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Restoration of Electrical Distribution Systems Using a Relaxed Mathematical Model

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Abstract

This paper proposes a relaxed mathematical model to solve the restoration problem of radial and balanced electrical distribution systems. The mathematical model is a mixed-integer linear programming formulation that can be efficiently solved by commercial solvers. After a restoration problem is solved using this model, the quality and feasibility of the corresponding solution can be verified by using a conventional radial power flow. The performance of the proposed relaxed model is evaluated through exhaustive tests and the solutions found are compared with the ones provided by an exact mathematical formulation. The results obtained demonstrate the efficiency of the proposed approach.

Keywords Electrical distribution systems · Mathematical programming · Restoration problem

1 Introduction

The restoration problem of the distribution system is one of the most relevant topics related to the efficient operation of electrical distribution systems (EDSs). This problem tries to restore the portion of the EDS that was de-energized after a permanent fault. Therefore, given a fault in the EDS, it is isolated, and three sections appear: (1) the section that remains energized and in normal operation (in-service customers); (2) the section related directly with the fault, which must remain isolated until the corresponding repair is carried out; and (3) the section that was disconnected downstream of the section in fault (out-of-service areas) but can be energized again through switching operations. In this context, the intention is to reestablish the service to the largest possible number of disconnected loads than can be restored (out-of-service areas).

There are several optimization methods to solve the restoration problem of radial EDSs. These optimization approaches can be classified within three types: (1) heuristic methods, (2) metaheuristic methods, and (3) classical optimization techniques. Different from heuristic and metaheuristic methods, classical optimization techniques require a complete mathematical model to solve the problem. A few mathematical models have been proposed for the restoration problem in the specialized literature, such as Romero et al. (2016); Ciric and Popovic (2000); Carvalho et al. (2007), which can be solved using exact techniques. The development of complete mathematical models for the restoration problem was deferred, mainly due to the difficulty of representing the radiality condition through simple algebraic relations. This issue was recently overcome by several proposals, such as the one developed in Lavorato et al. (2012).

The heuristic methods were widely used to solve the restoration problem. They have the advantage of finding good quality solutions within a low processing time and dealing properly with large-scale EDSs. Important contributions related to the application of heuristic methods to the solution of the restoration problem have been presented in Miu et al. (1998); Kleinberg et al. (2011); Shirmohammadi (1992); Wu et al. (1991); Hsu et al. (1992); Nahman and Strbac (1994); Dimitrijevic and Rajakovic (2011). The methods in Miu et al. (1998) and Kleinberg et al. (2011) are particularly interesting, because they consider priority customers and permit load shedding in the section energized after the fault, in order to reduce the load curtailment in the restored system. The proposals in Shirmohammadi (1992); Wu et al. (1991); Hsu et al. (1992); Nahman and Strbac (1994) also

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made significant contributions to improve the performance of heuristic methods. A logic based on branch exchange to reduce the number of switching operations was developed in Shirmohammadi (1992), while a heuristic that makes switching operations only in the boundary area of feeders and that can be applied to unbalanced EDSs is presented in Wu et al. (1991). Moreover, the proposal of restoring the largest portion of the disconnected demand appears clearly in Hsu et al. (1992), in which an attempt is made to reduce the number of switching operations in the boundary area between energized and disconnected sections. Similarly, the method in Nahman and Strbac (1994) uses a strategy that generates the minimal paths to the boundary branches connecting the out-of-service area. An intensive use of Prim's algorithm is suggested in Dimitrijevic and Rajakovic (2011) as a fast exact technique to solve the minimum spanning tree problem, which is an efficient alternative to implementing heuristic techniques in the optimization of the restoration problem.

In the last decades, metaheuristics were the optimization techniques that showed the largest potential to solve the restoration problem and, therefore, numerous proposals using these techniques have been developed, such as in Kumar et al. (2008); Nagata and Sasaki (2002); Augugliaro et al. (1988); Mendoza et al. (2014); Camillo et al. (2016). These proposals came up with significant contributions to efficiently solve the restoration problem. Nevertheless, some topics still have not been properly developed in the literature, such as solving the restoration problem in an efficient way when it is not possible to reconnect all the loads due to an unfeasible operation, then requiring load curtailment. For instance, the method presented in Kleinberg et al. (2011), which considers the possibility of curtailing the load of an energized node to supply a larger demand in the disconnected area, was not extended in recent works due to its high complexity. A detailed review on the methods related to the restoration problem can be found in Curcic et al. (1995); Sudhakar and Srinivas (2011). Finally, it should be noted that metaheuristics are still the most important alternatives to solve the restoration problem for large distribution systems within small processing times. Recent research related to the application of metaheuristics to this problem can be found in Li et al. (2014); Ren et al. (2012) and Sanches et al. (2014).

