

Nonlocal effects in the nucleus-nucleus fusion cross section

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Effects of the nonlocality of factorizable potentials are taken into account in the calculation of nucleus-nucleus fusion cross section through an effective mass approach. This cross section makes use of the tunneling factor calculated for the nonlocal barrier, without the explicit introduction of any result coming from coupled channel calculation, besides the approximations of Hill-Wheeler and Wong. Its new expression embodies the nonlocal effects in a factor which redefines the local potential barrier curvature. Applications to different systems, namely, $^{16}\text{O} + ^{59}\text{Co}$, $^{16,18}\text{O} + ^{58,60,64}\text{Ni}$, and $^{16,18}\text{O} + ^{63,65}\text{Cu}$ are presented, where the nonlocal range is treated as a free parameter.

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I. INTRODUCTION

It has been pointed out in the literature that the nucleus-nucleus fusion cross section presents an anomalous behavior at bombarding energies near and below the Coulomb barrier [1,2]. The measured fusion cross sections are much larger than the values predicted by the usual models which make use of barrier penetration calculations. In these models, in order to obtain the nucleus-nucleus fusion cross section, one has to calculate the barrier penetration factor for a nuclear plus Coulomb potential, assuming that the two nuclei will fuse when they have penetrated to some extent such a potential. Obviously there remains the problem of defining this potential for the colliding heavy ions, and it is frequently assumed that the real attractive potential is to be taken as the one which describes the fusion cross section at higher energies or the real part of an optical potential which is obtained from the fitting of the elastic scattering data at higher energies [3]. The enhancement of the measured fusion cross section is then to be ascribed to some other physical effects, and it has been pointed out the importance of the energy dependence of the nucleus-nucleus real potential in such a way to become more attractive at energies near the Coulomb barrier — this will lower that barrier — or by invoking the couplings to other channels, both inelastic and transfer [4]. In this last case, one can see that a net lowering can occur which will also enhance the barrier penetration when the incident energy is near the uncoupled barrier top. In both cases the enhancement effect is negligible when the energy of the colliding nuclei increases above the barrier.

In this paper we intend to draw attention to the nonlocal character of the attractive part of the nucleus-nucleus total potential, which is a fundamental characteristic not considered in the phenomenological simple barrier penetration models and show that it can produce an enhancement in the fusion cross section also. In this connection we must emphasize that this contribution to the fusion cross section will be always present since for any nuclear system the nonlocality, arriving from many-body quantum effects [5], just reflects a fundamental feature of the

nucleus-nucleus potential; therefore, in general, the total fusion cross section is to be considered as the result of a sum of the contributions coming from the tunneling of a nonlocal potential plus other contributions, for instance, from coupled channel calculations.

We start from the assumption that the nucleus-nucleus potential has to be taken as the sum of a real attractive nonlocal part plus the local Coulomb potential; here the nonlocality is not deduced from basic principles but is treated instead in a phenomenological way in a similar fashion as that presented several years ago in the description of neutron-nucleus elastic scattering [6]. This treatment leads naturally to the introduction of an effective mass, in the same sense as that of Frahn and Lemmer [7], which is now position dependent and presents an explicit dependence on the range of the nonlocality also. In a previous paper [8] the authors have already shown in a schematic nonlocal potential, using path integral techniques, that the enhancement in the barrier penetration in the WKB approximation is controlled by the presence of the effective mass in the tunneling factor. In the present case we derive the expression for the effective mass which depends now on the explicit form of the real potential and show how this contribution modifies the nucleus-nucleus fusion cross section also. To this end we have assumed the well known Christensen-Winther real potential [9] to be suited for our purposes, and we adapted it so as to become nonlocal and, furthermore, only as a plausible assumption, we have considered that the main change in the effective mass will occur when the nuclei are at the radius of the local barrier. It is then direct to obtain an extended version of Wong's formula [10] for the fusion cross section which embodies now the nonlocal effects of the real attractive potential. Since this expression is not obtained from basic principles, we end up with an expression involving a free parameter, namely, the parameter b which controls the range of the nonlocality. The fitting of this parameter was performed in several cases, and the results obtained indicate that for some heavy-ion fusion experimental data they are greater than the values of the nonlocality b which were also obtained in the nucleon-nucleus elastic scattering [6,11]. We also discuss the cases for which a departure of the expected

value of b is obtained.

This paper is organized as follows. In Sec. II we discuss how to include nonlocal effects in the nucleus-nucleus fusion cross section, and in Sec. III we present the results for $^{16}\text{O} + ^{59}\text{Co}$, $^{16,18}\text{O} + ^{58,60,64}\text{Ni}$, and $^{16,18}\text{O} + ^{63,65}\text{Cu}$ systems, and the conclusions.

