# Complex scalar dark matter in a $\boldsymbol{B}$ - $L$ model 

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#### Abstract

In this work, we implement a complex scalar dark matter (DM) candidate in a $U(1)_{B-L}$ gauge extension of the Standard Model. The model contains three right-handed neutrinos with different quantum numbers and a rich scalar sector, with extra doublets and singlets. In principle, these extra scalars can have vacuum expectation values ( $V_{\Phi}$ and $V_{\phi}$ for the extra doublets and singlets, respectively) belonging to different energy scales. In the context of $\zeta \equiv \frac{V_{\phi}}{V_{\phi}} \ll 1$, which allows one to obtain naturally light active neutrino masses and mixing compatible with neutrino experiments, the DM candidate arises by imposing a $Z_{2}$ symmetry on a given complex singlet, $\phi_{2}$, in order to make it stable. After doing a study of the scalar potential and the gauge sector, we obtain all the DM-dominant processes concerning the relic abundance and direct detection. Then, for a representative set of parameters, we find that a complex DM with mass around 200 GeV , for example, is compatible with the current experimental constraints without resorting to resonances. However, additional compatible solutions with heavier masses can be found in vicinities of resonances. Finally, we address the issue of having a light $C P$-odd scalar in the model showing that it is safe concerning the Higgs and the $Z_{\mu}$-boson invisible decay widths, and also astrophysical constraints regarding energy loss in stars.


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## I. INTRODUCTION

Currently, it is well established from several observations and studies on different scales that most of the Universe's mass consists of dark matter (DM) [1-5]. Although the nature of DM is still a challenging question, the solution based on the existence of new kinds of neutral, stable and weakly interacting massive particles (WIMPs) is both well motivated and extensively studied. This is mainly due to two reasons. The first reason is that WIMPs appearing in a plethora of models [6-16] "naturally" give the observed relic abundance, $\Omega_{\mathrm{DM}} h^{2}=0.1199 \pm 0.0027$ [5]. The second reason is that WIMPs may be accessible to direct detection. Currently, there is a variety of experiments involved in the search for direct signals of WIMPs which have imposed bounds on spin-independent WIMP-nucleon elastic scattering [17-19].

It is also well known that the Standard Model (SM)despite being tremendously successful in describing electroweak and strong interaction phenomena-must be extended. Physics beyond the SM has both theoretical and experimental motivations. For instance, the neutrino masses and mixing-which are required for a consistent explanation of the solar and atmospheric neutrino

[^0]anomalies-are some of the most compelling reasons to go beyond the SM. Another motivation is providing a satisfactory explanation of the nature of DM. This last reason is the focus of our work. The preferred theoretical framework which provides a DM candidate is supersymmetry [6-9]. However, many other interesting scenarios have been proposed [10-16]. In this paper, we focus on the possibility of having a viable scalar DM candidate in a $\mathrm{U}(1)$ gauge extension of the SM. In particular, this model, sometimes referred as the flipped $B-L$ model $[20,21]$ has a very rich scalar content, which allows us to obtain a complex scalar DM candidate.

The paper is organized as follows. In Sec. II we briefly summarize the model under consideration. In Sec. III we study the vacuum structure and the scalar sector spectrum that allows us to have a viable complex scalar DM candidate in the model. In particular, we consider the scalar potential in the context of $\zeta \equiv \frac{V_{\Phi}}{V_{\phi}} \ll 1$, where $V_{\Phi}$ and $V_{\phi}$ are vacuum expectation values (VEVs) of the doublets $\Phi_{1,2}$ and singlets $\phi_{1,3, X}$, respectively. In Sec. IV we present the gauge sector and choose some parameters that simplify the study of the DM candidates. In Sec. V we calculate the thermal relic density of the complex scalar DM candidate and present a set of parameters that are consistent with the current observations. In Sec. VI we summarize the main features of our study. Finally, in the Appendix, we show the general minimization conditions used to calculate the scalar mass spectrum.

## II. BRIEF REVIEW OF THE B - L MODEL

We briefly summarize here the model from Refs. [20,21]. It is an extension of the SM based on the gauge symmetry $\mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y^{\prime}} \otimes \mathrm{U}(1)_{B-L}$, where $B$ and $L$ are the usual baryonic and leptonic numbers, respectively, and $Y^{\prime}$ is a new charge different from the hypercharge $Y$ of the SM. The values of $Y^{\prime}$ are chosen to obtain the hypercharge $Y$ through the relation $Y=\left[Y^{\prime}+(B-L)\right]$, after the first spontaneous symmetry breaking. Assuming a generationindependent charge assignment and the nonexistence of mirror fermions, and by restricting ourselves to integer quantum numbers for the $Y^{\prime}$ charge, the anomaly cancellation constrains the number of right-handed neutrinos, $n_{R} \geq 3$ [20]. Considering $n_{R}=3$, there is an exotic charge assignment for the $Y^{\prime}$ charge where $Y_{n_{R 1}, n_{R 2}}^{\prime}=-4$ and $Y_{n_{R 3}}^{\prime}=5$ besides the usual one where $Y_{n_{R i}}^{\prime}=1$ with $i=1,2,3$. The model under consideration has this exotic $Y^{\prime}$ charge assignment. The respective fermionic charge assignment of the model is shown in Table I.
In the scalar sector the model has three $\operatorname{SU}(2)_{L}$ doublets, $H, \Phi_{1}, \Phi_{2}$, and four $\operatorname{SU}(2)_{L}$ singlets, $\phi_{1}, \phi_{2}$, $\phi_{3}, \phi_{X}$. The scalar charge assignments are shown in Table II. The $H$ doublet is introduced to give mass to the lighter massive neutral vector boson $Z_{1 \mu}$, the charged vector bosons $W_{\mu}^{ \pm}$, and the charged fermions, as in the SM. Besides giving mass to the extra neutral vector boson $Z_{2 \mu}$, which is expected to be heavier than $Z_{1 \mu}$, the other scalars are mainly introduced to generate mass for both the left- and the right-handed neutrinos. In order to be more specific, the other doublets $\Phi_{1}$ and $\Phi_{2}$ are introduced to

TABLE I. Quantum number assignment for the fermionic fields.

| Fermion | $I_{3}$ | $I$ | $Q$ | $Y^{\prime}$ | $B-L$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\nu_{e L}, e_{L}$ | $1 / 2,-1 / 2$ | $1 / 2$ | $0,-1$ | 0 | -1 |
| $e_{R}$ | 0 | 0 | -1 | -1 | -1 |
| $u_{L}, d_{L}$ | $1 / 2,-1 / 2$ | $1 / 2$ | $2 / 3,-1 / 3$ | 0 | $1 / 3$ |
| $u_{R}$ | 0 | 0 | $2 / 3$ | 1 | $1 / 3$ |
| $d_{R}$ | 0 | 0 | $-1 / 3$ | -1 | $1 / 3$ |
| $n_{1 R}, n_{2 R}$ | 0 | 0 | 0 | 4 | -4 |
| $n_{3 R}$ | 0 | 0 | 0 | -5 | 5 |

TABLE II. Quantum number assignment for the scalar fields.

| Scalar | $I_{3}$ | $I$ | $Q$ | $Y^{\prime}$ | $B-L$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $H^{0,+}$ | $\mp 1 / 2$ | $1 / 2$ | 0,1 | 1 | 0 |
| $\Phi_{1}^{0,-}$ | $\pm 1 / 2$ | $1 / 2$ | $0,-1$ | -4 | 3 |
| $\Phi_{2}^{0,-}$ | $\pm 1 / 2$ | $1 / 2$ | $0,-1$ | 5 | -6 |
| $\phi_{1}$ | 0 | 0 | 0 | -8 | 8 |
| $\phi_{2}$ | 0 | 0 | 0 | 10 | -10 |
| $\phi_{3}$ | 0 | 0 | 0 | 1 | -1 |
| $\phi_{\mathrm{X}}$ | 0 | 0 | 0 | 3 | -3 |

give Dirac mass terms at tree level through the renormalizable Yukawa interactions $\mathcal{D}_{i m} \bar{L}_{L i} n_{R m} \Phi_{1}$ and $\mathcal{D}_{i 3} \bar{L}_{L i} n_{R 3} \Phi_{2}$ in the Lagrangian. The $\phi_{1}, \phi_{2}$, and $\phi_{3}$ singlets are introduced to generate Majorana mass terms at tree level $\left[\mathcal{M}_{m n} \overline{\left(n_{R m}\right)^{c}} n_{R n} \phi_{1}, \quad \mathcal{M}_{33} \overline{\left(n_{R 3}\right)^{c}} n_{R 3} \phi_{2}\right.$, $\left.\mathcal{M}_{m 3} \overline{\left(n_{R m}\right)^{c}} n_{R 3} \phi_{3}\right]$. Finally, the $\phi_{X}$ singlet is introduced to avoid dangerous Majorons when the symmetry is broken down, as shown in Ref. [21]. These extra scalars allow the model to implement a seesaw mechanism at the $\mathcal{O}(\mathrm{TeV})$ energy scale, and the observed mass-squared differences of the neutrino are obtained without resorting to fine-tuning the neutrino Yukawa couplings [21]. Other studies regarding the possibility that the model accommodates different patterns for the neutrino mass matrix using discrete symmetries ( $S_{3}, A_{4}$ ) have been done [22,23].

