Can dark matter be a scalar field?

View the article online for updates and enhancements.

Can dark matter be a scalar field?

J.F. Jesus,[a] S.H. Pereira,[b] J.L.G. Malatrasi[a] and F. Andrade-Oliveira[c]

[a]Universidade Estadual Paulista "Júlio de Mesquita Filho",
 Campus Experimental de Itapeva — R. Geraldo Alckmin, 519, Itapeva, SP, Brazil

[b]Universidade Estadual Paulista "Júlio de Mesquita Filho",
 Departamento de Física e Química, Campus de Guaratinguetá,
 Av. Dr. Ariberto Pereira da Cunha, 333, 12516-410 — Guaratinguetá, SP, Brazil

[c]Institute of Cosmology and Gravitation, University of Portsmouth,
 Burnaby Road, PO1 3FX, Portsmouth, U.K.

 E-mail: jfjesus@itapeva.unesp.br, shpereira@gmail.com, malatrasi440@gmail.com,
 felipe.oliveira@port.ac.uk

Abstract. In this paper we study a real scalar field as a possible candidate to explain the dark matter in the universe. In the context of a free scalar field with quadratic potential, we have used Union 2.1 SN Ia observational data jointly with a Planck prior over the dark matter density parameter to set a lower limit on the dark matter mass as $m \geq 0.12 H_0^{-1}$ eV ($c = \hbar = 1$). For the recent value of the Hubble constant indicated by the Hubble Space Telescope, namely $H_0 = 73 \pm 1.8$ km s^{-1}Mpc^{-1}, this leads to $m \geq 1.56 \times 10^{-33}$ eV at 99.7% c.l. Such value is much smaller than $m \sim 10^{-22}$ eV previously estimated for some models. Nevertheless, it is still in agreement with them once we have not found evidences for a upper limit on the scalar field dark matter mass from SN Ia analysis. In practice, it confirms free real scalar field as a viable candidate for dark matter in agreement with previous studies in the context of density perturbations, which include scalar field self interaction.

Contents

1 Introduction

Nowadays the most accepted model in cosmology is known as ΛCDM model, where CDM stands for Cold Dark Matter and Λ is the cosmological constant term, the latter being the main candidate to explain the current phase of acceleration of the universe and the former having a central role for the structure formation in the standard cosmology. The pressureless matter represents about 30% of the total material content of the universe, where about 5% is baryonic matter and 25% is nonbaryonic dark matter (DM). The cosmological constant stands for the 70% remaining part, sometimes also associated to a dark energy (DE) exotic fluid (see [1–5] for a review).

The idea of an accelerating universe is indicated by type Ia Supernovae observations [6–9], in agreement with ΛCDM, but the nature and origin of the dark matter is still a mystery (see [10, 11] for a review and [12] for a ten-point test that a new particle has to pass in order to be considered a viable DM candidate).

The first candidate to DM as a scalar field is the axion, one of the solutions for the Charge-Parity problem in QCD [13, 14]. The axion is essentially a scalar field with mass of about 10^{-5} eV, which has its origin at 10^{-30} seconds after the big bang. This candidate is until now one of the most accepted candidates to DM particles. Among others candidates for DM particles in the universe, the so called scalar field dark matter (SFDM) model [15–23] is one of the most studied models in quantum field theory, and its applicability in cosmology to explain the different processes in the evolution of the universe has been investigated in the last decades (see [24–26] for a review and references therein). In [27] it is proposed a cosmological scalar field harmonic oscillator model, in agreement to ΛCDM model. Scalar fields are also used to drive the inflationary phase of the universe [28, 29]. One of the first applications of a complex scalar field for structure formation of the universe was given by Press and Madsen [30, 31]. Most recently, in [32] it was studied structure formation by assuming that dark matter can be described by a real scalar field. It was also investigated the symmetry breaking and possibly a phase transition of this scalar field in the early Universe. At low temperatures, scalar perturbations of SFDM leads to formation of gravitational structures. In [33–35] it has been shown that such model agree with rotation curves of dwarf galaxies and small and large low surface brightness (LSB) galaxies. The model was also extended to include the dark matter temperature and finite temperature corrections up to one-loop in the perturbative regime.

Gravitational constraints imposed to dark matter halos in the context of finite temperature SFDM has been investigated in [36]. In [37] it was studied the Sextans dwarf spheroidal galaxy, embedded into a scalar field dark matter halo. In [38] the quantum creation of scalar particles was studied in the context of an alternative accelerating model of the universe.

It is important to notice that some models [22, 23, 37] points to a mass of the scalar field of about 10^{-22} eV, while a recent work [27] points to 10^{-32} eV. As we shall see, our model is in agreement to the last one.

In this work we study a real scalar field evolving in a Friedmann-Robertson-Walker background as a possible candidate to explain the dark matter in the universe. In section II, we develop the dynamics of the model and analyse the stability of the resulting system of equations. In section III, we make an alternative derivation based on a change of variables, more suitable for numerical integration. In section IV, we constrain the free parameters of the model by using observational SN Ia data. We conclude in section V and deduce the scalar field motion equation in the appendix.

2 The dynamics of the model

We are interested in the study of the action of the form

$$S = S^{\text{grav}} + S^{\text{mat}}, \tag{2.1}$$

where

$$S^{\text{grav}} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}(R + 2\Lambda) \tag{2.2}$$

is the standard Einstein-Hilbert action for the gravitational field, R stands for the Ricci curvature scalar, Λ is the cosmological constant parameter and G is Newton's gravitational constant.

