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An analysis of context-based similarity tasks in textbooks from Brazil and the United States

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ABSTRACT

Three textbooks from Brazil and three textbooks from the United States were analysed with a focus on similarity and context-based tasks. Students' opportunities to learn similarity were examined by considering whether students were provided context-based tasks of high cognitive demand and whether those tasks included missing or superfluous information. Although books in the United States included more tasks, the proportion of tasks focused on similarity were about the same. Context-based similarity tasks accounted for 9%–29% of the similarity tasks, and many of these contextual tasks were of low cognitive demand. In addition, the types of contexts that were included in the textbooks were critiqued and examples provided.

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KEYWORDS

Context-based problems; similarity; tasks; textbook

1. Introduction

In this paper, we report on findings from a project involving two countries, Brazil and the USA. Our goal is to analyse geometry content in textbooks from both countries to share with a broader audience a critical analysis of the opportunities students have to learn a particular concept from different mathematics textbooks and descriptions of the different types of contextual problems that are presented in textbooks. This paper focuses on the geometrical concept of *similarity* and provides an analysis of contextual problems for that concept. Similarity often provides opportunities for students to investigate real-world problems. Even though each country throughout the world utilizes different textbooks, the results of our work can contribute to existing knowledge about this most used resource in the classroom [1,2].

Many researchers have examined textbooks; this line of research is not new. Fan [3] identified several aspects of textbooks that could be examined (e.g. the development of mathematics textbooks, the relationship between mathematics curriculum standards and textbooks, the role of textbooks in the teaching and learning of mathematics, students' use of textbooks), and several researchers have investigated mathematics content in textbooks. For example, Haggarty and Pepin [1], Schmidt, McKnight, Valverde, Houang, and Wiley

[4], and Wijaya, Van den Heuvel-Panhuizen, and Doorman [5] have all discussed opportunities students have to learn particular content. Deep analyses of the possibilities of textbook tasks to support students' learning have been conducted. In our work, we focus on opportunities to learn and will present results from the analysis of the ways in which three textbooks from Brazil and three textbooks from the USA present the concept of similarity and use context-based problems. Our results respond to the questions:

- (1) What Opportunities to Learn similarity are provided to students in a subset of textbooks from Brazil and the United States?
- (2) How are context-based similarity tasks presented in the textbooks?

With these questions, we analysed three different aspects of the textbooks' presentation of similarity: (a) the physical characteristics of the textbooks, (b) the purpose of context-based tasks, investigating the type of context used (e.g. what kind of reality is presented, the type of information provided in tasks), and (c) the cognitive demand of tasks.

To support our analysis, we conducted a review of the literature focused on mathematics tasks, considered how tasks have been described and analysed in textbooks, and reviewed the concept of Opportunity to Learn as researchers have used it to analyse textbooks.

2. Tasks

Mathematical tasks included in textbooks strongly influence the teaching and learning of mathematics. Many researchers have highlighted the importance of using challenging mathematical tasks to support students' mathematics learning [6–11].

When the goals of mathematics instruction are for students to build their abilities to reason with novel problems, fluently use multiple representations, and communicate and justify mathematical ideas, then the tasks given to students need to display those characteristics, so students can become proficient in those practices. ([11,p.2])

Tasks that require students to memorize definitions and procedures provide one kind of opportunity for students' mathematical thinking; tasks that ask students to engage in problem solving and that motivate them to make connections allow a different kind of opportunity for student thinking [10]. One way to examine the nature of students' opportunities to engage in different types of mathematical thinking is to examine the tasks that are included in mathematics textbooks frequently used in mathematics classrooms. With students' continuous and extensive experience as learners of mathematics, they can apprehend the nature of mathematics, realizing mathematics is not a fixed, static subject but something that can make sense personally. It is important for teachers to realize how long and how hard students have to work to come to this understanding.

2.1. Definition of task

Tasks may be defined in different ways. Some consider written questions tasks. Others consider the oral questions teachers pose during a lesson as tasks. Although others may only consider large multiday projects to be tasks, Doyle [8] was one of the first researchers to define a *mathematical task* and to describe it as consisting of four components: a product

that a student is expected to generate, mathematical operations students use to create the product, resources to support students' work on the task, and the weight or significance that is placed on the task.

Doyle created this definition of mathematical task based on his analyses of how students and teachers worked together to solve mathematics problems in elementary classrooms. On the other hand, White and Mesa [11], who examined college mathematics classrooms, defined a task as

- (1) A question posed by the instructor that students are expected to produce an answer for (i.e. the task as written);
- (2) The hypothetical operations used to produce the answer (i.e. cognitive demand);
- (3) The hypothetical resources available (e.g. time, study groups, internet);
- (4) The significance of the product in the course (i.e. grade weight). (p. 3)

Another useful definition provided by Stein and colleagues [10] described a mathematical task as 'a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea' (p. 460).

