

Grand unification and proton stability near the Peccei-Quinn scale

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We show that in an $SU(2) \otimes U(1)$ model with a Dine-Fischler-Srednicki-like invisible axion it is possible to obtain (i) the convergence of the three gauge coupling constants at an energy scale near the Peccei-Quinn scale; (ii) the correct value for $\sin^2 \hat{\theta}_W(M_Z)$; (iii) the stabilization of the proton by the cyclic $Z_{13} \otimes Z_3$ symmetries which also stabilize the axion as a solution to the strong CP problem. Concerning the convergence of the three coupling constants and the prediction of the weak mixing angle at the Z peak, this model is as good as the minimal supersymmetric standard model with $\mu_{\text{SUSY}} = M_Z$. We also consider the standard model with six and seven Higgs doublets. The main calculations were done in the 1-loop approximation but we briefly consider the 2-loop contributions.

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The convergence of the three gauge coupling constants $g_3, g_2, g_1 = \sqrt{5/3}g'$ and the prediction of the electroweak mixing angle are some of the motivations for grand unified theories (GUTs) [1–4]. Unfortunately the simplest and more elegant GUTs which break into a simple step to the standard model, like $SU(5)$ [1,2] and some of the $SO(10)$ and E_6 GUTs [3,4], were ruled out by two experimental results. The first one is concerned with the fact that using the value of the electroweak mixing angle measured at the Z peak by LEP, i.e., $\sin^2 \hat{\theta}_W(M_Z) = 0.231\,13(15)$ [5], the running coupling constants extrapolated from the values measured at low energies do not meet at a single point or, inversely, assuming the convergence of the three coupling constants, the prediction of the value of $\sin^2 \hat{\theta}_W(M_Z)$ [or, alternatively, we can obtain the convergence point of α_1 and α_2 and then the value of $\alpha_s(M_Z)$ can be predicted] does not agree with the experimental value. On the other hand, in the minimal supersymmetric extension of the standard model (MSSM for short) the coupling constants do intercept at a single point, with a remarkable precision, and the predicted value of the weak mixing angle is also in agreement with the observed value [6]. In fact, this has become one of the most important reasons for believing on the existence of supersymmetry at the TeV scale and, in particular, on the MSSM. In particular, a new scale for physics, μ_{SUSY} , is then required. The second trouble with the simplest GUTs is related to the nonobservation of the proton decay at the predicted lifetime. Recent data by Super-Kamiokande on $p \rightarrow K^+ \bar{\nu}$ imply that $\tau_P > 10^{33}$ yr [5,7]. However, this is also a trouble for supersymmetric (SUSY) $SU(5)$ since this theory has $d = 5$ effective operators induced by colored-Higgs triplets that produce a

rapid proton decay [8] and it is necessary to appeal to fermion mixing in order to keep (tight) agreement with data [9]. Thus, it appears natural to ask ourselves if there are options for SUSY yielding the convergence of the couplings, the observed value of the weak mixing angle at the Z pole, and an appropriately stable proton. The importance of the Higgs boson contributions to the convergence of the couplings has been emphasized recently [10], although exotic fermions can also lead to the gauge coupling unification even at the TeV range [11]. In this case, unification at a lower energy scale is possible even without supersymmetry and the proton stability is guaranteed by additional assumptions as extra dimensions or dynamical symmetry breaking [11], by the conservation of the baryon number in the gauge interactions as in the $[SU(3)]^3$ trinification [10], or in $[SU(3)]^4$ quartification where the proton decay is mediated only by Higgs scalars [12].