II. NONLOCAL EFFECTS IN THE NUCLEUS-NUCLEUS FUSION CROSS SECTION

As has been pointed out previously [8] the nonlocal effects, originated from the microscopic nuclear mean field, and initially not considered in the nucleus-nucleus real parametrized potentials used to describe fusion cross sections, can be introduced in a phenomenological fashion just by rewriting those attractive potentials as

$$V_{NL}(\vec{x}, \vec{x}') = \frac{V_0 \left(\frac{\vec{x} + \vec{x}'}{2} \right)}{\sqrt{2\pi b}} \exp \left[-\frac{(\vec{x} - \vec{x}')^2}{b^2} \right], \quad (2.1)$$

where we have assumed a Gaussian nonlocal dependence; the final results do not depend crucially on this particular choice — it could be assumed as any general normalized bell-shaped function — and will now exhibit that feature which is inherent to many-body systems. In this way we construct a total nucleus-nucleus potential as the sum of this nonlocal attractive contribution, a local term which describes the range repulsive part of the interaction and the kinetic energy term, namely,

$$H(\vec{x}, \vec{x}') = -\frac{\hbar^2}{2\mu} \delta''(\vec{x} - \vec{x}') + V_{NL}(\vec{x}, \vec{x}') + V_L(\vec{x}, \vec{x}') \delta(\vec{x} - \vec{x}'). \quad (2.2)$$

Using previous results [12] we see that a Hamiltonian in phase space can be written such that it is expressed as

$$H(\vec{p}, \vec{q}) = \sum_{n=0}^{\infty} \left(-\frac{i}{\hbar} \right)^n \vec{p}^n \mathcal{H}^{(n)}(\vec{q}), \quad (2.3)$$

where $\mathcal{H}^{(n)}(\vec{q})$ is the n th-moment of $H(\vec{x}, \vec{x}')$. These moments, bearing the physical contents necessary to describe the system, are then extracted from the Hamiltonian which has the nonlocal term. In general one gets

$$H(\vec{p}, \vec{q}) = \frac{\vec{p}^2}{2\mu} + \sum_{n=0}^{\infty} \frac{(-1)^n}{\hbar^n} \vec{p}^n V^{(n)}(\vec{q}) + V_L(\vec{q}), \quad (2.4)$$

where $V^{(n)}(\vec{q})$ are now the moments of the nucleus-nucleus nonlocal potential. The first two even values of n give the dominant semiclassical contributions. For $n = 0$ we will have a \vec{p} -independent term, which we interpret as the usual real local potential, and for $n = 2$ we have a term proportional to \vec{p}^2 , which will give rise to a quantum correction to the mass of the colliding system. Odd terms in \vec{p} will not occur due to our choice of the potential, moreover they correspond to dissipative terms and, consequently, they cannot be present in a Hamiltonian associated to a conservative system. Thus, keeping terms only to second order, we arrive at a Hamiltonian for the

nucleus-nucleus system which is suited for the description of low-energy processes

$$H(\vec{p}, \vec{q}) \cong \frac{\vec{p}^2}{2\mu(\vec{q}; b)} + V(\vec{q}). \quad (2.5)$$

The new effective mass is explicitly written as

$$\mu(\vec{q}; b) = \frac{\mu}{1 - \frac{2\mu}{\hbar^2} V^{(2)}(\vec{q}; b)}, \quad (2.6)$$

and the potential to this order is

$$V(\vec{q}) = V^{(0)}(\vec{q}) + V_L(\vec{q}), \quad (2.7)$$

which will be considered as being the total real local standard potential used to describe the nucleus-nucleus interaction.

For a nonlocal potential described by Eq. (2.1), one can easily show that the effective reduced mass then reads [7,8]

$$\mu(\vec{q}; b) = \frac{\mu}{1 + \frac{\mu b^2}{2\hbar^2} |V^{(0)}(\vec{q})|}. \quad (2.8)$$

Therefore, it is clear that the nonlocal character of the attractive part of the nucleus-nucleus potential gives rise to a partial reduction on the reduced mass thus leading to an enhancement in the transmission coefficient. In this description, the reduced mass is also dependent on the nonlocal range, b , which will be treated as a fitting parameter.

In order to calculate the nucleus-nucleus fusion cross section in this context, we would have to write the starting nonlocal potential. However, we refrain from studying a microscopic derivation of this potential and, instead, we will only try to introduce its main expected properties, namely, its short range character and the dependence of its form on the density profile of the colliding nuclei. To explore these features, the starting Hamiltonian, Eq. (2.5), is now rewritten in spherical coordinates such that a centrifugal potential also contributes to Eq. (2.7), namely,

$$V_i(r) = V^{(0)}(r) + V_{\text{Coul}}(r) + V_{\text{cent}}(r), \quad (2.9)$$

where

$$V_{\text{cent}}(r) = \frac{\hbar^2 l(l+1)}{2\mu(r; b)r^2}. \quad (2.10)$$

Now let us discuss briefly the effective reduced mass correction to the Hamiltonian. As is well known from calculations with momentum-dependent microscopic interactions [12,13], the effective mass depends on the density function, and here we also assume that our effective reduced mass depends on the density distribution profile of the colliding nuclei in a simple form. As a plausible assumption, we will consider that the effective reduced mass change to its new value only at the radius of the barrier, R_B , defined by Eq. (2.9) and, also that it will remain constant for $0 \leq r \leq R_B$. With these assumptions we see that our effective reduced mass then reads

$$\mu = \begin{cases} \mu, & r > R_B \\ \frac{\mu}{1 + \frac{\mu b^2}{2\hbar^2} |V^{(0)}(R_B)|}, & 0 \leq r \leq R_B \end{cases} \quad (2.11)$$

