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Semi-analytical study of the rotational motion stability of artificial satellites using quaternions

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Abstract.

This study aims performing the stability analysis of the rotational motion to artificial satellites using quaternions to describe the satellite attitude (orientation on the space). In the system of rotational motion equations, which is composed by four kinematic equations of the quaternions and by the three Euler equations in terms of the rotational spin components. The influence of the gravity gradient and the direct solar radiation pressure torques have been considered. Equilibrium points were obtained through numerical simulations using the softwares Matlab and Octave, which are then analyzed by the Routh-Hurwitz Stability Criterion.

1. Introduction

In this paper the stability analysis of the artificial satellite rotational motion is performed by applying the Routh-Hurwitz Criterion (RHC). This criterion allows the investigating of the static stability for systems of equations using the coefficients of the characteristic equation associated with the linearized system (in this paper, the system of equations for the rotational motion), without the need of determining the characteristic equation roots. Quaternions are used to describe the satellite spatial orientation (attitude) and the gravity gradient torque and the solar radiation torque are included in dynamic equations of rotational motion. These torques are discussed in section 3 and their components are determined using rotational matrices composed by the quaternion attitude. For the RHC application, the motion equations are linearized around each equilibrium point determined by an implemented program using the softwares Matlab and Octave. Applications are performed for satellites with similar characteristics to the American satellite PEGASUS and to the Brazilian satellites SCD-1 and SCD-2. These Brazilian satellites are spin-stabilized and the rotation axis is aligned in the direction of the greatest satellite principal moment of inertia axis. The RHC checks the static stability of motion, no matter how it's motion, but whether it oscillates in the desired condition. The eigenvalues from the Jacobian matrix associated to the linearized system are also obtained in order to verify if the real parts of eigenvalues were nulls or negatives and if the imaginary parts were not zero. In order to investigate the range around the equilibrium points, there are introduced small variations in the principal moments of inertia of the satellite. In this paper is assumed that the satellite is illuminated all the time.



2. Attitude Representation and Motion Equations

2.1. Quaternion Attitude

Quaternions were introduced by Hamilton in 1843 and the main idea was introduced as the extension of the complex numbers. They form a non-commutative algebra in the space \mathbb{R}^4 . Nowadays, quaternions are used in several research areas like Theory of Numbers, Theoretical Physics, High Energy Physics, Differential Geometry, Applied Mathematics and Aerospace Engineering. In the context of the space research, they can be used to represent the spatial rotations in a compact and convenient form. For more details, see [2, 5, 13]. If Φ is the rotation angle and $(n_1 \ n_2 \ n_3)^t$ is the unit vector aligned to the rotation axis, with t indicating the transposed matrix, the rotation quaternion is a vector (4x1) defined by [9, 13]

$$Q = (\vec{Q} \ q_4)^t = (q_1 \ q_2 \ q_3 \ q_4)^t \quad (1)$$

with $q_1 = n_1 \sin(\Phi/2)$, $q_2 = n_2 \sin(\Phi/2)$, $q_3 = n_3 \sin(\Phi/2)$ and $q_4 = \cos(\Phi/2)$, where $\vec{Q} = (q_1 \ q_2 \ q_3)$ is the quaternion vectorial component and q_4 the scalar quaternion component. The main properties of quaternions are its unitary module and the attitude matrix corresponding to a rotation Φ about the axis $\hat{n} = (n_1 \ n_2 \ n_3)^t$. This matrix can be seen in [9, 15]

2.2. Kinematic and Euler Equations

The rotational motion equations are given by the dynamic and kinematic equations. The dynamic equations for the artificial satellite rotational motion are described on satellite principal-axis-of-inertia system (Oxyz) which is called in this paper as Rotating System and is given by [9, 15]

$$I_x \dot{p} = N_x + (I_y - I_z)qr \quad (2)$$

$$I_y \dot{q} = N_y + (I_z - I_x)pr \quad (3)$$

$$I_z \dot{r} = N_z + (I_x - I_y)pq \quad (4)$$

where I_x , I_y e I_z are the principal moments of inertia of the satellite; p , q e r are the spin velocity components on Rotating System; N_x , N_y e N_z are the resulting torque components due to the external torques on Rotating System. The time dependence for the quaternion components are given by the kinematic equations [11]

$$\dot{q}_1 = 0.5[pq_4 - qq_3 + rq_2] \quad (5)$$

$$\dot{q}_2 = 0.5[qq_4 - rq_1 + pq_3] \quad (6)$$

$$\dot{q}_3 = 0.5[rq_4 - pq_2 + qq_1] \quad (7)$$

$$\dot{q}_4 = -0.5[pq_1 - qq_2 + rq_3] \quad (8)$$

By theses equations there are not any singularities associated to the null denominator or indefinitions in the variables.

