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ORIGINAL PAPER

Optimal siting and sizing of renewable energy sources, storage devices, and reactive support devices to obtain a sustainable electrical distribution systems

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Abstract This paper presents an integrated planning framework to optimally determine the location and allocation of renewable-based distributed generation (DG) units, energy storage systems (ESSs), and capacitor banks (CBs). This planning aim at improving the performance of electrical distribution systems (EDSs). In the proposed model, the cost of energy delivered by the substation and the investment costs are minimized. The environmental aspects are taken into account to obtain an efficient environmentally committed plan. The uncertainties due to PV generation and demand profile are considered via external uncertainty indexes in a deterministic environment. The proposed model is a mixed-integer nonlinear programming (MINLP) problem, which is recast to a mixed-integer linear programming (MILP) problem using appropriate linearization techniques. This approximated MILP model is implemented in the mathematical language AMPL, while the commercial solver CPLEX is used to obtain the global optimal solutions. The proposed model is validated by testing on a medium voltage distribution system with 135 nodes under different conditions and topologies.

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Keywords Capacitor banks · Energy storage systems · Environmental aspects · Planning framework · Renewable generation

Abbreviations

CB	Capacitor bank
DISCO	Distribution company
DG	Distributed generation
DoD	Depth of discharge
EDS	Electrical distribution system
ESS	Energy storage system
MILP	Mixed integer linear programming
MINLP	Mixed integer nonlinear programming
PV	Photovoltaic

Sets and indexes

Ω^{cb}	Set of CB capacities
D	Set of planning horizon (years)
L	Set of circuits
N	Set of nodes
T	Set of time intervals
b	Index of reactive power capacity of installed CB
d	Index of years
i, j	Index of nodes
ij	Index of circuits
k	Index of PV modules
t	Index of time intervals

Constants

$\zeta^{G}_{t,d}$	Energy cost at time interval t, at year d
$\zeta^{CB^{f/sw}}_{b}$	Installation cost of fixed/switchable CB with power capacity b
ζ^{PV}	Installation cost of PV modules
$\zeta^{o\&m^{PV}}_{d}$	Operation and maintenance cost of installed PV modules at year d
$\zeta^{D\&R}_{n}$	Disposal and recycling cost of ESS at present net value (n)
$\zeta^{o\&m^{ESS}}_{d}$	Operation and maintenance cost of installed ESS at year d
$\zeta^{PC^{ESS}}$	Investment charging and discharging power capacity cost of ESS
$\zeta^{RC^{ESS}}$	Investment storage reservoir cost of ESS
Δ_d	Duration of the year d (8760 h)
Δ_t	Duration of time interval t
$\underline{\Phi}^{S}, \overline{\Phi}^{S}$	Lower and upper limit of power factor for substation
Φ^{pv}	Power factor for PV-based DG units
η	Efficiency of the ESS

$\overline{E}^{ESS}$	Maximum energy reservoir capacity of ESS that can be installed
$\underline{E}^{ESS}$	Minimum energy reservoir capacity of ESS that can be installed
e^{p}	Emission coefficient
$f_{t,d}^{G^{pv}}$	PV generation factor; time t, year d
$\overline{I}_{ij}$	Maximum current magnitude allowed on the circuit ij
IC^{CB}	Investment cost limit of CB
IC^{PV}	Investment cost limit of PV-based DG units
IC^{ESS}	Investment cost limit of ESS
N^{PV}	Maximum number of PV plants that can be installed
$P_{i,t,d}^{Ld}$	Active power demand at node i, time t, year d
$\overline{P}^{ESS}$	Maximum power rating of ESS that can be installed
$\overline{P}^{PV}$	Active power capacity of PV module
$\overline{PE}_{d}$	Pollutant emission limit at year d
Q_{b}^{esp}	Nominal reactive power of the CB with capacity b
$Q_{i,t,d}^{Ld}$	Reactive power demand at node i, time t, and year d
R, X, Z_{ij}	Resistance, reactance, and impedance of the circuit ij
$\overline{V}, \underline{V}$	Upper and lower voltage magnitude limits
V^{nom}	Nominal voltage magnitude
$V_{i,t,d}^{*}$	Estimated voltage magnitude at node i, time t, and year d

Continuous variables

$\tilde{E}_{i}^{ESS}$	Energy reservoir of installed ESS at node i
$E_{i,t,d}^{ESS}$	State of charge of installed ESS at node i, time t, and year d
$I_{ij,t,d}^{sqr}$	Square of current flow magnitude of circuit ij in time t, and year d
$P_{ij,t,d}$	Active power flow of circuit ij in time t, and year d
$\tilde{P}_{i}^{ESS}$	Power rating of installed ESS at node i
$P_{i,t,d}^{ESS^{c}}$	Charging power of installed ESS at node i in time t, and year d
$P_{i,t,d}^{ESS^{d}}$	Discharging power of installed ESS at node i in time t, and year d
$P_{i,t,d}^{PV}$	PV active power generation at node i in time t, and year d
$P_{t,d}^{S}$	Active power supplied by substation in time t, and year d
$Q_{ij,t,d}$	Reactive power flow of circuit ij in time t, and year d
$Q_{i,d}^{CB}$	Reactive power delivered by installed CB at node i in year d
$Q_{i,t}^{PV}$	PV reactive power generated at node i in time t
$Q_{t,d}^{S}$	Reactive power supplied by substation in time t, and year d
$V_{i,t,d}^{sqr}$	Square of voltage at node i in time t, and year d

Binary and integer variables

$x_{i,b}^{f/sw}$	Binary variables that define the capacity b and CB type (fixed or switchable) to be installed at node i
y_{i}^{ESS}	Binary variable that define the ESS to be installed at node i
$z_{i,k}$	Binary variable that define the PV modules k to be installed at node i

$B_{i,d}$ Integer variable for the CB modules to be installed at node i in year d
$\overline{M}_i$ Integer variable for the maximum CB bank to be installed in node i