With the above matter content we can write the most general Yukawa Lagrangian respecting the gauge invariance as follows:

$$
\begin{align*}
-\mathcal{L}_{\mathrm{Y}}= & Y_{i}^{(l)} \bar{L}_{L i} e_{R i} H+Y_{i j}^{(d)} \bar{Q}_{L i} d_{R j} H+Y_{i j}^{(u)} \bar{Q}_{L i} u_{R j} \tilde{H} \\
& +\mathcal{D}_{i m} \bar{L}_{L i} n_{R m} \Phi_{1}+\mathcal{D}_{i 3} \bar{L}_{L i} n_{R 3} \Phi_{2} \\
& +\mathcal{M}_{m n} \overline{\left(n_{R m}\right)^{c}} n_{R n} \phi_{1}+\mathcal{M}_{33} \overline{\left(n_{R 3}\right)^{c}} n_{R 3} \phi_{2} \\
& +\mathcal{M}_{m 3} \overline{\left(n_{R m}\right)^{c}} n_{R 3} \phi_{3}+\text { H.c. }, \tag{1}
\end{align*}
$$

where $i, j=1,2,3$ are lepton/quark family numbers, $m, n=1,2$, and $\tilde{H}=i \tau_{2} H^{*}$ ( $\tau_{2}$ is the Pauli matrix). Also, we have omitted summation symbols over repeated indices.

From the Lagrangian in Eq. (1) we see that quarks and leptons obtain masses from the VEV of just one Higgs, $H$; thus, the Higgs interactions with quarks and leptons are diagonalized by the same matrices that diagonalize the corresponding mass matrices. In this case the neutral interactions are diagonal in flavor and there is no flavorchanging neutral current in the quark and lepton sector. This particular feature remains if we change from the symmetry basis to the mass eigenstate basis [24,25]. On the other hand, the terms proportional to $\mathcal{D}_{i m}$ and $\mathcal{D}_{i 3}$ can induce lepton flavor violation (LFV) at the loop level, due to the couplings of the charged leptons with right-handed neutrinos and charged scalars, coming from the doublets $\Phi_{1}$ and $\Phi_{2}$. We have already studied this issue in a previous work [21]. For the parameters we are using, the model is safe regarding the kinematically allowed LFV decays of the form $l_{i} \rightarrow l_{j}+\gamma$, where $i=2,3=\mu, \tau$ and $j=1,2=e, \mu$, respectively. In particular, we can give an estimate for the branching ratios $B(\mu \rightarrow e+\gamma)$ and $B(\tau \rightarrow \mu+\gamma)$ for $\mathcal{D}_{e i} \simeq 0.01, \quad \mathcal{D}_{\mu i} \simeq 0.6, \quad \mathcal{D}_{\tau i} \simeq 0.9$, $m_{n_{R}} \simeq 1 \mathrm{TeV}$, and $m_{\Phi^{ \pm}}=380 \mathrm{GeV}$, obtaining $\simeq 1.1 \times$ $10^{-12}$ and $\simeq 9.1 \times 10^{-9}$, respectively, in agreement with the experimental data [26,27]. The model does not present other sources of flavor violation since the interactions with neutral vector bosons are also diagonal.

Finally, the most general renormalizable scalar potential obtained by the addition of all the above-mentioned scalar fields is given by

$$
\begin{align*}
V_{B-L}= & -\mu_{H}^{2} H^{\dagger} H+\lambda_{H}\left|H^{\dagger} H\right|^{2}-\mu_{11}^{2} \Phi_{1}^{\dagger} \Phi_{1}+\lambda_{11}\left|\Phi_{1}^{\dagger} \Phi_{1}\right|^{2}-\mu_{22}^{2} \Phi_{2}^{\dagger} \Phi_{2}+\lambda_{22}\left|\Phi_{2}^{\dagger} \Phi_{2}\right|^{2}-\mu_{s \alpha}^{2}\left|\phi_{\alpha}\right|^{2}+\lambda_{s \alpha}\left|\phi_{\alpha}^{*} \phi_{\alpha}\right|^{2} \\
& +\lambda_{12}\left|\Phi_{1}\right|^{2}\left|\Phi_{2}\right|^{2}+\lambda_{12}^{\prime}\left(\Phi_{1}^{\dagger} \Phi_{2}\right)\left(\Phi_{2}^{\dagger} \Phi_{1}\right)+\Lambda_{H \gamma}|H|^{2}\left|\Phi_{\gamma}\right|^{2}+\Lambda_{H \gamma}^{\prime}\left(H^{\dagger} \Phi_{\gamma}\right)\left(\Phi_{\gamma}^{\dagger} H\right)+\Lambda_{H s \alpha}|H|^{2}\left|\phi_{\alpha}\right|^{2}+\left.\Lambda_{\gamma \alpha}^{\prime}\left|\Phi_{\gamma}\right|\right|^{2}\left|\phi_{\alpha}\right|^{2} \\
& +\Delta_{\alpha \beta}\left(\phi_{\alpha}^{*} \phi_{\alpha}\right)\left(\phi_{\beta}^{*} \phi_{\beta}\right)+\left[\beta_{123} \phi_{1} \phi_{2}\left(\phi_{3}^{*}\right)^{2}+\Phi_{1}^{\dagger} \Phi_{2}\left(\beta_{13} \phi_{1} \phi_{3}^{*}+\beta_{23} \phi_{2}^{*} \phi_{3}\right)-i \kappa_{H 1 X} \Phi_{1}^{T} \tau_{2} H \phi_{X}\right. \\
& \left.-i \kappa_{H 2 X}\left(\Phi_{2}^{T} \tau_{2} H\right)\left(\phi_{X}^{*}\right)^{2}+\beta_{X}\left(\phi_{X}^{*} \phi_{1}\right)\left(\phi_{2} \phi_{3}\right)+\beta_{3 X}\left(\phi_{X}^{*} \phi_{3}^{3}\right)+\text { H.c. }\right], \tag{2}
\end{align*}
$$

where $\gamma=1,2 ; \alpha, \beta=1,2,3, X$; and $\alpha \neq \beta$ in the $\Delta_{\alpha \beta}\left(\phi_{\alpha}^{*} \phi_{\alpha}\right)\left(\phi_{\beta}^{*} \phi_{\beta}\right)$ terms.

## III. THE VACUUM STRUCTURE AND THE SCALAR SECTOR SPECTRUM

In general, DM must be stable in order to provide a relic abundance in agreement with that measured by WMAP and Planck, $\Omega_{\mathrm{DM}} h^{2}=0.1199 \pm 0.0027$ [4,5]. Although the DM stability could result from the extreme smallness of its couplings to ordinary particles, we restrict ourselves to searching for a discrete (or continuous) symmetry, such as $Z_{2}$, or $\mathrm{U}(1)$, to protect DM candidates from decaying.

First, we consider the scalar potential in Eq. (2) by looking for an accidental symmetry that naturally stabilizes the DM candidate. Doing so, we find that the scalar potential has just the $\mathrm{SU}(2) \otimes \mathrm{U}(1)_{Y^{\prime}} \otimes \mathrm{U}(1)_{B-L}$ initial symmetry. However, none of these gauge groups can generate a stable neutral scalar when they are spontaneously broken down to $\mathrm{U}(1)_{Q}$. Therefore, we impose a discrete symmetry in the following way: $Z_{2}\left(\phi_{2}\right)=-\phi_{2}$, and the other scalar fields are even under this $Z_{2}$ symmetry. As a result, the $\beta_{23} \Phi_{1}^{\dagger} \Phi_{2} \phi_{2}^{*} \phi_{3}, \beta_{123} \phi_{1} \phi_{2}\left(\phi_{3}^{*}\right)^{2}$ and $\beta_{X}\left(\phi_{X}^{*} \phi_{1}\right)\left(\phi_{2} \phi_{3}\right)$ terms are prohibited from appearing in the scalar potential (2). Actually, when these terms are eliminated from Eq. (2), the true global symmetry in the potential is $\mathrm{SU}(2) \otimes \mathrm{U}(1)_{Y^{\prime}} \otimes \mathrm{U}(1)_{B-L} \otimes \mathrm{U}(1)_{\chi}$, where the last one is $\mathrm{U}(1)_{\chi}: \phi_{2} \rightarrow \exp \left(-i \chi_{\phi_{2}}\right) \phi_{2}$, where $\chi_{\phi_{2}}$ is the $\phi_{2}$ quantum number under the $\mathrm{U}(1)_{\chi}$ symmetry, and the rest of the fields are invariant. It is important to note that we have taken into account the simplicity criterion and some phenomenological aspects when choosing the $Z_{2}$ symmetry above. For example, if we impose $Z_{2}\left(\phi_{1}\right)=-\phi_{1}$ (with the other fields being even), the model has a massless righthanded neutrino, say $N_{R}$, at tree level. This poses a tension with the experimental data of the invisible $Z_{\mu}$ decay width [28], since $Z_{\mu} \rightarrow \bar{N}_{R}+N_{R}$ would be allowed to exist [29]. Other simple choices, such as $Z_{2}\left(\phi_{3}\right)=-\phi_{3}$ or $Z_{2}\left(\phi_{X}, \Phi_{1}\right)=-\phi_{X},-\Phi_{1}$, should be avoided due to the appearance of Majorons, $J s$, in the scalar spectra. As is well known, the major challenges to models with Majorons come from the energy loss in stars, through the process $\gamma+e^{-} \rightarrow e^{-}+J$, and the invisible $Z_{\mu}$ decay width, through $Z_{\mu} \rightarrow R J \rightarrow J J J$, where $R$ is a scalar [30].