In our research study, we define a task as the questions proposed in the textbook, emphasizing White and Mesa's [11] first two points. We examined all the similarity tasks proposed to the student in the geometry textbooks we selected.

2.2. Tasks in the curriculum

Curriculum can be described in four levels: intended, potentially implemented, implemented, and attained [2]. White and Mesa [11] characterized the literature on curriculum as a place to locate studies about tasks. They also adapted Rico, Castro, Coriat, and Segovia's [12] conceptualization of curriculum that included four dimensions: conceptual, cognitive, formative, and social. 'The conceptual dimension refers to the discipline, the cognitive dimension refers to the learner, the formative dimension refers to the teacher, and the social level refers to society's values' (p. 4). Connecting the four levels with the four dimensions, we can situate the current research study in a matrix (see Table 1):

The current research study is situated in the potentially implemented level, because textbooks can be considered a proposed curriculum. Teachers and students will be responsible for the way the proposed curriculum is enacted. In addition, our focus is on the contributions of context-based similarity tasks to student learning, which places our work on the cognitive dimension of learning.

2.3. Tasks in the textbooks

An examination of the research literature reveals a focus on mathematical tasks and a comparison of them across countries [13–15]. Jones and Fujita [16] studied the national curricula for mathematics in England and Japan by examining a sample of textbooks from major publishers. They focused on Geometry presented in Grade 8 textbooks because these topics 'raise two issues which are widely recognised as very important in mathematics education:

Table 1. Curricular levels x curricular dimensions (adapted from [11]).

	Conceptual	Cognitive	Formative	Social
Intended				
National/federal				
Regional/state				
District				
School				
Classroom				
Potentially implemented		X		
Textbooks				
Implemented				
Classroom				
Homework				
Attained				
Classroom assessments				
State assessments				
National assessments				
International assessments				

the teaching of mathematical reasoning and proof, and the teaching of problem-solving' (p. 671).

Jones and Tarr [9] analysed the levels of cognitive demand required by probability tasks in middle school textbooks using a historical perspective. They concluded Standards-based textbooks dispense more attention to probability. Of the six textbook series analysed, more than 85% required low levels of cognitive demand. The series with more high-level cognitive demand tasks were Standards-based textbooks.

Wijaya and colleagues [5] studied Indonesian textbooks investigating opportunities to learn by focusing on context-based mathematics tasks. They claimed that when students solve this type of task, they have difficulties in '(1) understanding what a problem is about, (2) distinguishing between relevant and irrelevant information, and (3) identifying the mathematical procedures required to solve the problem' (p. 42).

Following the work of these authors, we also describe contextual problems as the types of problems that present a situation referring to the real world or a scenario that can be imagined by students. These contexts and scenarios may include personal, scientific, occupational, or public information. However, just because a problem includes context does not mean that the problem will be of high cognitive demand. Thus, it was also important to consider the type of thinking that was required of students to solve these context-based problems. As discussed by Boston and Smith [7], different types of tasks require different kinds of thinking that can influence the opportunities students have to learn particular mathematical ideas.

3. The opportunity to learn

3.1. Conception of opportunity to learn

The idea of *Opportunity to Learn* (OTL) is well established. Carroll [17] described it as the amount of time students spend learning particular topics. Husén [18] defined it as 'one of the factors which may influence scores' and it 'is whether or not the students have had an opportunity to study a particular topic or learn how to solve a particular type of problem' (p. 162).

Floden [19] built on Husen's [18] definition of OTL to describe different methods that might be used to measure OTL. Liu [20] noted that usually 'OTL focuses on course content, instructional strategies, teacher background, class size, student readiness (i.e. initial achievement levels), and the availability of physical resources (such as books and equipment)' (p. 10). He also highlighted four OTL variables:

(a) content coverage – whether or not there is a match between the curriculum taught and the content tested, (b) content exposure – whether or not there is enough time spent on the content tested, (c) content emphasis – whether the teacher provides sufficient emphasis on the content tested, and (d) quality of instructional delivery – whether the teacher has taught the content adequately. (p. 10)

McDonnell [21] contextualized OTL as a research concept, as a policy instrument, and as an education indicator, describing research about curriculum, school, and classes. For her, OTL is used to explain the complexity of the schooling process and 'although designed as a technical concept to ensure valid cross-national comparisons, OTL has changed how researchers, educators, and policymakers think about the determinants of student learning' (p. 305).