1 Introduction

Nowadays, electrical power systems and specifically the electrical distribution system (EDS) are facing different technical, economic, and environmental challenges to meet the needs and preferences of its consumers. These challenges are accentuated due to increase in demands in the EDS, where the quality and reliability of the energy supplied is directly impaired. Hence, the distribution companies (DISCOs) should look for strategies to ensure proper performance of EDSs and thus ensure the quality and reliability of the energy supplied, considering futures operation scenarios. In order to find solutions to these problems, the capacitor banks (CBs) are installed on the EDS as a support to reduce energy losses, controlling the reactive power flow and improving the voltage profile. Recently, several optimization techniques have been applied to determine the optimal locations, sizes, and types of CBs to be installed. In [1], a genetic algorithm with a two-stage method was used to determine the optimal location of CBs by using the loss sensitivity technique. A sensitivity analysis was implemented in [2] to reduce the search space and to find an accurate solution for recognizing the location of CBs through the gravitational search algorithm. In [3], a two-stage procedure was presented to identify the locations and size of CBs; in the first stage, loss sensitivity indices were used to select the candidate CB locations, while at the second stage the ant colony optimization algorithm was used to find the optimal locations and sizes of CBs. A combination of genetic algorithm and fuzzy multi-objective approach was proposed in [4] aiming at optimal CBs placement to improve the substation power factor, reduce the energy losses, and decrease the burden on the substation. In [5], a comprehensive objective function was proposed for CBs placement to maximize the net saving from the perspective DISCO manager using particle swarm optimization (PSO). Another approach using PSO was proposed in [6] to find the optimal location and number of CBs while wind-based DG units were taken into account aiming at reducing the power losses and enhancing the voltage profile.

On the other hand, the electrical sector is responsible for a significant share of the CO_2 emissions. This emission, related with the generation of electrical energy, must be reduced in order to attend the global warming targets [7]. A feasible alternative to the planning framework of EDS to take into account these issues can be distributed generation (DG) installation. These technologies bring several benefits to improve the performance of EDSs. From the technical point of view, the DG units offer loss reduction, voltage control, reduction of the current flow enhancing the quality of the energy supplied [8]. Besides, these energy sources promote the pollutant emissions abatement at the distribution level; these effects are more evident when the DG is based on renewable energy sources, such as the wind and photovoltaic (PV) technologies. In the literature, for this problem, various planning strategies have been developed. A probabilistic method for determining the optimal mix of different renewable DG technologies was proposed in [9] to minimize energy losses. Reference [10] presents a DG allocation strategy for radial EDS under uncertainties of load and generation using an adaptive genetic algorithm, where the uncertainties are modeled using fuzzy-based approach. A hybrid of evolutionary programming and PSO was used for solving DG sizing problem in [11]. In this work the DG units were installed in fixed locations; thus finding their optimal locations was not considered. In addition, a stochastic two-stage multi-period mixed-integer linear programming was proposed in [12] to minimize the renewable investment costs and the operation and maintenance expenses in an EDS. In [13], to find the optimal site and size of renewable and dispatchable DG units a PSO-based approach was used where different technologies based on PV, wind, and biomass were taken into account considering technical, economic, and environmental performance indices. Moreover, in the literature, the simultaneous allocation methodologies of CBs and DG units have been proposed considering the requirements of the distribution systems. A hybrid method, based on imperialist competitive and genetic algorithm, for solving the optimal siting and sizing problem of DG units and shunt CBs was proposed in [14]. In the same way, a simultaneous integrate planning considering renewable and dispatchable DG units and CBs was proposed in [15], in which a hybrid method based on tabu search and genetic algorithms was used as the solution tool. In [16], a method for the allocation of CBs with renewable DG units was proposed. Two criterion functions modeled this optimization problem were optimized simultaneously by applying the non-dominated sorting genetic algorithm, where voltage profile was improved by optimal selection of the locations and installed powers of the shunt CBs. In [17] a strategic planning approach to find the best allocation for mix renewable energy sources and CBs, where benefits as: minimization of distribution power losses, enhancement of voltage stability, and reduction of carbon-di-oxide emission were considered in this proposed.

In recent years, energy storage systems (ESSs) have emerged as a solution to even out the power mismatch between renewable DG and power consumption. The ESS can store the surplus of power to be used during peak periods. In addition, the performance of these technologies has also proved the advantages that they bring to improve the economic and technical operation of the EDSs considering the optimal coordination with other devices [18,19]. The great challenge in the planning phase of an ESS is to determine the optimal site and size of these devices. In [20], a methodology for allocating an ESS in a distribution system with a high penetration of wind energy aiming at maximizing the benefits of DG owner and sizing the ESS to minimize the annual cost of energy was proposed. In this methodology, a formulation that solves siting and sizing problem simultaneously was not proposed. In [21], a mixed integer second-order cone programming formulation of the optimal power flow was used to define the optimal siting and sizing of ESSs in EDS considering technical and economic goals. However, their model of dispersed energy storage was able to support the EDS by both active and reactive power. In [22], a multi-objective procedure to find the optimal siting and sizing of distributed storage system to provide ancillary services was proposed. In this model the voltage support and minimizing the network losses along with minimization of the energy cost from EDS were taken into account. In [23], a heuristic method to find the optimal location and capacity ESS including transmission and distribution parts was presented. In the transmission storage part, a sensitive analysis was performed to detect the optimal location of ESS. Additionally, in the distribution storage part, the optimal ESS size was calculated to perform distribution grid

Table 1 Summary of the bibliography review

References	CBs	DG units	ESS	Environmental issues
[1–5]	X	–	–	–
[9–11]	–	X	–	–
[13]	–	X	–	X
[14–16]	X	X	–	–
[17]	X	X	–	X
[20–24]	–	X	X	–
Proposed approach	X	X	X	X

services such as peak load shaving and load curve smoothing. A stochastic model was proposed in [25] in which the optimal storage size along with the optimal generation outputs of the ESS were simultaneously determined where the uncertainty of wind power forecast was taking into account in the optimization problem. A methodology that explores the siting and sizing of ESS to reduce the voltage fluctuations caused by high PV penetration was studied in [24]. In this model, a genetic algorithm based on bi-level optimization method was developed, in which the siting of PV was not considered in the optimization problem.

Considering the above review shows that different methodologies have been implemented to siting and sizing of either CBs, DG units, and ESSs or a combination of some. This review is summarized in Table 1. This table shows the works that take advantage of each technology (marked with X) to maximize the efficiency of EDSs. It is worth noting that, in [1–5,14–16], and [20–24], the environmental issues were disregard, while [13] and [17] are the works that the environmental issues were taken into account.

Consequently, the primary objective of this paper is to develop an integrated planning framework for simultaneous siting and sizing of renewable-based DG units, ESS, and CBs in an EDS not only by taking into account the economic aspects but also the environmental issues. The resulting model that considers all the aforementioned devices and planning conditions to find an efficient environmentally committed plan is a highly complex mixed integer nonlinear programming (MINLP) model. Besides, considering the correlated uncertainties due to PV generation and demands in the proposed model via external uncertainty indexes results in a more complicated problem. Considering nonlinear and nonconvex terms may result in an intractable model. Thus, to achieve an appropriate tradeoff between modeling accuracy and computational tractability of the proposed integrated planning, the resulted uncertainty-based MINLP model using proper linearization techniques is recast to an approximated mixed-integer linear programming (MILP) model.