Here, we will show that in a multi-Higgs extension of the standard model we have the convergence of the three gauge coupling constants at an energy scale of the order of 10^{13} GeV, and the weak mixing angle in agreement with the measured value at the Z pole. The proton decay occurs only throughout dimension 8, 9, 10, and 11 effective operators because discrete $Z_{13} \otimes Z_3$ symmetries forbid all $d = 6, 7$ operators. This model was proposed independently of the issue of the gauge unification, and it has a Dine-Fischler-Srednicki-like invisible axion [13] stabilized against a semiclassical gravitational effect by those discrete symmetries [14]. The use of discrete gauge symmetries in the proton decay problem has been used recently in Ref. [15] in a model with a Z_6 symmetry, where baryon number violation low dimension dangerous effective operators are all forbidden. For other recent uses of symmetries like these see [16]. We compare our results with the usual MSSM showing that the convergence of the coupling constants and the prediction of the

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$\sin^2 \hat{\theta}_w(M_Z)$ is, in this model, as good as in the MSSM with $\mu_{\text{SUSY}} = M_Z$. We also consider the SM with seven Higgs doublets, and briefly comment on the SM with six Higgs doublets and $\mu_{\text{SUSY}} = 1$ TeV.

In Ref. [14] the representation content of the standard model was augmented, by adding scalar fields and right-handed neutrinos, in such a way that the discrete $Z_{13} \otimes Z_3$ symmetries could be implemented in the model. The particle content of the model is the following: $Q_L = (ud)_L^T \sim (2, 1/3)$, $L_L = (\nu l)_L^T \sim (2, -1)$ denote any quark and lepton doublet; $u_R \sim (1, 4/3)$, $d_R \sim (1, -2/3)$, $l_R \sim (1, -2)$, and $\nu_R \sim (1, 0)$ are the respective right-handed components. It was also assumed that each charge sector gains mass from a different scalar doublet, hence we have the following Higgs multiplets: four doublets Φ_u, Φ_d, Φ_l , and Φ_ν [all of them of the form $(2, +1) = (\varphi^+, \varphi^0)^T$ under $SU(2) \otimes U(1)$] which generate Dirac masses for u - and d -like quarks, charged leptons, and neutrinos, respectively; a neutral complex singlet $\phi \sim (1, 0)$, a singly charged singlet $h^+ \sim (1, +2)$, and, finally, a non-Hermitian triplet $\tilde{T} \sim (3, +2)$. With the discrete symmetries flavor changing neutral currents are also naturally suppressed at the tree level.

Next, let us consider the evolution equations of the three gauge coupling constants, at the 1-loop level,

$$\begin{aligned}\alpha_1^{-1}(M) &= \alpha^{-1}(M_Z) \frac{3}{5} \cos^2 \theta_w(M_Z) + \frac{b_1}{2\pi} \ln\left(\frac{M}{M_Z}\right), \\ \alpha_2^{-1}(M) &= \alpha^{-1}(M_Z) \sin^2 \theta_w(M_Z) + \frac{b_2}{2\pi} \ln\left(\frac{M}{M_Z}\right), \\ \alpha_3^{-1}(M) &= \alpha^{-1}(M_Z) + \frac{b_3}{2\pi} \ln\left(\frac{M}{M_Z}\right),\end{aligned}\quad (1)$$

where b_i are the 1-loop beta-function coefficients

$$b_i = \frac{2}{3} \sum_{\text{fermions}} T_{Ri}(F) + \frac{1}{3} \sum_{\text{scalars}} T_{Ri}(S) - \frac{11}{3} C_2(V). \quad (2)$$

For $SU(N)$, we have that $T_R = C_2 = N$, with $N \geq 2$, for fields in the adjoint representation, and $T_R(S, F) = 1/2$ for fields in the fundamental or antifundamental representation; for $U(1)$, $C_2(V) = 0$, and $T(S_a, F_a) = (3/5) \text{Tr}(Y_a^2/4)$ for a unification in $SU(5)$ [it is the same for the case of $SO(10)$]. Equation (2) is valid for Weyl spinors and complex scalar fields. Considering the extension of the standard model with N_g matter generations, N_H Higgs doublets, and N_T non-Hermitian scalar triplets, all of them considered relatively light,

$$\begin{aligned}b_1 &= \frac{4}{3} N_g + \frac{1}{10} N_H + \frac{3}{5} N_T, \\ b_2 &= \frac{4}{3} N_g + \frac{1}{6} N_H + \frac{2}{3} N_T - \frac{22}{3}, \quad b_3 = \frac{4}{3} N_g - 11.\end{aligned}\quad (3)$$