We adopted the description of OTL provided by Wijaya and colleagues [5] that considers

three aspects of OTL [that] are crucial to develop the competence of solving context-based tasks. The first aspect is giving students experience to work on tasks with real-world contexts and implicit mathematical procedures. The second aspect is giving students tasks with missing or superfluous information. The last aspect is offering students experience to work on tasks with high cognitive demands. (p. 46)

3.2. Opportunity to learn and textbooks

Investigating textbooks published in English, French, and German to understand the teaching and learning cultures in different countries, Haggarty and Pepin [1] observed the relation between the teachers, the learners, and the material they use: 'Each of these is influenced, and in some cases determined, by the educational and cultural traditions of the particular country in which the teachers and learning takes place' (p. 567). They also noted that learners 'are offered different mathematics and given different opportunities to learn that mathematics, both of which are influenced by textbook and teacher' (p. 567).

Haggarty and Pepin [1] also claimed that the textbook is an important part of the context of the work of teachers and students, and it is the most used source for the content covered. In addition, the textbook influences the pedagogical approach in the classroom, orientating teachers and influencing their work. They stated that it is well documented that textbooks reflect national curricular goals and legitimize national cultural traditions. Tornroos [22] complemented their evidence showing a fairly high correlation between content of textbooks and student performance on tests. On the other hand, a textbook can be one of the most controversial tools, especially when considering commercial revenues that are generated by their sales.

Wijaya and colleagues [5] added that there is a 'linkage between the findings of the error analysis and the textbook analysis suggests that the lacking opportunity-to-learn in Indonesian mathematics textbooks may cause Indonesian students' difficulties in solving

context-based tasks' (p. 41), which happens not just in Indonesia and strengthens the argument for the importance of textbooks in students' learning.

4. Context and method

4.1. Data set

For our project, six textbooks were selected for analysis: three textbooks from Brazil and three textbooks from the USA. Most textbooks from Brazil present mathematical ideas in an integrated manner. For example, similarity is included in a textbook that also includes topics such as algebra and statistics. It is presented in the 9th grade (for 14 year olds), which is the last grade of middle school in Brazil.

At the high school level, textbooks in the USA tend to present mathematics in two different ways. Some textbooks have a single-subject focus. For example, in 9th, 10th, and 11th grade students might use *Algebra I*, *Geometry*, and *Algebra II* textbooks, respectively. Similarity would be included in the *Geometry* textbook, and the main focus of the text would be on geometry topics. Other textbooks in the USA present topics in an integrated manner, similar to Brazil and other countries. Students in 9th, 10th, and 11th grade might use textbooks titled *Integrated Mathematics I*, *Integrated Mathematics II*, and *Integrated Mathematics III*. We selected textbooks from the USA that presented mathematical topics in an integrated manner, as Brazilian textbooks are structured. In these textbooks, similarity was included in an *Integrated Mathematics II* textbook. Students would typically use this textbook during the 9th or 10th grade when they are 14 or 15 years old.

The choice of the textbooks were based on two criteria. For Brazilian textbooks it was based on the fact that this research is a part of a larger research study involving Geometry and technology. It is relevant to explain that textbooks are freely distributed to all public school students in Brazil. In a Textbook National Program¹ (PNLD) the editors send the material to a government committee for evaluation. The approved textbooks are included in a guide that describes them to the teachers. The teachers can decide which book is most appropriate when considering their pedagogical proposal at school. After this, the government purchases the books and sends them to students.

In 2014 an innovation started in PNLD: Printed books could be accompanied by multimedia content. For our research we are interested in analysing this initiative in Brazil. At the middle school level (which includes similarity content), 10 books were approved, 3 of which included multimedia content. These three books cover the content carefully, and that is the reason they were chosen to be included in our collective project.

The American books were chosen based on their distribution in the United States. Two 'traditional' textbooks were selected that are commonly used across the USA. Another, 'reform' oriented textbook was selected that is also widely used across the USA. Similarity is introduced in middle school and studied in more depth at the high school level.

4.2. The three steps

After we chose the textbooks, we first identified the physical characteristics of the books (e.g. location, number of pages on similarity, similarity topics addressed and sequence of presentation, structure of lessons, number of contextual tasks, and their cognitive demands). Our second step was to analyse the OTL from the real-world tasks, noting how

Refer to the map at the right. 3rd Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.

Figure 1. Example of context-based task ([23,p.577])⁴.

they were presented in the problems, the missing or superfluous information of these problems, and the aspect of high cognitive demands. The third step was to identify the differences (and similarities) of context in the problems and study more carefully the relationship between high-level tasks and contextual problems.