To validate the mathematical representation and the solution technique, tests on a medium voltage distribution system with 135 nodes under different conditions are conducted, and results are considered in detail. The MILP model is implemented in the mathematical language AMPL, while the commercial solver CPLEX is used to obtain the global optimal solutions. The main contributions of this study can be summarized as follows:

- Proposing an integrated planning framework of EDSs considering different support alternatives that besides improving the quality and reliability of provided energy to the consumers, provides an environmentally committed approach controlling the pollutant emissions from the substation side throughout the planning horizon.
- Addressing and management environmental issues of an EDS, where the correlation between uncertain characteristics of renewable energy sources and demands are considered via external uncertainty indexes in the proposed deterministic model.
- Recasting the nonlinear representation of the problem to a solver-friendly flexible MILP model; this way, using a commercial MILP solver guarantees to find the global optimal solution with a high computational efficiency.

The rest of this paper is organized as follows. In Sect. 2, the mathematical formulation of MINLP model is presented and then the recast process to obtain an approximated MILP model is explained. Section 3 presents the assumptions and the case studies that are used to validate the proposed model. Subsequently, numerical results are reported and analyzed in the Sect. 4. Finally, the conclusions are drawn in Sect. 5.

2 Problem formulation and solution framework

This section presents the mathematical formulation of the proposed problem, which aims at obtaining the most efficient plan of EDSs via a flexible and multi-choice strategy by simultaneously location-allocation of PV-based DG units, CBs, and ESS. This model is naturally an MINLP problem; therefore, via appropriate linearization techniques, it is recast to an approximated MILP model that guarantees finding the global optimal solutions. It is worth mentioning that the proposed planning framework is related to a central planning context where all the considered technologies, in the scope of this paper, are owned and operated by DISCO.

2.1 PV power generation profile

In this work, the PV-based DG units are considered that have the capability of connecting to a medium-voltage network, operating with the desired power factor, and controlling their power output by smart power inverters. To obtain the uncertainty indexes related to the variation of PV generation, the deterministic optimization model requires forecast profiles along the horizon planning. The PV power generation profile is built using historical data that considers an estimation for the weather conditions and using the relationship between radiation and output power of a PV module with the corresponding PV power generation model [9,12].

Since in this work a tropical country such as Brazil is considered, a year is divided into two blocks with seasonal conditions (Hot and Mild weather) as shown in Fig. 1. Each season involves demand data, energy cost, and solar irradiation. Thereby,

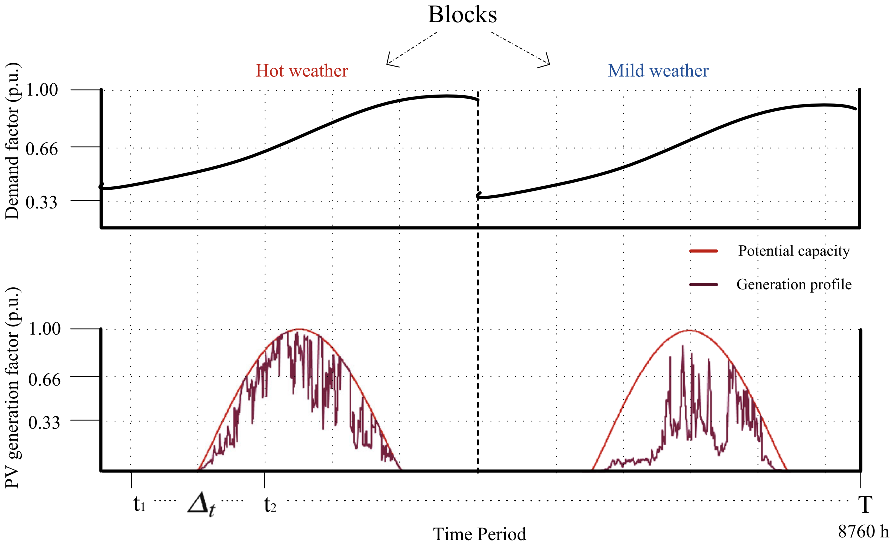

Fig. 1 Demand, and PV generation curves

the information of demand profile and solar irradiation are used to create demand factor and PV generation curves (p.u.), respectively. To represent these forecast profiles, a year with time T is divided into two blocks, and each block contains time intervals t that presents the correlation between the PV power generation and the demand factor curves. For a specific t, which represent the conditions of several typical days, a value of demand factor, energy cost, and PV generation factor are assigned. Finally, this correlation, between the demand and PV generation, is illustrated in the Fig. 1.

2.2 Energy storage systems

The ESS can be used for peak shaving in an EDS, in terms of scheduling, the ESS is operated in the charging mode during off-peak load time intervals and during the on-peak load time intervals in discharging mode. In addition, benefits of voltage regulation can be obtained during periods, since the power generation due to renewable energy sources can be increased or decreased and consequently the voltage magnitude and operational limits may violate. In this approach, the optimization model also determines the optimal siting and sizing of the ESS in the EDS. It is assumed that the ESSs are owned by the DISCO and can be controlled through the communication structure to define their state (charging or discharging). In order to integrate this technology in an EDS, through the analysis process the time intervals t in a year is considered to determine the operation state of an ESS that can be installed.

The integration of ESS in an EDS considers capital investment cost, operational and maintenance cost, and recycling and disposal expenditures. The investment cost includes two cost, the energy cost of storage elements and the power cost of power electronic rectifier/inverters in a battery storage system. These costs determine the total initial investment of an ESS. The operational and maintenance cost is considered to be fixed in the first year and may increase in the following years. These technologies contain polluting materials, in this paper, the ESS are considered to be recycled and a correct environmental treatment must be applied after whole life cycle. Therefore, in this work, disposal and recycling are not omitted in the EDS planning. In the initial planning phase, this disposal and recycling cost can be expressed by (1).

$$\zeta_n^{D\&R} = \frac{\zeta^{D\&R}}{(1+r^i)^{LC}} \tag{1}$$

where $\zeta^{D\&R}$ represents the future cost of disposal and recycling; and r^i is the nominal interest rate (%/year) considering the life cycle (LC) of the ESS.