The second term in (2.1) stands for the matter content of the universe, namely,

$$S^{\text{mat}} = \int d^4x \sqrt{-g}\mathcal{L}^{\text{sm}} + \int d^4x \sqrt{-g}\mathcal{L}^{\phi}, \tag{2.3}$$

which is composed of the Lagrangian density of the standard matter contributions (radiation, baryons, neutrinos etc.), $\mathcal{L}^{\text{sm}}$, in addition to the Lagrangian density of a real massive scalar field ϕ minimally coupled to gravity, $\mathcal{L}^{\phi}$, given by

$$\mathcal{L}^{\phi} = \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - V(\phi), \tag{2.4}$$

where $V(\phi)$ is the potential.

Variation of the action S with respect to ϕ leads to the equations of motion for the real scalar field. The equations of motion for the gravitational field are obtained by varying the action with respect to the metric $g^{\mu\nu}$, which leads to

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}, \tag{2.5}$$

where $T_{\mu\nu}$, the energy-momentum tensor of the matter fields, is defined as

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S^{\text{mat}}}{\delta g^{\mu\nu}} = T_{\mu\nu}^{\text{sm}} + T_{\mu\nu}^{\phi}. \tag{2.6}$$

Here we are assuming that the standard matter term in the above equation can be approximated by a perfect fluid characterized by its energy density $\rho^{\rm sm}$, pressure $p^{\rm sm}$ and four-velocity $u_\mu^{\rm sm}$. Therefore, we have

$$T_{\mu\nu}^{\rm sm} = (\rho^{\rm sm} + p^{\rm sm})u_\mu u_\nu - p^{\rm sm} g_{\mu\nu} \,. \tag{2.7}$$

On the other hand, the scalar field part of the energy-momentum tensor is

$$T_{\mu\nu}^\phi = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left[\partial^\alpha\phi\partial_\alpha\phi - V(\phi)\right], \tag{2.8}$$

which can be rewritten in the form of a perfect fluid (2.7) using the definitions

$$\rho_\phi \equiv \frac{1}{2}\partial^\alpha\phi\partial_\alpha\phi + V(\phi)\,, \qquad p_\phi \equiv \frac{1}{2}\partial^\alpha\phi\partial_\alpha\phi - V(\phi)\,, \qquad u_\mu \equiv (\partial_\mu\phi)/\sqrt{\partial^\alpha\phi\partial_\alpha\phi}\,. \tag{2.9}$$

In particular, assuming that the field is homogeneous ($\partial_i\phi = 0$), we have

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi)\,, \qquad p_\phi = \frac{1}{2}\dot\phi^2 - V(\phi)\,. \tag{2.10}$$

We restrict our treatment to a universe composed of a single baryonic component with energy density ρ_b, a cosmological constant term with energy density ρ_Λ, and the scalar field dark matter component ρ_ϕ. The equations of state of those components are respectively $p_b = 0$, $p_\Lambda = -\rho_\Lambda$, and $p_\phi = \omega_\phi\rho_\phi$, where

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)}\,. \tag{2.11}$$

The Friedmann equations obtained from this approach are (in units $c = \hbar = 1$).

$$\dot H = -\frac{\kappa^2}{2}\left(\dot\phi^2 + \rho_b\right), \tag{2.12}$$

$$\ddot\phi + 3H\dot\phi + \frac{dV}{d\phi} = 0, \tag{2.13}$$

$$\dot\rho_b + 3H\rho_b = 0, \tag{2.14}$$

$$\dot\rho_\Lambda = 0, \tag{2.15}$$

with the Friedmann constraint

$$H^2 = \frac{\kappa^2}{3}\left(\rho_\phi + \rho_b + \rho_\Lambda\right), \tag{2.16}$$

where $\kappa^2 \equiv 8\pi G$ and $H \equiv \dot a/a$ is the Hubble parameter.

In order to study the system of equations (2.12)–(2.15), we define the following dimensionless variables (accordingly to the variable transformations adopted in [21] to a similar set of equations):

$$x \equiv \frac{\kappa}{\sqrt{6}}\frac{\dot\phi}{H}, \quad u \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{V}}{H}, \quad b \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{\rho_b}}{H}, \quad l \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{\rho_\Lambda}}{H}. \tag{2.17}$$

Consequently, the Friedmann constraint (2.16) reduces to

$$x^2 + u^2 + b^2 + l^2 = 1\,. \tag{2.18}$$

Specializing to a quadratic scalar potential $V(\phi) = m^2\phi^2/2$, where m is the mass of the scalar field, the background evolution of the universe is given by the equations (2.12)–(2.15) as

$$x' = -3\,x - su + \frac{3}{2}\Pi\,x, \tag{2.19}$$

$$u' = sx + \frac{3}{2}\Pi\,u, \tag{2.20}$$

$$b' = \frac{3}{2}\,(\Pi - 1)\,b, \tag{2.21}$$

$$l' = \frac{3}{2}\Pi\,l, \tag{2.22}$$

$$s' = \frac{3}{2}\Pi\,s, \tag{2.23}$$

where a prime denotes derivatives with respect to the e-folding number $N = \ln a$,

$$s \equiv m/H \ , \tag{2.24}$$

and

$$\frac{3}{2}\Pi \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}(2x^2 + b^2). \tag{2.25}$$

In order to study the stability of the system (2.19)–(2.23) from a dynamical system approach [39, 40], we define the vector $\vec{v} = (x,\, u,\, b,\, l,\, s)$ and consider a linear perturbation of the form $\vec{v} \to \vec{v} + \delta\vec{v}$. Thus, the linearized system reduces to $\delta\vec{v}' = M\delta\vec{v}$, where M is given by

$$M = \begin{pmatrix} -3 + 9x^2 + \frac{3}{2}b^2 & -s & 3bx & 0 & -u \\ 6xu + s & \frac{3}{2}b^2 + 3x^2 & 3bu & 0 & x \\ 6xb & 0 & \frac{9}{2}b^2 + 3x^2 - \frac{3}{2} & 0 & 0 \\ 6xl & 0 & 3bl & \frac{3}{2}b^2 + 3x^2 & 0 \\ 6xs & 0 & 3bs & 0 & \frac{3}{2}b^2 + 3x^2 \end{pmatrix} \tag{2.26}$$

and represents the Jacobian of $\vec{v}'$.