Only the scalar singlet h^+ will be considered with mass

of the order of the unification scale. The evolution equations in Eq. (1) imply the unification condition $\alpha_1^{-1}(M_U) = \alpha_2^{-1}(M_U) = \alpha_3^{-1}(M_U) \equiv \alpha_U^{-1}$, which also defines the mass scale M_U :

$$M_U = M_Z \exp \left[2\pi \frac{\alpha^{-1}(M_Z) - \frac{8}{3} \alpha_3^{-1}(M_Z)}{\frac{5}{3} b_1 + b_2 - \frac{8}{3} b_3} \right]. \quad (4)$$

From Eqs. (3) with $N_g = 3$, $N_H = 1$, and $N_T = 0$, i.e., the standard model, give $(b_1, b_2, b_3) = (41/10, -19/6, -7)$. In this case, the evolution of g_i is shown in Fig. 1. In this figure (and below) we have used the inputs $M_Z = 91.1876$ GeV; $\alpha(M_Z) = 1/128$; and $\alpha_3(M_Z) = 0.1172$ [5]. It is clear that with only the representation content of the SM, there is no convergence of the three g_i at a given point [6].

On the other hand, with $N_g = 3$, $N_H = 4$, and $N_T = 1$ i.e., the model of Ref. [14], with Eqs. (3) we have $(b_1, b_2, b_3) = (5, -2, -7)$ and the evolution of the coupling constants in this case is shown in Fig. 2. We obtain that the three forces unify at the energy scale $M_U \simeq 2.8 \times 10^{13}$ GeV and $\alpha_U^{-1} \simeq 38$. The value predicted for $\sin^2 \hat{\theta}_w(M_Z)$, using the unification scale as an input, is

$$\sin^2 \hat{\theta}_w = \frac{3}{8} + \frac{5}{16\pi} \alpha(M_Z) (b_1 - b_2) \ln\left(\frac{M_Z}{M_U}\right), \quad (5)$$

and we obtain in this model $\sin^2 \hat{\theta}_w(M_Z) = 0.2311$, which coincides with the measured value as it should be since the unification occurs with a very good precision. In order to compare this result with others [10] we write at the 1-loop approximation, by eliminating the unification scale from Eq. (3) and also using Eq. (1),

$$\tilde{b} = \frac{11 + \frac{1}{2} N_H + 2 N_T}{22 - \frac{1}{5} (N_H + N_T)} = \frac{5}{3} \left[\frac{\sin^2 \hat{\theta}_w(M_Z) - \frac{\alpha(M_Z)}{\alpha_3(M_Z)}}{1 - \frac{8}{3} \sin^2 \hat{\theta}_w(M_Z)} \right], \quad (6)$$

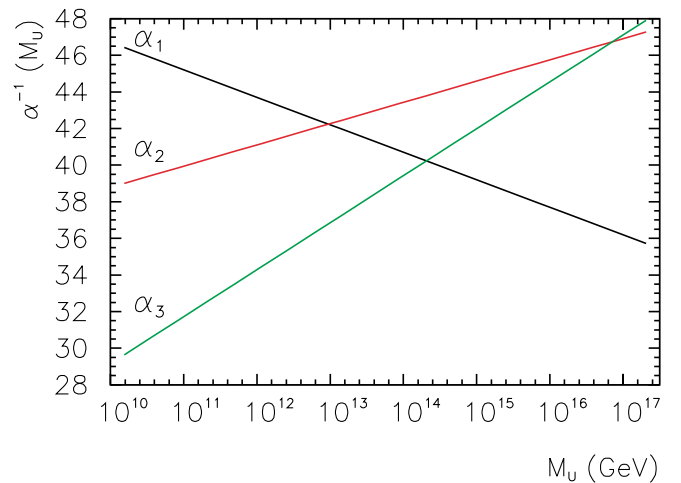


FIG. 1 (color online). Running couplings in the standard model.