4.3. The codification process

All similarity tasks (a total of 1445) in the six textbooks were coded. The codification process was conducted during a 12 week period, during which time adjustments were made in the way definitions for high- and low-cognitive demand tasks were operationalized. For example, a task that required a student to repeat the process used in the example above it was considered low level. It was assumed that the first time a student encountered a high-cognitive based task that it would be challenging. However, if the same type of task was presented a second time, then it was assumed the student could simply apply what they learned in the first task to the second. A task that challenged the student with a new way to use the content or used some content explored in chapters or grades before was considered high level.

All the tasks were coded by the first author to identify the cognitive demand. Afterwards, the reliability of the coding was checked by the second author by selecting a random sample of about 21% of the tasks. 92% agreement between the two coders was reached.

4.4. The contextual problems

After coding all tasks, we focused on those that included a real-life situation. In Figure 1, we show an example of this type of task. In Figure 2, we show an example of a task that involves context, but the student can solve the task without referring to the context because all of the relevant information is included in the drawing.

All tasks that mentioned some real-life situation or at least things people may reasonably use daily (doing some measuring, for example) were considered contextual problems. We analysed these tasks using the three components of OTL by following three steps. First the context-based tasks were organized in a table, counted, and their relation in the book was described. Second, we discussed whether the context was real or could be imagined, whether there was missing or superfluous information, and what the cognitive demand of

Figure 2. Example of irrelevant context ([23,p.568])⁴.

Table 2. Physical characteristics of textbooks.

	Total number of pages in the textbook	Number of pages focused on similarity	Percentage of the book focused on similarity
Book 1-BR	272	21	7.7%
Book 2-BR	270	28	10.3%
Book 3-BR	240	25	10.4%
Book 1-USA	1322	104	7.8%
Book 2-USA	941	76	8.1%
Book 3-USA	318	48	15.0%

the task was. Finally, we focused on the approach, comparing the textbooks and looking at the relation of high-level tasks and context-based problems.

5. Results

5.1. The physical characteristics of the textbooks

We began our analysis by considering the physical features of each book. This included the total number of pages and the number of pages focused on similarity (see Table 2). We noticed that even though there were more pages devoted to similarity in some of the books, the percentage of pages focused on similarity ranged between 7.7% and 15.0%.

In Brazil, the textbooks had almost the same sequence of topics. All of them consider similarity based on proportional sides and equal angles. They begin with a general intuitive introduction to the idea of similarity, then explore similar polygons. This is followed by similar triangles and ends with Theorem of Tales.² The concept of proportions is presented in the middle of the unit on similarity.

In the USA the approach of each textbook was different, and the topics examined in the selected textbooks were sequenced differently also. Book 1 assumed the definition of similarity based on dilations. The sequence of topics was dilation, proving figures are similar using transformations, corresponding parts of similar figures, then similarity of triangles. The proportionality idea is in the middle of unit focused on similarity. Book 2 started with ratios and proportions, then similarity of polygons (based on the proportional sides and equal angles), similarity of triangles, similarity transformations, finishing with scale drawing and models. Book 3 initially considered the concept of similarity based on proportional

Table 3. Similarity tasks in textbooks.

	Total tasks ¹	Total contextual tasks	Low level in contextual tasks	High level in contextual tasks
Book 1-BR	80	11 (13.8%)	5 (45.5%)	6 (54.5%)
Book 2-BR	140	12 (8.6%)	6 (50.0%)	6 (50.0%)
Book 3-BR	70	20 (28.6%)	9 (45.0%)	11 (55.0%)
Book 1-USA	360	58 (16.1%)	41 (70.7%)	17 (29.3%)
Book 2-USA	538	79 (14.7%)	64 (81.0%)	25 (19.0%)
Book 3-USA	257	38 (14.8%)	25 (65.8%)	13 (34.2%)

¹We considered each item as a task (for example, a task with three items was considered three tasks).

sides and equal angles, studying polygons and exploring theorems to provide sufficient conditions to prove triangles similar. After this section, dilation was investigated and then congruence and similarity were studied together, using a transformation approach.

The use of dilations and transformations to study similarity was unique to the US textbooks. Also, the use of coordinates in some of the problems was apparent in the US textbooks but not in the Brazilian textbooks. Both countries' textbooks utilized the definition of similarity involving congruent corresponding angles and proportional sides, although where this was introduced in the unit on similarity differed. Also, common to both sets of books was the use of contextual based tasks.

After examining the sequence of topics in the books, a closer investigation of the contextual tasks focused on similarity was conducted to determine whether these tasks were of high or low cognitive demand (see Table 3).

Looking at Table 3, we note that having more tasks does not necessarily suggest more high-level tasks (proportionally). More tasks, however, offer more options to the teacher, who can choose which tasks he or she wants to explore with students.