The equilibrium points or fixed points $\{x_c,\, u_c,\, b_c,\, l_c,\, s_c\}$ of the phase space are:

(I) $\{\pm 1,\, 0,\, 0,\, 0,\, 0\}$;

(II) $\{0,\, 0,\, \pm 1,\, 0,\, 0\}$;

(III) $\{0,\, \pm\sqrt{1 - l^2},\, 0,\, l,\, 0\}$;

(IV) $\{0,\, 0,\, 0,\, 1,\, s\}$.

According to the Friedmann constraint (2.18), the case (I) represents a totally kinetic energy scalar dominated universe, the case (II) characterizes a totally baryonic dominated universe, the case (III) represents a mixture of scalar potential energy contribution and cosmological constant and the case (IV) a totally cosmological constant dominated universe. The case (IV) is the only one that admits $s > 0$, leading to a mass $m > 0$ for the dark matter particles from (2.24). The eigenvalues of the matrix M at the fixed points above are:

(I) $\{6,\, \frac{3}{2},\, 3,\, 3,\, 3\}$;

(II) $\{3,\, -\frac{3}{2},\, \frac{3}{2},\, \frac{3}{2},\, \frac{3}{2}\}$;

(III) $\{0, 0, 0, -3, -\frac{3}{2}\}$;

(IV) $\{0, 0, -\frac{3}{2} + \frac{1}{2}\sqrt{9 - 4s^2}, -\frac{3}{2} - \frac{1}{2}\sqrt{9 - 4s^2}, -\frac{3}{2}\}$.

As it is well known from the dynamical system approach [39, 40], when the real parts of the eigenvalues are positive the fixed point is unstable, and when the real parts are negative the fixed point is stable. When positive and negative real terms are present, the fixed point is a saddle point. Finally, when some of the eigenvalues are null, nothing can be said about the stability, and more accurate methods should be applied, as the center manifold theory.

For the above set of eigenvalues, the most interesting one for us is the case (IV), where the stability is not completely discarded due to the null values of the two first eigenvalues. Beside that such case admits $s > 0$, leading to a non-null mass to the dark matter particle. Although such case corresponds to a completely cosmological constant dominated universe at the fixed point, it is possible to estimate the mass as $m \sim H_0 \sim 10^{-33}\,\mathrm{eV}$ if we assume $s \sim 1$. The case (III) is also interesting, since it also has some of the eigenvalues negative and others null, but it corresponds to $s = 0$, leading to $m = 0$. This case is also interesting since it admits the possibility of coexistence of cosmological constant and a potential part of ϕ.

The limit of large mass can also be studied. From (2.11), we have $\omega_\phi \to -1$ when $m \to \infty$ or equivalently $V(\phi) \gg \dot{\phi}^2/2$, which corresponds to $x \ll u$. Taking the limit $x = 0$, the fixed point of interest of the dynamical system is $\{0, 0, 0, 1, s\}$, indicating that the large mass limit corresponds to the ΛCDM model, with $l = 1$ and $s \to \infty$.

In order to study the above system quantitatively we should try to integrate the dynamical system (2.19)–(2.23). Nonetheless, we will do an alternative treatment in the next section, aiming the numerical integration.

3 Alternative derivation — change of variables

The system of differential equations derived in last section is useful for dynamical system analysis, as we had discussed in the last section. In order to constrain the SFDM mass we should solve these equations numerically. However, we must stress that this choice of variables may generate high frequency solutions in the variables x and u, as already presented in ref. [22] and shown in figure 1. For huge values of the mass m, these high frequency solutions require high computational efforts (as an illustration, one single evaluation for $m \sim 10^{10}H_0^{-1}$ takes more than 24 hours with an Intel i5 processor), being desirable to avoid such parametrization.

Given that the behaviour of Ω_ϕ and $\Omega_\phi = x^2 + u^2$ are smooth even for large values of the mass m, a reasonable choice of variables is $r \equiv \sqrt{\Omega_\phi}$, such that $r^2 = x^2 + u^2$, and the "angle" $\theta \equiv \tan^{-1}\frac{u}{x}$. Thus, the new variables are:

$$x = r\cos\theta \tag{3.1}$$
$$y = r\sin\theta\,. \tag{3.2}$$

In some sense we are changing from cartesian to polar coordinates. The resulting system of equations is now:

$$r' = -3r\cos^2(\theta) + \frac{3}{2}\Pi r, \tag{3.3}$$
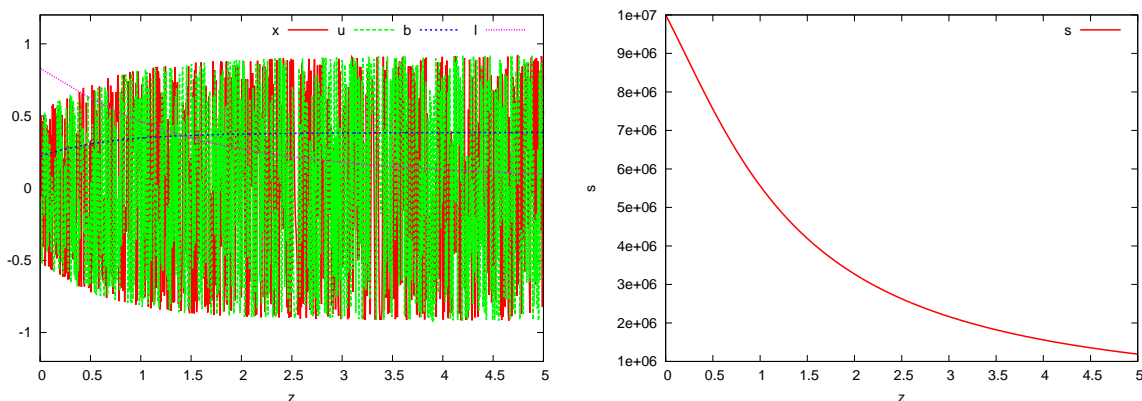
$$\theta' = s + \frac{3}{2}\sin^2(\theta), \tag{3.4}$$