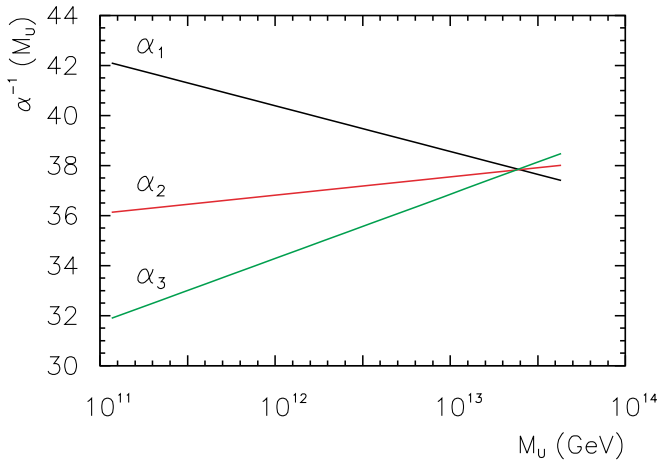


FIG. 2 (color online). Running couplings in the present model.

where $\tilde{b} = (b_3 - b_2)/(b_2 - b_1)$ [11]. For being more general we let N_H and N_T arbitrary to see if there are other values for them which could fit with unification. The theoretical ratio $\tilde{b}$ defined in the first line of Eq. (6), and which depends mainly on the scalar representation content (notice that, at the 1-loop level, $\tilde{b}$ does not depend on N_g), should coincide with the quantity defined in the second line which depends only on the experimental values of the coupling constants α and α_3 and $\sin^2 \hat{\theta}_W$ at the Z peak. The experimental input then implies $\tilde{b} = 0.714$ using the second line of Eq. (6). Using the first line of Eq. (6) the minimal standard model implies $\tilde{b} = 115/218 \approx 0.527$ (including the scalar contributions), so that it does not match and $\sin^2 \hat{\theta}_W(M_Z) = 0.204$ according to Eq. (5) and for this reason the unification of this model in nonsupersymmetric SU(5) was ruled out by LEP data. In the present model we obtain $\tilde{b} = 5/7 \approx 0.714$ and this value matches in Eq. (6) and gives, then, the observed value for $\sin^2 \hat{\theta}_W(M_Z)$ [5] as we pointed out above.

In the MSSM when $\mu_{\text{SUSY}} = M_Z$ we have $(b_1, b_2, b_3) = (33/5, 1, -3)$ and the respective evolution is shown in Fig. 3, with $M_U \approx 2.1 \times 10^{16}$ GeV and the inverse of the coupling constant at the unification scale, denoted in this case by α_5 , is $\alpha_5^{-1} \approx 24$. The weak mixing angle has also the correct value [17] and the same value for $\tilde{b}$ is obtained as in the present model. The case when the SUSY scale is of the order of M_Z is better than the case when that scale is of the order of 1 TeV but we do not show the latter case. As can be seen from Figs. 2 and 3 and from the value of $\tilde{b}$, concerning the unification and the prediction of the weak mixing angle, the model of Ref. [14] is as good as the MSSM and for this reason it was not necessary in the present work to take into account the theoretical and experimental uncertainties.

Finally, in the case when $N_g = 3$ but $N_H = 7$ and $N_T = 0$ we have $(b_1, b_2, b_3) = (47/10, -13/6, -7)$, $\tilde{b} =$

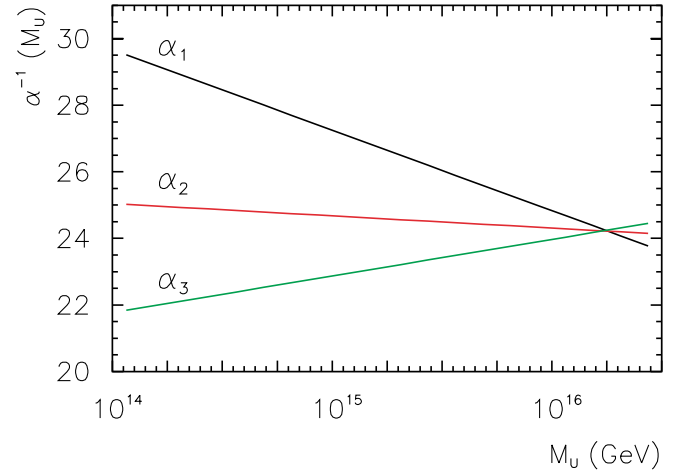


FIG. 3 (color online). Running couplings in the MSSM with $\mu_{\text{SUSY}} = M_Z$.