3. External torques

3.1. Gravity Gradient Torque

The gravitational force on one part of the spacecraft is different from the another, because the magnitude of the Earth gravitational force varies with the inverse of the square of the distance from the geocenter and this difference result in a torque called gravity gradient torque (GGT). The GGT model adopted in this paper is a classic one used in previous papers. Some applications were developed like analytical propagation of the rotational motion with Andoyer variables [12, 14] and numerical propagation with the quaternions [15]. The GGT is also included

in stability analysis [7, 12]. The GGT model [9, 13] for a circular cylinder shape satellite is given by

$$\vec{N}_G = 3 \frac{\mu}{r^3} [a_{21}a_{31}(I_z - I_y)\vec{i} + a_{11}a_{31}(I_x - I_z)\vec{j} + a_{11}a_{21}(I_y - I_x)\vec{k}] \quad (9)$$

where the elements a_{11} , a_{21} e a_{31} are given in terms of the attitude quaternions and orbital angular elements [10]. Therefore, if the satellite has a homogeneous mass distribution (equipments on board) or a spherical symmetry around of the center of mass (which is equivalent to $I_x = I_y = I_z$), the influence of this torque is null; otherwise, its influence becomes significant. The GGT can be used for the stabilization of satellites, for which it is necessary a good precision in its spatial orientation. This is possible due to the existence of two stable equilibrium points, which are lagged in an angle of 180° [3]. This torque decreases with the cube of altitude, making the gravity gradient stabilization of satellites in high altitude orbits unviable¹, due to the small magnitude of this torque.

3.2. Solar Radiation Torque

The Sun and the Earth are the most important radiation sources in space. The provided radiation from the Earth has two significant portions: the solar radiation reflected diffusely on the Earth surface (terrestrial albedo), and the terrestrial radiation, that is the spontaneous emission in the infrared range, proportional to the fourth power of the absolute terrestrial surface temperature. The albedo and the terrestrial radiation are respectively 90% and 93% smaller (at 700 km of altitude) than the direct solar radiation (at 700 km of altitude). Due to their magnitudes, we can ignore their contributions and then we can consider only the direct solar radiation as a good approximation for the total solar radiation received by the satellite surface. As the satellite-Sun distance is much greater than the satellite-Earth distance, this torque is practically independent of the satellite altitude for the satellites in little geocentric altitude [13]. The solar direct radiation pressure force is created by the continuous photon collisions with the satellite surface. The satellite surface can be able to absorb or to reflect this flow. The direct solar radiation pressure force arise from the total change of the momentum of all the incident photons on the satellite surface and it can produce a torque known as direct solar radiation pressure torque (DSRPT). It can cause considerable disturbances in spacecraft orbits when the satellite has certain characteristics such as great ratio, high altitude and great solar panels. The model for the DSRPT as developed by [14] has z component null ($\bar{N}_z = 0$) and the x and y components are

$$\bar{N}_x = -\frac{\bar{K}}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2\{a_s^2 R_y R_z + a_s r'(R_y a_{31} + R_z a_{21}) + r'^2 a_{21} a_{31}\} \quad (10)$$

$$\bar{N}_y = \frac{\bar{K}}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2\{a_s^2 R_x R_z + a_s r'(R_x a_{31} + R_z a_{11}) + r'^2 a_{11} a_{31}\} \quad (11)$$

where \bar{K} is a solar parameter and assume the value² $1.01 \times 10^{17} \text{ kgm/s}$, a_s is Sun-Earth distance and here assume the value $1.49597870 \times 10^{11} \text{ m}$, r' is the satellite geocentric distance, R_s is the Sun-satellite distance, β_i e γ_i , $i=1,2$, are the specular and total reflection coefficients, respectively, for each satellite surface (which assume constant values), a_{11} , a_{21} e a_{31} are direction cosines which relate the Orbital System and the Satellite System (which are composed by the quaternions [10]) and R_x , R_y and R_z give the Sun direction in the Satellite System.

