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$Y(3940)$ as a mixed charmonium-molecule state

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Using the QCD sum rules approach, we study the mass and the decay widths of the $Y(3940)$ state, assuming that it can be described by a mixed charmonium-molecule scalar state, i.e., a mixture between the χ_{c0} charmonium and $D^*\bar{D}^*$ molecule. Using a current with $J^{PC}=0^{++}$, we estimate for the mixing angle, $\theta=(76.0\pm 5.0)°$, resulting in a mass value of $M_Y=(3.95\pm 0.11)$ GeV, which is in reasonable agreement with the experimental mass of the $Y(3940)$ state. For the decay width, we evaluate the channels $Y\to J/\psi\omega$ and $Y\to\gamma\gamma$. We find the values $\Gamma_{Y\to J/\psi\omega}\approx(1.7\pm 0.6)$ MeV and $\Gamma_{Y\to\gamma\gamma}\approx(1.6\pm 1.3)$ KeV, respectively. We also study the decay process of this state into channels containing $D\bar{D}$ mesons in the final state. The result for the order of magnitude of the product $\Gamma_{Y\to\gamma\gamma}\times\Gamma_{Y\to J/\psi\omega}\sim\mathcal{O}(10^3)$ KeV^2 is also in reasonable agreement with the experimental data. We thus conclude that the present description of the $Y(3940)$ as a mixed charmonium-molecule state is a possible scenario to explain the structure of such a state.

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I. INTRODUCTION

In the last years, several states in the region of mass of about 3940 MeV have been observed in different processes of production and decay. The first one was the $Y(3940)$, observed by BELLE Collaboration in the decay $B\to(J/\psi\omega)K$, with a mass $m=3943\pm 11(\mathrm{stat})\pm 13(\mathrm{syst})$ MeV and decay width $\Gamma=87\pm 22(\mathrm{stat})\pm 26(\mathrm{syst})$ MeV [1]. Soon after, this state has been also observed in the process $B\to(J/\psi\omega)K$ by *BABAR* Collaboration, with a slightly smaller mass of $m=3914.6^{+3.9}_{-3.4}(\mathrm{stat})\pm 2.0(\mathrm{syst})$ MeV and width $\Gamma=34^{+12}_{-8}(\mathrm{stat})\pm 5.0(\mathrm{syst})$ MeV [2]. Between these two observations, BELLE Collaboration has reported the observation of a state in the process $\gamma\gamma\to D\bar{D}$, that is generally linked to the charmonium state $\chi_{c2}(2P)$ [3,4]. BELLE has called this state "$Z(3930)$," and the results for the mass and width are $m=(3929\pm 5(\mathrm{stat})\pm 2(\mathrm{sys}))$ MeV, $\Gamma=(29\pm 10(\mathrm{stat})\pm 2(\mathrm{sys}))$ MeV. Finally the state $X(3915)$ was observed by BELLE Collaboration in the process $\gamma\gamma\to J/\psi\omega$ [5], with a mass $m=(3915\pm 3\pm 2)$ MeV and total width $\Gamma=(17\pm 10\pm 3)$ MeV. These values are consistent with those of the $Y(3940)$, which is seen in the $J/\psi\omega$ final state [1,2], and close to those of the $Z(3930)$, which is seen in $\gamma\gamma\to D\bar{D}$ [3,4].

The proximity of the masses could indicate that all these states are connected to the same particle observed in different processes. There is evidence, however, that the two reported states, $Y(3940)$ and $X(3915)$, could be interpreted as molecular states. The $X(3915)$ state has a larger product of the two-photon width times the decay branching fraction than usually expected for charmonium states, as noted in Ref. [6]. Regarding the $Y(3940)$, the lower limit for the decay channel $J/\psi\omega$ has been estimated to be $\Gamma>1$ MeV, which is large for a channel that is OZI suppressed for conventional charmonium states [7,8]. These facts suggests that these states cannot be interpreted as a conventional $c\bar{c}$ state. In Ref. [9] these two states were interpreted as the same state, and it was called $X(3915)$. However, the Particle Data Group [10] associates the label "$X(3915)$" with the charmonium state $\chi_{c0}(2P)$. Therefore, to avoid misinterpretation, here we use the label "$Y(3940)$" to identify the state observed in the decay mode $J/\psi\omega$.

In Ref. [11], it was proposed that the $Y(3940)$ can be a molecular state $D^*\bar{D}^*$, with quantum numbers $J^{PC}=0^{++}$ or 2^{++}. It was also concluded that the $Y(3940)$ must be the molecular partner of the state $Y(4140)$, a $D_s^*\bar{D}_s^*$ molecule. This interpretation has been tested in several approaches, such as phenomenological Lagrangians [12] and vector-meson dominance [13]. In Ref. [14], the Y state was studied with the QCD sum rules (QCDSR) method [15–17] as a $D^*\bar{D}^*$ molecule with quantum numbers 0^{++} and the mass obtained was $m_{D^*\bar{D}^*}=(4.13\pm 0.10)$ MeV, failing to reproduce the experimental mass of the state.

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In the present work we revisit the study of the $Y(3940)$ within the QCDSR approach, using a mixed charmonium-molecule current. The prescription of a mixture of two- and four-quark states has been successfully implemented for other states in the framework of sum rules. Following the work of Ref. [18] that was applied in the light quark sector, the authors in Refs. [19–21] described the $X(3872)$ state as a molecule-charmonium state, implementing the mixing of the current and extending it to the charm sector. In these works the mass and decay width for the channels $J/\psi + (2\pi, 3\pi, \gamma)$ and the production in B-meson decays were estimated in good agreement with the experimental values. Another state that was studied as a mixture was the $Y(4260)$. In Ref. [22], the $Y(4260)$ was described as a tetraquark-charmonium mixed state, and the mass and decay width estimated were also consistent with the experimental values.

In the following sections we use the QCDSR approach to describe the $Y(3940)$ as a mixing between the χ_{c0} charmonium and the $D^*\bar{D}^*$ molecule, with $J^{PC} = 0^{++}$. We obtain the mass for this state and the decay width in the channel $Y \to J/\psi\omega$.