$145/206 \approx 0.704$, and $\sin^2 \hat{\theta}_W(M_Z) = 0.230$ according to Eq. (5) and the evolution is shown in Fig. 4 with $M_U \approx 5.8 \times 10^{13}$ GeV [3]. We have also studied the case of the SM with six Higgs doublets. In this case the unification of the coupling coincides with that of Ref. [10] and $\sin^2 \hat{\theta}_W(M_Z) = 0.226$. Moreover, the convergence of the three coupling constants in the seven Higgs scalar doublets is better than the case of six of such doublets [10].

We see that, in order to have the unification of the couplings and the correct value for the weak mixing angle at the Z pole it is not necessary to have low energy supersymmetry. On the other hand, four Higgs doublets and a non-Hermitian Higgs triplet are more precise, for the unification of the coupling constant, than just seven Higgs doublets. As we said before, only the singlet h^+ has been considered having a mass of the order of the unification scale. Assuming that this singlet is light we obtain the same value for M_U but $\sin^2 \hat{\theta}_W(M_Z) = 0.229$.

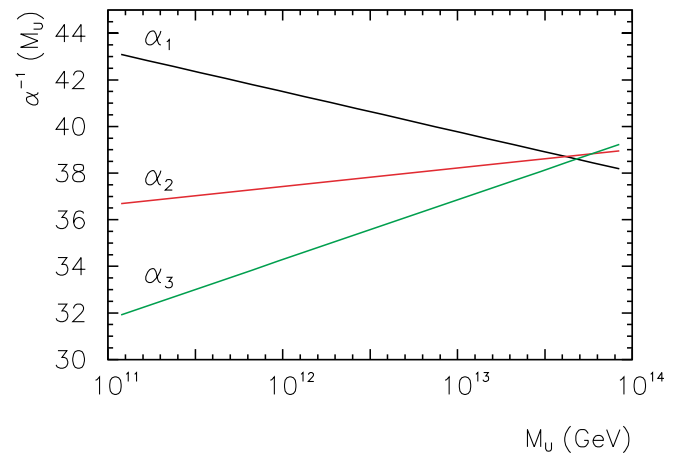


FIG. 4 (color online). Case of the standard model with seven scalar doublets.

The contribution of just one Higgs doublet is almost negligible compared to that of quarks and leptons when dealing with the renormalization group equations. But in models like the multi-Higgs extensions we have discussed here, the Higgs multiplets have a total contribution already important to unify the theories. However, we stress again, looking at Eq. (6), that the value of $\sin^2 \hat{\theta}_W(M_Z)$ depends strongly on N_H and N_T selecting in this way only a few possibilities. Moreover, the prediction of the weak mixing angle in this model is not an accident. To see that, with the experimental inputs in the second line of Eq. (6) we have $N_H = 7.324 - 3.333N_T$ and it is clear that the best solution for N_H and N_T integers is when $N_H = 4$ for $N_T = 1$. The seven doublets model $N_H = 7$ for $N_T = 0$ is the second best solution. The denominator in the exponent of Eq. (4) can be written as $(1/3)N_H + (5/3)N_T + 22$, and we see that a larger number of Higgs multiplets imply a

smaller value for M_U , however, we obtain the best match condition for $\tilde{b}$ with $N_H = 4$ and $N_T = 1$.