This paper proposes a mathematical model that is a relaxation of the exact formulation presented in Romero et al. (2016). The relaxed mathematical model has an advantage over the exact formulation as the former can be efficiently solved by commercial solvers within a low computational time, and therefore, it can be used to solve the restoration problem for larger EDSs. Nevertheless, in some situations, the solution obtained through the relaxed model can violate operational limits. In these cases, the viability of the practical implementation of the obtained solutions must be evaluated or they can be used as an initial solution in an exact formulation, which contributes toward reducing the computational effort required in the solution process.

Thus, the proposed relaxed mathematical model complements, in an appropriate way, the exact mathematical formulation presented in Romero et al. (2016). In both models, the objective function assumes a hierarchical form: supplying the demand is the main objective, but it is also possible to give priority to the operation of automatic switches, as a secondary objective, instead of manual ones. In addition, the models can fulfill other restoration requirements, such as priority loads and the EDS restoration after multiple faults, as discussed in Romero et al. (2016). Another fundamental strategy in both mathematical models is the use of a fictitious substation that can be connected to each load node through fictitious circuits that are initially open. This strategy makes it possible to find optimal solutions in which the restored system has a radial topology while keeping the initial state of the closed switches within the non-restored section.

In summary, the main contributions of this paper are the following: (1) a relaxed mathematical model for the restoration problem that provides adequate performance, in terms of quality solution and computational effort, as demonstrated in the tests presented; (2) the utility of the relaxed model as a complement to the exact formulation in Romero et al. (2016) is presented; and (3) the advantages of the proposed model are discussed.

2 Relaxed Mathematical Model for the Restoration Problem

The mixed-integer second-order cone programming (MISOCP) formulation for the restoration problem presented in Romero et al. (2016) (an exact formulation) can be used to derive a relaxed mathematical model that corresponds to a mixed-integer linear programming (MILP) problem. This model takes the following form:

$$\begin{aligned} min \;\; v = & \sum_{ij \in \Omega_{an}} \beta_{ij} x_{ij} + \sum_{ij \in \Omega_{ap}} \rho_{ij} x_{ij} + \sum_{ij \in \Omega_{fn}} \mu_{ij}(1 - x_{ij}) \\ & + \sum_{ij \in \Omega_{fp}} \lambda_{ij}(1 - x_{ij}) + \sum_{i \in \Omega_b} \alpha_i S_i^D y_i \end{aligned} \tag{1}$$

subject to:

$$\sum_{ki \in \Omega_l} S_{ki} - \sum_{ij \in \Omega_l} S_{ij} + S_i^G = S_i^D (1 - y_i) \quad \forall i \in \Omega_b \tag{2}$$

$$\sum_{ij \in \Omega_l \cup \Omega_h} x_{ij} = n_b - n_s \tag{3}$$

$$\sum_{ki \in \Omega_l \cup \Omega_h} H_{ki} - \sum_{ij \in \Omega_l \cup \Omega_h} H_{ij} + H_i^G = y_i \quad \forall i \in \Omega_b \tag{4}$$

$$H_i^G = 0 \qquad \forall i \in \Omega_b, i \neq S^f \tag{5}$$

$$|H_{ij}| \leq M\, x_{ij} \qquad \forall ij \in \Omega_l \cup \Omega_h \tag{6}$$

$$S^G_{S^f} = 0 \tag{7}$$

$$S^G_i \leq \overline{S}^G_i \qquad \forall i \in \Omega_s, i \neq S^f \tag{8}$$

$$|S_{ij}| \leq \overline{S}_{ij} x_{ij} \qquad \forall\ ij \in \Omega_l \tag{9}$$

$$\sum_{ij \in \Omega_l \cup \Omega_h} x_{ij} + \sum_{ki \in \Omega_l \cup \Omega_h} x_{ki} \geq 1 \quad \forall i \in \Omega_b \tag{10}$$

$$|y_i - y_j| \leq (1 - x_{ij}) \qquad \forall\ ij \in \Omega_l \tag{11}$$

$$x_{ij} \in \{0, 1\} \qquad \forall ij \in \Omega_l \cup \Omega_h \tag{12}$$

$$y_i \in \{0, 1\} \qquad \forall\ \ i \in \Omega_b \tag{13}$$

The most important variables of the mathematical model are the binary decision variables, x_{ij} and y_i. The variable $x_{ij} = 1$ indicates that the circuit ij is connected after the restoration, while $x_{ij} = 0$ means that the circuit is open. The variable y_i represents the decision of connecting or disconnecting node i after the restoration. If $y_i = 0$, then node i is served by a real substation, and therefore, it is energized. On the other hand, if $y_i = 1$, then the load in node i is not supplied by a real substation; therefore, the node is de-energized (it is connected to a fictitious substation, as will be shown below).