II. MIXED HADRONIC CURRENT

In order to evaluate the sum rule for the $Y(3940)$ state as a mixed $(\chi_{c0}) - (D^*\bar{D}^*)$ state, with $J^{PC} = 0^{++}$, one employs the following hadronic current,

$$j = a\cos\theta j_{\chi_{c0}} + \sin\theta j_{D^*D^*}, \tag{1}$$

where θ is an arbitrary mixing angle. The meson and molecule currents are, respectively, given by

$$j_{\chi_{c0}} = \bar{c}_k c_k \tag{2}$$

$$j_{D^*D^*} = (\bar{q}_i\gamma_\mu c_i)(\bar{c}_j\gamma^\mu q_j). \tag{3}$$

Notice that the normalization factor a is introduced in Eq. (3) for ensuring that the mixed current can be evaluated at the same Fock space. Usually, one sets [18–21]

$$a = -\frac{\langle\bar{q}q\rangle}{\sqrt{2}}. \tag{4}$$

Then, evaluating the two- and three-point correlation functions altogether with Eq. (1), one can estimate the mass and decay width of the mixed $(\chi_{c0}) - (D^*\bar{D}^*)$ state.

III. TWO-POINT CORRELATION FUNCTION

To obtain the mass of a hadronic state using the QCDSR approach, one has to evaluate the two-point correlation function

$$\Pi(q) = i\int d^4x e^{iq\cdot x}\langle 0|T[j(x)j^\dagger(0)]|0\rangle. \tag{5}$$

According to the quark-hadron duality principle, Eq. (5) can be evaluated in two ways: the phenomenological side and the QCD side. The phenomenological side is calculated by inserting, in Eq. (5), a complete set of intermediate states, Y, which couple to the hadronic current in Eq. (1). Parametrizing this coupling through a generic parameter λ_Y, one defines

$$\langle 0|j|Y\rangle = \lambda_Y. \tag{6}$$

Using Eq. (6) and after some algebraic manipulation, one can write the phenomenological side of Eq. (5) as

$$\Pi^{PHEN}(q) = \frac{\lambda_Y^2}{M_Y^2 - q^2} + \int_0^\infty ds\frac{\rho^{\rm cont}(s)}{s - q^2}, \tag{7}$$

where M_Y is the mixed $(\chi_{c0}) - (D^*\bar{D}^*)$ ground state mass, and the second term in the rhs of Eq. (7) denotes the continuum (or higher resonance) contributions. As usual in a QCDSR approach, it is assumed that the continuum contribution to the spectral density, $\rho^{\rm cont}(s)$ in Eq. (7), vanishes below a certain threshold s_0. Above this threshold, it is assumed that the result coincides with the one obtained in the OPE side. Therefore, one uses the ansatz [23]

$$\rho^{\rm cont}(s) = \rho^{OPE}(s)\Theta(s - s_0), \tag{8}$$

where $\Theta(s - s_0)$ is the Heaviside step function.

In the OPE side, one calculates the correlation function in terms of quark and gluon fields using the Wilson's operator product expansion (OPE). This is also called the OPE side. Then, inserting Eq. (1) into the above equation, one obtains

$$\Pi^{\rm OPE}(q) = i\int d^4x e^{iq\cdot x}\left\{\frac{1}{2}\langle\bar{q}q\rangle^2\cos^2\theta\Pi_{\chi_{c0}} + \sin^2\theta\Pi_{D^*D^*} - \frac{\langle\bar{q}q\rangle}{\sqrt{2}}\sin\theta\cos\theta[\Pi_{\rm mix} + \Pi^*_{\rm mix}]\right\}, \tag{9}$$

where the $\Pi_{\chi_{c0}}(x)$ and $\Pi_{D^*D^*}(x)$ functions are, respectively, the correlation functions of the χ_{c0} meson and the $D^*D^*(0^{++})$ molecular state, which have been calculated in other works [14,16]. Thus, one only has to calculate the $\Pi_{\rm mix}(x)$ and $\Pi^*_{\rm mix}(x)$ functions defined as follows:

$$\Pi_{\rm mix}(x) = \langle 0|T[j_{\chi_{c0}}(x)j^\dagger_{D^*D^*}(0)]|0\rangle = -{\rm Tr}[S^q_{ji}(0)\gamma_\mu S^c_{ik}(-x)S^c_{kj}(x)\gamma^\mu] \tag{10}$$

$$\Pi^*_{\rm mix}(x) = \langle 0|T[j_{D^*D^*}(x)j^\dagger_{\chi_{c0}}(0)]|0\rangle = -{\rm Tr}[S^q_{ji}(0)\gamma_\mu S^c_{ik}(x)S^c_{kj}(-x)\gamma^\mu], \tag{11}$$

where $S^c(x)$ and $S^q(x)$ are the charm- and light-quark propagators, respectively. The next step is to write the correlation function in terms of a dispersion relation, such that

$$\Pi^{\mathrm{OPE}}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s - q^2}, \tag{12}$$

where $\rho^{\mathrm{OPE}}(s)$ is given by the imaginary part of the correlation function: $\pi\rho^{\mathrm{OPE}}(s) = \mathrm{Im}[\Pi^{\mathrm{OPE}}(q^2 = s)]$. According to Eq. (9), the expression for the spectral density is

$$\rho^{\mathrm{OPE}}(s) = \frac{1}{2}\langle\bar{q}q\rangle^2\cos^2\theta\rho_{\chi_{c0}}(s) + \sin^2\theta\rho_{D^*D^*}(s) - \frac{\langle\bar{q}q\rangle}{\sqrt{2}}\sin\theta\cos\theta\rho_{\mathrm{mix}}(s). \tag{13}$$

One calculates the sum rule at leading order in α_s in the operators and considers the contributions from the condensates up to dimension eight in the OPE. The expressions for the spectral density are given in Appendix A.

To improve the matching between the two sides of the sum rule, one performs the Borel transform. After transferring the continuum contributions to the OPE side, the sum rule for the scalar charmonium-molecule, considered as a mixed scalar $(\chi_{c0}) - (D^*\bar{D}^*)$ state, can be written as

$$\lambda_Y^2 e^{-M_Y^2/M_B^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2}\rho^{\mathrm{OPE}}(s). \tag{14}$$

Therefore, one can estimate the ground state mass from the following ratio,

$$\mathcal{R} = \frac{\int_{4m_c^2}^{s_0} ds s e^{-s/M_B^2}\rho^{\mathrm{OPE}}(s)}{\int_{4m_c^2}^{s_0} ds e^{-s/M_B^2}\rho^{\mathrm{OPE}}(s)}, \tag{15}$$

where at the M_B^2-stability point, one obtains $M_Y \simeq \sqrt{\mathcal{R}}$.

A. Numerical analysis

The numerical values for the quark masses and condensates are listed in Table I. These values are consistent with the ones used in Refs. [20–22] for the QCDSR analysis on other mixed hadronic states.

TABLE I. QCD input parameters.