In the low-energy domain we expect the term $\frac{\mu b^2}{2\hbar^2} |V^{(0)}(R_B)|$ to produce only a small correction to the reduced mass, then we can expand

$$\mu(r; b) \simeq \mu \left[1 - \frac{\mu b^2}{2\hbar^2} |V^{(0)}(R_B)| \right], \quad 0 \leq r \leq R_B, \quad (2.12)$$

and also we can assume the centrifugal potential to be given by its dominant part for the low angular momenta l — which are the relevant contributions for the fusion below the barrier

$$\begin{aligned} \frac{\hbar^2 l(l+1)}{2\mu(r; b)r^2} &= \frac{\hbar^2 l(l+1)}{2\mu r^2} \left[1 + \frac{b^2 \mu}{2\hbar^2} |V^{(0)}(R_B)| \right] \\ &\simeq \frac{\hbar^2 l(l+1)}{2\mu r^2}. \end{aligned} \quad (2.13)$$

On the other hand, this standard expression for the centrifugal potential will be kept hereafter in the Hamiltonian because, in this form, it allows us to extend our description to higher energies also since then it does not introduce spurious repulsion effects in the centrifugal potential for higher l 's, which would come from the low-energy effective mass initial approximation.

Now considering our results [8] and taking the transmission coefficient to be written in the nonlocal approach by the WKB form [14]

$$T_l(E; b) = \frac{1}{1 + \exp \left\{ 2 \int_{r_1}^{r_2} K_l(r; b) dr \right\}}, \quad (2.14)$$

and keeping only the dominant terms in the potential we then write

$$K_l(E; b) = \sqrt{\frac{2}{\hbar^2} \mu(r; b) [V_l(r) - E]}. \quad (2.15)$$

In Eq. (2.14) r_1 and r_2 denote the turning points of the barrier present in $V_l(r)$ defined by Eq. (2.9) with the approximations of Eq. (2.13). Again we can use the approximations to the effective reduced mass, Eq. (2.12), to write

$$K_l(E; b) \simeq \sqrt{\frac{2\mu}{\hbar^2} [V_l(r) - E] \left[1 - \frac{b^2}{2} f(R_B) \right]}. \quad (2.16)$$

It is important to observe that for $b \rightarrow 0$, i.e., the local limit, we get again the standard transmission coefficient.

Now, by making the quadratic approximation to the total potential [15]

$$V_l(r) = V_l - \frac{\mu \omega_l^2}{2} (r - R_l)^2, \quad (2.17)$$

it is straightforward to obtain two contributions to the new transmission coefficient. The first term gives the standard contribution of the Hill-Wheeler approximation

$$2 \int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2} [V_l(r) - E]} dr = \frac{2\pi}{\hbar \omega_l} (V_l - E). \quad (2.18)$$

Due to our assumption to the effective reduced mass and the parabolic approximation to the barrier, the second term is then written as

$$\begin{aligned} \frac{b^2}{2} f(R_B) \int_{r_1}^{R_B} \sqrt{\frac{2\mu}{\hbar^2} [V_l(r) - E]} dr \\ \simeq \frac{1}{2} \frac{b^2}{2} f(R_B) \int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2} [V_l(r) - E]} dr, \end{aligned} \quad (2.19)$$

so that the new transmission coefficient in this approximation reads

$$T_l(E; b) = \frac{1}{1 + \exp \left\{ \frac{2\pi(V_l - E)}{\hbar \omega_l} \left[1 - \frac{b^2}{4} f(R_B) \right] \right\}}. \quad (2.20)$$

With this expression we can directly calculate the nuclear fusion cross section embodying the nonlocal effects:

$$\sigma_f(E; b) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \frac{2l+1}{1 + \exp \left\{ \frac{2\pi(V_l - E)}{\hbar \omega_l} \left[1 - \frac{b^2}{4} f(R_B) \right] \right\}}, \quad (2.21)$$

where k is the asymptotic wave number. Now, following Wong [10], we make the further approximations

$$V_l \simeq V_0 + \frac{\hbar^2 l(l+1)}{2\mu R_0^2}, \quad (2.22)$$

$$\hbar \omega_l \simeq \hbar \omega_0, \quad (2.23)$$

and replacing the sum in Eq. (2.21) by an integral we finally obtain

$$\begin{aligned} \sigma_f(E; b) &= \left[\frac{R_0^2 \hbar \omega_0}{2E \left[1 - \frac{b^2}{4} f(R_B) \right]} \right] \\ &\times \ln \left\{ 1 + \exp \left[\frac{2\pi(E - V_0)}{\hbar \omega_0} \right] \right. \\ &\times \left. \left(1 - \frac{b^2}{4} f(R_B) \right) \right\}. \end{aligned} \quad (2.24)$$