Figure 1. Panel **a)** Evolution of variables (x, u, b, l) in function of redshift for $\mu = 10^7$ and $u_0 = 0$. Panel **b)**. Evolution of variable s in function of redshift for $\mu = 10^7$ and $u_0 = 0$.

$$b' = \frac{3}{2} (\Pi - 1) \, b, \tag{3.5}$$

$$l' = \frac{3}{2} \Pi \, l, \tag{3.6}$$

$$s' = \frac{3}{2} \Pi \, s \tag{3.7}$$

where $\Pi = 2r^2 \cos^2(\theta) + b^2$. As can be seen, the equations are now even simpler on r and θ, and the system is invariant under the transformation $\theta \to \theta_1 + \pi$, in such a way that it is periodic over θ with a period π. Thus, we seek to solve this system of equations over the vector of variables $\vec{r} = (r, \theta, b, l, s)$ with the corresponding initial conditions at $N = 0$:

$$\vec{r}_0 = \left(\sqrt{\Omega_{\phi 0}}, \theta_0, \sqrt{\Omega_{b0}}, \sqrt{1 - \Omega_{\phi 0} - \Omega_{b0}}, \mu \right) \tag{3.8}$$

where θ_0 becomes a free parameter, $\theta_0 \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$, we have used the normalization condition $\Omega_m + \Omega_\phi + \Omega_\Lambda = 1$ and we have defined the parametrization of the mass:

$$\mu \equiv \frac{m}{H_0} \tag{3.9}$$

It is also interesting to notice that in these new variables, the ω_ϕ EOS assumes a nicely simple expression:

$$\omega_\phi = \cos(2\theta) \tag{3.10}$$

In figure 2, we may see that none of the "polar" variables present oscillations for higher mass values ($\mu = 10^7$ on figure 2).

4 Constraining dark matter mass

In order to obtain limits to the dark matter mass, we have to constrain the set of free parameters of the SFDM model, namely, $\mathbf{s} = (\Omega_{\phi 0}, \theta_0, \Omega_{b0}, \mu, H_0)$. Hence, even imposing spatial flatness, as predicted by inflation and indicated by CMB observations, we end up with 5 free parameters. Among these parameters, Ω_{b0} is the most well constrained to date, with compatible limits from BBN and CMB. We therefore choose to fix it at the value given by Planck and WMAP: $\Omega_{b0} = 0.049$ [41], reducing the number of free parameters to 4.
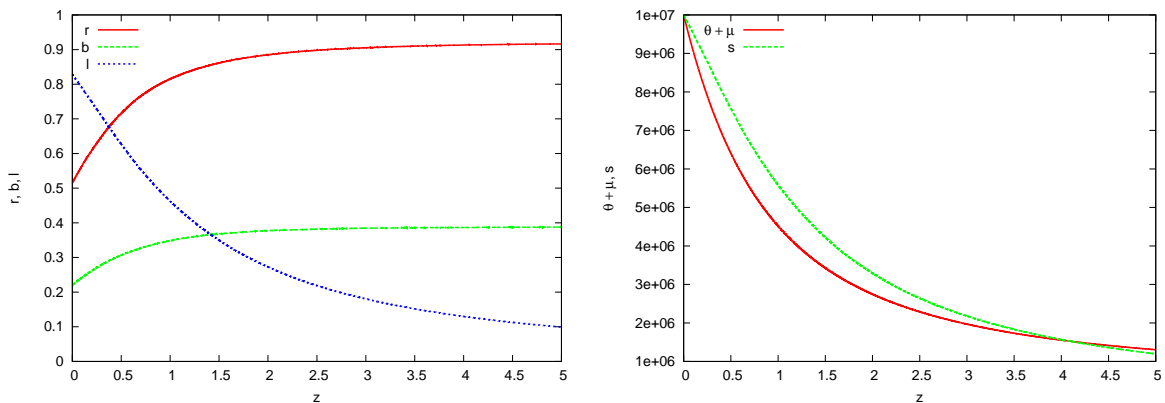
Figure 2. Panel **a)** Evolution of variables (r, b, l) in function of redshift for $\mu = 10^7$ and $\theta_0 = 0$. Panel **b)**. Evolution of variables (s, θ) in function of redshift for $\mu = 10^7$ and $\theta_0 = 0$. We have plotted $\theta + \mu$ here in order for both variables stand on same axis scale.

As a first attempt to constrain these free parameters we have considered the measurement of the Hubble parameter $H(z)$ in different redshifts. These kind of observational data are quite reliable because in general such observational data are independent of the background cosmological model, just relying on astrophysical assumptions. However, even using the current most complete compilation of $H(z)$ data, with 34 measurements [42], it was not enough to constrain the 4 free parameters, mainly because of the oscillatory behaviour of the model along with the relatively small number of degrees of freedom ($\nu = n - p = 30$).

Next, we considered a SNe Ia data sample, which, although being more dependent on the fiducial cosmological model, it consists of a large data sample and has passed recently through more refined and model independent methods of light curve fitting [43].