For the general case of the scalar potential with the $\mathrm{U}(1)_{\chi}$ symmetry, we have the minimization conditions given in the Appendix. In general, these conditions lead to different symmetry-breaking patterns and to a complex vacuum structure because the scalar potential has several free parameters. In this paper, however, we restrict ourselves to find a (or some) viable scalar DM candidate(s) and to study its (their) properties in a relevant subset of the parameter space.

First, we impose the conditions necessary for all real neutral components of the scalar fields (except $\phi_{2 R}$ ) to obtain nontrivial VEVs, i.e., $\left\langle H_{R}^{0}\right\rangle=V_{H},\left\langle\Phi_{1 R}^{0}\right\rangle=V_{\Phi_{1}}$, $\left\langle\Phi_{2 R}^{0}\right\rangle=V_{\Phi_{2}}, \quad\left\langle\phi_{1 R}\right\rangle=V_{\phi_{1}}, \quad\left\langle\phi_{2 R}\right\rangle=0, \quad\left\langle\phi_{3 R}\right\rangle=V_{\phi_{3}}$, and $\left\langle\phi_{X R}\right\rangle=V_{\phi_{X}}$. For the sake of simplicity, we set $V_{\Phi_{1}}=V_{\Phi_{2}}=V_{\Phi}$ and $V_{\phi_{1}}=V_{\phi_{3}}=V_{\phi_{X}}=V_{\phi}$. Thus, the $U(1)_{\chi}$ symmetry is not spontaneously broken and the model possesses two neutral stable scalars which are the real $(C P$-even) and imaginary ( $C P$-odd) parts of the $\phi_{2}$ field with the same mass, which are given by

$$
\begin{align*}
M_{\mathrm{DM}}^{2}= & \frac{1}{2}\left[\Lambda_{H s 2} V_{\mathrm{SM}}^{2}+\left(\Lambda_{12}^{\prime}+\Lambda_{22}^{\prime}-2 \Lambda_{H s 2}\right) V_{\Phi}^{2}\right. \\
& \left.+\left(\Delta_{12}+\Delta_{23}+\Delta_{2 X}\right) V_{\phi}^{2}-2 \mu_{s 2}^{2}\right] \tag{3}
\end{align*}
$$

where we have defined $V_{S M}^{2} \equiv V_{H}^{2}+V_{\Phi_{1}}^{2}+V_{\Phi_{2}}^{2}=$ $V_{H}^{2}+2 V_{\Phi}^{2}=(246)^{2} \mathrm{GeV}^{2}$. From here on, we work with $M_{\mathrm{DM}}^{2}$ as an input parameter, and thus we solve Eq. (3) for $\mu_{s 2}^{2}$,

$$
\begin{align*}
\mu_{s 2}^{2}= & \frac{1}{2}\left[\Lambda_{H s 2} V_{\mathrm{SM}}^{2}+\left(\Lambda_{12}^{\prime}+\Lambda_{22}^{\prime}-2 \Lambda_{H s 2}\right) V_{\Phi}^{2}\right. \\
& \left.+\left(\Delta_{12}+\Delta_{23}+\Delta_{2 X}\right) V_{\phi}^{2}-2 M_{\mathrm{DM}}^{2}\right] . \tag{4}
\end{align*}
$$

If we allow $\left\langle\phi_{2}\right\rangle \neq 0$, the real part of the $\phi_{2}$ field obtains mass and its imaginary part is massless and stable. In that case, the DM candidate would be the Goldstone boson related to the breakdown of the $U(1)_{\chi}$ symmetry. In general, such massless DM has severe constraints from
the big bang nucleosynthesis $[31,32]$ and the bullet cluster [14,33]. Here we do not consider this case.

Also, we work in the context of $\zeta \equiv \frac{V_{\Phi}}{V_{\phi}} \ll 1$. This assumption allows us to implement a stable and natural seesaw mechanism for neutrino masses at low energies, as shown in Ref. [21]. Once $V_{H}^{2}+2 V_{\Phi}^{2}=(246)^{2} \mathrm{GeV}^{2}$ and $V_{H}$ is mainly responsible for giving the top-quark mass at tree level, we have $V_{H}^{2} \gg V_{\Phi}^{2}$. Choosing $V_{\phi} \sim 1 \mathrm{TeV}$ and $V_{\Phi} \sim 1 \mathrm{MeV}$, as in Ref. [21], we have that the $\zeta$ parameter is $\sim 10^{-6}$. At first glance, the tiny value of $\zeta=V_{\Phi} / V_{\phi}$ could seem unnatural. However, let us remark that setting $V_{\Phi}$ to a tiny value is justified and natural because if it was in fact taken to be zero (keeping $V_{\phi}$ finite and different from zero, then $\zeta \rightarrow 0$ ) the symmetry of the entire Lagrangian would increase ('t Hooft's principle of naturalness). Furthermore, it can be shown that in that case the emergent $\mathrm{U}(1)$ global symmetry would prevent the active neutrinos from obtaining masses.

In general, we solve numerically the minimization conditions, and using standard procedures we construct numerically the mass-squared matrices for the charged, $C P$-even, and $C P$-odd scalar fields. We choose the parameters in the potential such that they satisfy simultaneously the minimization conditions, the positivity of the squared masses and the lower boundedness of the scalar potential. In order to satisfy this last condition, we choose the parameters such that the quartic terms in the scalar potential are positive for all directions. Although all these constraints are checked numerically, we here give some insight into some constraints coming from the minimization conditions and the positivity of the squared masses when we make some simplifying assumption about the parameters. First, we solve Eqs. (A1) and (A7) in the limit $\zeta \rightarrow 0$. Doing so, we have

$$
\begin{equation*}
\mu_{H}= \pm \sqrt{\lambda_{H} V_{\mathrm{SM}}^{2}+\frac{1}{2}\left(\Lambda_{H s 1}+\Lambda_{H s 3}+\Lambda_{H s X}\right) V_{\phi}^{2}}+\mathcal{O}(\zeta) \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\kappa_{H 1 X}=\mathcal{O}(\zeta), \quad \kappa_{H 2 X}=\mathcal{O}(\zeta) \tag{6}
\end{equation*}
$$

$$
\begin{align*}
\mu_{s 1}= & \pm \frac{\sqrt{\Lambda_{H s 1} V_{\mathrm{SM}}^{2}+\left(\Delta_{13}+\Delta_{1 X}+2 \lambda_{s 1}\right) V_{\phi}^{2}}}{\sqrt{2}}+\mathcal{O}(\zeta)  \tag{7}\\
\mu_{s 3}= & \pm \frac{\sqrt{\Lambda_{H s 3} V_{\mathrm{SM}}^{2}+\left(3 \beta_{3 X}+\Delta_{13}+\Delta_{3 X}+2 \lambda_{s 3}\right) V_{\phi}^{2}}}{\sqrt{2}} \\
& +\mathcal{O}(\zeta),  \tag{8}\\
\mu_{s X}= & \pm \frac{\sqrt{\Lambda_{H s X} V_{\mathrm{SM}}^{2}+\left(\beta_{3 X}+\Delta_{1 X}+\Delta_{3 X}+2 \lambda_{s X}\right) V_{\phi}^{2}}}{\sqrt{2}} \\
& +\mathcal{O}(\zeta) . \tag{9}
\end{align*}
$$

From Eq. (6), we see that $\kappa_{H 1 X} \rightarrow 0$ and $\kappa_{H 2 X} \rightarrow 0$ when $\zeta \rightarrow 0$ (and keeping $V_{\phi}$ finite). Thus, in our calculations we choose $\kappa_{H 1 X} \sim V_{\Phi}$ and $\kappa_{H 2 X} \sim V_{\Phi} / V_{\phi}$.

To simplify the squared masses and obtain useful analytical expressions, let us consider $\lambda_{11}=\lambda_{22}=$ $\lambda_{s 1}=\lambda_{s 3}=\lambda_{s X} ; \quad \Lambda_{H 1}=\Lambda_{H 2}=\Lambda_{H s 1}=\Lambda_{H s 3}=\Lambda_{H s X}=\Lambda_{H 1}^{\prime}=$ $\Lambda_{H 2}^{\prime} ; \Lambda_{11}^{\prime}=\Lambda_{13}^{\prime}=\Lambda_{1 X}^{\prime}=\Lambda_{21}^{\prime}=\Lambda_{23}^{\prime}=\Lambda_{2 X}^{\prime}=\lambda_{12}=\lambda_{12}^{\prime}=$ $\Delta_{13}=\Delta_{1 X}=\Delta_{3 X} ; \quad \Lambda_{12}^{\prime}=\Lambda_{22}^{\prime}=\Delta_{12}=\Delta_{23}=\Delta_{2 X}$, and the other parameters without restrictions. The previous constraints have been inspired by the similitude of the respective potential terms. We have left free the parameters that involve the DM candidates. Also, we have assumed that the $H$ scalar field is the Higgs-like field in this model. With these considerations, we have-apart from the Goldstone bosons that are eaten by the $W^{ \pm}$bosons-two charged scalars, $C_{1,2}^{ \pm}$, with masses given by

$$
\begin{align*}
m_{C_{1}^{ \pm}, C_{2}^{ \pm}}^{2}= & \frac{1}{4}\left[2 \Lambda_{H 1} V_{\mathrm{SM}}^{2}+(1+\sqrt{2}) V_{\mathrm{SM}} V_{\phi}\right. \\
& \mp V_{\phi}\left(\sqrt{(3-2 \sqrt{2}) V_{\mathrm{SM}}^{2}+4 \beta_{13}^{2} V_{\phi}^{2}}\right. \\
& \left.\left.+2 \beta_{13} V_{\phi}\right)\right]+\mathcal{O}(\zeta) . \tag{10}
\end{align*}
$$

