Traveling solitons in Lorentz and *CPT* breaking systems

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In this work we present a class of traveling solitons in Lorentz and *CPT* breaking systems. In the case of Lorentz violating scenarios, as far as we know, only static solitonic configurations were analyzed up to now in the literature. Here it is shown that it is possible to construct some traveling solitons which cannot be mapped into static configurations by means of Lorentz boosts due to explicit breaking. In fact, the traveling solutions cannot be reached from the static ones by using something similar to a Lorentz boost in those cases. Furthermore, in the model studied, a complete set of exact solutions is obtained. The solutions present a critical behavior controlled by the choice of an arbitrary integration constant.

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I. INTRODUCTION

The study of the problem of Lorentz symmetry breaking has appeared in the physics literature, motivated by the fact that the superstring theories suggest that Lorentz symmetry should be violated at higher energies [1]. Recently, many works considering the impact of some kind of Lorentz symmetry breaking have appeared in the literature. For instance, some years ago, Carrol, Field, and Jackiw [2] addressed the problem of *CPT* (charge conjugation-parity-time reversal) symmetry violation. On the other hand, some of the impacts on the standard model due to the breaking of Lorentz and *CPT* symmetries were discussed by Colladay and Kostelecky [3–5]. Another problem analyzed in the literature is the spontaneous breaking of the four-dimensional Lorentz invariance of QED [6]. At this point, it is interesting to mention that a space-time with torsion interacting with a Maxwell field by means of a Chern-Simons–like term was introduced in Ref. [6]. In this case, it is possible to explain the optical activity in the synchrotron radiation emitted by cosmological distant radio sources.

Recently motivated by the problem of Lorentz symmetry violating gauge theories in connection with gravity models, Boldo *et al.* [7] have analyzed the graviton excitations and Lorentz violating gravity with the cosmological constant. It is important to remark that considerable effort has been made experimentally to observe signs of Lorentz and *CPT* symmetry violation effects. In fact, in a very recent work, Maccione, Liberati, and Sigl [8] have shown that experimental data on the photon content of ultrahigh-energy cosmic rays lead to strong constraints over the Lorentz symmetry violations in stringy space-time foam models. This was done by studying the time delay between γ rays of different energies from extragalactic sources. Moreover, Gubitosi *et al.* [9] have analyzed the impact of Planck-scale modifications to electrodynamics characterized by a spacelike symmetry breaking vector. Last year, several studies involving Lorentz violation appeared in the literature [7–17].

Finally, it is important to remark that nonlinear models which have topological solutions are very interesting and important in many branches of physics [18–23]. In a recent work [24] it was shown that some nonlinear models in two-dimensional space-time, where two scalar fields interact in the Lorentz and *CPT* violating scenarios, present static solitonic configurations. This was done by generalizing a model presented by Barreto and collaborators [25]. Finally, in a very recent work, Bazeia *et al.* [26] also analyzed the effects of Lorentz violation on topological defects generated by two real scalar fields. In that case, the symmetry breaking is induced by a tensor with arbitrarily fixed coefficients that couple the two fields. In all of these examples, the presented solitonic configurations were static. In this work we are going to show that it is possible to find nontrivial traveling solutions in this kind of scenario. This is going to be done by taking as an example a generalization of some models recently discussed in the literature [24–29]. As a consequence, we present a class of traveling solitons in Lorentz and *CPT* breaking systems as well as some static configurations. Finally, it is shown that the static configurations are not the static limit of the traveling ones. This is done by using an approach developed to deal with some classes of nonlinear models in two-dimensional space-time of two interacting scalar fields which were presented in [30]. In this last reference it was shown that in these systems in $1+1$ dimensions, the so-called orbit equation can be cast into the form of a linear first-order differential equation, thus leading to general solutions of the system. We also show that the solutions present a critical behavior controlled by the choice of an arbitrary integration constant.