5.2. Three aspects of opportunity to learn

Considering OTL, we analysed the context-based similarity tasks using the three previously mentioned aspects [5]. First is the opportunity to work on tasks involving the real world. As Dietiker and Brakonieck [24] suggested, one must consider, what is the real world? Do the tasks require students to consider important aspects of reality? Do the tasks require students to take a critical perspective and use information about the real world that they know?

To look at the context-based task focusing on the real world, we considered three types of problems: (a) problems most students can relate to and make sense of, (b) problems students with particular experiences can relate to, and (c) problems that have contexts that students are not likely to encounter in their everyday lives.

A Type 1 task is shown in Figure 1. Most students have experiences with maps and addresses. We can indicate another group of contextual tasks, Type 2, for which the context may not be relevant to students. Tasks that focus on specifically rural issues, for example, might not be relevant to a group of students who have always lived in urban areas and vice versa [25]. It is important for the teacher to consider to what extent the context of this problem is relevant to the students in the class.

On the other hand, we found some tasks, Type 3, about which we could ask: Who experiences this reality? Or, is it possible to consider it as a reality? Consider the task of Figure 3³ that depicts a flying saucer. We do not find any problem with the mathematical question.

A person said he saw a flying saucer. To mark the disc position, he placed a broomstick on the floor, pointing to the flying saucer. He saw that the stick formed a horizontal angle of 30° . Another person said he saw the same object at the same time. But the flying saucer was straight over his head, vertically. The distance between the two persons was 600 meters.

Using a square, draw a triangle with a 30° angle and another at 90° . Note that it will be similar to that shown in the figure. Now, believing what the two people said, and using the measurements of the design, calculate the height the saucer was at.

Figure 3. Example of false-reality situation ([26,p.20]).

A scuba flag is used to indicate there is a diver below. In North America, scuba flags are red with a white stripe from the upper left corner to the lower right corner. Justify the triangles formed on the scuba flag are similar triangles.

Figure 4. Example of false-reality situation ([27,p.848])⁵.

Of course some people believe in life on other planets, *but is this a good example of a context-based task?* When our students solve this kind of problem, are they critical about the context?

We cannot change the textbook's tasks. One option for teachers is to skip this type of problem. Our reflection for the purposes of this paper is to think about how to use this kind of task as an opportunity for students to be critical. This is contextual and educative, teaching students not to accept everything without first thinking about it. Following Dietiker and Brakonieck [24], we can help students develop sophisticated ways to interpret problems and think geometrically if we encourage them to approach problems critically.

Another example of this type of task is the flag shown in Figure 4. Students are asked to determine if the triangles in the flag are similar. We wonder, *is this a good example of a contextual task?* Is this a usual problem in real life? Does this problem present a practical significance or is it a mathematical interest about similarity of triangles based in a real-world context?

The second aspect of OTL is considering tasks with missing or superfluous information. Often superfluous information means that we have more data than we need. This is important for students to learn to choose the information that is really important to the problem. In real life, we have to decide among all information available which pieces are important to solve the problem.

Here we are reflecting about something more. Again we propose some questions to be explored with students: What is considered superfluous? What kind of information do we not need?

To measure the height of a building, Mary did the following: tied a wire on top of the building; then set the other end of the wire on the ground 5 meters away from the base of the building.

Then, at a height of 5 meters from the ground, tied another wire, parallel to the first, fixing it in the ground, 2 meters away from the base of the building. Draw this situation and determine the building height.

Figure 5. Example to reflect about superfluity in tasks ([28,p.91]).

The task shown in Figure 5 seems much the same as those above: Mathematically, there is no problem with the given data, the expected solution uses similarity, the measurements are sufficient to calculate the height of the building, and so forth.

But why would Mary tie a wire on top of the building and the other end 5 meters away from the building? If she can have a wire on the top, why doesn't she keep the wire close to the building and measure the distance from the top to the foot of the building? We are inviting reflection here about the concept of superfluity. What is not necessary in this task is not the extra information. The math solution proposed is extra work to do in a real situation. It is important for students to critically analyse that the best solution in the real world is not always the solution of the textbook.

The last aspect of OTL is 'offering students experience to work on tasks with high cognitive demands' ([5,p.46]). When we read this, we think about the importance of offering students tasks that make them investigate, explore, think, and not simply repeat procedures. However, it is important that these tasks relate to what students are learning in class. The student needs to analyse the situation, see that it has more than one solution, and so on. But is this aligned with what students are learning in class?

How is similarity explored in this task? The relation with similarity is not explicit. When the student is solving this problem, is he or she asking why it is proposed to learn similarity? Is the teacher discussing this with students?