Because of the precise measurements in the electro-weak sector higher order corrections to the 1-loop calculations are important to verify if the solutions above can be stable against these corrections. Considering only the gauge coupling constants, their running $g_i(\mu)$ are now solutions of the corrected renormalization group equations

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = \frac{1}{2\pi} \left[b_i + \frac{1}{4\pi} \sum_{j=1}^3 b_{ij} \alpha_j(\mu) \right] \alpha_i(\mu)^2. \quad (7)$$

The general form of the coefficients b_{ij} is given in Ref. [18]. For the cases discussed in this paper, i.e., the only nontrivial scalar representations under SU(2) are doublets and triplets, they have the following form:

$$b_{ij} = \begin{pmatrix} \frac{19}{15}N_g + \frac{9}{50}N_H + \frac{36}{5}N_T & \frac{3}{5}N_g + \frac{9}{10}N_H + \frac{72}{5}N_T & \frac{44}{15}N_g \\ \frac{1}{5}N_g + \frac{3}{10}N_H + \frac{24}{5}N_T & \frac{49}{3}N_g + \frac{13}{3}N_H + \frac{56}{3}N_T - \frac{136}{3} & 4N_g \\ \frac{11}{30}N_g & \frac{3}{2}N_g & \frac{76}{3}N_g - 102 \end{pmatrix}. \quad (8)$$

The quark top Yukawa coupling being of order of unity, is the only one comparable with the α_i . However its contribution to Eq. (7) is unimportant when compared with the other contributions in Eq. (8) and does not affect the 2-loops running significantly for the values of N_H and N_T considered here. A complete treatment includes, of course, all scalar interactions (which in the present model include trilinear interactions) and it is much more complicated. However, as an illustration, we will consider only the corrections of the gauge coupling constants in Eq. (8). In this case, the numerical solutions to the system of equations in Eq. (7) can be found using Eq. (1) after making the simple substitution $b_i \rightarrow \Delta_i$, with the Δ_i extracted numerically. The values of Δ_i and the respective values for M_U and $\sin^2 \hat{\theta}_W(M_Z)$, at the 2-loop order, are given in Table I. We see from the table that in the cases with $N_H = 4$, $N_T = 1$ and $N_H = 7$, $N_T = 0$, the values of the weak mixing angle at the Z pole are a little above the experimental value (since this is only an illustration that considers just the evolution of the gauge coupling constants it is not necessary to take into account the experimental and theoretical uncertainties). This does not rule out these models since the correct value of $\sin^2 \hat{\theta}_W(M_Z)$

may be obtained at the 2-loop level by constraining the self-interactions couplings and the masses of the scalars fields. Hence, this partial analysis suggests that solutions in the 1-loop approximation might be stable under higher corrections since they are not drastically changed when 2-loop corrections are included.

Next, we come to the question of the proton stability in the present model. We have seen that the energy scale of the unification of the coupling constants is of the order of 10^{13} GeV, i.e., smaller than the scale of the non-SUSY SU(5) and near, by a 10–1000 factor, to the Peccei-Quinn (PQ) scale. This is, apparently, a disaster from the point of view of the nucleon decay. However, it is not because the model accepts discrete symmetries that forbid potentially dangerous effective operators of $d = 6, 7$. With the representation content of the model it is possible to impose the following Z_{13} symmetry:

$$\begin{aligned} Q &\rightarrow \omega_5 Q, & u_R &\rightarrow \omega_3 u_R, & d_R &\rightarrow \omega_5^{-1} d_R, \\ L &\rightarrow \omega_6 L, & \nu_R &\rightarrow \omega_0 \nu_R, & l_R &\rightarrow \omega_4 l_R, \\ \Phi_u &\rightarrow \omega_2^{-1} \Phi_u, & \Phi_d &\rightarrow \omega_3^{-1} \Phi_d, & \Phi_l &\rightarrow \omega_2 \Phi_l, \\ \Phi_\nu &\rightarrow \omega_6^{-1} \Phi_\nu, & \phi &\rightarrow \omega_1^{-1} \phi, & T &\rightarrow \omega_4^{-1} T, \\ & & h^+ &\rightarrow \omega_1 h^+, \end{aligned} \quad (9)$$

TABLE I. Values for Δ_i and $\sin^2 \hat{\theta}_W(M_Z)$ at the 2-loop level for multi-Higgs models. M_U is in units of 10^{13} GeV.

