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Stable matter-wave solitons in the vortex core of a uniform condensate

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Abstract

We demonstrate a stable, mobile, dipolar or nondipolar three-dimensional matter-wave soliton in the vortex core of a uniform nondipolar condensate. All intra- and inter-species contact interactions can be repulsive for a strongly dipolar soliton. For a weakly dipolar or nondipolar soliton, the intra-species contact interaction in the soliton should be attractive for the formation of a compact soliton. The soliton can propagate with a constant velocity along the vortex core without any deformation. Two such solitons undergo a quasi-elastic collision at medium velocities. We illustrate the findings using realistic interactions in a mean-field model of binary ^{87}Rb - ^{85}Rb and ^{87}Rb - ^{164}Dy systems.

Keywords: dipolar atom, soliton, vortex, cold atoms, mean-field model

(Some figures may appear in colour only in the online journal)

1. Introduction

A bright soliton is a self-bound object that travels at a constant velocity without deformation in one dimension (1D), due to a cancellation of nonlinear attraction and defocusing forces. A 1D dark soliton is a dip in uniform density, which also moves with a constant velocity maintaining its shape. 1D Solitons have been observed in nonlinear optics [4], and in Bose–Einstein condensate (BEC) [2]. Experimentally, bright matter-wave solitons were created in BECs of ^7Li [4] and ^{85}Rb atoms [4]. Dark solitons were also observed in BECs of ^{87}Rb [5] and ^{23}Na [6]. The 1D set up is obtained by putting confining traps in directions perpendicular to the motion of the soliton. However, a three-dimensional (3D) trap-less soliton cannot be formed for a cubic nonlinearity, generally encountered in BEC and nonlinear optics, due to collapse [7]. The collapse can be stopped in a weaker saturable [8], cubic-quintic [11] or quadratic [10] nonlinearity, or by an application of nonlinearity and/or dispersion management [11]. In nonlinear optics, 1D temporal solitons [12] as well as lattice solitons in arrays of nonlinear optical wave guide in 1D [13] and in two dimensions (2D) [14] and 3D [15], with modified dynamics/nonlinearity, have been observed.

Here we demonstrate the formation of a *trap-less* matter-wave soliton in the core of a quantized vortex of a uniform nondipolar BEC, which we call a binary nondipolar vortex-soliton, which is a 3D analogue of an 1D dark-bright soliton [4] studied previously. These solitons are shown to be stable and execute steady oscillation for very long time under a small perturbation. In the case of a 3D vortex-soliton all interactions can be repulsive except the intra-species interaction in the soliton. The soliton can swim freely with a constant velocity along the vortex core. Because of the strong localization of the soliton due to inter-species contact repulsion, the soliton can move without visible deformation. The collision between two integrable 1D solitons is truly elastic [1, 2]. However, at medium velocities the collision between two solitons is found to be quasi-elastic without visible deformation. The stability of the vortex-soliton is established through a controlled prolonged real-time dynamics of the initial imaginary-time profile upon a small perturbation introduced by (i) a small change in the intra-species interaction in the soliton, and by (ii) a small initial transverse displacement of the soliton relative to the vortex core. As no modification of the nonlinear interactions is suggested such a trap-less soliton can be realized in a laboratory.

The observation of dipolar BECs of ^{164}Dy [16], ^{168}Er [17] and ^{52}Cr [18] has initiated studies of new types of BEC

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solitons. For example, one can have a dipolar BEC soliton for a repulsive contact interaction [19], in 2D [20] or on optical-lattice potentials [21]. These new dipolar solitons were possible due to the peculiar nature of dipolar interaction. The dipolar BEC solitons of a large number of atoms stabilized by a long-range dipolar attraction, could be robust and less vulnerable to collapse in the presence of a short-range contact repulsion [19, 22]. Hence, we also consider a dipolar soliton in a nondipolar vortex core, which we call a dipolar vortex-soliton. Actually, a strong short-range inter-species contact repulsion between the atoms of the matter-wave soliton and the atoms of the vortex localizes the soliton in 3D.

In the present investigation we use a mean-field model described in section 2. First we present an 1D model in section 2.1 useful for an analytic understanding of the formation of a dark-bright soliton. The 3D mean-field model is presented in section 2.2. In section 3 we present numerical results for the formation of a binary vortex-soliton. A summary of our findings is given in section 4.