II. THE MODEL

Some years ago, Ref. [24] presented a two-field model in $1+1$ dimensions, where the Lorentz breaking Lagrangian

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density generalizes some results in the literature. That Lagrangian density contains vector functions with a dependence on the dynamical scalar fields. Moreover, the mentioned vector functions are responsible by the Lorentz symmetry breaking. On the other hand, in Ref. [26], the effects of the Lorentz violation on topological defects generated by two real scalar fields was analyzed, too; here, the Lagrangian density has a tensor which is the term that breaks the Lorentz and, eventually, the *CPT* symmetry. Thus, in this work we construct a generalized two-field model in $1 + 1$ dimensions which is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - G^\mu(\phi,\chi)\partial_\mu\phi - F^\mu(\phi,\chi)\partial_\mu\chi - \gamma k^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) - pk^{\mu\nu}\partial_\mu\phi\partial_\nu\chi - V(\phi,\chi), \tag{1}$$

with

$$k^{\mu\nu} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{pmatrix}, \tag{2}$$

where $\mu = 0, 1$; $G^\mu(\phi,\chi)$ and $F^\mu(\phi,\chi)$ are vector functions; and $V(\phi,\chi)$ is the potential. Furthermore, $k^{\mu\nu}$ is a constant tensor, here represented by a 2×2 matrix, where α_1, α_2, α_3, and α_4 are arbitrary parameters, in general. However, if one wants to keep the *CPT* symmetry, this matrix must be a real, symmetric, and traceless one [4,5]. A similar process of breaking the Lorentz symmetry was put forward by Anacleto *et al.* [17] in a recent work, where the tensor $k^{\mu\nu}$ is a 4×4 matrix; here, the authors studied the problem of acoustic black holes in the Abelian Higgs model with Lorentz symmetry breaking. Finally, it is possible to recover the Lorentz symmetry by imposing that $\alpha_4 = -\alpha_1$ and $\alpha_2 = 0 = \alpha_3$ [4,5].

Note that, from the Lagrangian density (1), we can recover the one presented in the work by Bazeia *et al.* [26] by choosing $\gamma = 0$, $G^0(\phi,\chi) = F^0(\phi,\chi) = 0$, $\alpha_1 = \alpha_4 = \beta$, $\alpha_2 = \alpha_3 = \alpha$, and $p = -1$. Furthermore, we can also recover the Lagrangian density presented in [24,25] by conveniently setting the above defined parameters. Therefore, we have a more general model, including vector functions and a constant tensor. It is important to remark that the more general model presented here can be used to bring more information about the impact of the Lorentz violation of important systems like, for instance, those presenting topological structures [24,25].

From the Lagrangian density (1), we can write the corresponding equations of motion

$$(1-\gamma\alpha_1)\ddot{\phi} - (1+\gamma\alpha_4)\phi'' - p(\alpha_1\ddot{\chi} + \alpha_4\chi'') + (F^0_\phi - G^0_\chi)\dot{\chi} + (F^1_\phi - G^1_\chi)\chi' - (\alpha_3+\alpha_2)(\gamma\dot{\phi}' + p\dot{\chi}') + V_\phi = 0, \tag{3}$$

$$(1-\gamma\alpha_1)\ddot{\chi} - (1+\gamma\alpha_4)\chi'' - p(\alpha_1\ddot{\phi} + \alpha_4\phi'') - (F^0_\phi - G^0_\chi)\dot{\phi} - (F^1_\phi - G^1_\chi)\phi' - (\alpha_3+\alpha_2)(\gamma\dot{\chi}' + p\dot{\phi}') + V_\chi = 0, \tag{4}$$

where the dots stand for the derivative with respect to time, while the prime represents the derivative with respect to x, $V_\phi \equiv \partial V/\partial\phi$ and $V_\chi \equiv \partial V/\partial\chi$. It can be seen that the two equations above are carrying information about the symmetry breaking of the model through the presence of the α_i parameters and the vector functions. Here one can see that, when looking for static solutions, the terms depending on the time derivative of the fields in the above nonlinear differential equation vanish. Thus, those solutions will not present a dependence on the parameters α_1, α_2, and α_3, and this will also be true in the Lorentz invariant case when the α_2 and α_3 parameters are explicitly zero, and $\alpha_2 = -\alpha_4$.

Furthermore, as a consequence of the model studied in this work, in general, we cannot analytically solve the above differential equations. However, one can still consider an interesting case for the field configurations, where one searches for traveling wave solutions. Configurations that exhibit traveling waves have an important impact when we study boundary states for D-branes and the supergravity fields in a D-brane [27–29].