Parameters	Values
$\bar{m}_c$	$(1.23 - 1.47)$ GeV
$\langle\bar{q}q\rangle$	$-(0.23 \pm 0.03)^3$ GeV^3
$\langle g_s^2 G^2\rangle$	(0.88 ± 0.25) GeV^4
$\langle g_s^3 G^3\rangle$	(0.58 ± 0.18)GeV^6
$m_0^2 \equiv \langle\bar{q}Gq\rangle/\langle\bar{q}q\rangle$	(0.8 ± 0.1) GeV^2
$\rho \equiv \langle\bar{q}q\bar{q}q\rangle/\langle\bar{q}q\rangle^2$	$(0.5 - 2.0)$

For reliable results in a sum rule calculation, one must establish a valid Borel window which guarantees the existence of a region with M_B^2 stability, a good OPE convergence, and pole dominance over continuum contributions. Nevertheless, another crucial point is the optimal choice of the continuum threshold s_0 and the mixing angle θ.

We start our analysis discussing the possible values of both parameters. Considering that we are interested in a mixed state with a mass $M_Y \sim 3.9$ GeV, a reasonable initial value for the continuum threshold would be $\sqrt{s_0} = 4.40$ GeV. In principle, the choice of the mixing angle seems to be arbitrary. Hence, for a fixed value of θ, we search for a continuum threshold which allows us to determine the best M_B^2 stability inside of a valid Borel window. After lengthy numerical calculations, we find that the optimal choice is

$$\sqrt{s_0} = (4.40 \pm 0.10)\ \mathrm{GeV} \tag{16}$$

$$\theta = (76.0 \pm 5.0)^\circ. \tag{17}$$

We notice that the OPE does not converge for θ values outside this range. Using these values, we analyze the relative contributions of the terms in the OPE, for $\sqrt{s_0} = 4.40$ GeV and $\theta = 76.0°$. As one can see in Fig. 1, the contribution of the dimension-eight condensate is smaller than 20% of the total contribution for values of $M_B^2 \geq 2.40$ GeV^2, which indicates the starting point for a good OPE convergence. In order to determine the maximum value of the Borel mass parameter, we must

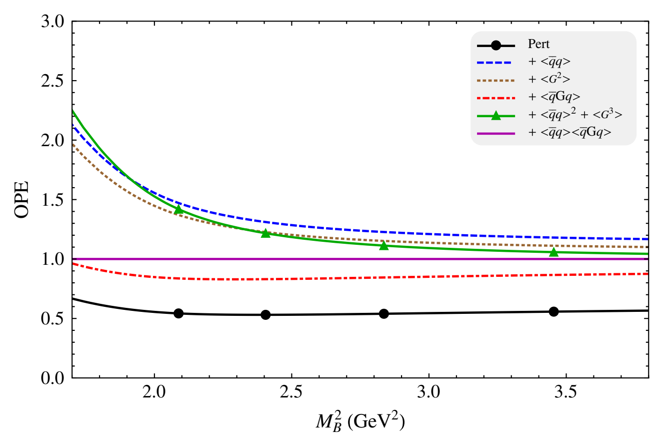

FIG. 1 (color online). OPE convergence in the region $1.7 \leq M_B^2 \leq 3.8$ GeV^2 for $\sqrt{s_0} = 4.40$ GeV and $\theta = 76.0°$. One plots the relative contributions starting with the perturbative contribution (line with circles), and each other line represents the relative contribution after adding of one extra condensate in the expansion: + $\langle\bar{q}q\rangle$ (dashed line), $+\langle G^2\rangle$ (dotted line), $+\langle\bar{q}Gq\rangle$ (dot-dashed line), $+\langle\bar{q}q\rangle^2 + \langle G^3\rangle$ (line with triangles), and $\langle\bar{q}q\rangle \cdot \langle\bar{q}Gq\rangle$ (solid line).

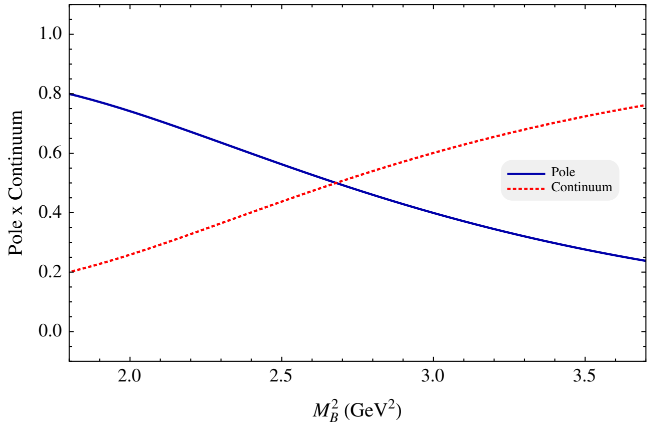

FIG. 2 (color online). The pole (solid line) and continuum (dotted line) contributions for $\sqrt{s_0} = 4.40$ GeV and $\theta = 76.0°$.

analyze the pole contribution. Since the QCDSR approach extracts information only from the ground state, we have to ensure that the pole contribution is greater than the continuum contribution. Thus, we fix the maximum value of the Borel mass parameter as the value for which the pole is greater than or equal to the continuum contribution. From Fig. 2, we can see that this condition is satisfied when $M_B^2 = 2.70\ \text{GeV}^2$. Therefore, the Borel window is set as $2.40 \le M_B^2 \le 2.70\ \text{GeV}^2$.

In Fig. 3, we plot the ground state mass as a function of M_B^2, considering three different values of $\sqrt{s_0}$. We conclude that there is a good M_B^2 stability in the determined Borel window.

Varying the value of the continuum threshold in the range $\sqrt{s_0} = (4.40 \pm 0.10)$ GeV, the mixing angle in the range $\theta = (76.0 \pm 5.0)°$, and the other parameters as indicated in Table I, we get

$$M_Y = (3.95 \pm 0.11)\ \text{GeV}. \tag{18}$$

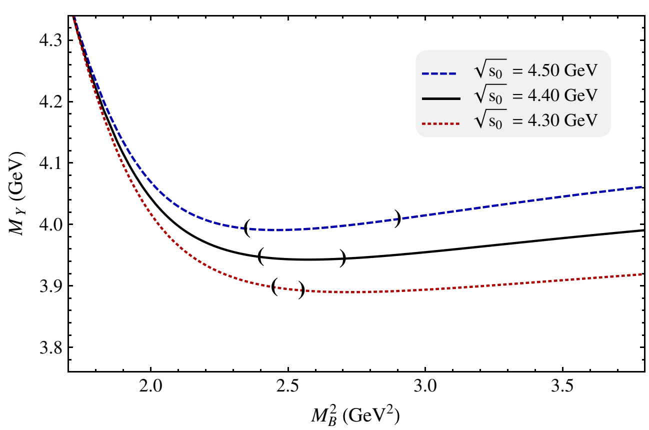

FIG. 3 (color online). The mass as a function of the sum rule parameter M_B^2 for $\sqrt{s_0} = 4.30$ GeV (dotted line), $\sqrt{s_0} = 4.40$ GeV (solid line) and $\sqrt{s_0} = 4.50$ GeV (dashed line). The respective parentheses indicate the valid Borel window.