In the $C P$-odd scalar sector, we have-besides the two Goldstone bosons which give mass to the $Z_{1 \mu}$ and $Z_{2 \mu}$ gauge bosons-the following mass eigenvalues:

$$
\begin{gather*}
m_{I_{3}}^{2}=\mathcal{O}(\zeta) ; \quad m_{I_{4}}^{2}=M_{\mathrm{DM}}^{2} ; \quad m_{I_{7}}^{2}=-5 \beta_{3 X} V_{\phi}^{2}+\mathcal{O}(\zeta)  \tag{11}\\
m_{I_{5}, I_{6}}^{2}=  \tag{12}\\
\frac{1}{4} V_{\phi}\left[(1+\sqrt{2}) V_{\mathrm{SM}}-2 \beta_{13} V_{\phi} \mp \sqrt{4 \beta_{13}^{2} V_{\phi}^{2}+(3-2 \sqrt{2}) V_{\mathrm{SM}}^{2}}\right]+\mathcal{O}(\zeta)
\end{gather*}
$$

Finally, in the $C P$-even scalar sector we have $m_{R_{4}}^{2}=M_{\mathrm{DM}}^{2}$, and

$$
\begin{equation*}
m_{R_{5}, R_{6}}^{2}=\frac{1}{4} V_{\phi}\left[(1+\sqrt{2}) V_{\mathrm{SM}}-2 \beta_{13} V_{\phi} \mp \sqrt{4 \beta_{13}^{2} V_{\phi}^{2}+(3-2 \sqrt{2}) V_{\mathrm{SM}}^{2}}\right]+\mathcal{O}(\zeta) \tag{13}
\end{equation*}
$$

the other mass eigenvalues are not shown for conciseness. As shown in the above expressions, in the $\mathcal{O}(\zeta)$ we have three degenerate mass eigenstates, i.e., $m_{R_{4}}^{2}=m_{I_{4}}^{2}, m_{R_{5}}^{2}=m_{I_{5}}^{2}$, and $m_{R_{6}}^{2}=m_{I_{6}}^{2}$. Imposing that all these masses are positive, we find the following conditions:

$$
\begin{gather*}
M_{\mathrm{DM}}>0 \wedge \beta_{3 X}<0, \\
\wedge\left[\left(\Lambda_{H 2}^{\prime}>0 \wedge \beta_{13} V_{\phi}+\sqrt{2} V_{\mathrm{SM}}<2 V_{\mathrm{SM}}\right) \vee\right. \\
\left.\left(V_{\phi}>-2(\sqrt{2}-1) \Lambda_{H 2}^{\prime} V_{\mathrm{SM}} \wedge \beta_{13}<\frac{V_{\mathrm{SM}}\left(\Lambda_{H 2}^{\prime} V_{\mathrm{SM}}+V_{\phi}\right)\left(\Lambda_{H 2}^{\prime} V_{\mathrm{SM}}+\sqrt{2} V_{\phi}\right)}{V_{\phi}^{2}\left(2 \Lambda_{H 2}^{\prime} V_{\mathrm{SM}}+(1+\sqrt{2}) V_{\phi}\right)} \wedge \Lambda_{H 2}^{\prime} \leq 0\right)\right] . \tag{14}
\end{gather*}
$$

Despite the fact that Eqs. (5)-(14) are only valid in the limit $\zeta \rightarrow 0$, these relations will be useful in our analysis, at least as a starting point. However, we would like to stress that in our numerical programs to analyze the scalar potential, the full constraints-i.e., the minimization conditions, the positivity of the squared masses, and the lower boundedness of the scalar potential-are rigorously taken into account.

## IV. GAUGE BOSONS

In this model the gauge symmetry breaking proceeds in two stages. In the first stage, the real components of the $\phi_{1}, \phi_{3}$, and $\phi_{X}$ fields obtain VEVs, say $V_{\phi}$, as discussed in
the previous section. Once this happens, the gauge symmetry is broken down to $\mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y}$, where $Y$ is the usual hypercharge of the SM. In the second stage, the electrically neutral components of the $H, \Phi_{1,2}$, obtain VEVs, $V_{H}$ and $V_{\Phi}$, respectively, thus breaking the symmetry down to $U(1)_{Q}$.

The mass terms for the three electrically neutral $\mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y^{\prime}} \otimes \mathrm{U}(1)_{B-L}$ gauge bosons $\left(W_{\mu}^{3}, B_{\mu}^{Y^{\prime}}\right.$, and $\left.B_{\mu}^{B-L}\right)$ arise from the kinetic terms for the scalar fields upon replacing $H, \Phi_{1,2}$, and $\phi_{1,2,3, X}$ by their respective VEVs $\left(\left\langle\phi_{2 R}\right\rangle=0\right)$. In general the mass-squared matrix for $W_{\mu}^{3}, B_{\mu}^{Y^{\prime}}$, and $B_{\mu}^{B-L}$ can be written as follows:

$$
\mathcal{M}_{\text {gauge bosons }}^{2}=\left[\begin{array}{ccc}
g^{2}(K+P+2 N) & -g g_{Y^{\prime}}(K+N) & -g g_{B-L}(P+N)  \tag{15}\\
-g g_{Y^{\prime}}(K+N) & g_{Y^{\prime}}^{2} K & g_{Y^{\prime}} g_{B-L} N \\
-g g_{B-L}(P+N) & g_{Y^{\prime}} g_{B-L} N & g_{B-L}^{2} P
\end{array}\right],
$$

where $g, g_{Y^{\prime}}$, and $g_{B-L}$ are the $\mathrm{SU}(2)_{L}, \mathrm{U}(1)_{Y^{\prime}}$, and $\mathrm{U}(1)_{B-L}$ coupling constants, respectively. $K, P$, and $N$ are defined by $K \equiv \frac{1}{4} \sum_{a} V_{a}^{2} Y_{a}^{\prime 2}, P \equiv \frac{1}{4} \sum_{a} V_{a}^{2}(B-L)_{a}^{2}$, and $N \equiv \frac{1}{4} \sum_{a} V_{a}^{2} Y_{a}^{\prime}(B-L)_{a}$, with $Y_{a}^{\prime}$ and $(B-L)_{a}$ being the quantum numbers given in Tables I and II. Considering our aforementioned assumptions, we have

$$
\begin{equation*}
K=\frac{1}{4}\left(V_{H}^{2}+41 V_{\Phi}^{2}+74 V_{\phi}^{2}\right), \quad P=\frac{1}{4}\left(45 V_{\Phi}^{2}+74 V_{\phi}^{2}\right), \quad N=-\frac{1}{4}\left(42 V_{\Phi}^{2}+74 V_{\phi}^{2}\right) . \tag{16}
\end{equation*}
$$

In order to obtain the relation between the neutral gauge bosons $\left(W_{\mu}^{3}, B_{\mu}^{Y^{\prime}}, B_{\mu}^{B-L}\right)$ and the corresponding mass eigenstates, we diagonalize $\mathcal{M}_{\text {gauge bosons. }}^{2}$. Doing so, we have

$$
\begin{align*}
\gamma_{\mu} & =\frac{1}{N_{\gamma}}\left[\frac{1}{g} W_{\mu}^{3}+\frac{1}{g_{Y^{\prime}}} B_{\mu}^{Y^{\prime}}+\frac{1}{g_{B-L}} B_{\mu}^{B-L}\right],  \tag{17}\\
Z_{1 \mu} & =\frac{1}{N_{Z_{1}}}\left[g\left(P g_{B-L}^{2}-N g_{Y^{\prime}}^{2}-M_{Z_{1}}^{2}\right) W_{\mu}^{3}-g_{Y^{\prime}}\left((P+N) g^{2}+P g_{B-L}^{2}-M_{Z_{1}}^{2}\right) B_{\mu}^{Y^{\prime}}+g_{B-L}\left((P+N) g^{2}+N g_{Y^{\prime}}^{2}\right) B_{\mu}^{B-L}\right],  \tag{18}\\
Z_{2 \mu} & \left.=\frac{1}{N_{Z_{2}}}\left[g\left(P g_{B-L}^{2}-N g_{Y^{\prime}}^{2}-M_{Z_{2}}^{2}\right) W_{\mu}^{3}-g_{Y^{\prime}}\left((P+N) g^{2}+P g_{B-L}^{2}-M_{Z_{2}}^{2}\right) B_{\mu}^{Y^{\prime}}\right]+g_{B-L}\left((P+N) g^{2}+N g_{Y^{\prime}}^{2}\right) B_{\mu}^{B-L}\right], \tag{19}
\end{align*}
$$

where $N_{\gamma}, N_{Z_{1}}$, and $N_{Z_{2}}$ are the corresponding normalization constants. Also, $\gamma_{\mu}$ corresponds to the photon, and $Z_{1 \mu}$ and $Z_{2 \mu}$ are the two massive neutral vector bosons of the model, and their squared masses are given by $M_{\gamma}^{2}=0$ and

$$
\begin{equation*}
M_{Z_{1 \mu}, Z_{2 \mu}}^{2}=\frac{1}{2} R \mp \frac{1}{2}\left[R^{2}-4\left(K P-N^{2}\right)\left(g^{2}\left(g_{Y^{\prime}}^{2}+g_{B-L}^{2}\right)+g_{Y^{\prime}}^{2}, g_{B-L}^{2}\right)\right]^{1 / 2}, \tag{20}
\end{equation*}
$$

with $R \equiv(K+P+2 N) g^{2}+K g_{Y^{\prime}}^{2}+P g_{B-L}^{2}$. Here, it is important to note that the matrix $\mathcal{M}_{\text {gauge bosons }}^{2}$ in Eq. (15) has the correct texture to naturally give the masses of the neutral gauge bosons. Furthermore, in the case that $V_{\phi} \gg V_{H}, V_{\Phi}$, it is straightforward to show that $M_{\mathrm{Z}_{1, ~}}^{2} \sim$ $\frac{g^{2}\left(V_{H}^{2}+2 V_{\bar{W}}^{2}\right)}{4 \cos ^{2} \theta_{\mathrm{w}}}=\frac{M_{W}^{2}}{\cos ^{2} \theta_{\mathrm{w}}}$ and $M_{Z_{2 \mu}}^{2} \sim \frac{74}{4}\left(g_{Y^{\prime}}^{2}+g_{B-L}^{2}\right) V_{\phi}^{2}$, where the angle $\theta_{\mathrm{W}}$ is defined below.