Again we note here that in the local limit, i.e., $b \rightarrow 0$, this expression for the fusion cross section gives back the usual Wong's formula. On the other hand, it is interesting to observe that the nonlocal effects of the potential manifest themselves in the form of the expression $1 - \frac{b^2}{4} f(R_B)$, which can be interpreted as a redefinition factor of the fitted curvature $\hbar \omega_0$. It is then evident that for sub-Coulomb collisions, $E < V_0$, we have $\sigma_f(E; b) > \sigma_f(E)$, since

$$\frac{\hbar \omega_0}{1 - \frac{b^2}{4} f(R_B)} > \hbar \omega_0, \quad \text{for } b > 0. \quad (2.25)$$

Furthermore, for $E > V_0$, $\sigma_f(E; b) \simeq \sigma_f(E)$ due to the

competition between the two terms in Eq. (2.24).

Having obtained the new expression for the fusion cross section we can now study its behavior for low and high energies, respectively. First, one observes that for values of E such that $E \ll V_0$, Eq. (2.24) gives

$$\sigma_f(E; b) \simeq \left[\frac{R_0^2 \hbar \omega_0}{2E \left[1 - \frac{b^2}{4} f(R_B) \right]} \right] \times \exp \left[\frac{2\pi(E - V_0)}{\hbar \omega_0} \left(1 - \frac{b^2}{4} f(R_B) \right) \right], \quad (2.26)$$

and the fusion cross section decreases exponentially. In the local limit, $b \rightarrow 0$, we get

$$\sigma_f(E; b = 0) \rightarrow \frac{R_0^2 \hbar \omega_0}{2E} \exp \left[\frac{2\pi(E - V_0)}{\hbar \omega_0} \right], \quad (2.27)$$

which is the corresponding Wong's expression for $E \ll V_0$ [10].

On the other hand, when the incident energy is well above the top of the barrier one can see that, after a straightforward calculation, Eq. (2.24) reduces to the simple form

$$\sigma_f(E; b) \simeq \pi R_0^2 \left(1 - \frac{V_0}{E} \right), \quad (2.28)$$

which is the well-known geometrical classical fusion cross section [16]. Moreover, it does not depend on b , thus indicating that for $E \gg V_0$ the nonlocal effects are not important since in this case the transmission coefficient is already approximately one regardless of the additional nonlocal effects. Therefore, it is clear from our model that these nonlocal effects are important for energies below or around the top of the barrier, and it is based on these results that we can argue about the importance of the nonlocality as, at least in part, a possible answer to the well-known problem of the discrepancies in the sub-barrier fusion cross section.

III. RESULTS AND CONCLUSIONS

In order to calculate the fusion cross section we must first of all estimate the value of $f(R_B)$. In this connection we have assumed the well-established Christensen and Winther potential [9] to describe the tail of the nucleus-nucleus potential

$$|V^{(0)}(R_0)| = 50 \left[\frac{R_1 R_2}{R_1 + R_2} \right] \exp \left[\frac{R_1 + R_2 - R_0}{0.63} \right], \quad (3.1)$$

stressing, however, that this particular choice is not essential to the final results. In fact any potential with a similar behavior gives similar results, at least in respect to the region of the potential which governs the fusion process. Furthermore, we calculate $f(r)$ at the point R_0 since we are working with the same approximations as those of Wong [10].

The parameter controlling the nonlocality range, b , is

to be adjusted so as to produce the best fit to the experimental data. But, in order to look for this best choice for the nonlocality range, so as to describe only the desired nonlocal contributions of the nucleus-nucleus potential, Eq. (2.1), we must look for experimental data for which a calculation taking into account all possible known channels were performed. This choice is necessary in order to extract a value for b which does not contain information of the eventual coupled channels effects relevant for the reaction, otherwise that value of b will represent an effective global value taking into account all the effects we want to separate. In this connection we have considered the experimental data for $^{16}\text{O} + ^{59}\text{Co}$ as presented in Ref. [17], where the stripping channels $-\alpha$, $-1p$, $-2p$, $-d$ were considered the most important for the transfer channels calculation. Furthermore, running the CCFUSB code, the authors selected the channels $-\alpha$ and $-p$ as those that have dominant influence in the fusion cross section. It was also verified that the dynamical deformation of the nuclei or the coupling of inelastic channels are similar and give a very small contribution to the fusion cross section. This was obtained with those two transfer channels using a correction factor of $1.0 \leq f \leq 2.0$ for the transfer cross section; we adopted here the $f = 2.0$ case which provides then the maximum expected fusion cross section.