Then, let us begin our search for traveling wave solutions in the forms $\phi = \phi(u)$ and $\chi = \chi(u)$, with $u = Ax + Bt$. Thus, Eqs. (3) and (4) take the form

$$-\phi_{uu} + \tilde{\beta}\chi_{uu} - \tilde{\alpha}\chi_u + \tilde{V}_\phi = 0, \tag{5}$$

$$-\chi_{uu} + \tilde{\beta}\phi_{uu} + \tilde{\alpha}\phi_u + \tilde{V}_\chi = 0, \tag{6}$$

with the definitions

$$\tilde{\beta} \equiv -\frac{p[(\alpha_2+\alpha_3)AB + \alpha_4A^2 + \alpha_1B^2]}{(1+\gamma\alpha_4)A^2 - (1-\gamma\alpha_1)B^2 + AB\gamma(\alpha_2+\alpha_4)}, \tag{7}$$

$$\tilde{\alpha} \equiv -\frac{B(F^0_\phi - G^0_\chi) + A(F^1_\phi - G^1_\chi)}{(1+\gamma\alpha_4)A^2 - (1-\gamma\alpha_1)B^2 + AB\gamma(\alpha_2+\alpha_4)}, \tag{8}$$

$$\tilde{V}_\phi \equiv \frac{V_\phi}{(1+\gamma\alpha_4)A^2 - (1-\gamma\alpha_1)B^2 + AB\gamma(\alpha_2+\alpha_4)}, \tag{9}$$

$$\tilde{V}_\chi \equiv \frac{V_\chi}{(1+\gamma\alpha_4)A^2 - (1-\gamma\alpha_1)B^2 + AB\gamma(\alpha_2+\alpha_4)}. \tag{10}$$

In this case, one can observe that the terms which do not contribute to a static configuration are still present in this variable, from which one can conclude that the solutions in the traveling variable case are essentially different from those in the static configuration. In other words, the traveling wave solutions depend explicitly on the variables α_1, α_2, and α_3, in contrast with what happens for static configurations, as stated above.

In order to decouple the pair of second order differential equations, we multiply Eq. (5) by ϕ_u and Eq. (6) by χ_u. Thus, it is not difficult to conclude that, after adding the two equations, one can write

$$\frac{d}{du}\left[-\frac{1}{2}(\phi_u^2+\chi_u^2)+\tilde{\beta}\phi_u\chi_u+\tilde{V}(\phi,\chi)\right]=0. \quad (11)$$

In this case, we have

$$-\tfrac{1}{2}(\phi_u^2+\chi_u^2)+\tilde{\beta}\phi_u\chi_u+\tilde{V}(\phi,\chi)=b_0. \quad (12)$$

In the above equation, the arbitrary constant b_0 should be set to zero in order to get solitonic solutions. It can be noticed that this condition allows the field configuration to go asymptotically to the vacua of the field potential, where the derivative of the field configuration and the fields' potential vanish simultaneously. Otherwise, one obtains oscillating (in space) or complex solutions depending on the chosen value of the constant b_0. For instance, one can see such a feature in a recent work [31], where some properties of soliton configurations in twisted orbifolds are addressed.

Note that in the above equation (with $b_0=0$), the dependence on $\tilde{\alpha}$ has disappeared. However, the dependence of the system on the Lorentz breaking parameters is still present but it is implicit. Now, in order to decouple the above equation, we apply the rotation

$$\begin{pmatrix}\phi(u)\\ \chi(u)\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\theta(u)\\ \varphi(u)\end{pmatrix}. \quad (13)$$

Thus, Eq. (12) is rewritten as

$$-\tfrac{1}{2}(1-\tilde{\beta})\theta_u^2-\tfrac{1}{2}(1+\tilde{\beta})\varphi_u^2+\tilde{V}(\theta,\varphi)=0. \quad (14)$$

Furthermore, performing the dilations

$$\theta(u)=\frac{\sigma(u)}{\sqrt{1-\tilde{\beta}}}, \qquad \varphi(u)=\frac{\rho(u)}{\sqrt{1+\tilde{\beta}}}, \quad (15)$$

one gets

$$-\tfrac{1}{2}\sigma_u^2-\tfrac{1}{2}\rho_u^2+\tilde{V}(\sigma,\rho)=0. \quad (16)$$