For future discussion, it is convenient to define the following basis:
$Z_{\mu}=\cos \theta_{W} W_{\mu}^{3}-\sin \theta_{W} \sin \alpha B_{\mu}^{Y^{\prime}}-\sin \theta_{W} \cos \alpha B_{\mu}^{B-L}$,
$Z_{\mu}^{\prime}=\cos \alpha B_{\mu}^{Y^{\prime}}-\sin \alpha B_{\mu}^{B-L}$,
and $\gamma_{\mu}$ is defined as in Eq. (17). The angle $\alpha$, defined as $\tan \alpha \equiv g_{B-L} / g_{Y^{\prime}}$, can be understood as the parameter of a particular $\mathrm{SO}(2)$ transformation on the two gauge bosons, $B_{\mu}^{Y^{\prime}}$ and $B_{\mu}^{B-L}$, that rotates the $\mathrm{U}(1)_{Y^{\prime}} \otimes \mathrm{U}(1)_{B-L}$ gauge group into the $\mathrm{U}(1)_{Y} \otimes \mathrm{U}(1)_{Z}$ gauge group. In the last expression $\mathrm{U}(1)_{Y}$ is the usual hypercharge gauge group. Also, we have that $g^{2} \sin ^{2} \theta_{\mathrm{W}}=e^{2}=\left(1 / g^{2}+1 / g_{Y^{\prime}}^{2}+\right.$ $\left.1 / g_{B-L}^{2}\right)^{-1}$. The $\mathrm{U}(1)_{Z}$ can be understood as the gauge group with the coupling $g_{Z}^{2}=g_{Y^{\prime}}^{2}+g_{B-L}^{2}$. Using Eqs. (21) and (22), we can write the two massive gauge bosons $Z_{1 \mu}$ and $Z_{2 \mu}$ in terms of $Z_{\mu}$ and $Z_{\mu}^{\prime}$ as follows:

$$
\begin{gather*}
Z_{1 \mu}=\cos \beta Z_{\mu}+\sin \beta Z_{\mu}^{\prime}  \tag{23}\\
Z_{2 \mu}=-\sin \beta Z_{\mu}+\cos \beta Z_{\mu}^{\prime} \tag{24}
\end{gather*}
$$

where

$$
\begin{equation*}
\tan \beta=\frac{\sqrt{g^{2}\left(g_{Y^{\prime}}^{2}+g_{B-L}^{2}\right)+g_{Y^{\prime}}^{2} g_{B-L}^{2}}\left(g_{B-L}^{2} P-g_{Y^{\prime}}^{2} N-M_{Z_{2}}^{2}\right)}{g^{2}\left(g_{Y^{\prime}}^{2}+g_{B-L}^{2}\right)(P+N)+g_{Y^{\prime}}^{2}\left(g_{B-L}^{2}(P+N)-M_{Z_{2}}^{2}\right)} . \tag{25}
\end{equation*}
$$

From Eqs. (20), (23), and (24), we can see that $\tan \beta=0$ when $V_{\phi} \rightarrow \infty$ or $V_{H}^{2}=\left(g_{Y^{\prime}}^{2}+3 g_{B-L}^{2}\right) V_{\Phi}^{2} / g_{Y^{\prime}}^{2}$. In the first case, as $V_{\phi}$ approaches infinity the numerator of Eq. (25) is $\propto V_{H}^{2}$. However, the denominator is $\propto V_{\phi}^{2}$, and hence $\tan \beta \rightarrow 0$, meaning that $Z_{2 \mu}$ becomes so heavy that it decouples. The last solution is not allowed since in our case we have $V_{H} \gg V_{\Phi}$ and $\mathcal{O}\left(g_{Y^{\prime}}\right) \sim \mathcal{O}\left(g_{B-L}\right)$.

In this work, we use the gauge couplings $g \simeq 0.65$ and $g_{Y^{\prime}}=g_{B-L} \simeq 0.505$, such that $\tan \beta \simeq 4 \times 10^{-4}$. Doing so, we have $Z_{1 \mu} \simeq Z_{\mu}$ and $Z_{2 \mu} \simeq Z_{\mu}^{\prime}$. In general, the angle $\beta$ must be quite small $\left(\beta \lesssim 10^{-3}\right)$ to be in agreement with
precision electroweak studies [34-36] since a new neutral boson $Z_{2 \mu}$ which mixes with the $\mathrm{SM} Z_{\mu}$ distorts its properties, such as couplings to fermions and masses relative to electroweak inputs. Using these parameters for the gauge couplings and the VEVs discussed in the previous section, we obtain $M_{Z^{\prime}} \simeq 3.1 \mathrm{TeV}$ besides the already known masses for the SM gauge bosons. In general, a new neutral vector boson must have a mass on the order of a few TeV , or be very weakly coupled to the known matter to maintain consistency with the present phenomenology [34-39]. Doing a phenomenological study of the bounds on the parameter space imposed by data coming from LEP II, Tevatron, and the LHC in the present model is out of the scope of this work. However, we see that the $M_{Z^{\prime}}$ value above is consistent with the relation $M_{Z^{\prime}} / g_{B-L} \simeq 6.13 \gtrsim$ $6 \mathrm{TeV}[37,38]$.

Finally, the charged gauge bosons $W_{\mu}^{ \pm}$are not affected by the presence of one additional neutral gauge boson $Z_{2 \mu}$. These have the same form as in the SM, $W_{\mu}^{ \pm}=\frac{1}{\sqrt{2}}\left(W_{\mu}^{1} \mp i W_{\mu}^{2}\right)$, with masses given by $M_{W^{ \pm}}^{2}=$ $\frac{1}{4} g^{2} V_{\text {SM }}^{2}=\frac{1}{4} g^{2}\left(V_{H}^{2}+2 V_{\Phi}^{2}\right)$.

## V. DARK MATTER

## A. Thermal relic density

In order to calculate the present day DM mass density, $\Omega_{\mathrm{DM}} h^{2}$, arising from $R_{\mathrm{DM}}$ and $I_{\mathrm{DM}}$ scalars freezing out from thermal equilibrium, we follow the standard procedure from Refs. [40,41]. Thus, we should find the solution to the Boltzmann equations for $Y_{R_{\mathrm{DM}}}$ and $Y_{I_{\mathrm{DM}}}$, which are defined as the ratio of the number of particles ( $n_{R_{\mathrm{DM}}}$ and $n_{I_{\mathrm{DM}}}$ ) to the entropy, $Y_{i} \equiv n_{i} / s\left(i=R_{\mathrm{DM}}, I_{\mathrm{DM}}\right)$, with $s$ being the total entropy density of the Universe. Usually, $s$ is written in terms of the effective degrees of freedom $h_{\text {eff }}(T)$ as follows: $s=\frac{2 \pi^{2}}{45} h_{\text {eff }}(T) T^{3}$, where $T$ is the photon temperature and $h_{\text {eff }}$ is calculated as in Ref. [40]. Actually, in our case, due to the $U(1)_{\chi}$ symmetry introduced in Sec. III, $M_{I_{\mathrm{DM}}}=M_{R_{\mathrm{DM}}}=M_{\mathrm{DM}}, Y_{R_{\mathrm{DM}}}=$ $Y_{I_{\mathrm{DM}}} \equiv Y$, and $\Omega_{\mathrm{DM}} h^{2}=\Omega_{R_{\mathrm{DM}}} h^{2}+\Omega_{I_{\mathrm{DM}}} h^{2}=2 \Omega_{R_{\mathrm{DM}}} h^{2}=$ $2 \Omega_{I_{\mathrm{DM}}} h^{2}$. Therefore, the Boltzmann equation that we have to solve is
$\frac{d Y}{d x}=-\left(\frac{45}{\pi} G\right)^{-1 / 2} \frac{g_{*}^{1 / 2} M_{\mathrm{DM}}}{x^{2}}\left\langle\sigma v_{\text {Moller }}\right\rangle_{\text {ann }}\left[Y^{2}-Y_{\text {eq }}^{2}\right]$,
where $x=M_{\mathrm{DM}} / T, G$ is the gravitational constant, and $Y_{\text {eq }}=n_{\text {eq }} / s . n_{\text {eq }}$ is the thermal equilibrium number density and when $M_{\mathrm{DM}} / T \gg 1$, it is $n_{\mathrm{eq}}=g_{i}\left(\frac{M_{\mathrm{DM}} T}{2 \pi}\right)^{3 / 2} \exp \left[-\frac{M_{\mathrm{DM}}}{T}\right]$, where $g_{i}=1$ is the internal degree of freedom for the scalar


FIG. 1. Main annihilation processes that contribute to $\left\langle\sigma v_{\text {Moller }}\right\rangle_{\text {ann }}$.
dark matter. The $g_{*}$ parameter in Eq. (26) is calculated as in Ref. [40].