The parameters dependent distance modulus for a supernova at a redshift z can be computed through the expression

$$\mu_{SN}(z|\vec{p}) = m_{SN} - M_{SN} = 5 \log d_L + 25, \tag{4.1}$$

where m_{SN} and M_{SN} are respectively the apparent and absolute supernova magnitudes, $\vec{p} \equiv (\Omega_{\phi 0}, \theta_0, \mu, H_0)$ is the set of free parameters of the model and d_L is the luminosity distance in units of Megaparsecs.

Since we have no analytic expression for $H(z)$, it is necessary to define d_L through a differential equation. The luminosity distance d_L can be written in terms of a dimensionless comoving distance D by:

$$d_L = (1 + z)\frac{H_0}{c}D. \tag{4.2}$$

On its turn, the comoving distance can be related to $H(z)$, for a spatially flat Universe, by the following relation [44]:

$$\frac{dD}{dz} \equiv \frac{1}{E(z)}, \tag{4.3}$$

where $E(z) \equiv \frac{H(z)}{H_0}$. Changing to the independent variable N and given that $E = \frac{\mu}{s}$, we arrive to:

$$D'(N) = -\frac{e^{-N}s}{\mu} \tag{4.4}$$

which can be seen as a sixth equation to be solved simultaneously with the system (3.3)–(3.7) with the initial condition for the comoving distance, $D(0) = 0$. In order to constrain the free parameters of the model we considered the Union 2.1 SN Ia dataset from Suzuki et al. [43]. The best-fit set of parameters $\vec{p}$ was estimated from a χ^2 statistics with

$$\chi^2_{SN} = \sum_{i=1}^{n} \frac{[\mu_{SN}(z_i|\vec{p}) - \mu_{SN,o,i}]^2}{\sigma_i^2} \tag{4.5}$$

where $\mu_{SN}(z_i|\vec{p})$ is given by (4.1), $\mu_{SN,o,i}$ is the corrected distance modulus for a given SNe Ia at z_i being σ_i its corresponding individual uncertainty and $n = 580$ for the Union 2.1 data compilation.

As usual on this analysis, we marginalize over the H_0 dependence by rewriting the distance modulus:

$$\mu_{SN}(z) = 5 \log D_L(z) + M_* \tag{4.6}$$

where $D_L = (1+z)D$ is dimensionless luminosity distance and $M_* \equiv 25 + 5 \log \frac{c}{H_0}$ comprises all the dependence over H_0. Then, we marginalize the likelihood over M_*:

$$\tilde{\mathcal{L}}(\Omega_{\phi 0}, \theta_0, \mu) = \int_{-\infty}^{+\infty} \mathcal{N} \exp\left[-\frac{1}{2} \chi^2(M_*, \Omega_{\phi 0}, \theta_0, \mu) \right] dM_* \tag{4.7}$$

where $\mathcal{N}$ is a normalization constant. The corresponding $\tilde{\chi}^2 = -2 \ln \left(\frac{\tilde{\mathcal{L}}}{\mathcal{N}} \right)$ is given by:

$$\tilde{\chi}^2 = C - \frac{B^2}{A} \tag{4.8}$$

where $A = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}$, $B = \sum_{i=1}^{n} \frac{5 \log[D_L(z_i)] - \mu_{o,i}}{\sigma_i^2}$, $C = \sum_{i=1}^{n} \left\{ \frac{5 \log[D_L(z_i)] - \mu_{o,i}}{\sigma_i} \right\}^2$.

Since we are mainly interested on the constraints over the dark matter mass, we have marginalized $\tilde{\mathcal{L}}$ over the parameter θ_0. The result of this analysis can be seen in figure 3, where we have plotted the statistical confidence contours on the plane $\log_{10} \mu$-$\Omega_{\phi 0}$.

As one may notice, there is a strong degeneracy for $\Omega_{\phi 0}$ in the range of low DM mass $(10^{-1} \lesssim \mu \lesssim 10)$. For higher masses, $\mu \gtrsim 10^2$, the degeneracy changes to prevent a mass determination.

In this case, restricting $\Omega_{\phi 0}$ to the quite wide range $0.04 < \Omega_{\phi 0} < 0.6$ (flat prior on this interval), we have found a $\chi^2_{\min} = 560.854$, which corresponds to $\chi^2_\nu = 0.974$. We have found the best fit $\Omega_{\phi 0} = 0.22^{+0.38+0.38+0.38}_{-0.15-0.17-0.18}$ at $\Delta\chi^2 = 2.30$, 6.17 and 11.83, respectively. The best fit for μ was 19.95 and we may infer lower limits to μ as 0.94, 0.51 and 0.13 at the same χ^2 levels.

In order to alleviate the degeneracy over $\Omega_{\phi 0}$, we use a prior over $\Omega_{\phi 0}$ based on Planck results in order to constrain the mass m, or, at least, give it an inferior limit. As Planck (2013) [41] yields the limit $\Omega_{dm0} = 0.265 \pm 0.011$, at 1σ, we use a Gaussian prior of $\Omega_{\phi 0} = 0.265 \pm 0.022$, which is approximately 2σ of the Planck analysis. The result is shown on figure 4.