With this example we wonder about the meaning of high-level cognitive demand. It is important, as discussed earlier with regard to our theoretical framework (e.g. [10]), to support students to improve their mathematical knowledge. It is expected that this kind of task makes students think, investigate, and explore different solutions. And on the task of Figure 6 it is very interesting that it is happening in a contextual situation that is truly real. Some high-level tasks are just 'math problems' that do not provide opportunities for students to see where the content can be applied (of course we know that 'math problems' are important too). But it is fundamental that students realize what they are doing and know why the tasks they are doing are relevant to them.

5.3. Comparison of the similarity tasks

The six textbooks' similarity problems were analysed. We wondered about the type of context that would be presented in the textbooks. What type of realistic situations are students studying? Are the situations particularly close to the culture of the country?

Edward and Lia were furnishing a room with two arrangements. To do this, they chose the following furniture: a sofa that measures 2m long and 1m meter wide, an armchair measuring 1m long and 1m wide, and a square base rack that measures 75 cm per side. For the dining room they chose a table that measures 1.5m long by 75cm wide and six chairs.

Each chair occupies a square area of 0.25 m^2 .

Look at the floor plan for Eduard and Lia's room.

a) will the couple's room hold this furniture?

b) what would be the way to arrange them?

Figure 6. High-cognitive-demand sample task ([28,p.97]).

(a) A boy who is 4 feet 11 inches tall observed his shadow and a building's shadow at the same time of the day. He didn't have any measuring instrument so he took a string to measure both shadows. He notice that the building's shadow was 10 times greater than his. What is the building height?

(b) Jenny is 5 feet 2 inches tall. To find the height h of a light pole, she measured her shadow and the pole's shadow. What is the height of the pole?

(c) At a certain time of the day, a person who is 5 feet 4 tall projected a shadow of 3 feet 9 tall. At the same time, a tree projected a shadow of 45 feet 8 tall. What is the height of the tree?

Figure 7. Examples of common context on similarity tasks (a – [28,p.98]; b – [27,p.908]⁵; c – [29,p.142]).

Although there were differences in the approach to similarity in some of the textbooks, a set of posed problems were similar. For example, all six textbooks included problems that depict an object and its shadow. Students are required to use similar triangles to find the height of the object (Figure 7).

Another common context was the use of maps to find the distance between two points when provided with the distance between other points on the map. In both cases, finding the heights of objects indirectly and using similarity to determine the distance between two locations from a map are important real-life skills students might encounter for which they would need to use mathematics (Figure 8).

Other contexts were common to a subset of textbooks but not to all textbooks. For example, the use of dance and dancers as well as tennis and ping pong were found in the US textbooks but not in the books in Brazil. (Figure 9).

The US textbooks explore many situations involving sports (and games). We can find soccer, billiards, and other sports. Textbook 2, in particular, seemed to have a sports theme throughout each chapter in the book. The authors probably wanted to appeal to the real experiences and interests of young people, which often includes sports.

Figure 8. Examples of common context on similarity tasks (a – [27,p.888]⁵; b – [29,p.129]; c – [23,p.578]⁴; d – [28,p.94]).

Other examples of situations that involve the real world were present in only one of the textbooks. Below we show some examples of contexts related to everyday experiences that were unique to their respective textbooks (Figure 10).

It is very interesting to analyse how many different kinds of situations can be used as contexts for similarity problems. It is a rich context for the teacher to exemplify to the students how mathematics can be used in the real world.

Looking at these different contextual problems, we considered the cognitive demand of these tasks. Did the authors select the context to offer students access to tasks that are high in cognitive demand? We conclude for most of them we could answer ‘no,’ even though research on problem solving (e.g. [30]) has suggested this type of task should be a challenge to the student, associating it with a real problem, in a real situation. However, in the textbooks analysed we noticed that most contextual problems were similar to (a) a ‘math problem,’ that does not involve any context (e.g. Figures 11a and 11b); or (b) the ‘explanation provided by the authors’ or an ‘example’ that illustrates how to solve the problem (e.g. Figures 11c, 11d, and 11e). We note that US Book 3 is an exception in that it offered no explanations or examples before the tasks.

In this textbook, the lessons start with tasks the student explores in order to build the concepts. Some short summarizing is presented by the author just at the end of the lessons.