At this point, one can verify that the above equations allow one to write two first-order coupled differential equations. In this case it is usual to impose that the potential must be written in terms of a superpotential like

$$\bar{V}(\sigma,\rho)=\frac{1}{2}\left(\frac{\partial W(\sigma,\rho)}{\partial\sigma}\right)^2+\frac{1}{2}\left(\frac{\partial W(\sigma,\rho)}{\partial\rho}\right)^2, \quad (17)$$

which leads to the following set of equations:

$$\frac{d\sigma}{du}=\pm W_\sigma, \qquad \frac{d\rho}{du}=\pm W_\rho, \quad (18)$$

where $W_\sigma\equiv\partial W(\sigma,\rho)/\partial\sigma$ and $W_\rho\equiv\partial W(\sigma,\rho)/\partial\rho$, and this will lead us to the solitonic solutions we are looking for.

In order to analyze the energy of the configurations obtained, we write the energy-momentum tensor in the form

$$T^{\mu\nu}=\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial^\nu\phi+\frac{\partial\mathcal{L}}{\partial(\partial_\mu\chi)}\partial^\nu\chi-g^{\mu\nu}\mathcal{L}. \quad (19)$$

Therefore, the energy density for the Lagrangian (1) is given by

$$\begin{aligned}T^{00}=&\frac{\dot{\phi}}{2}+\frac{\dot{\chi}^2}{2}+\left(\frac{1}{2}+\gamma\alpha_4\right)(\phi'^2+\chi'^2)+G^1(\phi,\chi)\phi'\\&+F^1(\phi,\chi)\chi'+2-p\alpha_1\dot{\phi}\,\dot{\chi}+\gamma(\alpha_2+\alpha_3)\phi'\dot{\phi}\\&+\gamma(\alpha_2+\alpha_3)\chi'\dot{\chi}+p\alpha_4\phi'\chi'+p\alpha_2\dot{\phi}\chi'\\&+p\alpha_3\phi'\dot{\chi}+V(\phi,\chi).\end{aligned} \quad (20)$$

For the traveling wave solutions, the energy density is written in the form

$$\begin{aligned}T^{00}_{\text{traveling}}=&\left[\frac{B^2}{2}+\left(\frac{1}{2}+\gamma\alpha_4\right)A^2+\gamma(\alpha_2+\alpha_3)AB\right]\phi_u^2\\&+\left[\frac{B^2}{2}+\left(\frac{1}{2}+\gamma\alpha_4\right)A^2+\gamma(\alpha_2+\alpha_3)AB\right]\chi_u^2\\&+p[\alpha_4A^2-\alpha_1B^2+(\alpha_2+\alpha_3)AB]\phi_u\chi_u\\&+A[G^1(\phi,\chi)\phi_u+F^1(\phi,\chi)\chi_u]+V(\phi,\chi).\end{aligned} \quad (21)$$

Now, we choose the superpotential that was used in [30], which is written as

$$W(\sigma,\rho)=-\lambda\sigma+\frac{\lambda}{3}\sigma^3+\mu\sigma\rho^2. \quad (22)$$

In this case, the solutions presented in Ref. [30], with $\lambda=\mu$, are given by

$$\begin{aligned}\sigma_+(u)&=\frac{(c_0^2-4)e^{4\mu(u-u_0)}-1}{[c_0e^{2\mu(u-u_0)}-1]^2-4e^{4\mu(u-u_0)}},\\ \sigma_-(u)&=\frac{4-c_0^2+e^{4\mu(u-u_0)}}{[e^{2\mu(u-u_0)}-c_0]^2-4},\\ \rho_+(u)&=\frac{4e^{2\mu(u-u_0)}}{[c_0e^{2\mu(u-u_0)}-1]^2-4e^{4\mu(u-u_0)}},\\ \rho_-(u)&=\frac{4e^{2\mu(u-u_0)}}{[e^{2\mu(u-u_0)}-c_0]^2-4},\end{aligned} \quad (23)$$

where we must impose that $c_0\leq-2$ in both solutions. The resulting fields are illustrated in Fig. 1, where the