Other variables of the mathematical model are S_{ij} and S^G_i, which correspond to the apparent power flow through the circuit ij and the apparent power generated in the node i (only the real substations generate apparent power). Artificial variables are incorporated in the model to satisfy the requirement of radial operation in the non-restored portion of the EDS and to keep the status of the switches unchanged in that portion. These variables are H_{ij} and H^G_i, which correspond to the artificial power flow through the circuit ij and the artificial power generated in the node i (there is artificial generation only in the fictitious substation, represented by S^f). Finally, it is necessary to mention that the mathematical models represents a modified electrical network. The original EDS is modified by the addition of a fictitious substation and fictitious circuits connecting it with each load node. This strategy facilitates the generation of feasible solutions, as discussed below (Romero et al. 2016).

The input data for the problem are represented by the apparent power S^D_i demanded in node i, the apparent power flow limit $\overline{S}_{ij}$ in circuit ij, and the upper limit $\overline{S}^G_i$ for the generation in the substation i. Furthermore, the model uses the following parameters: α_i is the curtailment cost of the load at node i; β_{ij} and ρ_{ij} represent the priorities of operating manual and automatic switches, respectively, that are originally open; μ_{ij} and λ_{ij} are related to the priorities of operating manual and automatic switches, respectively, that are originally closed; n_b is the number of nodes of the EDS, which includes the real substations and the fictitious substation, but excludes the node at fault; and n_s is the number of substations, which includes the fictitious substation. Let n_l be the number of circuits in the EDS that are available for the restoration process. Then there are $n_l + n_b - n_s$ available circuits in the modified problem ($n_b - n_s$ is the number of fictitious circuits incorporated in the problem).

The sets of the models are as follows: Ω_{ap} and Ω_{an} represent the set of automatic and manual switches, respectively, that are originally open; Ω_{fp} and Ω_{fn} represent the set of automatic and manual switches, respectively, that are originally closed; Ω_b is the set of nodes of the EDS, which includes the real substations and the fictitious substation and excludes the nodes in the area under fault (isolated nodes that are not subject to restoration); Ω_l is the set of real circuits of the EDS, which excludes the circuits that isolate the area under fault (they are disregarded in the restoration process); and Ω_h is the set of fictitious circuits intended to connect the fictitious substation with each load node. All of these sets are known at the beginning of the restoration process.

The objective function (1) has five terms. The first two are related to the operation of manual and automatic switches, respectively, that are initially open; the third and the fourth correspond to the operation of manual and automatic switches, respectively, that are initially closed; the last term represents the cost of the load curtailment. A suitable choice of the parameters α_i, β_{ij}, ρ_{ij}, μ_{ij}, and λ_{ij} makes it possible for the parts of the objective function to reflect a hierarchical structure. Therefore, the strategy adopted in the proposed relaxed model is to adjust the parameters in such a way that the optimization process preferably finds a solution without load curtailment and with the lowest possible number of switching actions. If there are alternative solutions that restore all of the loads, the optimization process chooses the solution with the lowest number of switching actions. Therefore, in this hierarchical framing, solutions without load curtailment are preferable to solutions that curtail part of the load, and if there are alternative solutions regarding this aspect, the solutions with the lowest modifications and the switch statuses are chosen. In addition, it is possible that the solution process finds a solution that prioritizes the operation of automatic switches rather than manual switches; however, in this work, that distinction is not considered.

Constraint (2) represents the apparent power balance in each node of the EDS. If $y_i = 0$, the apparent power balance for an energized node is satisfied as usual; if $y_i = 1$, the apparent power balance is artificially satisfied for a de-energized node. When $y_i = 1$, the load at node i is curtailed and separated from the portion of the EDS served by the set of real substations, i.e., the node is disconnected from the energized portion of the EDS and stays de-energized. Each of the disconnected nodes demands one unit of artificial power (represented by $y_i = 1$) in (4), which corresponds to the artificial power balance in each node. Thus, if $y_i = 1$, there must be a path between the node i (de-energized) and the fictitious substation (the only one that generates artificial power); if $y_i = 0$, which means that the node i is energized, then the artificial power balance is zero, i.e., it becomes irrelevant for an energized node.