¹ For example: the geostationary orbits

² According G M Georgevic: "The Solar Radiation Pressure Force and Torques Model", The Jour. Astron. Scienc., XX, 5, p. 257 (1973).

4. Algorithm for Stability Analysis

The chosen criterion for a preliminary stability analysis of the rotational motion is the Routh-Hurwitz Criterion (RHC) [4, 8], which allows for investigating the systems of equations static stability using the coefficients of the characteristic equation associated with the linearized system, without the need to determine the roots of this characteristic equation. For applying the RHC is required the linearization of the system of motion equations, which corresponds here, the three dynamic equations in terms of spin components, where are included GGT and DSRPT components, and the four kinematic equations related to the four quaternion attitude components [15]. The procedure for RHC is described below: 1) Write the characteristic equation for the linearized system of equations as follow [10]

$$a_n \lambda^n + a_{n-1} \lambda^{n-1} + \dots + a_2 \lambda^2 + a_1 \lambda + a_0 = 0 \quad (12)$$

In this paper, the coefficients a_l , $l = n, n-1, \dots, 0$ are dependent on the satellite moments of inertia, the semimajor axis of the orbit, the longitude of the ascending node, the argument of pericenter, the orbital inclination, and initial conditions for the attitude quaternions, the true anomaly and the spin velocity. 2) If one coefficient is zero or negative in the presence of another positive coefficient, then there is at least one root with positive real part, and then the system is not stable [4]. 3) If all coefficients are positive, must be constructed a table (Routh table), whose elements are associated with the coefficients of the characteristic equation in rows and columns: with a_{n+2-2j} e a_{n+1-2j} being the characteristic equation coefficients³ [10]. In

$\begin{matrix} \diagdown \\ j \\ k = i - 1 \end{matrix}$	1	2	3	...	j
0	$\lambda^n : a_n$	a_{n-2}	a_{n-4}	a_{n+2-2j}
1	$\lambda^{n-1} : a_{n-1}$	a_{n-3}	a_{n-5}	a_{n+1-2j}
2	$\lambda^{n-2} : b_{n-2}$	b_{n-4}	b_{n-6}	b_{n-2j}
3	$\lambda^{n-3} : c_{n-3}$	c_{n-5}	c_{n-7}	c_{n-1-2j}
\vdots	\vdots	\vdots	\vdots		\vdots
k	$\lambda^{n-k} : Z_{n-k}$	Z_{n-k-2}	Z_{n-k-4}	$Z_{n-k+2-2j}$
\vdots	\vdots	\vdots	\vdots		\vdots
n - 4	$\lambda^4 : \alpha_4$	α_2	α_0		
n - 3	$\lambda^3 : \beta_3$	β_1			
n - 2	$\lambda^2 : \gamma_2$	γ_0			
n - 1	$\lambda^1 : \delta_1$				
n	$\lambda^0 : \varepsilon_0$				

Figure 1. Scheme for the Routh table general form. [10]

this paper are considered seven variables (the four quaternions components and the three spin velocity components) and the Routh table for the case $n=7$ variables is represented in the scheme in Figure 2 [10].

5. Results obtained by numerical simulations

The RHC was applied to analyze the rotational motion stability of three satellites: 1) A medium sized satellite (MS) with similar data⁴ to the north-American satellite PEGASUS. 2) Two small sized satellite (SS-1 e SS-2) with similar data to the Brazilian Data Collection satellites, SCD1 and SCD2. Simulations were performed for each satellite, using the softwares Matlab and Octave, considering different initial conditions for spin velocity and quaternions and for the satellites in

³ The Routh table elements can be seen with details in [10]

⁴ These satellites data can be seen in [10]

$$\begin{aligned}
\lambda^7 &: a_7 \quad a_5 \quad a_3 \quad a_1 \\
\lambda^6 &: a_6 \quad a_4 \quad a_2 \quad a_0 \\
\lambda^5 &: b_5 \quad b_3 \quad b_1 \\
\lambda^4 &: c_4 \quad c_2 \quad c_0 \\
\lambda^3 &: d_3 \quad d_1 \\
\lambda^2 &: e_2 \quad e_0 \\
\lambda^1 &: f_1 \\
\lambda^0 &: g_0
\end{aligned}$$

Figure 2. Scheme for the Routh table in the case $n=7$ variables. [10]

Satellite	10^{-5}	10^{-6}	10^{-7}	10^{-9}	Total
MS	9	2	2	1	14

Table 1. Order of magnitude of the eigenvalues with real part less than 10^{-5} .