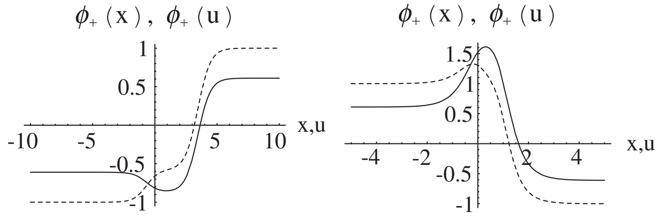

FIG. 1. Traveling soliton solutions and static solutions for $\lambda = \mu = 1$ and $c_0 = -2.001$. The dashed line corresponds to the static case with $\beta = 0.5$, $A = 1$. The thin continuous line corresponds to the traveling wave case for $\beta = 0.5$, $\alpha = 0.4$, $A = 1$, $B = -1.5$.

difference between the static and the traveling wave configurations can be seen. On the other hand, in the case where $\lambda = 4\mu$, the exact solutions are written as

$$\sigma_+(u) = \frac{4 + (16c_0 - 1)e^{8\mu(u-u_0)}}{[2 + e^{4\mu(u-u_0)}]^2 - 16c_0 e^{8\mu(u-u_0)}},$$
$$\sigma_-(u) = \frac{16c_0 + 4e^{8\mu(u-u_0)} - 1}{[1 + 2e^{4\mu(u-u_0)}]^2 - 16c_0},$$
$$\rho_+(u) = -\frac{2e^{2\mu(u-u_0)}}{\sqrt{[(1/2)e^{4\mu(u-u_0)} + 1]^2 - 4c_0 e^{8\mu(u-u_0)}}}, \tag{24}$$
$$\rho_-(u) = -\frac{4e^{2\mu(u-u_0)}}{\sqrt{[1 + 2e^{4\mu(u-u_0)}]^2 - 16c_0}}.$$

In this case, we impose that $c_0 \leq 1/16$. It is important to remark that, making the exchange of $\sigma \rightarrow \rho$ and $\rho \rightarrow \sigma$ in the case where $\lambda = \mu$, the equation of motion (16) remains invariant. Thus, the solutions in which kinks become lumps and vice versa shall appear, and this is used in order to generate the orbits appearing in Fig. 2. In fact, this symmetry is important for the generation of all possible classes of orbits connecting the vacua [18].

Thus, the fields $\phi(u)$ and $\chi(u)$ are given by

$$\phi_\pm(u) = \frac{1}{\sqrt{2}}\left[\frac{\sigma_\pm(u)}{\sqrt{1-\tilde{\beta}}} - \frac{\rho_\pm(u)}{\sqrt{1+\tilde{\beta}}}\right],$$
$$\chi_\pm(u) = \frac{1}{\sqrt{2}}\left[\frac{\sigma_\pm(u)}{\sqrt{1-\tilde{\beta}}} + \frac{\rho_\pm(u)}{\sqrt{1+\tilde{\beta}}}\right]. \tag{25}$$

Now, using the solutions presented in Ref. [30], which are represented here by (23) and (24), we have the complete set of solutions with position and time dependence. At this point, it is important to explain what we mean by a complete set of solutions.

Since we are not reproducing here all the steps followed in order to obtain the solutions presented in Ref. [30], it is important to say that in that work, the nonlinear differential orbit equation was mapped into a linear one and, as a consequence, a general solution was obtained. This first-order linear differential equation naturally has a solution depending on an arbitrary integration constant, c_0, which is such that the solutions present a kind of degeneracy, since the energy of the configuration is insensitive to its value. Furthermore, there exists a critical value of c_0, such that the behavior of the solitons changes drastically, as it can be seen in Fig. 2.

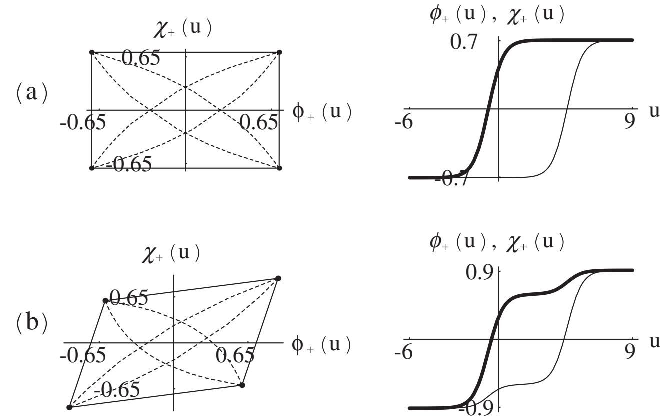

FIG. 2. Orbits (left) and solutions (right). (a) The orbit with $\lambda = \mu = 1$ and $\tilde{\beta} = 0$. (b) The orbit with $\lambda = \mu = 1$ and $\tilde{\beta} = 0.5$. (a) Solutions (right) to the case where $c_0 = -2.0001$ and $\tilde{\beta} = 0$. (b) Solutions (right) to the case where $c_0 = -2.0001$ and $\tilde{\beta} = 0.5$.