Constraints (3) and (4), along with (2), guarantee that every feasible solution, and therefore, the optimal one, is connected and has radial topology, according to Lavorato et al. (2012). Therefore, the energized portion of the EDS forms a connected and radial topology, including the real substations, while the de-energized portion forms a connected and radial topology, including the fictitious substation and the addition of fictitious circuits (making it possible to maintain the status of initially closed switches).

Constraint (5) establishes that only the fictitious substation can generate artificial power; (6) limits the artificial flow in closed circuits with a relatively big value M (e.g., $M = n_b$, because the artificial flow in a circuit cannot be larger than n_b); (7) indicates that the fictitious substation can not generate apparent power; (8) guarantees that a real substation has a limit for the supply of apparent power; and (9) requires that the flows in the circuits remain within the upper limit.

Furthermore, surrogate constraints are also incorporated, i.e., constraints that are irrelevant for the original mathematical model, but help in the solution process executed by optimization techniques such as branch and bound, which are implemented by commercial solvers for mathematical optimization. Thus, (10) is a surrogate constraint that indicates that at least one circuit must be connected to a node, and (11) establishes that if the circuit ij is closed, the variables y_i and y_j must have the same value: $y_i = y_j = 1$ if the nodes are energized and $y_i = y_j = 0$ otherwise. Finally, (12) and (13) represent the binary nature of the decision variables, x_{ij} and y_i.

The mathematical model (1)–(13) is a mixed-integer linear programming formulation that can be efficiently solved using commercial solvers such as CPLEX. As shown in the tests, this formulation has the advantage in that it can be solved with a very low computational effort, and in many cases, it is possible to find the same solution obtained by the exact model, i.e., the optimal solution. Obviously, as the model does not consider Kirchhoff's voltage law, unfeasible solutions (from the point of view of the exact model) could be obtained.

Compared with the exact model, the proposed relaxed model has the following characteristics: (i) the active and reactive powers were replaced by the corresponding apparent power in the objective function; (ii) the active and reactive power balance equations for each node in the exact formulation were replaced by the apparent power balance equation (2) for each node; (iii) the equations related to the Kirchhoff's voltage law for each fundamental loop of the system, as well as the voltage limits, are not shown in the relaxed model; (iv) the current limits in each circuit were replaced by constraints for the apparent power capacity; (v) the constraints related to the artificial substation and the corresponding artificial circuits remain unchanged; (vi) the necessary radiality constraint (3) remains the same in both models; (vii) the constraints that enforce the active and reactive power values to be equal to zero if the circuit is open were replaced by constraints that enforce the apparent power to be zero if the circuit is open (9); and (viii) all surrogate constraints remain unchanged on both models.

Some important aspects of the restoration problem have not been addressed in this work since the main objective here is to develop a relaxed version of the complete model, which can be quickly solved. Thus, a problem that is not addressed in this work is to take into account the cold load pick-up (CLPU) effect, which increases the level of the demand in the nodes de-energized by the fault, especially if the restoration process is slow. This important topic is addressed in El-Zonkoly (2012); Kumar et al. (2010) and Rodriguez et al. (2016).

Finally, both models can simulate different criteria that can be considered in the restoration process such as priority loads, differentiation between manual and automatic switches, all types of faults in components of a distribution system, and multiple faults. In particular, the fault may be of any nature and it may occur in one or more components. Thus, the model analyzes without difficulties if the fault is in a node, a transformer, a line or a combination of these devices. In this way, once the fault or simultaneous faults are produced, a procedure makes unavailable the faulted devices and the devices connected directly to them. Therefore, once a set of devices is unavailable, the energized and the de-energized (with the possibility of being energized) portions of the system are found, and therefore the solution for the restoration process can be obtained with the proposed mathematical model.