This mass is compatible with the experimental mass of the $Y(3940)$ state observed by BELLE Collaboration [1]. Therefore, from a QCD sum rule point of view, a mixed scalar $(\chi_{c0}) - (D^*\bar{D}^*)$ state could be a good candidate to explain the $Y(3940)$ state.

After the determination of the mass, we can use this result in Eq. (14) to estimate the coupling parameter, defined in Eq. (6). Therefore, considering the same values of s_0, θ and the Borel window used for the mass calculation, we obtain

$$\lambda_Y = (2.1 \pm 0.6) \times 10^{-2}\ \text{GeV}^5. \tag{19}$$

IV. THE $Y(3940) \to J/\psi\omega$ DECAY WIDTH

In order to provide more evidence to support the conclusion reached at the end of the previous section, that the $Y(3940)$ can be explained as a scalar mixed state, we now use the QCDSR to compute the form factor associated with the vertex $YJ/\psi\omega$ and to estimate the width of the channel $Y(3940) \to J/\psi\omega$. For this purpose, we start writing the three-point function defined as

$$\Pi_{\mu\nu}(p, p', q) = \int d^4x d^4y e^{ip'\cdot x} e^{iq\cdot y} \Pi_{\mu\nu}(x, y), \tag{20}$$

where $p = p' + q$ and $\Pi_{\mu\nu}(x, y)$ is given by

$$\Pi_{\mu\nu}(x, y) = \langle 0|T\{j_\mu^\psi(x) j_\nu^\omega(y) j^\dagger(0)\}|0\rangle. \tag{21}$$

The interpolating currents for the J/ψ meson and the mixed $(\chi_{c0}) - (D^*\bar{D}^*)$ state used in Eq. (21) are defined in Sec. II, while the interpolating current associated with the ω meson is defined by

$$j_\nu^\omega = \frac{1}{6}(\bar{u}_a\gamma_\nu u_a + \bar{d}_a\gamma_\nu d_a). \tag{22}$$

In the same manner that it was done for the two-point correlation function, we again invoke the quark-hadron duality principle to calculate the three-point function in two ways. We match both sides after performing the Borel transform. In the phenomenological side, one has to insert the intermediate states for the J/ψ, ω and $Y(3940)$ mesons in Eq. (20). Using the following relations,

$$\begin{aligned}\langle 0|j_\mu^\psi|J/\psi(p')\rangle &= M_\psi f_\psi \epsilon_\mu(p'),\\ \langle 0|j_\nu^\omega|\omega(q)\rangle &= M_\omega f_\omega \epsilon_\nu(q),\\ \langle Y(p)|j|0\rangle &= \lambda_Y,\end{aligned} \tag{23}$$

we obtain the expression

$$\Pi_{\mu\nu}^{PHEN}(p, p', q) = \frac{\lambda_Y M_\psi f_\psi M_\omega f_\omega g_{Y\psi\omega}(q^2)}{(p^2 - M_Y^2)(p'^2 - M_\psi^2)(q^2 - M_\omega^2)} \times [q_\mu p'_\nu - (p'\cdot q) g_{\mu\nu}] + \cdots, \tag{24}$$

where the dots stand for the contribution of all possible excited states. The form factor, $g_{Y\psi\omega}(q^2)$, is defined by the generalization of the on-shell mass matrix element, $\langle J/\psi\omega|Y\rangle$, for an off-shell ω meson,

$$\langle J/\psi\omega|Y\rangle = g_{Y\psi\omega}(q^2)[(p'\cdot\epsilon^*(q))(q\cdot\epsilon^*(p')) - (p'\cdot q)(\epsilon^*(p')\cdot\epsilon^*(q))], \tag{25}$$

which can be extracted from the effective Lagrangian that describes the coupling between two vector mesons and one scalar meson,

$$\mathcal{L} = \frac{i}{2} g_{Y\psi\omega} V_{\alpha\beta}\Psi^{\alpha\beta}Y, \tag{26}$$

where $V_{\alpha\beta} = \partial_\alpha\omega_\beta - \partial_\beta\omega_\alpha$ and $\Psi^{\alpha\beta} = \partial^\alpha\psi^\beta - \partial^\beta\psi^\alpha$ are the tensor fields of the ω and ψ fields, respectively.

In the OPE side, we calculate the correlation function at leading order in α_s and we consider condensates up to dimension seven. Notice that the three-point function includes a number of different Lorentz structures and the most suitable one for our purposes seems to be the $q_\mu p'_\nu$. The reasons for the choice of this structure are (a) it has the larger number of momenta and (b) the OPE leading term decreases as $1/Q^2$ as $Q^2 \to \infty$, which is an expected behavior for QCD form factors. In general, for any given structure, the sum rule method is inapplicable at large Q^2, where the power corrections become large and uncontrollable. At small Q^2, the situation is even worse since when approaching the physical region the operator expansion stops working. In this sense, one has to consider that the sum rule is valid up to a rather small Q^2, and the extrapolation from the values of Q^2 to the physical region can be obtained with good accuracy.

Matching both sides of the sum rule, taking the approximation $p^2 \simeq p'^2 = -P^2$ and doing the Borel transform to $P^2 \to M_B^2$, we get the following expression in the $q_\mu p'_\nu$ structure,

$$\frac{\lambda_Y M_\omega f_\omega M_\psi f_\psi g_{Y\psi\omega}(Q^2)}{(M_Y^2 - M_\psi^2)(Q^2 + M_\omega^2)}(e^{-M_Y^2/M_B^2} - e^{-M_\psi^2/M_B^2}) + H(Q^2)e^{-s_0/M_B^2} = \Pi^{\mathrm{OPE}}(M_B^2, Q^2), \tag{27}$$

where $Q^2 = -q^2$, and $H(Q^2)$ function represents the contribution to the pole-continuum transitions [19,24–26]. The $\Pi^{\mathrm{OPE}}(M_B^2, Q^2)$ function is

$$\Pi^{\mathrm{OPE}}(M_B^2, Q^2) = \sin\theta \int_{4m_c^2}^{+\infty} ds e^{-s/M_B^2}\rho(s, Q^2), \tag{28}$$

and $\rho = \rho^{\mathrm{pert}} + \rho^{\langle\bar{q}q\rangle} + \rho^{\langle G^2\rangle} + \rho^{\langle\bar{q}Gq\rangle} + \rho^{\langle\bar{q}q\rangle\langle G^2\rangle}$ is given explicitly by