When contextual problems are similar to problems that precede it, we consider that the contextual problem becomes a low-level task. The students do not need to really think about the problem, consider how to solve it, or create a strategy. They need to find a way to relate the problem with the math task they solved previously or the example or explanation given by the author because usually the solution is very similar, and students apply something they should already know. We found few tasks that were different from previous ones, making the students go beyond what had already been presented, requiring a different strategy, or

(a) A choreographer uses a number line to position dancers from ballet. Dancers A and B have coordinates 5 and 23, respectively. In Exercises 1-4, find the coordinate for each of the following dancers based on the given locations.

1. Dancer C stands at a point that is $\frac{5}{6}$ of the distance from Dancer A to Dancer B.
2. Dancer D stands at a point that is $\frac{1}{3}$ of the distance from Dancer A to Dancer B.
3. Dancer E stands at a point that divides the line segment from Dancer A to Dancer B in a ratio of 2 to 1.
4. Dancer F stands at a point that divides the line segment from Dancer A to Dancer B in a ratio of 1 to 5.

(b) Tonisha drew the line of dancers shown below for her perspective projects in art class. Each of the dancers is parallel. Find the lower distance between the first two dancers.

(c) The dimensions of a regulation tennis court are 27 feet by 78 feet. The dimensions of a table tennis are 152.5 centimeter by 274 centimeters. Is a table tennis table a dilation of a tennis court? If so, what is the scale factor? Explain.

(d) The dimensions of a standard tennis court are 36 feet x 78 feet with a net that is 3 feet high in the center. The court is modified for players aged 10 and under such that the dimensions are 27 feet x 60 feet and the same net is used. Use similarity to determine if the modified court is similar to the standard one.

Figure 9. Examples of common context on similarity tasks in the USA⁴ (a – [27,p.896]⁵; b – [23,p.578]; c – [23,p.596]⁴; d – [27,p.847])⁵.

(a) There are sodas in bigger bottles of 1L or smaller bottles of 290ml, for example. Are the bigger bottle and the smaller bottle similar? Why?

(b) A joiner wants to build a trapezoidal ladder with 6 steps, respecting the measures indicated in the figure. Every step will be obtained cutting a piece of wood measuring, at least, in centimeters:

(c) Consider this model of a train locomotive when answering the next two questions.

1. If the model is 18 inches long and the actual locomotive is 72 feet long, what is the similarity transformation to map from the model to the actual locomotive? Express the answer using the notations $x \rightarrow ax$, where x is a measurement on the model and ax is the corresponding measurement on the actual locomotive.
2. If the diameter of the front wheels on the locomotive is 4 feet, what is the diameter of the front wheels on the model? Express the answer in inches.

Figure 10. Examples of everyday experiences (a – [26,p.14]; b – [28,p.97]; c – [27,p.858])⁵.

<p>(a) Find the length of each segment.</p> <p>3. $\overline{KL}$</p> <p>4. $\overline{XZ}$</p> <p>5. $\overline{MN}$</p> <p>6. $\overline{AB}$ and $\overline{CD}$</p> <p>7. $\overline{MN}$ and $\overline{OR}$</p> <p>8. $\overline{WX}$ and $\overline{DE}$</p>	<p>(c) In these triangles, Tales measured RS (a stick), ST (the shadow of the stick) and BC (the shadow of the pyramid plus half of his base). Let's suppose that he obtained the following data: BC = 225m; RS = 2m; ST = 3m.</p> <p>Using the similarity, Tales has calculated the height of the piramide, AB:</p> $\frac{AB}{2} = \frac{225}{3}$ <p>So: $3 \cdot (AB) = 2 \cdot 225 \rightarrow AB = 150$</p> <p>Therefore, 150m. This was the way Tales has calculated the height of the piramide. He gained much prestige with his mathematical achievements and this example of application using similarity of triangles made history.</p>
<p>(b) On the map, 1st Street and 2nd Street are parallel. What is the distance from City Hall to 2nd Street along Cedar Road?</p>	<p>(d) Observe the landscape and calculate tree height.</p>
<p>(e) Real-World Example 4 – Use Proportions to Make Predictions</p> <p>Car Ownership: Fernando conducted a survey of 50 students driving to school and found that 28 owned cars. If 755 students drive to his school, predict the total number of students who own cars.</p> <p>Write and solve a proportion that compares the number of students who own cars to the number who drive to school.</p> $\frac{28}{50} = \frac{x}{755}$ <p>← students owning cars ← students driving to school</p> <p>$28 \cdot 755 = 50 \cdot x$ Cross Products Property</p> <p>$21,140 = 50x$ Simplify</p> <p>$422 = x$ Divide each side by 50</p> <p>Based on Fernando's survey, about 423 students at his school own cars</p> <p>Guided Practices</p> <p>Biology: In an experiment, students netted butterflies, recorded the number with tags on their wings, and then released them. The students netted 48 butterflies and 3 of those had tagged wings. Predict the number of butterflies that would have tagged wings out of 100 netted.</p>	

Figure 11. Contextual tasks similar to context-free math tasks or preview examples (a – [27,p.887]⁵; b – [27,p.887]⁵; c – [26,p.23]; d – [26,p.23]; e – [23,p.545]).

making the task⁴ a real problem to them (a problem should require finding the solution, independent of whether it is presented in a real-world context).