3 Tests and Results

To demonstrate the good performance of the proposed model, tests are performed using two EDSs with 53 and 417 nodes. The exact MISOCP model and the proposed relaxed MILP model were implemented in the mathematical programming language AMPL (Fourer et al. 2003) and solved using the commercial solver CPLEX (IBM ILOG CPLEX 2009) in a computer with an Intel i7-4770 processor and 16 GB of RAM. All of the tests were executed using the following parameters: $\alpha_i = 0.1$, $\beta_{ij} = 1$, $\rho_{ij} = 1$, $\mu_{ij} = 1$, $\lambda_{ij} = 1$, $M = 54$ for the 53-node EDS and $M = 418$ for the 417-node EDS.

3.1 53-Node System

A 53-node test distribution system with 61 circuits was used to evaluate the performance of the proposed relaxed model.

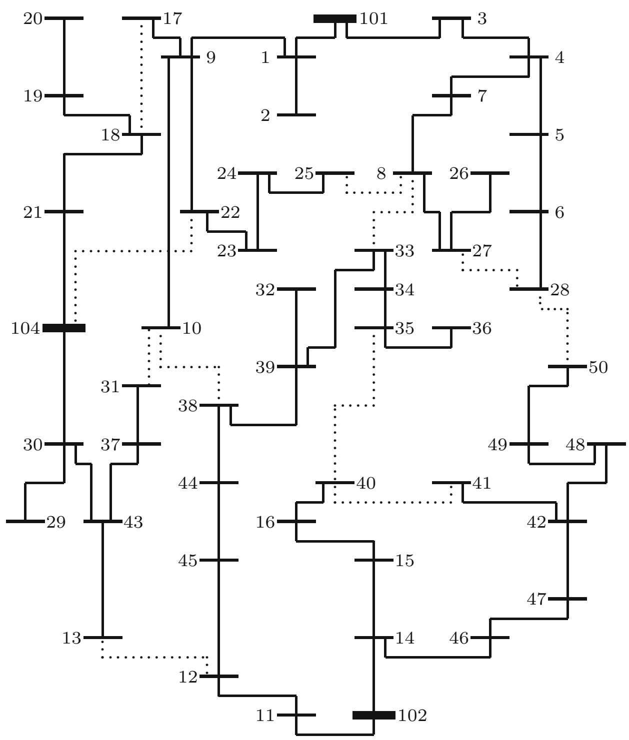

Fig. 1 Topology of the 53-node test system before the fault

The complete data of this test distribution system are available in Romero et al. (2016). The 53-node test system has 3 substations operating at a nominal voltage of 13.8 kV and it supplies 50 load nodes, which, under normal conditions, have a demand of 45,668.7 kW of active power and 22,118.24 kVAr of reactive power (50,743 kVA). The topology of the 53-node test system is shown in Fig. 1. In Fig. 1, nodes 101, 102 and 104 represent substations.

To study the performance of the proposed relaxed mathematical model, tests were carried out simulating faults in each of the 50 load nodes of the test system. The results were compared with those obtained using the exact MISOCP model presented in Romero et al. (2016). The quality of the solutions, as well as the required computational time, were compared. A large number of tests can illustrate the performance of the relaxed model when compared to the exact formulation.

The results found with the exact MISOCP formulation are shown in Table 1, while the results obtained with the relaxed MILP model are shown in Table 2. For each test, the switches that were open, the switches that were closed, the apparent power of the load (kVA) that was not restored, and the required computational time are presented. For instance, for a fault at node 3, in Table 1, the solution is to open switches 5–4, 27–8, 26–27, 28–6, and 34–33 and to close switches 28–27, 8–33, 35–40, and 28–50, leading to a load curtailment of 3,465.00 kVA with a processing time of 41 s. Additional information about the solution proposals is not shown in Tables 1 and 2, but it can easily be acquired, considering the topology of the system in Fig. 1 and the system's data. For example, note that, for the scenario of fault at node 3, the information related to node 3 and circuits 101–3 and 4–3 (used to isolate the fault and not available for the restoration) is not shown; also note that the loads at nodes 5, 6, and 26 were not restored and that they form two isolated areas: one is formed by node 26 and the other is formed by nodes 5 and 6, which are connected by circuit 5–6; thus, finally, fictitious circuits are used to connect nodes 5 and 26 with the fictitious substation. In Table 2, the results for this fault scenario are different only according to the computational time, which for the relaxed model, was just 2 s.