$$\rho^{\mathrm{pert}}(s, Q^2) = \rho^{\langle\bar{q}q\rangle}(s, Q^2) = 0, \tag{29}$$

$$\rho^{\langle G^2\rangle}(s, Q^2) = -\frac{\langle g_s^2G^2\rangle}{3^2\cdot 2^{10}\pi^4}\int_0^1 d\alpha\delta\left[s - \frac{m_c^2}{\alpha(1-\alpha)}\right] \times (3 - 3\alpha + \alpha^2), \tag{30}$$

$$\rho^{\langle\bar{q}Gq\rangle}(s, Q^2) = \frac{m_c\langle\bar{q}Gq\rangle}{72\pi^2Q^2}\int_0^1 d\alpha\delta\left[s - \frac{m_c^2}{\alpha(1-\alpha)}\right],$$

$$\rho^{\langle\bar{q}q\rangle\langle G^2\rangle}(s, Q^2) = \frac{m_c\langle\bar{q}q\rangle\langle g_s^2G^2\rangle}{3^3\cdot 2^5\pi^2Q^4}\int_0^1 d\alpha\delta\left[s - \frac{m_c^2}{\alpha(1-\alpha)}\right] \times \frac{(1 - 3\alpha(1-\alpha))}{\alpha(1-\alpha)}. \tag{31}$$

As observed in previous works [19–22], the charmonium part of the mixed current defined in Eq. (1) contributes to the three-point function uniquely with disconnected diagram. Hence, only the molecule part contributes to the decay channel $Y(3940) \to J/\psi\omega$. This fact is evident due to the presence of the sine function in Eq. (28).

We follow the usual procedure in order to extract the value of the coupling constant associated with the $Y \to J/\psi\omega$ process. First, we must determine the form factor of the $YJ/\psi\omega$ vertex, which can be done by isolating the function $g_{Y\psi\omega}(Q^2)$ in Eq. (27), then we divide Eq. (27) by its derivative with respect to $1/M_B^2$ in order to eliminate the unknown function $H(Q^2)$. Therefore, we are left with a function for the form factor $g_{Y\psi\omega}(Q^2)$ to be determined numerically.

In the numerical analysis we use the experimental values of the meson masses and decay constants (in GeV): $M_\psi = 3.10$, $f_\psi = 0.405$, $M_\omega = 0.782$, $f_\omega = 0.046$. For the Y mass, we use the experimental value in Ref. [1] and the meson-current parameter λ_Y, which has been evaluated in the previous section [see Eq. (19)].

In Fig. 4, we show a plot of the form factor $g_{Y\psi\omega}(Q^2)$ as a function of M_B^2 and Q^2. Note that a reliable sum rule must be independent of the choice of Borel mass parameter. As one can see, we obtain a good stability in the Borel mass parameter at $M_B^2 \geq 1.8\ \mathrm{GeV}^2$. Here we work at the interval $1.8\ \mathrm{GeV}^2 \leq M_B^2 \leq 4.0\ \mathrm{GeV}^2$. The form factor dependence in Q^2 can be evaluated by taking the average of the M_B^2 values inside this stability region. The results are shown in Fig. 5.

As mentioned above, the sum rule is not reliable at very large and very small values of Q^2. Here we find that the results are reliable for $1.2 \leq Q^2 \leq 2.4\ \mathrm{GeV}^2$.

Once we have determined the form factor behavior, we can now extract the coupling constant by using the momentum value at the ω meson pole, $Q^2 = -M_\omega^2$. For this purpose, we have to extrapolate the form factor to the region of Q^2 where the QCDSR is not valid. This extrapolation can be done by parametrizing the QCDSR results shown in Fig. 5 for $g_{Y\psi\omega}(Q^2)$ using a monopolar function,

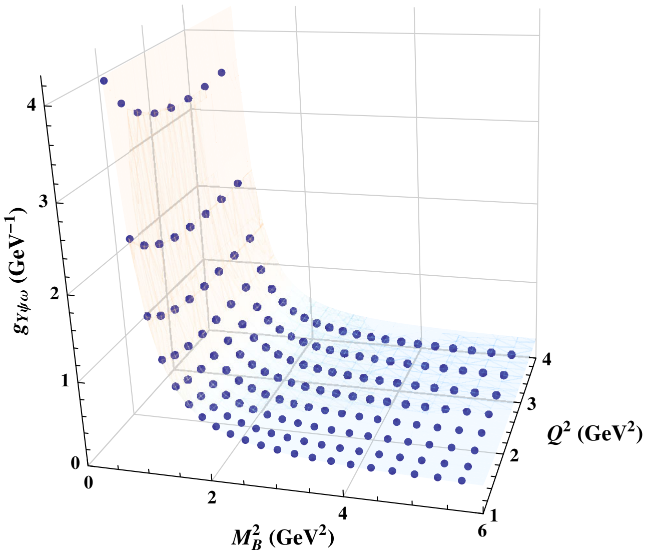

FIG. 4 (color online). The form factor $g_{Y\psi\omega}(Q^2)$ as a function of the momentum Q^2 and Borel mass parameter M_B^2.

$$g_{Y\psi\omega}(Q^2) = \frac{g_1}{g_2 + Q^2}, \tag{32}$$

and the results for the fitting parameters are

$$g_1 = (4.0 \pm 1.0)\ \text{GeV},$$
$$g_2 = (7.4 \pm 0.2)\ \text{GeV}^2. \tag{33}$$

Considering that the monopolar function could not be the optimal choice, we have also used an exponential function for fitting the data. It is noteworthy that both fits presented basically equivalent results. Therefore, we have used these two fits to estimate the error in the extrapolation.

The theoretical errors are evaluated considering errors on the following parameters: $\sqrt{s_0} = 4.40 \pm 0.10$ GeV, $\theta = 76.0^\circ \pm 5.0^\circ$, and also the error on the meson coupling parameter λ_Y, given by Eq. (19). We notice that the results do not depend much on the parameters $\sqrt{s_0}$ and θ, while the theoretical errors are mainly affected by the meson coupling λ_Y.