N_H	N_T	Δ_1	Δ_2	Δ_3	M_U	$\sin^2 \hat{\theta}_W(M_Z)$
4	1	5.102	-1.847	-7.089	1.5	0.235
6	0	4.659	-2.213	-7.089	5.0	0.231
7	0	4.762	-2.035	-7.089	3.4	0.234

with $\omega_k = e^{2\pi i k/13}$, $k = 0, 1, \dots, 6$. Moreover, in order to have an automatic PQ symmetry [19], it is also necessary to impose a Z_3 with parameters denoted by $\tilde{\omega}_0, \tilde{\omega}_1$, and $\tilde{\omega}_1^{-1}$ with Φ_l transforming with $\tilde{\omega}_1^{-1}$; Φ_ν, ν_R, l_R with $\tilde{\omega}_1$, while all other fields transform trivially under Z_3 . For details see Ref. [14].

We search for effective operators that are $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ and $Z_{13} \otimes Z_3$ invariant. No grand unified model is assumed here. Effective operators with $d = 6$ which induce a rapid proton decay for $M_U < 10^{16}$ GeV, are given by [20]

$$\begin{aligned}
O_{abcd}^{(1)} &= \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{(d_R)_{ia}^c} (u_R)_{jb} \overline{(Q_L)_{kc}^c} (L_L)_{\beta d}, \\
O_{abcd}^{(2)} &= \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{(Q_L)_{ia}^c} (Q_L)_{jb} \overline{(u_R)_{kc}^c} (l_R)_d, \\
O_{abcd}^{(3)} &= \epsilon_{ijk} \epsilon_{\alpha\beta} \epsilon_{\gamma\rho} \overline{(Q_L)_{ia}^c} (Q_L)_{jb} \overline{(Q_L)_{kc}^c} (L_L)_{\rho d}, \\
O_{abcd}^{(4)} &= \epsilon_{ijk} (\tilde{\tau} \epsilon)_{\alpha\beta} \cdot (\tilde{\tau} \epsilon)_{\gamma\rho} \overline{(Q_L)_{ia}^c} (Q_L)_{jb} \overline{(Q_L)_{kc}^c} (L_L)_{\rho d}, \\
O_{abcd}^{(5)} &= \epsilon_{ijk} \overline{(d_R)_{ia}^c} (u_R)_{jb} \overline{(u_R)_{kc}^c} (l_R)_d, \\
O_{abcd}^{(6)} &= \epsilon_{ijk} \overline{(u_R)_{ia}^c} (u_R)_{jb} \overline{(d_R)_{kc}^c} (l_R)_d, \\
O_{abcd}^{(7)} &= \epsilon_{ijk} \overline{(u_R)_{ia}^c} (d_R)_{jb} \overline{(d_R)_{kc}^c} (\nu_R)_d, \\
O_{abcd}^{(8)} &= \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{(d_R)_{ia}^c} (Q_L)_{jb} \overline{(\nu_R)_c} (Q_L)_{k\beta d},
\end{aligned} \tag{10}$$

where i, j , and k are $SU(3)$ indices; α, β, γ , and ρ are $SU(2)$ indices; and a, b, c , and d are generation indices. From Eq. (10), using Fierz transformations, it is possible to obtain all vector and tensor Dirac matrices [20]. All operators in Eq. (10) are forbidden by the Z_{13} symmetry in Eq. (9); $d = 7$ operators formed with those of Eq. (10) and the singlet ϕ (or ϕ^*) are also forbidden. Notice that $O_{abcd}^{(2,5,6,7,8)}$ are also forbidden by Z_3 . However, there are others $B - L$ conserving operators allowed by all the symmetries of the model as

$$O_{abcd}^{(1)} \Phi_l^\dagger \Phi_u, \quad O_{abcd}^{(1)} \phi^4, \quad O_{abcd}^{(3,4)} \phi^5, \tag{11}$$

of $d = 8, 10, 11$, respectively, that may induce the proton decay, via four fermion interactions, after the spontaneous symmetry breaking. Let us write the proton lifetime as

$$\tau_P \propto \tau_P^5 \left(\frac{\alpha_5}{\alpha_U} \right) \left(\frac{M_U}{M_5} \right)^4 |\xi|^{-2}, \tag{12}$$