2.3 Capacitors banks

In this analysis, two types of CBs such as fixed and switchable are considered. The fixed CBs are formed by units that are connected in the network, with the same capacity for each load level, while the switchable CBs are units that can be partially operated in the EDS. The proposed methodology considers these different types of CBs, and each type can be of different reactive support capacity. In order to have a more realistic model, for each type with its respective capacity, an installation cost is associated.

2.4 The MINLP model

The planning framework minimizes overall total energy cost delivered by the substation and total investment cost related with PV-based DG units, ESS, and reactive support devices. This formulation simultaneously determines (a) location of DG units, ESS, and CBs, and (b) size, capacity, and quantity of PV modules, ESSs, and CBs to be installed. The location, size, and capacity of PV-based DG units, and CBs are determined by binary variables z, x^f and x^{sw}, respectively. On the other hand, the ESS location is determined by the binary variable y^{ESS}, and the size of the storage reservoir and the capacity of charging and discharging are determined by $\tilde{E}^{ESS}$ and $\tilde{P}^{ESS}$, respectively.

The objective function (2) of this MINLP model considers (a) energy cost delivered by the substation (1st term), (b) installation costs of fixed and switchable , (2nd term) and (3rd term), respectively, (c) investment costs, and operation and maintenance costs of PV modules to be installed in an EDS, (4th term) and (5th term), respectively, (d) investment power cost and operation and maintenance costs of ESS (6th term), and (e) storage reservoir costs and disposal and recycling costs of ESS (7th term). These costs are minimized aiming at obtaining an appropriate planning, which meets both the needs of the DISCO and the consumers.

$$\min \sum_{d\in D}\sum_{t\in T} \Delta_d \zeta_{t,d}^{G} P_{t,d}^{S} + \sum_{b\in\Omega^{cb}}\sum_{i\in N} \zeta_b^{CB^f} x_{i,b}^{f} + \sum_{b\in\Omega^{cb}}\sum_{i\in N} \zeta_b^{CB^{sw}} x_{i,b}^{sw}$$

$$+\sum_{k=1}^{\overline{PV}}\sum_{i\in N}\zeta^{PV}kz_{i,k}+\sum_{d\in D}\sum_{k=1}^{\overline{PV}}\sum_{i\in N}\zeta_d^{o\&m^{PV}}kz_{i,k}$$

$$+\sum_{i\in N}\left(\zeta^{PC^{ESS}}+\sum_{d\in D}\zeta_d^{o\&m^{ESS}}\right)\tilde{P}_i^{ESS}+\sum_{i\in N}\left(\zeta^{RC^{ESS}}+\zeta_n^{D\&R}\right)\tilde{E}_i^{ESS} \tag{2}$$

Subject to :

$$\sum_{b\in\Omega^{cb}}\sum_{i\in N}\zeta_b^{CB^f}x_{i,b}^f+\sum_{b\in\Omega^{cb}}\sum_{i\in N}\zeta_b^{CB^{sw}}x_{i,b}^{sw}\leq IC^{CB} \tag{3}$$

$$\sum_{k=1}^{\overline{PV}}\sum_{i\in N}\zeta^{PV}kz_{i,k}\leq IC^{PV} \tag{4}$$

$$\sum_{i\in N}\zeta^{PC^{ESS}}\tilde{P}_i^{ESS}+\sum_{i\in N}\zeta^{RC^{ESS}}\tilde{E}_i^{ESS}\leq IC^{ESS} \tag{5}$$

$$\sum_{ji\in L}P_{ji,t,d}-\sum_{ji\in L}\left(P_{ij,t,d}+R_{ij}I_{ij,t,d}^{sqr}\right)+P_{t,d}^S+P_{i,t,d}^{PV}+P_{i,t}^{ESS^d}-P_{i,t}^{ESS^c}=P_{i,t,d}^{Ld} \tag{6}$$

$$\sum_{ji\in L}Q_{ji,t,d}-\sum_{ji\in L}\left(Q_{ji,t,d}+X_{i,j}I_{ij,t,d}^{sqr}\right)+Q_{t,d}^S+Q_{i,t,d}^{PV}+Q_{i,d}^{CB}=Q_{i,t,d}^{Ld} \tag{7}$$

$$V_{i,t,d}^{sqr}-2\left[(R_{ij}P_{ij,t,d}+X_{ij}Q_{ij,t,d})-Z_{ij}^2I_{ij,t,d}^{sqr}\right]-V_{j,t,d}^{sqr}=0 \tag{8}$$

$$V_{j,t,d}^{sqr}I_{ij,t,d}^{sqr}=P_{ij,t,d}^2+Q_{ij,t,d}^2 \tag{9}$$

$$\underline{V}^2\leq V_{i,t,d}^{sqr}\leq\overline{V}^2 \tag{10}$$

$$0\leq I_{ij,t,d}^{sqr}\leq\overline{I}_{ij}^2 \tag{11}$$

$$-P_{t,d}^S\tan\left(\cos^{-1}\underline{\Phi}^S\right)\leq Q_{d,t}^S\leq P_{t,d}^S\tan\left(\cos^{-1}\overline{\Phi}^S\right) \tag{12}$$

$$\sum_{t\in T}e^p\Delta_dP_{t,d}^S\leq\overline{PE}_d \tag{13}$$

$$Q_{i,d}^{CB}=B_{i,d}Q_b^{esp} \tag{14}$$

$$0\leq\overline{M}_i\leq\sum_{b\in\Omega^{cb}}bx_{i,b}^f+\sum_{b\in\Omega^{cb}}bx_{i,b}^{sw} \tag{15}$$

$$\overline{M}_i\leq B_{i,d}+\sum_{b\in\Omega^{cb}}bx_{i,b}^{sw} \tag{16}$$

$$0\leq B_{i,d}\leq\overline{M}_i \tag{17}$$

$$\sum_{b\in\Omega^{cb}}x_{i,b}^f+\sum_{b\in\Omega^{cb}}x_{i,b}^{sw}\leq 1 \tag{18}$$

$$\sum_{b\in\Omega^{cb}}x_{i,b}^f\leq 1 \tag{19}$$

$$\sum_{b\in\Omega^{cb}} x_{i,b}^{sw} \leq 1 \tag{20}$$

$$P_{i,t,d}^{PV} = \sum_{k=1}^{\overline{PV}} k z_{i,k} \overline{P}^{PV} f_{t,d}^{G^{pv}} \tag{21}$$

$$\left|Q_{i,t,d}^{PV}\right| \leq P_{i,t}^{PV} \tan\left(\cos^{-1}(\Phi^{pv})\right) \tag{22}$$