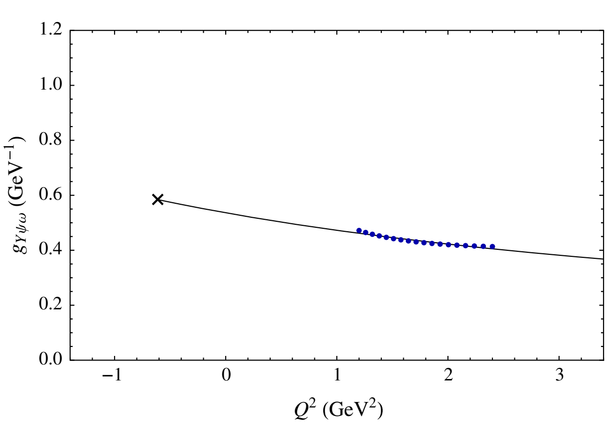

FIG. 5 (color online). QCDSR results for the form factor $g_{Y\psi\omega}(Q^2)$, for $\sqrt{s_0} = 4.40$ GeV (circles). The solid line gives the parametrization of the QCDSR results through Eq. (32). The cross is the value of the coupling constant.

In order to see how well the parametrization works, the solid line in Fig. 5 represents the Eq. (32) with values given by Eq. (33). The coupling constant, $g_{Y\psi\omega}$, is given by using the momentum value $Q^2 = -M_\omega^2$ in Eq. (32). Then, we get

$$g_{Y\psi\omega} = g_{Y\psi\omega}(-M_\omega^2) = (0.58 \pm 0.14)\ \text{GeV}^{-1}. \tag{34}$$

The decay width for this process $Y(3940) \to J/\psi\omega$ is given by

$$\Gamma_{Y(3940)\to J/\psi\omega} = \frac{g_{Y\psi\omega}^2}{3}\frac{p(M_Y, M_\omega, M_\psi)}{8\pi M_Y^2} \times \left(M_\psi^2 M_\omega^2 + \frac{1}{2}(M_Y^2 - M_\psi^2 - M_\omega^2)^2\right), \tag{35}$$

where

$$p(a,b,c) \equiv \frac{\sqrt{a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2}}{2a}. \tag{36}$$

Therefore, we obtain the decay width inserting the value obtained for the coupling constant (34) in (35):

$$\Gamma_{Y(3940)\to J/\psi\omega} = (1.7 \pm 0.6)\ \text{MeV}. \tag{37}$$

This result is consistent with the experimental width of the state and the lower limit for the process $Y \to J/\psi\omega$ [1,2,7,8]. It is also of the same order as other available theoretical evaluations [12,13].

V. THE $Y(3940) \to D\bar{D}$ DECAY WIDTH

Establishing the $Y(3940)$ as a mixed state, it seems that the main decay channel would be into D mesons, due to the charmonium part of the current. However, the approach used here does not allow us to evaluate such decay, since one can only use the QCDSR approach to study properties of the low-lying state. Therefore, a charmonium $J^{PC} = 0^{++}$ current can only be used to study the decay of $\chi_{c0}(1P)$. If one tries to use the charmonium current to study the $\chi_{c0}(2P)$ decay into $D\bar{D}$, one would get promptly a number different from zero, but this number is meaningless because the approach can not be used to study this resonant state. In addition we have verified that the molecular part of the current is not allowed to decay into the channel containing $D\bar{D}$ mesons.

VI. THE $Y(3940) \to \gamma\gamma$ DECAY WIDTH

As done in Sec. IV, we can estimate the decay width of the channel $Y(3940) \to \gamma\gamma$, through the three-point function (20). For this vertex $Y\gamma\gamma$, we must consider the following function,

$$\Pi_{\mu\nu}(x, y) = \langle 0|T\{j^{\gamma}_{\mu}(x) j^{\gamma}_{\nu}(y) j^{\dagger}(0)\}|0\rangle, \tag{38}$$

where the interpolating current for the photon is given by

$$j^{\gamma}_{\mu} = \frac{2}{3} e(\bar{u}_a \gamma_\mu u_a + \bar{c}_a \gamma_\mu c_a) - \frac{1}{3} e(\bar{d}_a \gamma_\mu d_a + \bar{s}_a \gamma_\mu s_a). \tag{39}$$

In the phenomenological side, we have the expression

$$\Pi^{PHEN}_{\mu\nu}(p, p', q) = -\frac{e^2 \lambda_Y g_{Y\gamma\gamma}(q^2)}{(p^2 - M_Y^2)} \times [q_\mu p'_\nu - (p' \cdot q) g_{\mu\nu}] + \cdots, \tag{40}$$

where the p' and q are the momenta related to the two-photon vertex. The form factor $g_{Y\gamma\gamma}(q^2)$ is defined by the transition matrix of the process $Y \to \gamma\gamma$ [27,28],

$$\mathcal{M}_{\mu\nu} = e^2 g_{Y\gamma\gamma}(q^2)[g_{\mu\nu}(p' \cdot q) - q_\mu p'_\nu]. \tag{41}$$

The matching of both sides of the sum rule is done in the same way as for the channel $J/\psi\omega$, and we get in the structure $q_\mu p'_\nu$ the following expression,

$$e^2 \lambda_Y g_{Y\gamma\gamma}(Q^2) e^{-M_Y^2/M_B^2} + F(Q^2) e^{-s_0/M_B^2} = \Pi^{\mathrm{OPE}}(M_B^2, Q^2), \tag{42}$$

and the $F(Q^2)$ function represents the contribution to the pole-continuum transitions. For this decay channel, the $\Pi^{\mathrm{OPE}}(M_B^2, Q^2)$ function is given by

$$\Pi^{\mathrm{OPE}}(M_B^2, Q^2) = \frac{8}{3} e^2 [\Pi^{\mathrm{OPE}}_{J/\psi\omega}(M_B^2, Q^2) + \Pi^{\mathrm{OPE}}_{\gamma\gamma}(M_B^2, Q^2)], \tag{43}$$

where $\Pi^{\mathrm{OPE}}_{J/\psi\omega}(M_B^2, Q^2)$ is the same function obtained in the $J/\psi\omega$ channel given in Eq. (28). The $\Pi^{\mathrm{OPE}}_{\gamma\gamma}(Q^2, M_B^2)$ function can be found in the Appendix.