As one may see, $\Omega_{\phi 0}$ is now better constrained in the interval $0.19 < \Omega_{\phi 0} < 0.34$, at 3σ. We have found, from this analysis, $\chi^2_{\min} = 561.897$, $\chi^2_\nu = 0.976$. The best fit is $\Omega_{\phi 0} = 0.252^{+0.041+0.065+0.086}_{-0.027-0.045-0.061}$. The degeneracy remains, however, over the dark matter mass, whose likelihood now presents two maxima, one for low mass and another for high mass. The best fit is $\mu = 19.68$, with inferior limits given by 1.12, 0.64 and 0.12 at $\Delta\chi^2 = 2.30$, 6.17 and 11.83, respectively.
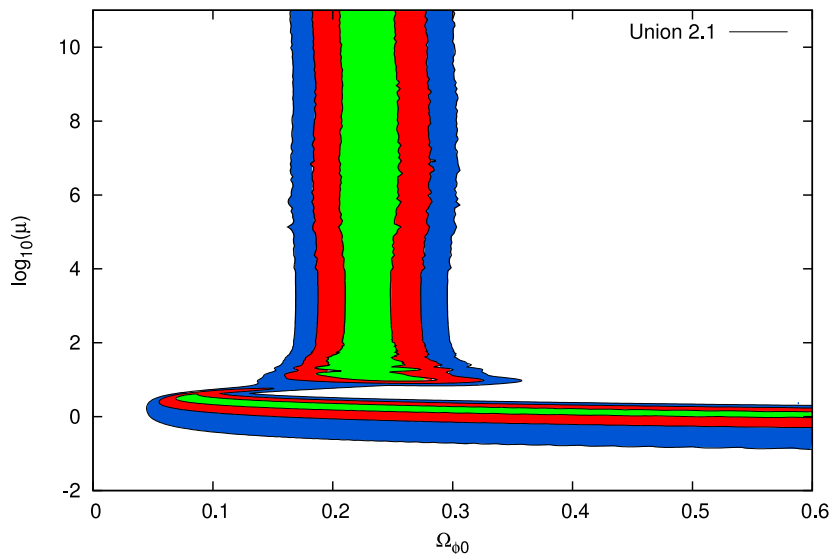
Figure 3. Confidence contours of SFDM from Union 2.1 SN data. The regions correspond to 68.3%, 95.4% and 99.7% c.l. (green, red and blue regions, respectively).
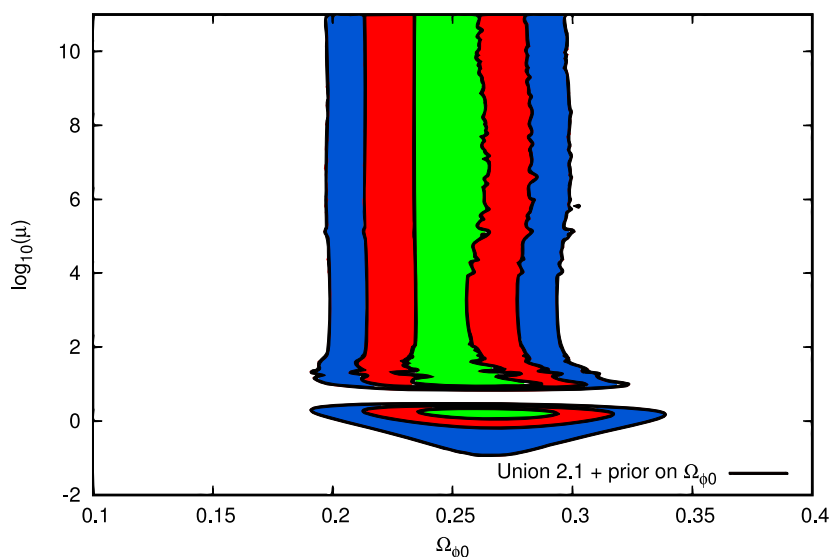

Figure 4. Confidence contours of SFDM from Union 2.1 SN data + Planck prior over $\Omega_{\phi 0}$. The regions correspond to 68.3%, 95.4% and 99.7% c.l. (green, red and blue regions, respectively).

The lowest limit on this context, $\mu = 0.12$, corresponds to a dark matter mass $m = 0.12H_0^{-1}$, which corresponds to $m = (1.557 \pm 0.038) \times 10^{-33}\,\mathrm{eV}$ for a Hubble constant of $H_0 = 73 \pm 1.8\,\mathrm{km\,s^{-1}Mpc^{-1}}$, as indicated by the most recent analysis from the Hubble Space Telescope [45].

5   Concluding remarks

In this work, we investigated the hypothesis of the dark matter as a scalar field, where we have analysed the dynamics of a quadratic potential for a single parameter free scalar field.

Considering the SN Ia observational data, we have found a very low inferior limit to the dark matter mass, namely, $m \geq 1.56 \times 10^{-33}$ eV. Although such lower limit is much smaller than the constraints obtained in [22, 23, 37] (of about 10^{-22} eV), we have shown that any dark matter mass greater than this one is also compatible with SN Ia observations. Our constraints are also much similar to the value recently obtained in [27] ($m \sim 10^{-32}$ eV), which opens the question if the mass of the scalar field dark matter could be so small.

A very small mass may be in conflict with the structure formation of the universe, since it consists of ultra hot dark matter. In the framework of structure formation, hot dark matter is disfavoured since in this case the galaxy-size density fluctuations would get washed out by free-streaming leading to an earlier superclusters formation, while observations indicate early galaxy formation.

Experiments involving dark matter particles detections, e.g DAMA [46], search for particles with mass in the range (~ 15–120 GeV). More recently [47], an upper bound limit for dark matter mass of ~ 197 TeV was established, based on relic abundance of thermal dark matter particles annihilating via a long-range interaction.

Studies of the density perturbation evolution should be made in order to obtain an upper limit on the scalar field dark matter mass. We may perform this analysis in a future work. Another forthcoming study is trying to test this result against different potential dependencies of the scalar field, including self-interaction which could eventually furnish an upper limit to dark matter mass.

