	0.1437	180	1	22.1	1	127	66	0.0201	750	0	65.3	1
57	61	0.0568	180	2	8.7	1	128	54	0.0527	390	2	1000.0	0
61	62	0.1063	180	1	16.6	1	24	129	0.0160	750	0	15.0	1
62	63	0.0506	180	1	7.9	1	129	15	0.0172	750	0	55.9	1
63	64	0.0146	450	2	4.3	1	127	130	0.0075	750	0	24.5	1
64	65	0.0253	450	1	4.0	1	128	130	0.0160	750	0	15.0	1
65	42	0.0249	700	1	145.3	1	131	27	0.0160	750	0	15.0	1
65	72	0.0202	700	1	110.6	1	131	130	0.0343	750	0	111.4	1
72	73	0.0052	700	1	63.3	1	127	122	0.0251	750	0	81.6	1
73	74	0.0132	700	1	40.8	1	127	32	0.0251	750	0	81.6	1
74	75	0.0280	750	1	20.0	1	127	66	0.0201	750	0	65.3	1

Source: adapted from the Peruvian system operator

Table 25 - Peruvian system - data for existing generation units

Unidad	π ($ / MWh)	nodo i	$\bar{g}_k$ MW	Unidad	π_k ($ / MWh)	nodo i	$\bar{g}_k$ MW
Pariac	0.0	20	2	Santa Rosa UTI 5	35.6	38	50
Yanapampa	0.0	27	2	Santa Rosa UTI 6	37.8	38	50
Marcara	0.0	20	3	Chimay	0.0	126	70
Caña Brava	0.0	10	5	Marañon	0.0	78	70
Las Pizarras	0.0	10	5	Rapay	0.0	27	70
Macon	0.0	50	5	Pisco	32.3	57	70.7
Manta	0.0	20	6	Aguaytia TG1	29.2	87	80
Shima	0.0	91	6	Aguaytia TG2	29.4	87	80
Curumuy	0.0	4	7	Huanza	0.0	49	80
Gera	0.0	93	7	Quitaracsa	0.0	22	80
El carmen	0.0	83	8	Santa Teresa	0.0	117	80
Esperanza	0.0	83	8	Tulumayo	0.0	50	80
Mushcapata	0.0	83	8	Yaupi	0.0	128	80
Tumbes TG1	203.9	2	9.17	Machupicchu	0.0	115	87
Tumbes TG2	190.0	2	9.17	Machupicchu II	0.0	115	90
8 de agosto	0.0	83	10	Malacas TG4	32.0	3	90
Cola	0.0	13	10	San Gaban II	0.0	103	90
Pias	0.0	20	10	Charcani V	0.0	98	100
Poechos	0.0	4	10	Yuncan	0.0	128	100
Santa Cruz	0.0	20	11	Cañon del Pato	0.0	20	120
Las Cruces	0.0	76	14	El Faro CC	19.3	63	120
Eolica Marcona	0.0	63	15	Matucana	0.0	39	120
Eolica Talara	0.0	5	15	Santa Rosa TG7	31.5	38	121
Gallito Ciego	0.0	12	15	Nueva Esperanza	25.8	2	135
Renovaandes	0.0	41	15	Cheves	0.0	28	140
Malacas TG1	45.0	3	16	Pucara	0.0	112	140
Charcani	0.0	96	18	Ilo 2	44.2	76	141
Rucuy	0.0	55	18	Belo Horizonte	0.0	81	150
Vilcanota	0.0	117	18	Curibamba	0.0	41	150
Cahua	0.0	27	20	Huinco	0.0	37	150
Pelagatos	0.0	20	20	Malacas TGD5	305.0	3	177
Runatullo	0.0	50	20	Las Flores TG1	27.2	43	192
Aipsa	0.0	27	23	Santa Rosa TG8	27.8	38	199
Independencia	24.5	57	23	Quillabamba	30.1	117	200
Tablazo	18.7	4	29	Restitucion	0.0	52	200
Callahuanca	0.0	39	30	Platanal	0.0	44	210
Chancay 2	0.0	55	30	ETEN	305.0	11	223
Yanango	0.0	126	30	Termochilca CC	18.5	42	286
Maple	0.0	4	37.5	Chaglla	0.0	55	360
El Angel	0.0	103	40	RF Ilo	333.6	76	460
Eolica Cupisnique	0.0	12	40	Ventanilla CC	18.9	33	480
RF Pucallpa	323.3	87	40	Cerro del Aguila	0.0	67	500
Olmos	0.0	9	45	Fenix CC	20.6	42	534
Tres hermanas	0.0	63	45	Mantaro	0.0	52	650
Carhuaquero	0.0	10	50	Chilca CC	18.6	43	811
El Tambo	0.0	76	50	Kallpa CC	18.2	44	857

Source: adapted from the Peruvian system operator

Table 26 - Peruvian system - data for the investment projects in generation units

Project	ε (\$/MW)	π_k (\$ / MWh)	Node	$\bar{g}_k$ MW	$\bar{w}$
Santa Rita	550	0.0	16	240	1
Santa Maria	1600	0.0	66	720	1
Molloco	600	0.0	77	280	1
TG Sur 1	400	17.5	74	500	1
TG Sur 2	400	17.5	74	500	1
TG Sur 3	400	17.5	74	500	1
Lluclla	560	0.0	71	240	1
Lluta	560	0.0	71	260	1
San Gaban I	400	0.0	112	180	1
San Gaban III	400	0.0	112	160	1

Source: adapted from the Peruvian system operator