If we were to fit the experimental data as if all the enhancement of the cross section (with respect to the theoretical predictions) was due to the nonlocal effects only, we would find an effective value, $b_{\text{eff}} = 1.65$ fm, Fig. 1. Now, by performing a best fit for the coupled channels fusion cross section, as obtained in Ref. [17], using our expression, we obtain a nonlocal range of $b_{\text{cc}} = 1.45$ fm, Fig. 1. The use of this value in our expression allows us to

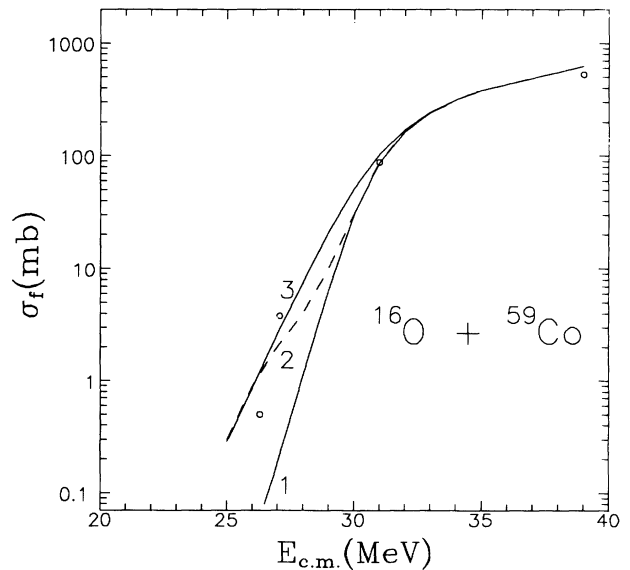


FIG. 1. Plot of excitation function of $^{16}\text{O} + ^{59}\text{Co}$ system. Open circles are experimental data; curve 1 is the local standard quantum tunneling predictions, curve 2, dashed line, is the result of channel-couplings calculation, and curve 3 is the nonlocal predictions. The values of parameter b and channels are presented in the text.

simulate the enhancement of the fusion cross section due to all relevant channels considered. We can now assign the still remaining difference of the experimental fusion cross section (to the coupled channels calculation) to the nonlocal effects.

Since we must look for the best value of b , associated to the nonlocal effects, which accounts for that difference, we must start from the fitted curve for the coupled channels fusion cross section and find the value of b which fits the experimental data; in the present case we obtained

$b_f = 0.94$ fm, Fig. 1, which is the result to be associated to our pure nonlocal effects. In this way, we have been able to separate, at least within the precision of the calculations presented in Ref. [17], the contribution of the nonlocal effects which will be present in every fusion reaction; in order to test this value we have also compared the results obtained by our approach to the experimental data for the fusion cross section of the systems $^{16}\text{O} + ^{63,65}\text{Cu}$ [18], $^{18}\text{O} + ^{63,65}\text{Cu}$ [19], $^{16}\text{O} + ^{60}\text{Ni}$, $^{18}\text{O} + ^{58}\text{Ni}$ [20], $^{16}\text{O} + ^{58,64}\text{Ni}$, and $^{18}\text{O} + ^{60,64}\text{Ni}$ [21]. The data were prepared using the dimensionless reduced quantities defined as

$$\sigma_{\text{red}} = \frac{2E}{R_0^2 \hbar \omega_0} \sigma_f(E; b) \quad (3.2)$$

