Comment on "Hawking Radiation, Unruh Radiation, and the Equivalence Principle"

In a recent Letter "Hawking Radiation, Unruh Radiation, and the Equivalence Principle" [1] the authors study massless scalar field in the two-dimensional analog of Schwarzschild spacetime. In particular, they study the response rates of the Unruh-DeWitt detector in the Boulware, Unruh, and Hartle-Hawking vacua in this spacetime and compare them with those of the accelerated Unruh-DeWitt detector in two-dimensional Minkowski spacetime with the same acceleration. (In fact the calculations in this Letter can be found in Sections 4.4 and 8.3 of Ref. [2], where many of the conclusions of this Letter, e.g., the vanishing of the response in the Boulware vacuum, are given though it contains some errors.)

The authors work in two dimensions "to simplify the calculations," but some of the quantities they calculate, e.g., the response rate for the Unruh vacuum, depend crucially on the dimensions as we see below. The authors only compare the temperatures and omit constants of proportionality (see Eqs. (8) and (13) of Ref. [1]). By restoring the constants of proportionality one finds the following results. The response function (2) in Ref. [1] with $E_0 = 0$ for an accelerated detector with acceleration a given by Eq. (8) should read [3]

$$\mathcal{F}_{\rm RM}(E) = \frac{1}{E(e^{E/k_B T_{\rm RM}} - 1)}, \tag{1}$$

where $k_B T_{\rm RM} = a/2\pi$. On the other hand the response function in the Unruh vacuum [Eq. (13)] should read

$$\mathcal{F}_{\rm SU}(E) = \frac{1}{2E(e^{E/k_B T_{\rm SU}} - 1)}, \tag{2}$$

where $k_B T_{\rm SU} = [8\pi M\sqrt{1-2M/R}]^{-1}$. (The corresponding equation in Ref. [2] is wrong by a factor of 2. The integral necessary to derive Eq. (2) can be found in Ref. [3].) Here M is the black hole mass and R is the Schwarzschild r coordinate of the detector. Finally, as the authors note, the response function for the Hartle-Hawking vacuum is twice that for the Unruh vacuum. (The authors find the factor of 2 "unimportant," but it is crucial as we see below.)

The authors observe from these equations (without the constants of proportionality) that the detector static in two-dimensional Schwarzschild spacetime with acceleration $a = M/R^2\sqrt{1-2M/R}$ responds differently from that in two-dimensional Minkowski spacetime with the same acceleration. This observation, violation of the equivalence principle, is certainly not surprising as the authors themselves note: a static detector far away from the black hole, with negligible acceleration, responds to the Hawking radiation of temperature $1/8\pi M k_B$.

Now, the authors also claim that the equivalence principle as the detector approaches the horizon—we call this the *horizon equivalence principle* below—holds in the Unruh vacuum simply by observing that the temperature for the Planckian spectrum approaches that for the accelerated detector in two-dimensional Minkowski spacetime with the same acceleration. This is wrong because the detector in the Unruh vacuum will detect only particles coming from the horizon. Thus, its response rate tends to half of that for the detector with the same acceleration in two-dimensional Minkowski spacetime as it approaches the horizon. On the other hand, the horizon equivalence principle holds for the Hartle-Hawking vacuum thanks to the factor of 2.

The situation is different in the more realistic four-dimensional case. The limit of the response rate as the detector approaches the horizon has been calculated by Candelas. He finds that in either the Unruh or Hartle-Hawking vacuum the horizon equivalence principle holds (see Table IV of Ref. [4]).

The authors of Ref. [1] also make a misleading statement: "The character of the vacuum is determined by the form of the metric...". The form of the metric has nothing to do with the vacuum state chosen. For example, the positive-frequency modes corresponding to the usual Minkowski vacuum can be constructed in Rindler coordinates (see Eq. (2.18) of Ref. [5]).

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