The numerical analysis of the sum rule (42) provides the form factor $g_{Y\gamma\gamma}(Q^2)$. In Fig. 6, we show a plot of the form factor $g_{Y\psi\omega}(Q^2)$ as a function of M_B^2 and Q^2. As one can see, we obtain a good stability in Borel mass parameter and we consider a confidence region at $4.0\ \mathrm{GeV}^2 \le M_B^2 \le 7.0\ \mathrm{GeV}^2$. Using again the monopolar function given in Eq. (32), we can extrapolate the QCDSR results and estimate the coupling constant for the process $Y \to \gamma\gamma$. Therefore, in Fig. 7 we present such extrapolation from where we obtain, at $Q^2 = 0$:

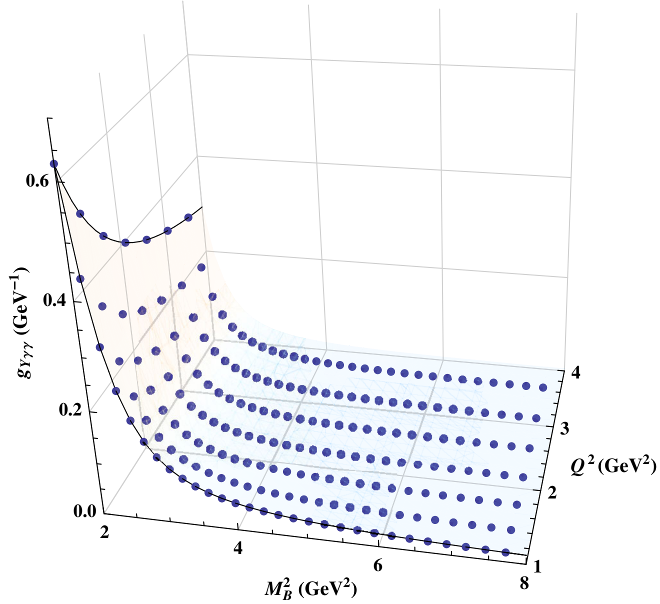

FIG. 6 (color online). The form factor $g_{Y\gamma\gamma}(Q^2)$ as a function of the momentum Q^2 and Borel mass parameter M_B^2.

$$g_{Y\gamma\gamma} = (0.025 \pm 0.010)\ \mathrm{GeV}^{-1}, \tag{44}$$

and the results for the fitting parameters are given by $g_1 = (0.08 \pm 0.05)$ GeV and $g_2 = (3.13 \pm 0.22)\ \mathrm{GeV}^2$.

The decay width into $\gamma\gamma$ can be evaluated by the expression [28]

$$\Gamma_{Y(3940)\to\gamma\gamma} = \frac{\pi}{4} \alpha_{em}^2 M_Y^3 g_{Y\gamma\gamma}^2, \tag{45}$$

where $\alpha_{em} \simeq 1/137$ is the fine structure constant. Replacing the value of the coupling constant given above we then obtain

$$\Gamma_{Y(3940)\to\gamma\gamma} = (1.6 \pm 1.3)\ \mathrm{KeV}. \tag{46}$$

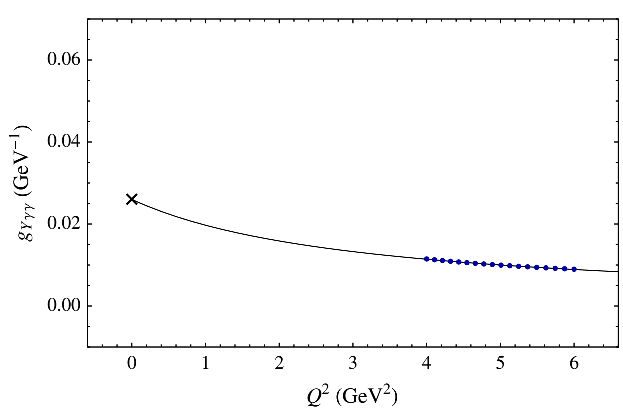

FIG. 7 (color online). QCDSR results for the form factor $g_{Y\gamma\gamma}(Q^2)$, for $\sqrt{s_0} = 4.40$ GeV (circles). The solid line gives the parametrization of the QCDSR results. The cross is the value of the coupling constant, at $Q^2 = 0$.

Based on this decay width value and the one obtained for the channel $J/\psi\omega$ in Eq. (37), the product of the two partial widths of the $Y(3940)$ is given by

$$\Gamma_{\gamma\gamma} \times \Gamma_{J/\psi\omega} \sim \mathcal{O}(10^3)\ \text{KeV}^2. \tag{47}$$

The result for this product is in reasonable agreement with one predicted by BELLE and *BABAR* Collaborations in Refs. [5,6].

VII. SUMMARY AND CONCLUSIONS

In summary, we have used the QCDSR approach to study the two-point and three-point functions of the $Y(3940)$ state, by considering it as a mixed charmonium-molecule state. We evaluated the mass working with the two-point function at leading order in α_s and considering the contributions from the condensates up to dimension seven in the OPE. We obtained a mass which is in reasonable agreement with the experimental value for the $Y(3940)$ state, and we found a mixing angle around $\theta = (76.0 \pm 5.0)^0$.

To evaluate the decay width of the channel $Y(3940) \to J/\psi\omega$, we worked with the three-point function also at leading order in α_s, and we considered the contributions from the condensates up to dimension seven. The obtained value of the width is $\Gamma_{J/\psi\omega} = (1.7 \pm 0.6)$ MeV, which is smaller than the total experimental width [1,2], but is consistent with the lower limit for this channel $\Gamma > 1$ MeV [12,13]. We also estimated the decay width of $Y(3940)$ into two photons as $\Gamma_{\gamma\gamma} = (1.6 \pm 1.3)$ KeV. These results allowed us to estimate the order of magnitude of the product of the two partial widths, $\Gamma_{\gamma\gamma} \times \Gamma_{J/\psi\omega} \sim \mathcal{O}(10^3)$ KeV^2, which is also in reasonable agreement with the experimental data.

Thus, according to the available experimental data, we can conclude that a mixing between the χ_{c0} charmonium and the $D^*\bar{D}^*$ molecule, with $J^{PC} = 0^{++}$ quantum numbers, could be a good candidate to explain the $Y(3940)$ state.

ACKNOWLEDGEMENTS

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APPENDIX A: SPECTRAL DENSITIES FOR THE TWO-POINT CORRELATION FUNCTION

Next, we list the spectral densities for the mixed scalar $(\chi_{c0}) - (D^*\bar{D}^*)$ state described by the current in Eq. (1). We consider the OPE contributions up to dimension-eight condensates and keep terms at leading order in α_s. In order to retain the heavy quark mass finite, we use the momentum-space expression for the heavy quark propagator. We calculate the light quark part of the correlation function in the coordinate-space and use the Schwinger parametrization to evaluate the heavy quark part of the correlator. For the d^4x integration in Eq. (5), we use again the Schwinger parametrization, after a Wick rotation. Finally, the result of these integrals are given in terms of logarithmic functions through which we extract the spectral densities. The same technique can be used for evaluating the condensate contributions.