Here, once more, we call attention to the fact that the static solutions for Eqs. (3) and (4) are different from the traveling wave ones. This difference can be seen from an inspection of the static field differential equations

$$-\phi'' + \bar{\beta}\chi'' + \bar{\alpha}\chi' + \bar{V}_\phi = 0, \tag{26}$$

$$-\chi'' + \bar{\beta}\phi'' - \bar{\alpha}\chi' + \bar{V}_\chi = 0, \tag{27}$$

where now one has

$$\bar{\beta} \equiv \frac{-p\alpha_4}{(1+\gamma\alpha_4)}, \qquad \bar{\alpha} \equiv \frac{(F^1_\phi - G^1_\chi)}{(1+\gamma\alpha_4)},$$
$$\bar{V}_\phi \equiv \frac{V_\phi}{(1+\gamma\alpha_4)}, \quad \text{and} \quad \bar{V}_\chi \equiv \frac{V_\chi}{(1+\gamma\alpha_4)}. \tag{28}$$

Note that the constants α_1, α_2, and α_3, which are present in the Lagrangian density of the system and in the traveling wave equations, disappear completely in the static case. In particular, if $\gamma = 0$, $G^\mu(\phi, \chi) = F^\mu(\phi, \chi) = 0$, $\alpha_1 = \alpha_4 = \beta$, $\alpha_2 = \alpha_3 = \alpha$, and $p = -1$, one recovers the model presented by Bazeia *et al.* [26]. In the static case, the equations of motion presented by the authors are given by

$$-\phi'' + \beta\chi'' + V_\phi = 0, \tag{29}$$

$$-\chi'' + \beta\phi'' + V_\chi = 0. \tag{30}$$

At this point, it is interesting to note that the pair of equations presented in [26] for the static solutions takes on a different form compared with Eqs. (26) and (27). In fact, we can recover the equations of motion presented in the work of Bazeia [26] by conveniently choosing the symmetry breaking parameters. But the general static configurations are given by Eqs. (26) and (27), which are carrying more information about the terms of the Lorentz breaking of the model through the presence of the parameter α, which is totally absent in the static solution.

III. CONCLUSIONS

In this work we have shown that a class of traveling solitons in Lorentz violating systems can be analytically obtained, which happens despite the fact that there is no Lorentz symmetry; consequently, one cannot recover the traveling solutions from the static one, just performing Lorentz boosts. This feature has been illustrated in some nonlinear models of interacting scalar fields in two-dimensional space-time, which were presented in [30]. Furthermore, in the model studied, a complete set of solutions was obtained. That complete set of solutions is characterized by the presence of an arbitrary integration constant for the orbit equation. The solutions present a critical behavior controlled by the choice of an arbitrary integration constant. The change in the behavior of the solution, when one is working with c_0 close to its critical value, is illustrated in the Fig. 2. There one can see that when β is far from zero, the soliton configuration develops a kind of double step profile. This happens due to the fact that the orbit connecting the vacua passes close to an intermediate vacuum, since the position of some vacua is deformed by the symmetry breaking. Finally, the solutions obtained in this work are valid for arbitrary values of the speed of the configuration. The speed of the configuration will be given by $v = B/A$, as can be verified from the inspection of the variable used in order to write the nonlinear differential equation of the model analyzed. Comparing the variable used in this work, $u = Ax + Bt$, with the usual boosted variable $u_L = \gamma(x + vt)$, one can verify that the parameters A and B can be chosen in a range larger than the corresponding ones in the boosted variable, allowing the appearance of superluminal solitons. It is interesting to note that some superluminal solutions in Lorentz symmetric cases were discussed a long time ago by Aharonov *et al.* [32].

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