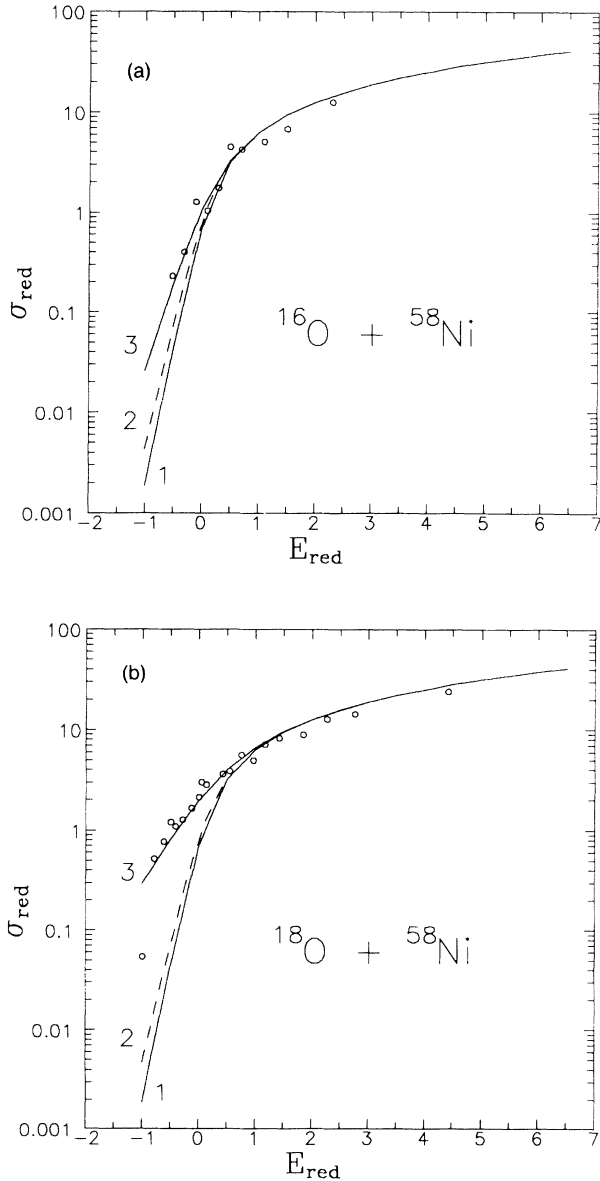


FIG. 2. (a), (b) Plot of dimensionless reduced excitation functions for $^{16}\text{O} + ^{58}\text{Ni}$ and $^{18}\text{O} + ^{58}\text{Ni}$ systems. Open circles are experimental data, curve 1 is the local standard quantum tunneling predictions, curve 2, dashed line, represents the nonlocal predictions for $b_f = 0.94$ fm, and curve 3 represents the nonlocal predictions for b_{eff} . The nonlocal range b_{eff} is given in Table I.

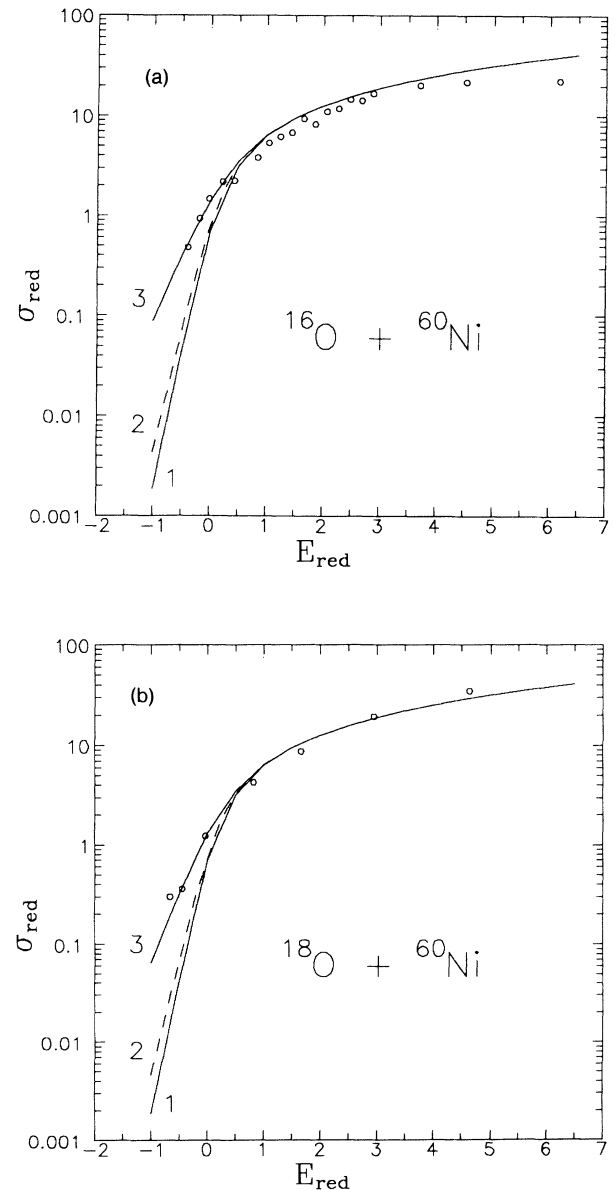


FIG. 3. (a), (b) The same as in Figs. 2(a) and 2(b) for $^{16}\text{O} + ^{60}\text{Ni}$ and $^{18}\text{O} + ^{60}\text{Ni}$ systems.

and

$$E_{\text{red}} = \frac{E - V_0}{\hbar\omega_0}. \quad (3.3)$$

Figures 2–6 show the fitted fusion cross sections; the values of b_{eff} which simulate the presence of coupled channels *plus* the nonlocal effects are presented in Table I.

It is interesting to observe that in all cross sections the calculation with the value $b_f = 0.94$ fm already predicts an enhancement of the fusion process at the barrier energy. We also expect the model to overestimate the fusion cross section for energies well below the barrier, since we assume a parabolic approximation which clearly underestimates the barrier width for lower energies. However, to the present values of the energy we can assume that

approximation to be a reasonable one. As we have previously said, for energies above the barrier our description coincides with that of Wong. In this way, the original potential also gives the curve described by the Wong's approach while the contributions coming from the nonlocal character of the potential, in our description, only correct that curve in the region of energy where the small quantum effects are important. It comes out that the value of b does in fact affect the fusion cross sections for energies below the barrier and are irrelevant for energies above.