Acknowledgments

A The motion equation

Another way to investigate the SFDM model is through the field motion equation. While it may be numerically intensive, this approach can be useful for dynamical interpretations. The motion equation for a scalar field is well known and can be obtained from the continuity equation as (2.13):

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \,. \tag{A.1}$$

Writing it in terms of the e-folds number, $N = \ln a$, which is more suitable for numerical integration, we have:

$$\phi'' + \left(\frac{H'}{H} + 3 \right) \phi' + \frac{V'(\phi)}{H^2} = 0 \,, \tag{A.2}$$

where the primes at ϕ and H denotes derivation with respect to N. Using the fact that ρ_ϕ can be written as $\rho_\phi(N) = \frac{1}{2}H^2\phi'(N)^2 + V(\phi)$, we can write the Hubble parameter as

$$H^2 = \frac{8\pi G \left[V(\phi) + \rho_{b0}e^{-3N} + \rho_\Lambda \right]}{3 - 4\pi G\phi'(N)^2} \,, \tag{A.3}$$

where ρ_Λ =constant. Inserting this result into the motion equation, we arrive at an equation involving only field derivatives. However, in order to simplify the equations, we apply the free scalar field potential $\left(V(\phi) = \frac{1}{2}m^2\phi^2\right)$ and define the dimensionless quantities

$$\Phi \equiv \sqrt{\frac{8\pi G}{3}}\phi \tag{A.4}$$

$$\mu \equiv \frac{m}{H_0}. \tag{A.5}$$

With these definitions, the Friedmann equation can now be written

$$\left(\frac{H}{H_0}\right)^2 = \frac{\frac{1}{2}\mu^2\Phi(N)^2 + \Omega_{b0}e^{-3N} + \Omega_{\Lambda 0}}{1 - \frac{1}{2}\Phi'(N)^2}. \tag{A.6}$$

With this expression we may find the motion equation as

$$\Phi'' + \frac{1}{2E^2}\left[3\mu^2\Phi^2\Phi' + \left(3\Omega_{b0}e^{-3N} + 6\Omega_{\Lambda 0}\right)\Phi' + 2\mu^2\Phi\right] = 0, \tag{A.7}$$

where $E^2 \equiv \left(\frac{H}{H_0}\right)^2$ is given by (A.6). This equation can be solved to find the free scalar field evolution. Furthermore, we have the dark matter density parameter

$$\Omega_\phi = \frac{8\pi G\rho_\phi}{3H^2} = \frac{\Phi'^2}{2} + \frac{\mu^2}{2E^2}\Phi^2. \tag{A.8}$$

Thus:

$$\Omega_{\phi 0} = \frac{\Phi_0'^2}{2} + \frac{\mu^2\Phi_0^2}{2}. \tag{A.9}$$

References

[1] T. Padmanabhan, *Cosmological constant: the weight of the vacuum*, *Phys. Rept.* **380** (2003) 235 [hep-th/0212290] [INSPIRE].

[2] P.J.E. Peebles and B. Ratra, *The cosmological constant and dark energy*, *Rev. Mod. Phys.* **75** (2003) 559 [astro-ph/0207347] [INSPIRE].

[3] S.M. Carroll, W.H. Press and E.L. Turner, *The cosmological constant*, *Ann. Rev. Astron. Astrophys.* **30** (1992) 499 [INSPIRE].

[4] J.A.S. Lima, *Alternative dark energy models: an overview*, *Braz. J. Phys.* **34** (2004) 194 [astro-ph/0402109] [INSPIRE].

[5] V. Sahni and A. Starobinsky, *Reconstructing dark energy*, *Int. J. Mod. Phys.* **D 15** (2006) 2105 [astro-ph/0610026] [INSPIRE].

[6] SUPERNOVA SEARCH TEAM collaboration, A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201] [INSPIRE].

[7] SUPERNOVA COSMOLOGY PROJECT collaboration, S. Perlmutter et al., *Measurements of Ω and Λ from 42 high redshift supernovae*, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133] [INSPIRE].

[8] A.G. Riess et al., *New Hubble space telescope discoveries of type Ia supernovae at $z \geq 1$: narrowing constraints on the early behavior of dark energy*, *Astrophys. J.* **659** (2007) 98 [astro-ph/0611572] [INSPIRE].

[9] SNLS collaboration, P. Astier et al., *The supernova legacy survey: measurement of Ω_M, Ω_Λ and W from the first year data set*, *Astron. Astrophys.* **447** (2006) 31 [astro-ph/0510447] [INSPIRE].

[10] M. Davis, G. Efstathiou, C.S. Frenk and S.D.M. White, *The evolution of large scale structure in a universe dominated by cold dark matter*, *Astrophys. J.* **292** (1985) 371 [INSPIRE].

[11] G. Bertone, *Particle dark matter: observations, models and searches*, Cambridge University Press, Cambridge U.K. (2010).

[12] M. Taoso, G. Bertone and A. Masiero, *Dark matter candidates: a ten-point test*, *JCAP* **03** (2008) 022 [arXiv:0711.4996] [INSPIRE].

[13] E.W. Kolb and M.S. Turner, *The early universe*, Westview Press, Boulder CO U.S.A. (1990).

[14] S. Weinberg, *Cosmology*, Oxford University Press, New York NY U.S.A. (2008).

[15] Y.M. Cho and Y.Y. Keum, *Dilatonic dark matter and unified cosmology: a new paradigm*, *Class. Quant. Grav.* **15** (1998) 907 [INSPIRE].

[16] R. Dick, *The dilaton as a candidate for dark matter*, hep-th/9609190 [INSPIRE].

[17] T. Matos, F.S. Guzmán and L.A. Ureña-López, *Scalar field as dark matter in the universe*, *Class. Quant. Grav.* **17** (2000) 1707 [astro-ph/9908152] [INSPIRE].

[18] T. Matos and F.S. Guzmán, *Scalar fields as dark matter in spiral galaxies*, *Class. Quant. Grav.* **17** (2000) L9 [gr-qc/9810028] [INSPIRE].

[19] F.S. Guzmán, T. Matos and H. Villegas-Brena, *Dilatonic dark matter in spiral galaxies*, *Rev. Mex. Astron. Astrofis.* **37** (2001) 63 [astro-ph/9811143] [INSPIRE].