Also, we have that the thermal average of the annihilation cross section times the Moller velocity, $\left\langle\sigma v_{\text {Moller }}\right\rangle_{\text {ann }}$, has the following form:

$$
\begin{align*}
\left\langle\sigma v_{\mathrm{Moller}}\right\rangle_{\mathrm{ann}}= & \frac{1}{8 M_{\mathrm{DM}}^{4} T K_{2}^{2}\left(M_{\mathrm{DM}} / T\right)} \int_{4 M_{\mathrm{DM}}^{2}}^{\infty} \sigma_{\mathrm{ann}}\left(s-4 M_{\mathrm{DM}}^{2}\right) \\
& \times \sqrt{s} K_{1}(\sqrt{s} / T) d s \tag{27}
\end{align*}
$$

where $K_{i}$ are the modified Bessel functions of order $i$. The variable $s$ in the integral above is the Mandelstam variable. Finally, once $Y$ is numerically calculated for the present time, $Y_{0}$, we can obtain $\Omega_{\mathrm{DM}} h^{2}=2.82 \times 10^{8} \times$ $\left(2 \times Y_{0}\right) \times \frac{M_{\mathrm{DM}}}{\mathrm{GeV}}$.

In order to calculate $\sigma_{\mathrm{ann}}$, we have taken into account all dominant annihilation processes, which are shown in Fig. 1. In our case, the dominant annihilation contributions come from the scalar exchange. This is due to the fact that our DM candidates, $R_{\mathrm{DM}}$ and $I_{\mathrm{DM}}$, couple neither to $Z_{\mu}$ nor to $W_{\mu}^{ \pm}$gauge bosons at tree level, since they are SM singlets. Also, we have found that contributions coming from $Z_{\mu}^{\prime}$ exchange are negligible for the parameter set considered here.

Taking into account all considerations above, we solve Eq. (26) numerically for a representative set of parameters. Although the scalar potential in this model has many free parameters, we find that the most relevant parameters for determining the correct DM relic density and satisfying the currently direct experimental limits are $\Lambda_{H s 2}, \Delta_{\alpha 2}$ (with
$\alpha=1,3, X$ ), and $\Lambda_{\gamma 2}^{\prime}$ (with $\gamma=1,2$ ). The $\Lambda_{H s 2}$ coupling strongly controls the direct detection signal, since in our case the Higgs-like scalar is almost totally the neutral $C P$-even component of the $H$ field, and (as discussed below) direct detection is mainly mediated by the $t$-channel Higgs exchange. In order to obtain the correct direct detection limits without resorting to resonances, we find that $\Lambda_{H s 2} \sim 10^{-4}$. The $\Delta_{\alpha 2}$ and $\Lambda_{\gamma_{2}}^{\prime}$ parameters are also crucial in obtaining the correct $\Omega_{\mathrm{DM}} h^{2}$ because they mostly control the $\mathrm{DM}-\mathrm{DM}-R_{i}\left(I_{i}\right)-R_{i}\left(I_{i}\right)$ and $\mathrm{DM}-\mathrm{DM}-$ $R_{i}$ couplings and, therefore, $\sigma_{\mathrm{ann}}$. The latter is not allowed to vary in a wide range since, roughly, $\Omega_{\mathrm{DM}} h^{2} \sim$ $1 /\left\langle\sigma v_{\text {Moller }}\right\rangle_{\text {ann }}$ and we aim to obtain values close to $\Omega_{\mathrm{DM}} h^{2} \sim 0.11$. In other words, as the $\Delta_{\alpha 2}$ and $\Lambda_{\gamma 2}^{\prime}$ parameters increase, $\Omega_{\mathrm{DM}} h^{2}$ decreases. In Fig. 2, we have used $\Lambda_{\gamma 2}^{\prime} \simeq 10^{-2}$ and $\Delta_{\alpha 2} \simeq 9 \times 10^{-2}$. It is also important to mention here that the dominant process is the $\mathrm{DM}+$ $\mathrm{DM} \rightarrow I_{3}+I_{3}$ annihilation, where $I_{3}$ refers to the lightest $C P$-odd scalars, as in Sec III. Although the other parameters in the scalar potential are not as critical in determining $\Omega_{\mathrm{DM}} h^{2}$, they give the other quantitative characteristics appearing in Fig. 2. In order to be more specific, we have chosen the other parameters such that the mass scalar spectrum is given by $1437.6,1016.9,631.7,544.9,379.6$, 125 GeV , and 707.1, 544.9, 379.6, $2.3 \times 10^{-6} \mathrm{GeV}$ for the $C P$-even and $C P$-odd scalars, respectively. The $C P$-even scalars with masses $1437.6,1016.9$, and 631.7 GeV have components only in the singlets $\phi_{1,3, X}$, and the $C P$-even scalars with 544.9 and 379.6 GeV have components only in the scalar doublets $\Phi_{1,2}$. The $C P$-even scalar with 125 GeV


FIG. 2 (color online). The total thermal relic density of $I_{\mathrm{DM}}$ and $R_{\mathrm{DM}}$ as a function of $M_{\mathrm{DM}}$. We have used three different parameters for $\Lambda_{H s 2}=3 \times 10^{-5}, 1 \times 10^{-4}, 5 \times 10^{-4}$.
has a component in the $H$ doublet and it is the Higgs-like scalar in our model. In Fig. 2, we can also observe three resonances at $\simeq 315.8,508.5$, and 718.8 GeV , corresponding to the $s$-channel exchange of $C P$-even scalars with components in the singlets. Let us also mention that the processes via the $s$-channel due to the exchange of the $C P$-even scalars with masses of $125,379.6$, and 544.9 GeV are highly suppressed because of the smallness of their couplings. Thus, their resonances do not appear in Fig. 2.

## B. Direct detection

Despite being weakly coupled to baryons, WIMPs can scatter elastically with atomic nuclei, providing the opportunity for direct detection. Currently, there are several experiments which aim to directly observe WIMP dark matter [17-19]. The signal in these experiments is the kinetic energy transferred to a nucleus after it scatters off a DM particle. The energies involved are less than or of the order of 10 keV . At these energies the WIMP sees the entire nucleus as a single unit, with a net mass, charge, and spin. In general, the WIMP-nucleus interactions can be classified as either spin independent or spin dependent. In our case, these interactions are spin independent because the two DM candidates are scalars. The relevant WIMP-nucleus scattering process for direct detection in the case considered here takes place mainly through the $t$-channel elastic scattering due to Higgs exchange: $\left(I_{\mathrm{DM}}, R_{\mathrm{DM}}\right)+N \rightarrow$ $\left(I_{\mathrm{DM}}, R_{\mathrm{DM}}\right)+N$ ( $N$ refers to the atomic nucleus). The spin-independent cross section is given by

$$
\begin{equation*}
\sigma_{\chi N}^{\mathrm{SI}}=\frac{4}{\pi} \frac{M_{\mathrm{DM}}^{2} m_{N}^{2}}{\left(M_{\mathrm{DM}}+m_{N}\right)^{2}}\left[Z f_{p}+(A-Z) f_{n}\right]^{2} \tag{28}
\end{equation*}
$$

where the effective couplings to protons and neutrons, $f_{p, n}$, are

$$
\begin{align*}
f_{p, n}= & \sum_{q=u, d, s} \frac{G_{\mathrm{eff}, q}}{\sqrt{2}} f_{T q}^{(p, n)} \frac{m_{p, n}}{m_{q}} \\
& +\frac{2}{27} f_{T G}^{(p, n)} \sum_{q=c, b, t} \frac{G_{\mathrm{eff}, q}}{\sqrt{2}} \frac{m_{p, n}}{m_{q}} \tag{29}
\end{align*}
$$

By using $f_{T q}^{(p, n)}$ and $f_{T G}^{(p, n)}$ given in Ref. [42] and the fact that, in our case, $G_{\text {eff }, q}=G_{0} \times m_{q} \equiv \frac{C_{\mathrm{DM}^{2} H}}{V_{H} M_{\mathrm{Higgs}}} \times m_{q}$ (with $C_{\mathrm{DM}^{2} H}$ being the coupling DM-DM-Higgs, which depends on the parameters of the model), we arrive at a cross section per nucleon of

$$
\begin{equation*}
\sigma_{\chi, p}^{\mathrm{SI}} \approx 2.7 \times 10^{7} \times \frac{M_{\mathrm{DM}}^{2} m_{N}^{2}}{\left(M_{\mathrm{DM}}+m_{N}\right)^{2}} \times G_{0}^{2} \mathrm{pb} \tag{30}
\end{equation*}
$$

Recently, the Large Underground Xenon (LUX) experiment [19] has reported its first results, setting limits on spin-independent WIMP-nucleon elastic scattering with a minimum upper limit on the cross section of $7.6 \times 10^{-10} \mathrm{pb}$ at a WIMP mass of $33 \mathrm{GeV} / c^{2}$. We have found that by choosing $\Lambda_{H s 2} \sim 10^{-4}$ we obtain the LUX bound without resorting to resonances. It is clear that larger values of $\Lambda_{H s 2}$ can be considered. However, we have chosen this conservative value for $\Lambda_{H s 2}$. Our results are shown in Fig. 3. The parameters are the same as in Fig. 2.