$$\sum_{k=1}^{\overline{PV}} z_{i,k} \leq 1 \tag{23}$$

$$\sum_{i\in N}\sum_{k=1}^{\overline{PV}} z_{i,k} \leq N^{PV} \tag{24}$$

$$\underline{E}^{ESS} y_i^{ESS} \leq \tilde{E}_i^{ESS} \leq \overline{E}^{ESS} y_i^{ESS} \tag{25}$$

$$0 \leq \tilde{P}_i^{ESS} \leq \overline{P}^{ESS} y_i^{ESS} \tag{26}$$

$$E_{i,t}^{ESS} = \eta \Delta_t P_{i,t,d}^{ESS^c} - \frac{1}{\eta}\Delta_t P_{i,t,d}^{ESS^d} + E_{i,t-1,d}^{ESS} \quad : \forall t > 1 \tag{27}$$

$$\tilde{E}_i^{ESS} DoD \leq E_{i,t,d}^{ESS} \leq \tilde{E}_i^{ESS} \tag{28}$$

$$-\tilde{P}_i^{ESS} \leq P_{i,t,d}^{ESS^d} - P_{i,t,d}^{ESS^c} \leq \tilde{P}_i^{ESS} \tag{29}$$

$$\forall b \in \Omega^{cb}, i \in N, ij \in L, t \in T, d \in D$$

This MINLP model is subject to investment limits, steady state operation constraints, EDS operational limits, environmental constraint, and operational constraints for every investment alternative. These constraints are described as follow:

Constraints (3)–(5) represent the investment limits for each investment alternative such as CB allocation, PV modules allocation, and ESS allocation. These set can be define by the DISCO.

The set of Eqs. (6)–(9) represent the steady state operation of an EDS. The active and reactive power balance is shown in (6) and (7), respectively. The voltage magnitude is determined by (8), while (9) defines the relation between the voltage and current flow with active and reactive power flows in the EDS.

The operational limits that must be fulfilled in an EDS in order to supply a high-quality service are considered in equations (10) and (11), which define the voltage magnitude and current limits, respectively. The substation power factor is controlled by (12), which limits the reactive power delivered by the substation, considering the relationship between the active power delivered and the desired power factor.

The pollutant emissions of EDS can be controlled by the environmental sector aiming at reducing the environmental impact. In this work, in the year d, the pollutant emission must be equal or less than a required emission limit, as is shown in (13).

The operation of CBs is represented by the mathematical model in (14)–(20) where constraint (14) represents the reactive power injection for each CB module to be installed and its respective capacity; the maximum number of modules to be installed at node i are determined by (15), in which the product bx^f and bx^{sw} describes the CB capacity of the installed type (fixed/switchable). On the other hand, to choose the type of CB to be installed, constraint (16) and (17) are used, in which if $\overline{M} = B$, the CB is fixed, and if $B < \overline{M}$, the CB is switchable. In addition, constraint (18) guarantees that only one type of CB can be installed at node i and its capacity is determined by (19) and (20).

The allocation of PV modules is presented by (21)–(24). Constraint (21) determines the available active power generation where $f^{G^{pv}}$ represents the generation factor parameter, which depends on the PV forecast profile. The investment in PV modules is associated with the product of kz with the maximum active power capacity. The injected reactive power is shown in the constraint (22), which depends on P^{PV} and the desired power factor. Constraint (23) stands for the maximum numbers of PV modules (size of DG units) that can be installed at node i, and (24) determines the maximum number of PV plants to be installed in the EDS.

The mathematical model for the siting and sizing of ESS is represented by the constraints set (25)–(29). The constraints (25) and (26) define the maximum of total power rating and energy reservoir capacity of an ESS to be installed in the EDS. This maximum limits, power ($\overline{P}^{ESS}$), and energy ($\overline{E}^{ESS}$) can be defined by the DISCO considering technical specifications to determine the appropriated size and capacity of ESS. Constraint (27) refers to amount of stored energy; this stored energy depends on the previous state of charge (E^{ESS}_{t-1}) and also the charging and discharging power multiplied by the ESS efficiency and the duration of each time interval. It worth mentioning that, in the initial state ($t = 0$) the value of the previous state of charge is the minimum energy available in the ESS reservoir, considering the depth of discharge (DoD) as an impact on the life expectancy. Equations (28) and (29) define the capacity limits of the ESS reservoir as well as the real power rating of the ESS installed.

2.5 MILP model approximation

The aforementioned MINLP model can be solved by three groups of solving techniques such as: (1) nonlinear solvers, (2) heuristic techniques, and (3) global optimization algorithms. However, the solving techniques in the first and second groups do not guarantee finding the global solution, while finding the global solution via the third group's technique hinges on a finite computer arithmetic and mild conditions [26]. Therefore, in this work, to obtain a tractable problem with high computational efficiency while guaranteeing finding the global solution, this nonlinear formulation is recast to a MILP model. In other words, this recast facilitates the utilization of classical optimization methods and commercial solvers and a global solution obtained.

This section shows linearization techniques applied to obtain the MILP model. Therefore, the non linearity in constraint (9), which is a result of the product of two continuous variables (on the left side), and the quadratic terms (on the right side), is alleviated by recasting to linear equation set. To recast the product of these variables, the approximated value $(V^*)^2$ is used, V^{sqr}. This approximation is done by the nominal voltage magnitude ($V^* = V^{nom}$) corresponding to the initial operating point. Therefore, the left term of (9) is substituted as is shown by (30).

$$V^{sqr}_{j,t,d} I^{sqr}_{ij,a,t,d} \approx \left(V^{*}_{j,t,d}\right)^2 I^{sqr}_{ij,a,t,d} \tag{30}$$

On the other hand, the quadratic terms in (9) are divided into pieces that are approximated by straight segments, and can be approximated using piecewise linear functions. This linearization technique has been used in multiple works [12,18,19,27], and [28] to find a linear approximation of quadratic terms. The linearization process is shown in (31)–(38).