where $\tau_P^5 = M_5^4 \alpha_5^{-1} m_P^{-5}$ with M_5 as the unification scale in the context of the MSSM, $M_5 \simeq 2.1 \times 10^{16}$ GeV; α_5 is the respective coupling constant at that unification scale with $\alpha_5^{-1} = 24$; m_P is the proton mass; and α_U is the coupling constant at M_U in this model with $\alpha_U^{-1} = 38$; finally, ξ is a factor depending on the effective operator. Although the $d = 8$ operator is suppressed by $1/M_U^4$, after the spontaneous symmetry breaking, it induces a four fermion interaction proportional to $\xi = v_l^* v_u / M_U^2$. Since $|v_l v_u| \lesssim (246 \text{ GeV})^2$ we have that $|\xi| < 7.7 \times 10^{-23}$ and since $M_U/M_5 \simeq 1.3 \times 10^{-3}$, $\alpha_5/\alpha_U = 38/24 \simeq 1.6$, in Eq. (12) there is a factor $\gtrsim 8 \times 10^{32}$ with respect to τ_P^5 . The $d = 10$ operator is suppressed by $M_U^{-2} \Lambda^{-4}$ and it

induces four fermion interactions like $M_U^{-2} (v_\phi/\Lambda)^4 O^{(1)}$ where Λ is a mass scale connecting the field ϕ with the four fermion effective operators $O^{(i)}$, Λ may be M_U (or the PQ scale) or the Planck scale. In this case there is a factor $|\xi|^{-2} = (\Lambda/v_\phi)^8$ in Eq. (12). The enhancement on the proton lifetime depends on the scales Λ and v_ϕ . Assuming $v_\phi = 10^{12}$ GeV and $\Lambda = M_{\text{Planck}} = 10^{19}$ GeV, we have an enhancement factor of 5×10^{44} with respect to τ_P^5 . If, instead of M_{Planck} we use $\Lambda = M_U$ but $v_\phi = 10^9$ GeV we still obtain an enhancement factor 1.7×10^{24} in the proton lifetime. Finally, if $v_\phi = 10^{12}$ GeV and $\Lambda = M_U$ the proton lifetime is raised by a factor of 2 with respect to τ_P^5 . A similar analysis follows for the $d = 11$ effective operators. Hence, this model survives the proton decay problem since with the natural values of the parameters we have that the proton has a lifetime which is compatible with the no observation of its decays at the present experimental level. Moreover, notice that the $d = 5$ effective operator $M_U^{-1} LL \Phi_\nu \Phi_\nu$ is allowed by the Z_{13} symmetry but forbidden by Z_3 . However the $d = 10$ operator $M_U^{-1} \Lambda^{-5} LL \Phi_\nu \Phi_\nu \phi^5$ gives a Majorana mass to the neutrinos with an upper limit of 2 eV, obtained when $\Lambda = v_\phi$ and $\langle \Phi_\nu \rangle = 246$ GeV.

Summarizing, we have obtained a multi-Higgs extension of the standard model with $Z_{13} \otimes Z_3$ symmetries that imply an automatic PQ, B and L symmetries at the tree level. The axion is stabilized against semiclassical gravitational effects by those symmetries and they also stabilize the nucleon allowing, at the same time, the unification of the three gauge coupling constants at an energy near the PQ scale. Last but not least, the correct value of the weak mixing angle at the Z peak is obtained. Although we can always implement a larger Z_N by adding more matter multiplets, concerning the unification of the coupling constants, a larger number of multiplets or higher dimensional representation of $SU(2)$ affect the running of the couplings. Only a limited set of representations is allowed in this respect. We should mention that a unification scale near the PQ scale is also obtained in an $[SU(3)]^4$ model [12] but this model has no PQ symmetry in its minimal version. The present model cannot be supersymmetric at low energy (of the order of TeVs), since the fermion superpartners of the Higgs scalars would upset the unification of the gauge couplings; however it is possible to have supersymmetry if $\mu_{\text{SUSY}} \gtrsim M_U$. It would be interesting to search what sort of non-SUSY GUT embeds this model.

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