From the results shown in Tables 1 and 2, important conclusions can be formulated. The first one is that the solution proposals (the switches that need to be open or closed) are exactly the same for 42 of the 50 tests. The different results occurred for the faults at nodes 1, 4, 15, 16, 38, 44, 46, and 47. The results for the faults at nodes 1, 16, 38, 44, 46, and 47 represent alternative optimal solutions with the same quality as the solutions provided by the exact model, although the topology is different. The relaxed model found different results that were operationally unfeasible only for the faults at nodes 4 and 15. For a fault at node 4, the exact formulation did not restore the demand at nodes 5 and 26 (2926.00 kVA), while the relaxed model did not restore the load at nodes 5 and 7 (2772.00 kVA). It should be note that, when the solution obtained by the relaxed model was checked using a radial power flow method, unfeasibility appeared only in the current through circuit 33–39: the current in that circuit was 251.2 A, while the current limit is 250 A (a violation of 0.47%). Nevertheless, that solution can be implemented in the practice because it corresponds to a very low violation percentage in the current limit. That solution proposal has the advantage of restoring an additional 154 kVA when compared with the solution provided by the exact formulation.

For the fault at node 15, both mathematical models found solution proposals without load curtailment, completely restoring the EDS (the portion of the system that could be restored). However, the solution provided by the relaxed model was also operationally unfeasible, due to the current through circuit 46–47 (254.6 A with a limit of 250 A and a violation of 1.8%). As the violation is very low, this solution can also be implemented in the practice, without compromising the operation of that circuit. Additionally, it should be highlighted that for a fault at node 15, the solution given by the exact formulation is an alternative solution for the relaxed model, i.e., the relaxed model can also find the solution provided by the exact formulation. In other words, the solver CPLEX can provide the optimal solution in Table 1 in the following cases: (1) if another test is carried out, changing the starting conditions for CPLEX; (2) using other solvers;

Table 1 Results for the 53-node EDS obtained with the MISOCP model

Fault at node	Switches open	Switches closed	Load shedding (kVA)	Time (s)
1	25–24	104–22, 8–25	1155	2
2			0	1
3	5–4, 27–8, 26–27, 28–6, 34–33	28–27, 8–33, 35–40, 28–50	3465	41
4	6–5, 26–27, 34–33	8–33, 35–40, 28–50	2926	15
5		28–50	0	1
6		28–50	0	2
7	27–8	8–25, 28–27	0	2
8		28–27	0	1
9		104–22, 18–17, 10–38	0	2
10			0	1
11	45–12, 39–38, 34–35	13–12, 8–33, 35–40, 10–38	0	8
12	39–38, 35–34	8–33, 35–40, 10–38	0	2
13			0	1
14	36–35, 16–40, 42–41, 47–42	40–41, 35–40, 28–50	4928	28
15	49–48	40–41, 28–50	0	1
16		40–41	0	1
17			0	1
18			1694	2
19			616	2
20			0	1
21		18–17	0	1
22		8–25	0	1
23		8–25	0	1
24		8–25	0	2
25			0	1
26			0	1
27			924	1
28			0	1
29			0	1
30	31–37	10–31, 13–12	1078	1
31			0	1
32			0	1
33		35–40	0	1
34		35–40	0	1
35			231	2
36			0	1
37		10–31	0	1
38	33–34	8–33, 35–40	0	1
39		8–33	1309	1
40			0	1

Table 1 continued

Fault at node	Switches open	Switches closed	Load shedding (kVA)	Time (s)
41			0	1
42		40–41, 28–50	0	1
43		10–31, 13–12	0	2
44	33–39	8–33, 10–38	0	2
45	39–38, 33–34	8–33, 35–40, 10–38	0	4
46	42–41, 47–42	40–41, 28–50	770	39
47	49–48	40–41, 28–50	0	2
48		28–50	0	1
49		28–50	0	1
50			0	1

Table 2 Results for the 53-node EDS obtained with the MILP model

Fault at node	Switches open	Switches closed	Load shedding (kVA)	Time (s)
1	17–9	104–22, 18–17	1155	<1
2			0	<1
3	5–4, 27–8, 26–27, 28–6, 34–33	28–27, 8–33, 35–40, 28–50	3465	2
4	8–7, 6–5, 34–33	8–33, 35–40, 28–50	2772	1
5		28–50	0	<1
6		28–50	0	1
7	27–8	8–25, 28–27	0	1
8		28–27	0	1
9		104–22, 18–17, 10–38	0	1
10			0	<1
11	45–12, 39–38, 34–35	13–12, 8–33, 35–40, 10–38	0	1
12	39–38, 35–34	8–33, 35–40, 10–38	0	1
13			0	1
14	36–35, 16–40, 42–41, 47–42	40–41, 35–40, 28–50	4928	1
15	50–49	40–41, 28–50	0	1
16		35–40	0	1
17			0	<1
18			1694	1
19			616	<1
20			0	1
21		18–17	0	1
22		8–25	0	<1
23		8–25	0	<1
24		8–25	0	1
25			0	<1
26			0	<1