$$P^2_{i,j,t,d} = \sum_{r \in R} m_{i,j,r} \Delta^P_{i,j,t,d,r} \tag{31}$$

$$Q^2_{i,j,t,d} = \sum_{r \in R} m_{i,j,r} \Delta^Q_{i,j,t,d,r} \tag{32}$$

$$P^+_{i,j,t,d} + P^-_{i,j,t,d} = \sum_{r \in R} \Delta^P_{i,j,t,d,r} \tag{33}$$

$$Q^+_{i,j,t,d} + Q^-_{i,j,t,d} = \sum_{r \in R} \Delta^Q_{i,j,t,d,r} \tag{34}$$

$$0 \le \Delta^P_{i,j,t,d,r} \le \Delta^S_{i,j} \tag{35}$$

$$0 \le \Delta^Q_{i,j,t,d,r} \le \Delta^S_{i,j} \tag{36}$$

$$m_{i,j,r} = (2r - 1)\Delta^S_{i,j} \tag{37}$$

$$\Delta^S_{i,j} = \frac{V^{nom}\overline{I}}{R} \quad \forall ij \in L, t \in T, d \in D, r \in R \tag{38}$$

where (31) and (32) approximate the square value of P and Q using variables Δ^P_r and Δ^Q_r, respectively, to discretize its value. The parameter m_r represents the slope of the rth straight segment of the linearization method. Partitions over the closed interval $[0, V^{nom}\overline{I}]$ are generated through of blocks Δ^P_r and Δ^Q_r, which denote the lengths of the discretized segments. The summation of these segments is equal to the sum of two corresponding auxiliary non-negative variables P^+, P^- and Q^+, Q^-, as can be seen in (33) and (34). The Δ^P_r and Δ^Q_r, limited by (35) and (36), are filled sequentially in ascending order considering the number of segments r. This condition is guaranteed by implication of the minimization of P^2 and Q^2 according to the objective function (2) where P^S is minimized. Considering this condition, equation (9) can be represented by a set of linear equations [27]. Finally, the bound Δ^S_r, in (37) represents the length of each discretized segment r.

This linearization represents an equidistant approximation of these terms, which yields the tightest approximation for the quadratic expressions [29]. It is worth mentioning that the quality of the solution obtained from the approximated MILP model depends on the suitable operating point and the number of discretization segments.

The proposed methodology that obtains a sustainable EDS via an integrated planning by taking into account the optimal siting and sizing of CBs, ESSs, and PV-based DG units can be summarized in Fig. 2. As can be seen, this figure consists of three parts such as obtaining external uncertainty indexes, optimization problem, and the

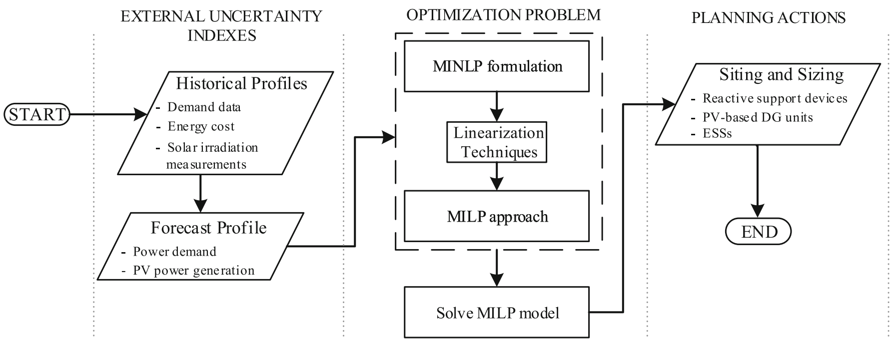

Fig. 2 Flowchart of the proposed methodology

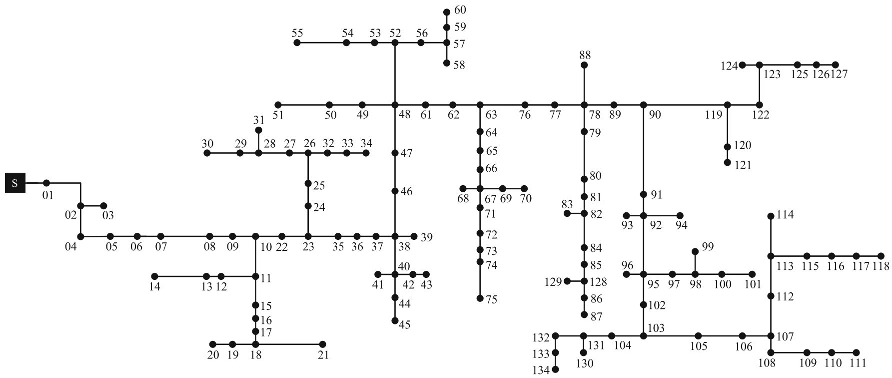

Fig. 3 135-nodes distribution system test case

optimal planning actions carrying out to maximize the efficiency of the EDS considering economic and environmental issues. In the first part, the uncertainty indexes are determined using historical data and forecast profiles. These indexes are used as input data in the MILP model. In other words, the proposed MILP model is solved considering a forecast profile for the parameters related to estimated demand factors, and PV power generation. Finally, the optimal solution obtained from the proposed MILP model is applied to improve the operation of the EDS in an environmentally committed plan.

3 Case studies

To evaluate the effectiveness of the proposed planning framework, several tests are conducted on the 135-node system shown in Fig. 3. This section presents the assumptions, system data, and test cases applied to this EDS.

Table 2 Installation costs and capacities of different types of CBs

Q^{esp} (kVAr)	Fixed ($)	Switched ($)
CB installation cost		
300	4950.00	7450.00
600	5150.00	7650.00
900	6550.00	9550.00
1200	7500.00	10,150.00
1500	8075.00	10,950.00

3.1 System data

The data of 135-node distribution system is derived from [30]. The nominal voltage level of the network is 13.8 kV, and upper and lower limit are 1.05 and 0.95 p.u, respectively. The upper and lower limits of substation power factor ($\underline{\Phi}^S$, $\overline{\Phi}^S$) are 0.95 and 1.00, respectively.

Two types of CBs, fixed and switchable, are considered in the optimization problem. In Table 2, the reactive power capacity (kVAr) by each CB type is specified, and a respective cost which depends on the capacity and type of CBs is associated.

This paper considers renewable generation based on PV technology. The generation capacity of each PV module is 20 kW with the power factor of 0.98. The maximum number of candidate PV modules that can be installed at node i is 50 with the investment cost of $10k per module; each module had operation and maintenance costs in the first year of $375, these costs increase 5% for the first 3 years and 8% for the last years.

On the other hand, ESS based on lead-acid technology is considered in this analysis. The typical lifetime is considered in 3–15 years (2000 cycles) [31]. The maximum power rating ($\overline{P}^{ESS}$) is 300 kW, minimum/maximum energy reservoir capacity ($\underline{E}^{ESS}$; $\overline{E}^{ESS}$) is 1800 and 6000 kWh, respectively of a given ESS units that can be installed at node i. The capital energy cost is $500 per kWh, the capital power cost is $175 per kW, and the annual operation and maintenance costs is $50 per kW which increases 5% for the first three years and 8% for the last years. The disposal and recycling cost is estimated considering a cycle lifetime of 10 years; the value is defined in $100 per kWh in the initial planning phase. Finally, the charging and discharging efficiency is considered to be 98% with DoD of 0.3.