[20] A. Suárez, V.H. Robles and T. Matos, *A review on the scalar field/Bose-Einstein condensate dark matter model*, *Astrophys. Space Sci. Proc.* **38** (2014) 107 [arXiv:1302.0903] [INSPIRE].

[21] T. Matos, A. Vázquez-González and J. Magaña, ϕ^2 *as dark matter*, *Mon. Not. Roy. Astron. Soc.* **393** (2009) 1359 [arXiv:0806.0683] [INSPIRE].

[22] J. Magana and T. Matos, *A brief review of the scalar field dark matter model*, *J. Phys. Conf. Ser.* **378** (2012) 012012 [arXiv:1201.6107] [INSPIRE].

[23] B. Li, T. Rindler-Daller and P.R. Shapiro, *Cosmological constraints on Bose-Einstein-condensed scalar field dark matter*, *Phys. Rev.* **D 89** (2014) 083536 [arXiv:1310.6061] [INSPIRE].

[24] E.J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of dark energy*, *Int. J. Mod. Phys.* **D 15** (2006) 1753 [hep-th/0603057] [INSPIRE].

[25] L.A. Ureña-López, *Bose-Einstein condensation of relativistic scalar field dark matter*, *JCAP* **01** (2009) 014 [arXiv:0806.3093] [INSPIRE].

[26] L.A. Ureña-López and M.J. Reyes-Ibarra, *On the dynamics of a quadratic scalar field potential*, *Int. J. Mod. Phys.* **D 18** (2009) 621 [arXiv:0709.3996] [INSPIRE].

[27] H.I. Ringermacher and L.R. Mead, *Observation of discrete oscillations in a model-independent plot of cosmological scale factor vs. lookback time and a scalar field model*, *Astron. J.* **149** (2015) 137 [arXiv:1502.06140] [INSPIRE].

[28] A.R. Liddle and D.H. Lyth, *Cosmological inflation and large-scale structure*, Cambridge University Press, Cambridge U.K. (2000).

[29] V. Mukhanov, *Physical foundations of cosmology*, Cambridge University Press, Cambridge U.K. (2005).

[30] W.H. Press, B.S. Ryden and D.N. Spergel, *Single mechanism for generating large scale structure and providing dark missing matter*, *Phys. Rev. Lett.* **64** (1990) 1084 [INSPIRE].

[31] J. Madsen, *Bose condensates, big bang nucleosynthesis and cosmological decay of a 17 keV neutrino*, *Phys. Rev. Lett.* **69** (1992) 571 [INSPIRE].

[32] J. Magaña, T. Matos, A. Suárez and F.J. Sánchez-Salcedo, *Structure formation with scalar field dark matter: the field approach*, *JCAP* **10** (2012) 003 [arXiv:1204.5255] [INSPIRE].

[33] V.H. Robles and T. Matos, *Flat central density profile and constant DM surface density in galaxies from scalar field dark matter*, *Mon. Not. Roy. Astron. Soc.* **422** (2012) 282 [arXiv:1201.3032] [INSPIRE].

[34] C.G. Böhmer and T. Harko, *Can dark matter be a Bose-Einstein condensate?*, *JCAP* **06** (2007) 025 [arXiv:0705.4158] [INSPIRE].

[35] V.H. Robles and T. Matos, *Exact solution to finite temperature SFDM: natural cores without feedback*, *Astrophys. J.* **763** (2013) 19 [arXiv:1207.5858] [INSPIRE].

[36] V.H. Robles and T. Matos, *Strong lensing with finite temperature scalar field dark matter*, *Phys. Rev.* **D 88** (2013) 083008 [arXiv:1302.5944] [INSPIRE].

[37] V. Lora and J. Magaña, *Is sextans dwarf galaxy in a scalar field dark matter halo?*, *JCAP* **09** (2014) 011 [arXiv:1406.6875] [INSPIRE].

[38] J.F. Jesus and S.H. Pereira, *CCDM model from quantum particle creation: constraints on dark matter mass*, *JCAP* **07** (2014) 040 [arXiv:1403.3679] [INSPIRE].

[39] S.H. Strogatz, *Nonlinear dynamics and chaos*, Perseus Books Publishing, U.S.A. (1994).

[40] L. Perko, *Differential equations and dynamical systems*, third edition, Springer, New York NY U.S.A. (2001).

[41] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, *Astron. Astrophys.* **571** (2014) A16 [arXiv:1303.5076] [INSPIRE].

[42] G.S. Sharov and E.G. Vorontsova, *Parameters of cosmological models and recent astronomical observations*, *JCAP* **10** (2014) 057 [arXiv:1407.5405] [INSPIRE].

[43] N. Suzuki et al., *The Hubble Space Telescope cluster supernova survey: V. Improving the dark energy constraints above $z > 1$ and building an early-type-hosted supernova sample*, *Astrophys. J.* **746** (2012) 85 [arXiv:1105.3470] [INSPIRE].

[44] C. Clarkson, B. Bassett and T.H.-C. Lu, *A general test of the Copernican principle*, *Phys. Rev. Lett.* **101** (2008) 011301 [arXiv:0712.3457] [INSPIRE].

[45] A.G. Riess et al., *A 2.4% determination of the local value of the Hubble constant*, *Astrophys. J.* **826** (2016) 56 [arXiv:1604.01424] [INSPIRE].

[46] DAMA collaboration, R. Bernabei et al., *First results from DAMA/LIBRA and the combined results with DAMA/NaI*, *Eur. Phys. J.* **C 56** (2008) 333 [arXiv:0804.2741] [INSPIRE].

[47] B. von Harling and K. Petraki, *Bound-state formation for thermal relic dark matter and unitarity*, *JCAP* **12** (2014) 033 [arXiv:1407.7874] [INSPIRE].