Table 2 continued

Fault at node	Switches open	Switches closed	Load shedding (kVA)	Time (s)
27			924	<1
28			0	<1
29			0	<1
30	31–37	10–31, 13–12	1078	1
31			0	<1
32			0	<1
33		35–40	0	1
34		35–40	0	<1
35			231	<1
36			0	1
37		10–31	0	<1
38	35–34	8–33, 35–40	0	<1
39		8–33	1309	1
40			0	<1
41			0	<1
42		40–41, 28–50	0	<1
43		10–31, 13–12	0	<1
44	34–33	8–33, 35–40	0	1
45	39–38, 33–34	8–33, 35–40, 10–38	0	<1
46	48–42, 47–42	40–41, 28–50	770	1
47	42–41	40–41, 28–50	0	<1
48		28–50	0	<1
49		28–50	0	<1
50			0	<1

or (3) when it is specified to store alternative optimal solutions. This matter was confirmed using the aforementioned first case.

The second conclusion is that the computational times are significantly lower when the tests are carried out using the relaxed model, as can be confirmed by Tables 1 and 2. It is particularly interesting to consider the performance of the relaxed model for a fault at node 3 (in which both models found the same solution). The relaxed model required a very low processing time for all of the executed tests. Actually, the computational times are relatively low for most of the analyzed faults with the exact model. Indeed, only for 5 fault scenarios (faults at nodes 3, 4, 11, 14, and 46), the times required by the exact formulation are relatively high.

Another important observation is that alternative optimal solutions can appear in the restoration problem with a high frequency when compared with other optimization problems in the operation of EDSs. For instance, this particularity does not appear with the same frequency in the reconfiguration problem of EDSs, which is relatively close to the restoration problem. This issue is due to the fact that, if it is possible to restore all of the demand, then there are several possible combinations with the same number of switching actions that enable this optimal restoration. The same situation happens when there is a load curtailment in the restored system. Then, it could be very convenient to solve the restoration problem in a way in which all of the alternative optimal solutions are found, which allows the EDS operator to have more than one option to execute the restoration.

3.2 417-Node System

In this case, tests using a large distribution system are presented to evaluate the performance of the proposed relaxed model. This EDS, initially proposed by Bernal (1998), has 417 nodes (3 substations and 414 demand nodes) and 473 circuits (all of them can be operated). The substations are represented by nodes 415, 416, and 417 and they operate with a nominal voltage of 10.0 kV. The lower and upper voltage magnitude limits are 0.90 p.u. and 1.0 p.u., respectively. Under normal conditions, the demand supplied is 30,406.81 kVA (27,372.60 kW and 13,237.00 kVAr). The pre-fault topology has 708.94 kW of active power losses. Other authors have used this system to evaluate their methods, as can be seen in Souza et al. (2016) for solving the reconfiguration problem to minimize losses. However, there

Table 3 Results for the 417-node EDS obtained with the MISOCP model

Fault at node	Switches open	Switches closed	Load shedding (kVA)	Time (s)
91	2–19, 57–19	19–32, 87–88, 84–243	23.26	33,400
222	76–77	124–127, 358–250	0	3162
345		159–179	0	17

Table 4 Results for the 417-node EDS obtained with the MILP model

Fault at node	Switches open	Switches closed	Load shedding (kVA)	Time (s)
91	2–19, 57–19	383–67, 19–32, 87–88	23.26	5
222	382–369	124–127, 358–250	0	1
345		159–179	0	1

is no available information about the current capacity of the circuits. Thus, maximum current data were defined for this system considering the base configuration of the system adopted in the literature. Complete data for this test EDS can be accessed online in http://www.feis.unesp.br/#!/lapsee (Downloads section).

Tables 3 and 4 show the results of the tests performed. Table 3 shows the results obtained with the exact mathematical formulation while Table 4 shows the results obtained using the proposed relaxed mathematical model. The fault scenarios at nodes 91, 222, and 345 were chosen for the analysis because they allows specific analysis about the performance of the exact and relaxed models. Node 91 is directly connected to substation 417 and its isolation de-energizes 47 other nodes. Node 222 is connected to the substation 415 and is located 5 nodes after this substation; the isolation of node 222 de-energizes 21 other nodes. Finally, node 345 is connected to substation 416, 10 nodes from it; the isolation of node 345 de-energizes 32 other nodes. All these scenarios are critical because the mathematical model needs to consider the complete electrical system to solve the restoration problem.