For the χ_{c0} meson contribution, the spectral densities are written below [16],

$$\begin{aligned}
\rho^{\text{pert}}_{\chi_{c0}}(s) &= -\frac{3m_c^2}{8\pi^2} v\left(4 - \frac{1}{x}\right),\\
\rho^{\langle G^2\rangle}_{\chi_{c0}}(s) &= \frac{\langle g_s^2 G^2\rangle}{2^5\pi^2 M_B^2} v\left(2 + \frac{2}{x} - \frac{m_c^2/M_B^2}{x^2}\right)\\
\rho^{\langle G^3\rangle}_{\chi_{c0}}(s) &= -\frac{\langle g_s^3 G^3\rangle}{3\cdot 2^7\pi^2 M_B^4}\frac{v}{x}\left[49 + \frac{6}{x} + (x - m_c^2\tau)\right.\\
&\quad\left.\times\left(28 + \frac{49}{x} + \frac{3}{x^2}\right)\right].
\end{aligned} \tag{A1}$$

For the $D^*\bar{D}^*(0^{++})$ molecular state [14],

$$\begin{aligned}
\rho^{\text{pert}}_{D^*D^*}(s) &= \frac{m_c^8}{5\cdot 2^{12}\pi^6}\left[v\left(480 + \frac{1460}{x} - \frac{274}{x^2} - \frac{38}{x^3} + \frac{1}{x^4}\right)\right.\\
&\quad\left. + 120\mathcal{L}_v\left(8x - 1 - 6\text{Log}(x) - \frac{8}{x} + \frac{2}{x^2}\right) - 1440\mathcal{L}_+\right]\\
\rho^{\langle\bar{q}q\rangle}_{D^*D^*}(s) &= \frac{m_c^5\langle\bar{q}q\rangle}{64\pi^4}\left[v\left(6 - \frac{5}{x} - \frac{1}{x^2}\right) + 6\mathcal{L}_v\left(2x - 2 + \frac{1}{x}\right)\right]\\
\rho^{\langle G^2\rangle}_{D^*D^*}(s) &= \frac{m_c^4\langle g_s^2 G^2\rangle}{3\cdot 2^{10}\pi^6}\left[v\left(6 - \frac{5}{x} - \frac{1}{x^2}\right) + 6\mathcal{L}_v\left(2x - 2 + \frac{1}{x}\right)\right]\\
\rho^{\langle\bar{q}Gq\rangle}_{D^*D^*}(s) &= \frac{3m_c^3\langle\bar{q}Gq\rangle}{128\pi^4}\left(\frac{v}{x} - 2\mathcal{L}_v\right)\\
\rho^{\langle\bar{q}q\rangle^2}_{D^*D^*}(s) &= \frac{m_c^2\rho\langle\bar{q}q\rangle^2}{4\pi^2} v\\
\rho^{\langle G^3\rangle}_{D^*D^*}(s) &= \frac{m_c^2\langle g_s^3 G^3\rangle}{3\cdot 2^{12}\pi^6}\left[v\left(6 - \frac{25}{x} + \frac{1}{x^2}\right)\right.\\
&\quad\left. + 6\mathcal{L}_v\left(2x + 2 + \frac{1}{x}\right)\right]\\
\rho^{\langle 8\rangle}_{D^*D^*}(s) &= -\frac{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}{8\pi^2} v\left(\frac{m_c^4/M_B^4}{x}\right).
\end{aligned} \tag{A2}$$

Finally, for the mixed term, we have

$$\begin{aligned}
\rho^{\langle\bar{q}q\rangle}_{\text{mix}}(s) &= \frac{m_c^2\langle\bar{q}q\rangle}{4\pi^2} v\left(4 - \frac{1}{x}\right),\\
\rho^{\langle\bar{q}Gq\rangle}_{\text{mix}}(s) &= 0.
\end{aligned} \tag{A3}$$

In all these expressions we have used the following definitions:

$$x = m_c^2/s \tag{A4}$$

$$v = \sqrt{1-4x} \tag{A5}$$

$$\mathcal{L}_v = \mathrm{Log}\left(\frac{1+v}{1-v}\right) \tag{A6}$$

$$\mathcal{L}_+ = \mathrm{Li}_2\left(\frac{1+v}{2}\right) - \mathrm{Li}_2\left(\frac{1-v}{2}\right). \tag{A7}$$

APPENDIX B: THE THREE-POINT CORRELATION FUNCTION FOR THE $Y(3940) \to \gamma\gamma$ VERTEX

According to Eq. (43), the three-point correlation function for the channel into two photons can be written in terms of the function related to the $J/\psi\omega$ channel. Thus, the second term of the Eq. (43) is given by

$$\Pi_{\gamma\gamma}^{\mathrm{OPE}}(M_B^2, Q^2) = \sin\theta \sum_d \Pi_{\gamma\gamma}^{\hat{\mathcal{O}}_d}(M_B^2, Q^2), \tag{B1}$$

where $\hat{\mathcal{O}}_d$ are the local field operators with d dimension in the OPE. Considering the OPE contributions up to dimension-seven condensates and keeping terms at leading order in α_s, we obtain the following expressions for the $\Pi_{\gamma\gamma}^{\mathrm{OPE}}(M_B^2, Q^2)$ function,

$$\begin{aligned}
\Pi_{\gamma\gamma}^{\mathrm{pert}}{}_{(M_B^2,Q^2)} &= \Pi_{\gamma\gamma}^{\langle\bar{q}q\rangle}{}_{(M_B^2,Q^2)} = 0,\\
\Pi_{\gamma\gamma}^{\langle G^2\rangle}{}_{(M_B^2,Q^2)} &= -\frac{\langle g_s^2 G^2\rangle}{3^2\cdot 2^{10}\pi^4}\left[1-\frac{2m_c^2/Q^2}{v_Q}\mathrm{Log}\left(\frac{v_Q+1}{v_Q-1}\right)\right]\\
\Pi_{\gamma\gamma}^{\langle\bar{q}Gq\rangle}{}_{(M_B^2,Q^2)} &= \frac{m_c\langle\bar{q}Gq\rangle}{72\pi^2 Q^2}\left[1-\frac{2m_c^2/Q^2}{v_Q}\mathrm{Log}\left(\frac{v_Q+1}{v_Q-1}\right)\right]\\
\Pi_{\gamma\gamma}^{\langle\bar{q}q\rangle\langle G^2\rangle}{}_{(M_B^2,Q^2)} &= -\frac{m_c\langle\bar{q}q\rangle\langle g_s^2 G^2\rangle}{3^3\cdot 2^5\pi^2 M_B^2 Q^2}\left[3-2\left(v_Q-\frac{m_c^2/Q^2}{v_Q}\right)\right.\\
&\left.\quad\times\mathrm{Log}\left(\frac{v_Q+1}{v_Q-1}\right)\right],
\end{aligned} \tag{B2}$$

where $v_Q = \sqrt{1+4m_c^2/Q^2}$.

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