In the proposed analysis, a planning horizon of 5 years is considered where each year is represented in two blocks (Hot and Mild), and each block, for study purposes, is divided into six intervals of time. In summary, 12 intervals are considered for each year d, in which each t involves expected energy cost, demand, and generation forecast profile. The Table 3 shows demand factors and energy cost for the first year, and PV generation factors for each interval during the horizon planning.

Table 3 Demand factor,energy cost, and PV generation factor per time interval

Time interval t	Demand factor (p.u.)	Energy cost ζ_1^G ($/MWh)	PV generation factor $f^{G^{pv}}$ (p.u.)				
			d_1	d_2	d_3	d_4	d_5
1	0.30	36.00	0.15	0.11	0.17	0.14	0.16
2	0.60	48.00	0.65	0.71	0.62	0.68	0.72
3	0.85	68.00	1.00	1.00	0.96	0.94	0.98
4	1.00	73.00	0.79	0.84	0.77	0.81	0.85
5	0.75	58.00	0.68	0.73	0.72	0.75	0.73
6	0.45	41.00	0.25	0.33	0.28	0.31	0.27
7	0.35	37.00	0.05	0.07	0.09	0.03	0.06
8	0.52	45.00	0.33	0.39	0.35	0.41	0.37
9	0.79	63.00	0.68	0.76	0.70	0.74	0.78
10	0.96	70.00	0.48	0.56	0.46	0.51	0.49
11	0.67	52.00	0.35	0.38	0.30	0.36	0.33
12	0.41	39.00	0.08	0.11	0.06	0.09	0.04

3.2 Assumptions

In this analysis, some assumptions are taking into account to efficiently evaluate the mathematical formulation presented in Sect. 2. These assumptions are:

- The demand increase is 15% across the planning horizon, this increase is considered proportionally in each year.
- The value of energy cost, presented in Table 3, increase in 2% for the last year of planning.
- The emission coefficient e^p is considered to be 2.17 Ton/kWh; the pollutant emission limit from substation side is annually 300 MTon.
- The maximum number of PV plants is limited to 5 distributed in the EDS.
- Investment costs of PV modules is limited to $1000 k.
- Investment costs of ESS is limited to $3000 k.
- Investment cost of CB allocation is limited to $20 k.

3.3 Case studies

To validate the proposed model, the 135-node system is considered under three different conditions. These three case studies are as follows.

- Case I: Before planning. This case is presented to determines the initial steady state operating point of the EDS. It means that the allocation of CBs, PV modules, and ESS are not considered in this case. A conventional power flow is used to determines the initial conditions, in the first year, of the 135-node distribution system.

- Case II: Considering DG units and reactive support devices. In this case, the location-allocation of two investment alternatives such as PV-based DG units and reactive power support devices are considered.
- Case III: Considering DG units, reactive support devices, and energy storage system. In this case, this siting and sizing of three alternatives, mentioned in the proposed model, are considered. This case is used to evaluate the complete proposed model.

4 Results and analysis

This section presents the results of three case studies for the 135-node distribution system. The proposed MILP model presented in the Sect. 2.5 is implemented in the mathematical modeling language AMPL [32] and solved using the commercial solver CPLEX [33].

4.1 Case I: Before planning

In this case, to evaluate the initial condition of the 135-nodes distribution system at the first year of the planning horizon, a conventional power flow is used. The operational cost corresponds to the energy losses is \$1030.33 k, the pollutant emission from substation side is 327.262 MTon, considering an emission coefficient of 2.17 Ton/kWh. In Fig. 4 the voltage magnitude profile of 135-nodes system is presented. This profile corresponds to the interval with maximum loading (peak) in the first year. The maximum substation power factor found during the analyzing period is 0.9166. Under this initial condition, the result of this power flow reveals violations in the voltage magnitude profile, the substation power factor, and environmental limits.

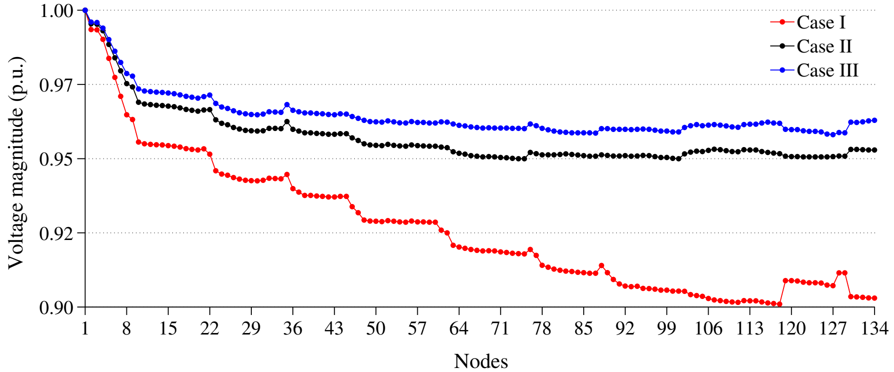

Fig. 4 Voltage magnitude profile for 135-nodes system in the peak loading time interval, for cases I, II, and III

4.2 Case II: Considering DG units and reactive support devices

This case aims at evaluating the impacts of neglecting the ESS in distribution system planning. This way, the results of this case are used for comparing with the Case III in which the ESS are considered. The investment alternatives for this case are CBs and PV modules. The total investment cost obtained by the MILP model is \$119676.93k. This cost contains (a) allocation of a fixed CB at node 82 and a switchable CB at node 104 with costs of \$6.55 k and \$10.95k, respectively, and (b) allocation of 100 PV modules at nodes 75, 127, 130, 131, and 134 with a total cost of \$1000 k and with a total operation and maintenance costs of \$80.20k. The energy supplied by the substation has a cost of \$11,8579.230 k along the planning horizon and the total pollutant emissions for each planning year from the substation side are 251.537, 257.228, 277.153, 286.764, and 304.193 MTon. On the other hand, the cost of energy losses is \$497.41, \$526.39, \$640.75, \$703.09, and \$1151.62 k for each planning year.

This case shows that, with the allocation of CBs and PV generation, the cost of energy losses, comparing with case I, are reduced about 51.75, 48.91, 37.81, and 31.76% in four years. However, in the last year, with the integration of this alternatives the cost of energy losses was greater than the first years. The pollutant emission compared with the value obtained in case I was reduced about 23.14, 21.4, 15.31, 12.37, and 7.05% in each year.