Real Part	Imaginary Part
1.251×10^{-4}	1.610×10^{-3}
1.251×10^{-4}	-1.610×10^{-3}
1.060×10^{-3}	0
-7.029×10^{-4}	7.373×10^{-4}
-7.029×10^{-4}	-7.373×10^{-4}
4.800×10^{-5}	7.448×10^{-4}
4.800×10^{-5}	-7.448×10^{-4}

Table 2. Eigenvalues associated to the MS equilibrium point x_0 .

different positions around their orbits (values for the true anomaly and the initial rotation angle Φ were 0° , 90° , 180° and 270°). The results did not show any stable equilibrium points by the RHC. However, analyzing the eigenvalues associated to the obtained equilibrium points, it is possible to observe many eigenvalues with a very small order of magnitude in their real part (values less than 10^{-5}). These ones produced solutions with small changes during long times⁵. There were obtained 14 points for MS, 56 points for SS1 and 49 points for SS2 with theses characteristics. The results for the satellite MS are presented in Table 1.

The values for the first column of the Routh table for each equilibrium point were obtained by numerical simulations. To illustrate this application for the MS equilibrium point $x_0 = (p, q, r, q_1, q_2, q_3, q_4) = (6.534 \times 10^{-4}, 6.121 \times 10^{-4}, 6.140 \times 10^{-4}, 6.019 \times 10^{-1}, 9.001 \times 10^{-2}, 9.001 \times 10^{-1}, 6.079 \times 10^{-12})$, we obtained the following first column elements: $a_7 = 1.000$, $a_6 = 4.499 \times 10^{-18}$, $b_5 = -2.281 \times 10^7$, $c_4 = -2.290 \times 10^{-10}$, $d_3 = 1.133 \times 10^{-10}$, $e_2 = -2.690 \times 10^{-15}$, $f_1 = 3.190 \times 10^{-19}$ and $g_0 = -1.597 \times 10^{-21}$. The eigenvalues for this equilibrium point are presented in Table 2, where we observed the small order for the eigenvalues real part (see footnote 5).

In order to investigate the range around the equilibrium points, there are introduced small variations in the principal moments of inertia of the satellite [6]. For each initial moment of inertia (I_{x0}, I_{y0}, I_{z0}) were introduced variation between $0.9I_{x0}$ and $1.1I_{x0}$, $0.9I_{y0}$ and $1.1I_{y0}$ and

⁵ Eigenvalues with positives real part equal to 10^{-4} , 10^{-5} , 10^{-6} , 10^{-7} and 10^{-9} produce an increase of approximately 2.7 times in periods of 2.78 hours, 27.80 hours, 11.60 days, 3.86 months and 3.17 years respectively

$0.9I_{z0}$ and $1.1I_{z0}$. There were investigate 305 equilibrium points in order to get RHC stable points. Various RHC stable points were obtained: 647 for MS, 297 for SS-1 and 647 for SS-2. For example, for the MS satellite with $I_{x0}=3.1599\times 10^5$ Kg.m² and $I_{y0}=3.6461\times 10^5$ Kg.m², we have three RHC equilibrium points with $I_{z0}=9.2754\times 10^4$ Kg.m², 1.0175×10^5 Kg.m² and 1.1071×10^5 Kg.m².

6. Conclusions

In this paper is presented an approach to analyze the stability of rotational motion by the Routh-Hurwitz Criterion. Many simulations were developed and RHC stable regions were obtained around the equilibrium points. The goal of this paper is the use of quaternions in determining the DSRPT and GGT components on Rotating System for the stability analysis. This approach is valid for cylindrical satellites in the illuminated phase of the trajectory and can be useful for analyzing the stability of future missions of the Brazilian space program.

However due the equations of rotational motion are non linearized and others stability analysis methods could be applied in order to get better results. We can highlight among them the Phase Space Method, the Liapunov Direct Method and the Kovalev-Savchenko Theorem [7,12]. Others approaches could include the Earth's shadow in the DSRPT and also include the others external torques.

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