From Figs. 2 and 3, we see that for a DM candidate with mass around 200 GeV and $\Lambda_{H s 2}=0.3 \times 10^{-4}, 1 \times 10^{-4}$, the two conditions- $\Omega_{\mathrm{DM}} h^{2}$ and direct detection-are satisfied outside the resonance regions. We also have verified that this is a general characteristic of this model. Due to the existence of the light $I_{3}$ scalar the annihilation process $\mathrm{DM}+\mathrm{DM} \rightarrow I_{3}+I_{3}$ [Fig. 1(a)] is the dominant one, so we do not have to appeal to resonances to get compatibility with experiments. Other $M_{\text {DM }}$ values which


FIG. 3 (color online). The spin-independent elastic scattering cross section, $\sigma_{\chi, p}^{\mathrm{SI}}$, off a proton $p$ as a function of $M_{\mathrm{DM}}$ for the same parameters as in Fig. 2, appropriately scaled to the relic density. We also show the XENON100 and LUX exclusion limits [17,19].
satisfy the experimental bounds are shown in Figs. 2 and 3. Specifically, $M_{\mathrm{DM}} \approx 319,410,511,590,737 \mathrm{GeV}$ are also possible solutions. However, these are within regions with resonances.

We now make some important remarks about the impact of the existence of $I_{3}$ in this model. First of all, we have a tree-level contribution to the Higgs invisible decay, $\Gamma_{h}^{\mathrm{Inv}}$, due to the coupling of the Higgs field with the light pseudoscalar field, $c_{h I_{3} I_{3}}$, which comes from the Lagrangian terms of the form $|H|^{2}\left|\phi_{1,2, X}\right|^{2}$, and gives $\Gamma_{h I_{3} I_{3}}^{\mathrm{Inv}}=c_{h I_{3} I_{3}}^{2} / 32 \pi m_{\mathrm{h}}$ for $m_{I_{3}} \ll m_{\mathrm{h}}$. Actually, when $2 M_{\mathrm{DM}}<m_{\mathrm{h}}$ the $h \rightarrow I_{\mathrm{DM}} I_{\mathrm{DM}}$ and $h \rightarrow R_{\mathrm{DM}} R_{\mathrm{DM}}$ decays are also allowed, thus further contributing to $\Gamma_{h}^{\mathrm{Inv}}$ according to $\Gamma_{h \mathrm{DMDM}}^{\mathrm{Inv}}=\Gamma_{h I_{\mathrm{DM}} I_{\mathrm{DM}}}^{\mathrm{Inv}}+\Gamma_{h R_{\mathrm{DM}} R_{\mathrm{DM}}}^{\mathrm{Inv}}=2 \times c_{h \mathrm{DMDM}}^{2} /\left(32 \pi m_{\mathrm{h}}\right) \times$ $\sqrt{1-4 M_{\mathrm{DM}}^{2} / m_{\mathrm{h}}^{2}}$, with $c_{h \mathrm{DMDM}} \approx \Lambda_{H s 2} V_{H}$. The current limit on the branching ratio into invisible particles of the Higgs, $B R_{h}^{\text {Inv }}$, is around $10-15 \%$ [43,44]. A stronger bound of $B R_{h}^{\operatorname{Inv}}<5 \%$ at the 14 TeV LHC has been claimed [45]. From the set of parameters used to obtain Figs 2 and 3 we have that $B R_{h}^{\mathrm{Inv}}=\left(\Gamma_{h I_{3} I_{3}}^{\mathrm{Inv}}+\Gamma_{h \mathrm{DMDM}}^{\mathrm{Inv}}\right) /\left(\Gamma_{h}^{\mathrm{Vis}}+\Gamma_{h I_{3} I_{3}}^{\mathrm{Inv}}+\right.$ $\left.\Gamma_{h \mathrm{DMDM}}^{\mathrm{Inv}}\right) \simeq 3.78 \%$ for $M_{\mathrm{DM}}=50 \mathrm{GeV}$. For different $M_{\mathrm{DM}}$ values we have found $B R_{h}^{\text {Inv }}<5 \%$. Also, we have used $\Gamma_{h}^{\mathrm{Vis}}=4.07 \mathrm{MeV}$ for $m_{\mathrm{H}}=125 \mathrm{GeV}$. The model is also safe regarding the severe existing constraints on the invisible decay width of the $Z_{\mu}$ boson since there is no process like $Z_{\mu} \rightarrow R I_{3} \rightarrow I_{3} I_{3} I_{3}$ [29] due to the fact that $I_{3}$ only has components in the SM singlets. (It would be kinetically forbidden anyway once all real scalar fields of the model are heavier than the $Z_{\mu}$ boson.) For the same reason, there is no issue with the astrophysical constraints regarding energy loss in stars since there is no tree-level coupling inducing the $\gamma+e^{-} \rightarrow e^{-}+I_{3}$ [30]. Finally, some last comments are necessary. First, note that the $I_{3}$ light scalar does not affect the stability of the DM candidate since the $Z_{2}$ symmetry introduced in Sec. III forbids processes such as $R_{\mathrm{DM}} \rightarrow I_{3}+I_{3}$ and $I_{\mathrm{DM}} \rightarrow I_{3}+R_{3}$. Furthermore, in general, $I_{3}$ could also contribute to $\Omega_{\mathrm{DM}} h^{2}$ because it is massive. However, the $I_{3}$ pseudoscalar is not stable. It decays mainly in active neutrinos, $\nu$, with $\Gamma_{I_{3} \rightarrow \nu \nu} \approx \frac{m_{I_{3}}}{16 \pi} \frac{\sum_{i} m_{\nu i}^{2}}{V_{\phi}^{2}}$ [46]. For the parameter set used here, we have $\tau_{I_{3}} \simeq 1 / \Gamma_{I_{3} \rightarrow \nu \nu} \approx 10^{9} \mathrm{~s}$, where we have used $\sum_{i} m_{\nu i}^{2} \lesssim 0.01 \mathrm{eV}^{2}$. With $\tau_{I_{3}}$ given here and $t_{U} \simeq 4.3 \times$ $10^{17} \mathrm{~s}$ (the age of the Universe), $\Omega_{I_{3}} h^{2} \simeq$ $\frac{m_{I_{3}}}{1.25 \mathrm{keV}} \exp \left(-t_{\mathrm{U}} / \tau_{I_{3}}\right) \simeq 0$. In the last expression for $\Omega_{I_{3}} h^{2}$ we have considered that $T_{D I_{3}}>175 \mathrm{GeV}$ (where $T_{D I_{3}}$ is the decoupling temperature of $I_{3}$ ). There is also a constraint which comes from the observed large-scale structure of the Universe [47,48]. Roughly speaking, this last condition imposes $r_{I_{3}} \frac{m_{I_{3}}}{1 \mathrm{keV}}\left(\frac{\tau_{I_{3}}}{1 \mathrm{~s}}\right)^{1 / 2} \lesssim 4 \times 10^{3}$ [47]. In the last expression $r_{I_{3}}=g_{\text {eff }}\left(T_{0}\right) / g_{\text {eff }}\left(T_{D I_{3}}\right) \approx 1 / 25$, where $g_{\text {eff }}$ is the effective number of the relativistic degrees
of freedom. With our parameter set this condition is satisfied.