It is worth mentioning that, in this case, due to the limited financial resources, there no exist a plan to satisfy the emission limit, especially for the last year of planning. Consequently, to find a feasible solution the emission limit is relaxed. This provide more information for the planned to either increase the financial resources, which is not a favourable option, or to consider more alternative tools.

4.3 Case III: Considering DG units, reactive support devices, and energy storage system

In this case, all the alternatives considered in the optimization problem. This case is used to show not only to consider the impacts of ESS but also to show the effectiveness of the proposed MILP model in finding the optimal solution for such problem with a lot of decision variables corresponds with the siting and sizing of alternative investment options. The total cost obtained for this case is \$118,184.21 k that contains the cost of the energy supplied by the substation, \$114,656.48 k, and the investment cost in sizing and siting of PV modules, CBs, and ESSs, \$3527.73 k. This investment cost corresponds to (a) allocation of 100 PV modules at nodes 119, 128, 130, 133, and 134 with total cost of \$1000 k while the operation and maintenance costs are \$80.20 k, (b) allocation of a fixed and a switchable CB at nodes 76 and 107 with costs of \$7.5 and \$9.55 k, respectively, and (c) allocation of two ESSs at nodes 74 and 116 with cost of \$2430.48 k, where \$2160 k is related to the energy reservoir cost (including disposal and recycling costs), and \$270.480 k is related to the power rating cost (including operation and maintenance costs). The total pollutant emission per year from the substation is 251.913, 257.581, 277.364, 286.880, and 297.829 MTon. Considering all the investment alternatives, to improve the operation of 135-nodes

Table 4 Proposed investment in renewable DG units, CBs, and ESSs for every case

Case	Proposed investment node (installed capacity)			
	Fixed CBs	Switchable CBs	PV modules	ESS units
I	82 (900 kVAr)	104 (1500 kVAr)	75 (140 kW)	X
			127 (280 kW)	
			130 (1000 kW)	
			131 (340 kW)	
			134 (240 kW)	
II	76 (1200 kVAr)	107 (900 kVAr)	119 (340kW)	74
			128 (320 kW)	Energy (1800 kWh)
			130 (680 kW)	Power (300 kW)
			133 (140 kW)	116
			134 (520 kW)	Energy (1800 kWh)
				Power (300 kW)

EDS, the operational cost due to energy losses for each year is \$435.23, \$460.36, \$559.53, \$611.34, and \$693.02 k.

This case evidence, with optimal planning scheme, how overall energy losses can be reduced by almost 57.76, 55.32, 45.69, 40.66, and 32.74% in each year, compared with the first case. About the pollutant emission, compared with the first case, about 23.02, 21.29, 15.25, 12.34, and 8.99% reduction in each year are obtained. The installation of ESS units in the EDS reveals several benefits both in the reduction of pollutant emissions and cost of the energy supplied by the substation. The first significant result is the flexibility that the ESS brings to the system and consequently can address the drawback of case II. Comparing the results of this case with the results of case II reveals a reduction of 3.3% in the cost of the energy supplied by the substation while the reduction in the costs of energy losses are 12.50, 12.54, 12.67, 13.0, and 39.82% in each year. On the other hand, in the last year, the emission limit has been satisfied with a reduction of 2.09%.

For every case, the optimization model, proposed in this work, found necessary actions to fulfill technical, operational, and environmental conditions in order to maximize the efficiency and guarantee the quality and reliability of the 135-nodes distribution system. This set of actions can be summarized as (a) fixed and switchable CB that can be allocated at node i with installed reactive power (Q^{esp}), (b) PV modules that can be allocated in the EDS at node i with installed active power ($\overline{P}^{PV}$), and (c) ESS units that can be installed at node i with energy reservoir capacity ($\tilde{E}^{ESS}$) and power rating ($\tilde{P}^{ESS}$). This set of planning alternatives is summarized in the Table 4, where nodal allocation and installed capacity of PV modules, reactive support, and ESS units are presented.

Figure 4 shows the voltage magnitude profile for the 135-nodes EDS in the peak loading time interval in each case (Case I—Red; Case II—Black; Case III—Blue). This figure demonstrates the fulfillment of voltage magnitude limits in the cases II and III. In these cases, multiples alternatives were considered to improve the operation

Table 5 ESS status at each time interval for the first year

Time interval (t)	ESS status
1	Charging
2	Charging
3	Discharging
4	Discharging
5	X
6	Charging
7	Charging
8	Charging
9	Discharging
10	Discharging
11	Discharging
12	X

of the EDS by finding the optimal siting and sizing of renewable generation, reactive support, and energy storage systems. This results in a substantial improvement in the system voltage profile comparing with the Case I, before planning.

It is worth to note that the costs of energy are different in each time interval based on the year representation, an expected ESS status operation in the first year is shown in Table 5. This table shows the optimal state (charging/discharging) at each t for each installed ESS, this information would be very useful for controlling the ESS operation to achieve the maximum arbitrage benefit. Analogously, the other years can be represented by the information provided by this status scheme.

5 Conclusions

A planning framework that optimizes the location and size of renewable generation, capacitor banks, and energy storage systems was proposed in this manuscript. To ensure the quality of the supplied energy and avoid technical and environmental violations, the proposed methodology performs a set of actions that can be considered by guarantee the operation of the EDS. A commonly used 135-node distribution system was chosen to validate the developed mathematical formulation. For this system, the solution founded by this model required a set of actions that are summarized as (a) location and capacity of renewable DG units based on PV technology, (b) location and capacity of CBs to be installed, and (c) location, energy storage capacity, and power capacity of ESS units to be installed in the EDS. Therefore, unlike the presented approaches in the literature, the proposed integrated planning framework considers technologies such as ESS, CBs, and DG units to maximize the efficiency of ESD while simultaneously controlling the pollutant emissions from the substation side.

The proposed model by providing the possibility of optimal location and allocation of several technologies such as DG units, CBs, and ESSs enhances the operation of the distribution system. Besides the technical, operational, and environmental targets, this model can be used by distribution companies to attend the needs and preferences of the consumers.

The results of the test system under three different conditions show the effectiveness of the proposed model in decreasing the operating cost, reducing the emission, and improving the voltage profile. However, the key role of ESSs in enhancing the operation of electrical distribution systems is undeniable while the renewable-based DGs also play a fundamental role in reaching the targets of future energy supply and addressing the environmental concerns.

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