The values of the nonlocality parameter b_{eff} (which simulates all the effects present in the fusion process) clearly indicate two kinds of processes. In one set we find the fusion cross sections for $^{16}\text{O} + ^{58,60,64}\text{Ni}$, $^{16}\text{O} + ^{63}\text{Cu}$,

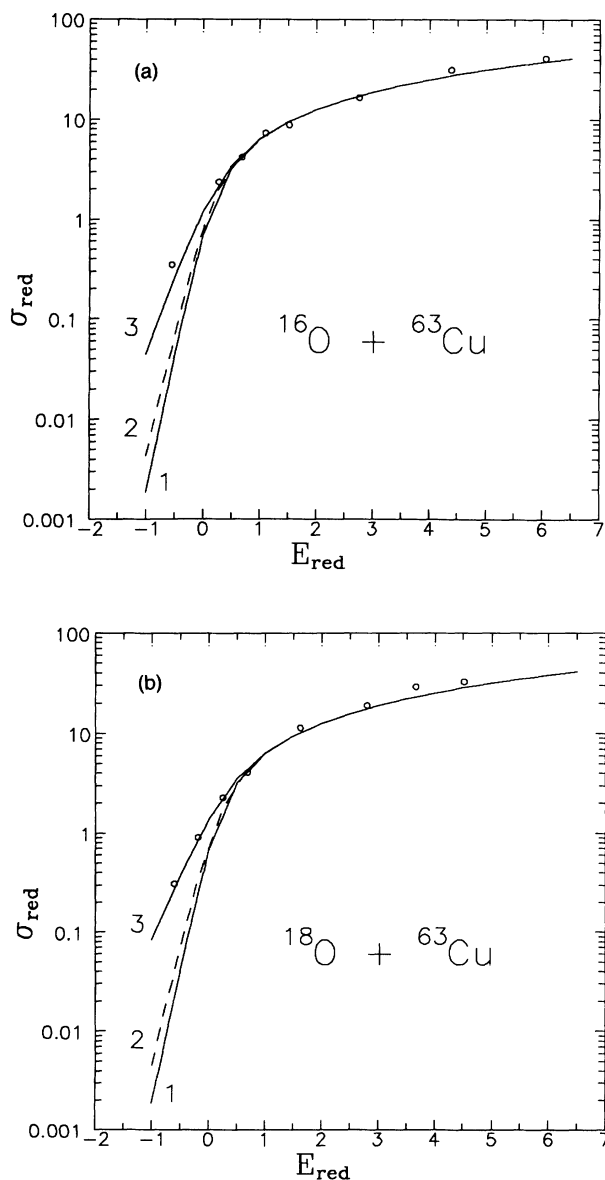


FIG. 4. (a), (b) The same as in Figs. 2(a) and 2(b) for $^{16}\text{O} + ^{63}\text{Cu}$ and $^{18}\text{O} + ^{63}\text{Cu}$ systems.

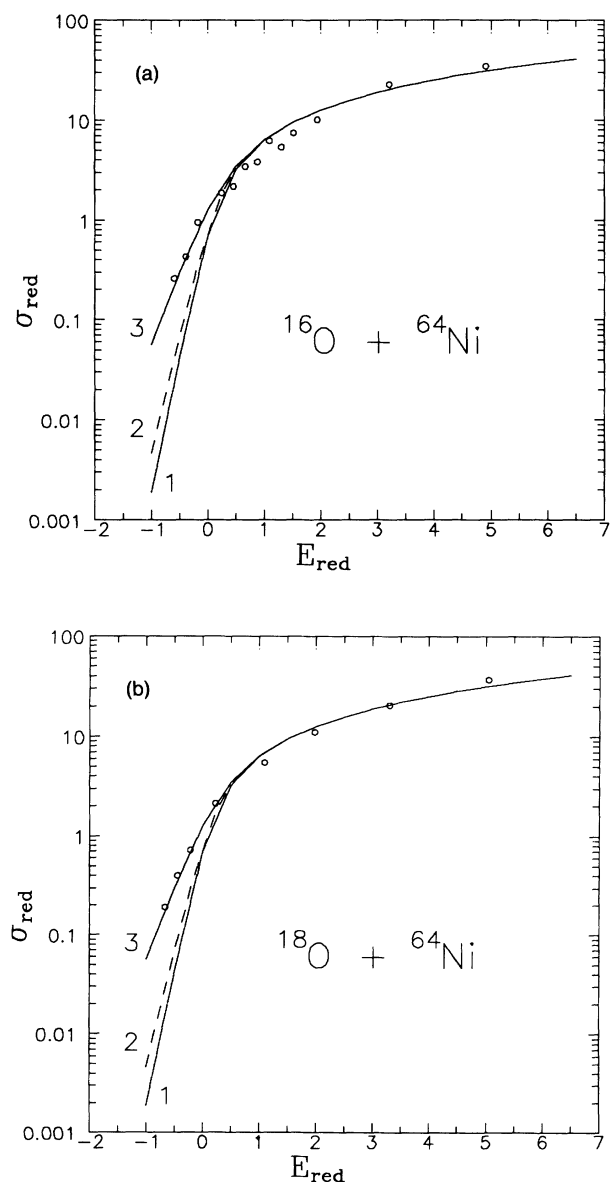


FIG. 5. (a), (b) The same as in Figs. 2(a) and 2(b) for $^{16}\text{O} + ^{64}\text{Ni}$ and $^{18}\text{O} + ^{64}\text{Ni}$ systems.