## VI. CONCLUSIONS

We have discussed in this work a scenario where a complex DM candidate is possible. In particular, the model studied here is a gauge extension of the SM based on a $\mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y^{\prime}} \otimes \mathrm{U}(1)_{B-L} \quad$ symmetry group. This model contents three right-handed neutrinos and some extra scalars, doublets, and singlets, with different quantum numbers. In principle, these scalars are introduced to generate Majorana and Dirac mass terms at the tree level and to allow the implementation of a seesaw mechanism at the TeV scale, as shown in Ref. [21]. The nonstandard doublets and singlets introduce two new energy scales, besides the electoweak one given by $V_{H}=246 \mathrm{GeV}$ : $V_{\Phi}$ (the VEVs of the extra doublet neutral scalars) and $V_{\phi}$ (the VEVs of the extra singlet neutral scalars). If $\zeta \equiv V_{\Phi} / V_{\phi} \ll$ 1 the seesaw mechanism becomes natural [21]. In this context, we have studied the scalar spectrum and imposed a $Z_{2}$ symmetry on the $\phi_{2}$ singlet scalar [which accidentally became a $U(1)_{\chi}$ symmetry, $\phi_{2} \rightarrow \exp \left(-i \chi_{\phi_{2}}\right) \phi_{2}$ ] in order to allow a complex DM candidate. Before studying the constraints coming from the thermal relic density $\left(\Omega_{\mathrm{DM}} h^{2}\right)$ and direct detection experiments on this DM candidate, we performed a brief analysis of the gauge sector concerning the $Z_{\mu}, Z_{\mu}^{\prime}$ mixing angle ( $\tan \beta \simeq 4 \times 10^{-4}$ ) which satisfies the $\beta \lesssim 10^{-3}$ electroweak precision constraint, and we have verified that the $Z_{\mu}^{\prime}$ mass emerging from the model is consistent with the relation $M_{Z^{\prime}} / g_{B-L} \simeq 6.13 \gtrsim 6 \mathrm{TeV}$. Then, we chose some parameters that simultaneously allowed us to have a compatible $\Omega_{\mathrm{DM}} h^{2}$ and satisfy the direct detection experiments. Although the scalar potential has many parameters, we have found that the $\Lambda_{H s 2}, \Delta_{\alpha 2}$ (with $\alpha=1,3, X$ ), and $\Lambda_{\gamma 2}^{\prime}$ (with $\gamma=1,2$ ) parameters mostly control these two constraints. The $\Lambda_{H s 2}$ parameter is fundamental in satisfying the limits coming from direct detection, since in our case it takes place through the $t$-channel elastic scattering due to the Higgs exchange. Choosing $\Lambda_{H s 2} \sim 10^{-4}$ roughly satisfies the bounds from the LUX experiment and allows for a $\Omega_{\mathrm{DM}} h^{2}$ that is in agreement with the WMAP and Planck experiments. The $\Delta_{\alpha 2}$ and $\Lambda_{\gamma_{2}}^{\prime}$ parameters control $\sigma_{\text {ann }}$ mostly and, therefore, $\Omega_{\mathrm{DM}} h^{2}$. As an example, we have shown $\Omega_{\mathrm{DM}} h^{2}$ and $\sigma_{\chi, p}^{\mathrm{SI}}$, for $\Lambda_{\gamma 2}^{\prime} \simeq 10^{-2}$ and $\Delta_{\alpha 2} \simeq 9 \times 10^{-2}$, in Figs. 2 and 3. It is interesting to note that this model, for the same set of fixed parameters (except the $M_{\mathrm{DM}}$ 's), has several $M_{\mathrm{DM}}$ values that satisfy the experimental bounds. In other words, we have found solutions in the regions outside and inside the resonances for the same parameters by only varying $M_{\mathrm{DM}}$. As previously mentioned, the presence of a light scalar, $I_{3}$, in this model makes the process $\mathrm{DM}+\mathrm{DM} \rightarrow I_{3}+I_{3}$ dominant for $\Omega_{\mathrm{DM}} h^{2}$. However, $I_{3}$ may bring some potential problems, so we have discussed some constraints imposed on $I_{3}$ coming from the Higgs and $Z_{\mu}$ invisible
decay widths, the energy loss in stars, and the observed large-scale structure of the Universe. We have found that in our context all of these constraints are satisfied. Finally, we would like to point out the recent work that studied the possibility of a Majoron DM candidate [49].

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## APPENDIX: THE MINIMIZATION CONDITIONS

The general minimization conditions coming from $\partial V_{B-L} / \partial R_{i}=0$, where $V_{B-L}$ is the scalar potential with $U(1)_{\chi} \quad$ symmetry $\quad$ and $\quad R_{i}=\left\{H_{R}^{0}, \Phi_{1 R}^{0}, \Phi_{2 R}^{0}, \phi_{1 R}, \phi_{2 R}\right.$, $\left.\phi_{3 R}, \phi_{X R}\right\}$ are the neutral real components of the scalar fields, can be written as follows:

$$
\begin{align*}
0= & V_{H}\left(2 \lambda_{H} V_{H}^{2}+\Lambda_{\mathrm{H} 1} V_{\Phi_{1}}^{2}+\Lambda_{\mathrm{H} 2} V_{\Phi_{2}}^{2}+\Lambda_{\mathrm{Hs} 1} V_{\phi_{1}}^{2}+\Lambda_{\mathrm{H} 2} V_{\phi_{2}}^{2}+\Lambda_{\mathrm{Hs} 3} V_{\phi_{3}}^{2}+\Lambda_{\mathrm{HsX}} V_{\phi_{X}}^{2}-2 \mu_{H}^{2}\right) \\
- & \sqrt{2} \kappa_{\mathrm{H} 1 X} V_{\Phi_{1}} V_{\phi_{X}}-\kappa_{\mathrm{H} 2 \mathrm{X}} V_{\Phi_{2}} V_{\phi_{X}}^{2},  \tag{A1}\\
0= & V_{\Phi_{1}}\left(\Lambda_{\mathrm{H} 1} V_{H}^{2}+2 \lambda_{11} V_{\Phi_{1}}^{2}+\left(\lambda_{12}^{\prime}+\lambda_{12}\right) V_{\Phi_{2}}^{2}+\Lambda_{11}^{\prime} V_{\phi_{1}}^{2}+\Lambda_{12}^{\prime} V_{\phi_{2}}^{2}+\Lambda_{13}^{\prime} V_{\phi_{3}}^{2}+\Lambda_{1 X}^{\prime} V_{\phi_{X}}^{2}-2 \mu_{11}^{2}\right) \\
- & \sqrt{2} \kappa_{\mathrm{H} 1 X} V_{H} V_{\phi_{X}}+\beta_{13} V_{\Phi_{2}} V_{\phi_{1}} V_{\phi_{3}},  \tag{A2}\\
0= & V_{\Phi_{2}}\left(\Lambda_{\mathrm{H} 2} V_{H}^{2}+\left(\lambda_{12}+\lambda_{12}^{\prime}\right) V_{\Phi_{1}}^{2}+2 \lambda_{22} V_{\Phi_{2}}^{2}+\Lambda_{21}^{\prime} V_{\phi_{1}}^{2}+\Lambda_{22}^{\prime} V_{\phi_{2}}^{2}+\Lambda_{23}^{\prime} V_{\phi_{3}}^{2}+\Lambda_{2 X}^{\prime} V_{\phi_{X}}^{2}-2 \mu_{22}^{2}\right) \\
& -\kappa_{\mathrm{H} 2 \mathrm{X}} V_{H} V_{\phi_{X}}^{2}+\beta_{13} V_{\Phi_{1}} V_{\phi_{1}} V_{\phi_{3}},  \tag{A3}\\
0= & V_{\phi_{1}}\left(\Lambda_{\mathrm{Hs} 1} V_{H}^{2}+\Lambda_{11}^{\prime} V_{\Phi_{1}}^{2}+\Lambda_{21}^{\prime} V_{\Phi_{2}}^{2}+2 \lambda_{\mathrm{s} 1} V_{\phi_{1}}^{2}+\Delta_{12} V_{\phi_{2}}^{2}+\Delta_{13} V_{\phi_{3}}^{2}+\Delta_{1 X} V_{\phi_{X}}^{2}-2 \mu_{\mathrm{sl}}^{2}\right) \\
+ & \beta_{13} V_{\Phi_{1}} V_{\Phi_{2}} V_{\phi_{3}},  \tag{A4}\\
0= & V_{\phi_{2}}\left(\Lambda_{\mathrm{H} 2} V_{H}^{2}+\Lambda_{12}^{\prime} V_{\Phi_{1}}^{2}+\Lambda_{22}^{\prime} V_{\Phi_{2}}^{2}+\Delta_{12} V_{\phi_{1}}^{2}+2 \lambda_{\mathrm{s} 2} V_{\phi_{2}}^{2}+\Delta_{23} V_{\phi_{3}}^{2}+\Delta_{2 X} V_{\phi_{X}}^{2}-2 \mu_{\mathrm{s} 2}^{2}\right),  \tag{A5}\\
0= & V_{\phi_{3}}\left(\Lambda_{\mathrm{Hs} 3} V_{H}^{2}+\Lambda_{13}^{\prime} V_{\Phi_{1}}^{2}+\Lambda_{23}^{\prime} V_{\Phi_{2}}^{2}+\Delta_{13} V_{\phi_{1}}^{2}+\Delta_{23} V_{\phi_{2}}^{2}+2 \lambda_{\mathrm{s} 3} V_{\phi_{3}}^{2}+\Delta_{3 X} V_{\phi_{X}}^{2}+3 \beta_{3 X} V_{\phi_{3}} V_{\phi_{X}}\right. \\
& \left.-2 \mu_{\mathrm{s} 3}^{2}\right)+\beta_{13} V_{\Phi_{1}} V_{\Phi_{2}} V_{\phi_{1}},  \tag{A6}\\
0= & V_{\phi_{X}}\left(\Lambda_{\mathrm{HsX}} V_{H}^{2}+\Lambda_{1 X}^{\prime} V_{\Phi_{1}}^{2}+\Lambda_{2 X}^{\prime} V_{\Phi_{2}}^{2}+\Delta_{1 X} V_{\phi_{1}}^{2}+\Delta_{2 X} V_{\phi_{2}}^{2}+2 \lambda_{\mathrm{sx}} V_{\phi_{X}}^{2}-2 \kappa_{\mathrm{H} 2 X} V_{H} V_{\Phi_{2}}-2 \mu_{\mathrm{sx}}^{2}\right) \\
& -\sqrt{2} \kappa_{\mathrm{H} 1 \mathrm{X}} V_{H} V_{\Phi_{1}}+\beta_{3 X} V_{\phi_{3}}^{3}+\Delta_{3 X} V_{\phi_{3}}^{2} V_{\phi_{X}} . \tag{A7}
\end{align*}
$$

In Eqs. (A1)-(A7) above, $V_{H}, V_{\Phi_{1}}, V_{\Phi_{2}}, V_{\phi_{1}}, V_{\phi_{2}}, V_{\phi_{3}}, V_{\phi_{X}}$ are the VEVs of $H_{R}^{0}, \Phi_{1 R}^{0}, \Phi_{2 R}^{0}, \phi_{1 R}, \phi_{2 R}, \phi_{3 R}, \phi_{X R}$, respectively.
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