Make a conjecture: Builders and architects use scale models to help them and build new buildings. An architecture student builds a model of an office building in which the height of the

model is $1/400$ of the height of the actual building. The model includes several triangles. Describe how a triangle in this model could be similar to the corresponding triangle in the actual building, then describe how a triangle in this model might not be similar to the corresponding triangle in the actual building. Use a similarity theorem to support each answer. ([27,p.871])

Some tasks, when we analyse them by themselves, are interesting, challenging the students, but the textbook authors present several similar tasks in a sequence. After some challenge, when students read the next task they may realize the procedure is the same as what they just used before. We believe that because of this, it is not a challenge anymore.

We will not focus on this point here deeply because it is somewhat tangential to the goals of this paper, but we invite the reader to reflect on aspects like the function of the textbook and function of the teacher. Is the author doing this to give more options to the teacher, who can decide not to ask students to solve all these similar tasks, but choose which one he/she thinks is more relevant to her/his class? Is the author doing this because he/she believes students need to do the same kind of task more than one time to practice? Does the teacher realize the questions are similar? Does he/she realize the importance of choosing the tasks? Or are they asking for similar questions because they do believe students need to practice or because he/she does not realize they are similar? Are the teachers making critical choices when they create their lesson plans?

Considering the previous analysis, we can look at the description of how similarity is addressed in the textbooks in Table 3, and it is obvious that only a small percentage of similarity tasks propose contextual problems. What is even more disappointing is that the few tasks that are context-based are of low cognitive demand. This situation devalues the opportunity for student investigations, making them less proactive as problem solvers.

Most of the contextual problems are not classified as high cognitive demand because students are asked to repeat what they have 'learned' in the explanation of the text. Some students became good 'repeaters' of procedures but do not improve their capability to investigate, to create their own ways of finding solutions.

6. Discussion

The principle function of context-based tasks is to prepare students to solve problems in real life. What we could note in evaluating textbooks of Brazil and the USA is that most textbooks have few contextual problems. More than this, we can reflect about how students can become better solvers with problems that do not make students think about real-world problems (like the flying saucer task). Students know the difference between tasks that are real or not. It is important to provide students opportunities to work on tasks that they perceive as real.

Another important question is whether we prepare students to be critical with the situations they are given. Students need to think about the situation they are provided and determine whether the scenario makes sense. When presented a problem situation in real life they will need to determine the most reasonable and efficient manner in which to solve the problem. With the work of Dietiker and Brakonieck [24] in mind, it is reasonable to assume that critical discussion and reflection could help students develop sophisticated ways to interpret problems and think about geometry, becoming good problem solvers in real situations.

Although teachers cannot change what is presented in existing textbooks, they can do more than just find the answer or skip the task; they can use this type of task to discuss critical aspects of contextual tasks with the students. They can bring up issues related to the realness of the situation and discuss whether the approach to the problem seems reasonable. They also need to ensure that students work on tasks with high-level cognitive demand. They cannot rely only on the textbook to provide appropriate tasks. Teachers may find different strategies that they can use to modify tasks that are presented in the textbook to convert them to high cognitive demand. Dan Meyer in his well-known TED talk, *Math Class Needs A Makeover* (<https://www.youtube.com/watch?v=NWUFjb8w9Ps>) has demonstrated how teachers can strip away the guiding questions that are included with a context-based task to increase its cognitive demand. As shown in our study, current textbooks have few contextual problems of this level.

Most researchers cited in this paper and others writing about OTL theory classify tasks as high and low, but we believe that it is necessary to do more than that. To identify high levels of cognitive demand is important so that teachers select tasks that contribute to students' active learning, but as we show, it is not enough. Some of the textbook tasks have the potential to help students think and reflect, but maybe not to the necessary degree without some modification to improve their context and rigor.

Notes

1. Programa Nacional do Livro Didático (PNLD).
2. When two parallel lines are cut by two transversals, measures of segments of the first transversal are proportional to the measures of the corresponding segments determined on the second transversal ([28], p. 92).
3. The tasks from Brazilian textbooks were translated to English.
4. From Integrated Mathematics Course 2, Teacher Edition. Copyright © 2012 by McGraw-Hill Education. All rights reserved. Reprinted by permission of McGraw-Hill Education.
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