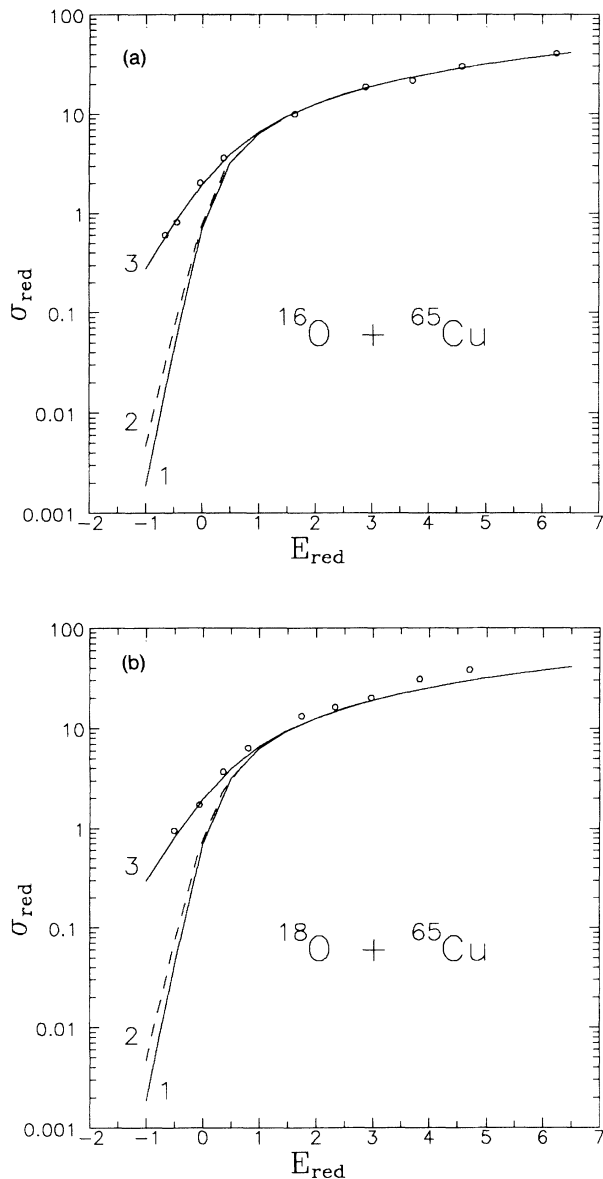


FIG. 6. (a), (b) The same as in Figs. 2(a) and 2(b) for $^{16}\text{O} + ^{65}\text{Cu}$ and $^{18}\text{O} + ^{65}\text{Cu}$ systems.

$^{18}\text{O} + ^{60,64}\text{Ni}$, and $^{18}\text{O} + ^{63}\text{Cu}$ for which $b_{\text{eff}} < 2$ fm. The other set includes the reactions $^{16}\text{O} + ^{65}\text{Cu}$, $^{18}\text{O} + ^{58}\text{Ni}$, and $^{18}\text{O} + ^{65}\text{Cu}$ for which $b_{\text{eff}} > 2$ fm. It is interesting to observe that in the latter case there is also a strong indication of a neutron pair transfer inducing an enhancement in the fusion cross section [19,20]. It is worth notic-

TABLE I. Values of b_{eff} obtained from the fitting for the various systems.

Systems	b_{eff} (fm)	Systems	b_{eff} (fm)
$^{16}\text{O} + ^{58}\text{Ni}$	1.66	$^{18}\text{O} + ^{58}\text{Ni}$	2.21
$^{16}\text{O} + ^{60}\text{Ni}$	1.97	$^{18}\text{O} + ^{60}\text{Ni}$	1.90
$^{16}\text{O} + ^{63}\text{Cu}$	1.87	$^{18}\text{O} + ^{63}\text{Cu}$	1.88
$^{16}\text{O} + ^{64}\text{Ni}$	1.92	$^{18}\text{O} + ^{64}\text{Ni}$	1.88
$^{16}\text{O} + ^{65}\text{Cu}$	2.23	$^{18}\text{O} + ^{65}\text{Cu}$	2.19

ing that our value for b_f is slightly greater than the value $b = 0.85$ fm obtained by Perey and Buck [6] in a somewhat different approach. Guided by this comparison we see that our results suggest that the present value for b_f indeed is a good estimate for the value of the nonlocality range which must account for the corresponding part in fusion cross sections, although, to be more confident in our result, the present approach must be checked also in other systems for which realistic coupled channels calculations cannot account for the total fusion cross section. Recently, for the sake of completeness, a schematic model for the study of the interplay between nonlocal effects and coupled channels has been discussed by the authors, and the results will appear in a future paper. Also a study of elastic scattering is being under investigation within the present approach with $b_f = 0.94$ fm in order to test the importance of the inclusion of the nonlocal effects in heavy ion reactions.

Finally we want to stress that the present scheme does not substitute other different approaches which take into account other nuclear degrees of freedom, its main virtue is to draw attention to the very important role of nonlocal effects — occurring concomitant to the other contributions, although introduced in a very simple form — to the fusion cross section at energies around and below the barrier.

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