In the case of a fault at node 345, a single switching operation was required to reconnect all the de-energized nodes. In this scenario, the exact and relaxed models found the same optimal solution within very small computational times: 17 s and less than 1 s, respectively. This solution is feasible and the minimum voltage magnitude in the new configuration is 0.9301 p.u. (at node 30).

For a fault at node 91, alternative optimal solutions were found. In this scenario, the load shedding was 23.26 kVA and 5 switching operations were proposed in both solutions. The demand not supplied corresponds to the load connected at node 86, which does not have alternative adjacent circuits after node 91 is isolated. By comparing both solutions, it can be verified that of the five proposed switching operations, only one of them is different. Both are feasible and alternative optimal solutions, and in both cases, the minimum voltage magnitude in the new configuration is 0.9301 p.u. (at node 30). The main difference is that in this fault scenario, the exact mathematical model demanded a high processing time (33400 s), while the relaxed model solved the problem in just 5 s.

Finally, in the case of a fault at node 222, alternative optimal solutions were also found. In this scenario, all the de-energized nodes were reconnected with three switching operations. By comparing the two solutions, it can be verified that of the three proposed switching operations, only one of them is different. Both are again feasible and alternative optimal solutions. In the solution presented by the exact model, the minimum voltage magnitude is 0.9095 p.u. (at node 70) and, in the solution presented by the relaxed model, the minimum voltage magnitude is 0.9014 p.u. (at node 60). The exact mathematical model required a high processing time (3162 s), while the relaxed model solved the problem in just 1 s.

The tests performed show how much the proposed relaxed mathematical model can be advantageous for the exact resolution of the restoration problem. In this large system, it was found that the exact model can solve the problem efficiently in less critical cases, as it can be observed in the simulated fault test at node 345. However, due to the great complexity of the problem, in very critical fault cases, the exact model required a prohibitive resolution time. On the other hand, the relaxed model was efficient in all the cases. In the simulated tests with the relaxed model, there was no violation of the current capacity limit of the circuits and minimum voltage magnitude at the nodes. This verification was performed through the analysis of the results obtained with a radial power flow calculation algorithm. Despite the efficiency of the relaxed model in the three cases presented, the verification of the feasibility of the solution is necessary.

4 Additional Comments About the Relaxed Model

Further tests were carried out to verify some conjectures discussed in the previous section. An additional test using the relaxed model for a fault at node 15 was executed. It was verified that the solver CPLEX found the same solution as the one obtained with the exact model, because it is an alternative optimal solution for the relaxed model. Another important fact is that the unfeasible solution found by the relaxed model can be used as an initial solution in the hot start of the solver for the exact model. This test was done for the fault at node 3. Thus, using the hot start in CPLEX, the exact model found the same solution with a computation time of 12 s, and therefore, the solution process was accelerated in comparison with the required time in the initial test (41 s). Consequently, the proposed relaxed model can be used along with the exact formulation to produce an efficient strategy to solve the restoration problem. That strategy takes the following form:

1. Solve the restoration problem using the relaxed model and generating all of the alternative optimal solutions.
2. If one of the alternative optimal solutions is feasible for the exact model, it is the optimal solution for the restoration problem. Otherwise, verify if the solution with the lowest unfeasibility can be implemented in the practice (even with a low unfeasibility). If this is not possible, go to the step 3.
3. Solve the restoration problem using the exact model and taking the lowest unfeasible solution identified in the step 2 as the initial solution.

5 Conclusions

A relaxed mathematical model has been proposed in this paper to solve the restoration problem of radial electrical distribution systems. The mixed-integer linear programming model can be efficiently solved using commercial solvers. Tests carried out simulating single faults in each load node of a test distribution system demonstrate the excellent performance of the proposed relaxed model.

It was verified that the computational times required by the relaxed model are very low when compared with the times necessary to solve the problem using the exact formulation, particularly for some critical faults. Nevertheless, occasionally, the relaxed model can find operationally unfeasible solutions according to the exact model. In these cases, the relaxed model still could be useful, because the solutions could present a low level of feasibility violation, which in turn, does not negate their practical implementation. In addition, an unfeasible proposal can be used as an initial solution in the hot start option, in order to reduce the computational time required to solve the exact formulation.

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