2. Mean-field model

2.1. One Dimension

A vortex in a uniform BEC bears similarity with an 1D dark soliton in having a hole along the axial z direction and is often called a 3D dark soliton [4]. A binary vortex-soliton is the 3D analogue of the well-known 1D dark-bright soliton. Hence, to understand how a 3D vortex-soliton can appear, we consider the following integrable binary 1D dark-bright soliton model of atoms of same mass m , same inter- and intra-species scattering lengths a and same number of atoms N [23]

$$i\hbar \frac{\partial \phi_j(x, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + g \sum_{i=1}^2 |\phi_i|^2 \right] \phi_j(x, t), \quad (1)$$

where $i, j = 1, 2$, $g = 2a\hbar^2 N / (md_p^2)$, $d_p = \sqrt{\hbar/(m\omega)}$, where ω is the frequency of a strong harmonic trap in the binary mixture in the transverse directions. Scaling the wave functions by $|\psi_i|^2 = g|\phi_i|^2$ we obtain

$$i \frac{\partial \psi_j(x, t)}{\partial t} = \left[-\frac{1}{2} \frac{\partial^2}{\partial x^2} + \sum_i |\psi_i|^2 \right] \psi_j(x, t), \quad (2)$$

in units $\hbar = m = 1$. Equation (2) with *all-repulsive* interactions has the analytic dark-bright soliton [4]

$$\psi_1(x, t) = \beta \tanh[\alpha(x - vt)] e^{i\alpha x - i(v^2/2 + \beta^2)t}, \quad (3)$$

$$\psi_2(x, t) = \gamma \operatorname{sech}[\alpha(x - vt)] e^{i\alpha x + i[(\alpha^2 - v^2)/2 - \beta^2]t}, \quad (4)$$

where α and β ($\beta > \alpha$) are constants which control the intensity and width of the solitons, $\gamma = \sqrt{\beta^2 - \alpha^2}$ and v is the velocity. The appearance of bright soliton (4) in the all-repulsive equation (2) is counterintuitive and is possible due to the coupled dark soliton (3). Without losing generality we impose the normalization $\int |\psi_2(x, t)|^2 dx = 1$, yielding $\beta =$

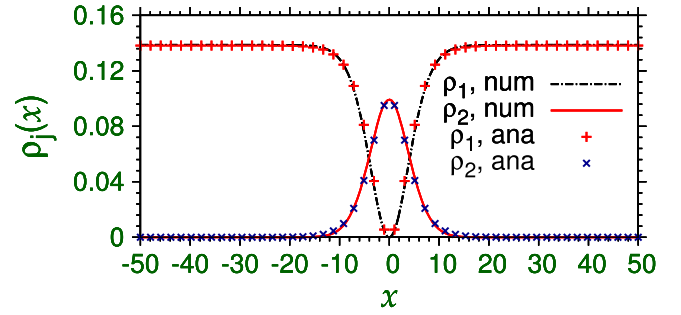


Figure 1. Densities $\rho_j(x) = |\psi_j(x)|^2$ from analytic (ana) model (5)

and (6) with $\alpha = 0.198$, $\beta = \sqrt{a^2 + a/2} = .372$ compared with those of numerical (num) simulation by imaginary-time propagation in the space domain $x = \pm L = \pm 50$. The normalization of the wave functions are $\int_{x=-L}^L \rho_1(x) dx = 12.42$ and $\int_{x=-L}^L \rho_2(x) dx = 1$.

$\sqrt{a^2 + a/2}$. For $v = 0$ the solutions have the form

$$\psi_1(x, t) = \sqrt{a^2 + a/2} \tanh(ax) \exp[-i(a^2 + a/2)t], \quad (5)$$

$$\psi_2(x, t) = \sqrt{a/2} \operatorname{sech}(ax) \exp[-i(a^2 + a)t/2]. \quad (6)$$

To solve (2) numerically the simulation was performed in a 1D box in the domain $x = \pm 50$. In figure 1 we plot the numerical matter-wave densities of the dark-bright soliton and the analytic results (5) and (6). The bright soliton sits in the central hole of the dark soliton and the inter-species repulsion between the (outer) dark and (inner) bright solitons confines the latter. Similarly, the soliton of a vortex-soliton can be confined in the radial $x - y$ plane by the inter-species repulsion between the vortex and soliton. The confinement along the z direction is obtained by the intra-species dipolar attraction with the dipoles polarized along z axis and/or by intra-species contact attraction.

2.2. Three Dimensions

We will consider a matter-wave vortex-soliton in the form of a vector soliton without any trapping potential and present the binary BEC model appropriate for this study. The first component ($j = 1$), with the vortex, is nondipolar and the second component ($j = 2$), with the soliton, can be dipolar or nondipolar. As in the 1D dark-bright soliton, the intra-species repulsion prevents the matter-wave soliton from escaping radially. The dipolar interaction or intra-species contact attraction prevents the soliton from escaping in the axial direction. The mass, number of atoms, and scattering length for the two species are m_j , N_j , a_j , respectively. The intra- (V_j) and inter-species (V_{12}) interactions for atoms at \mathbf{r} and \mathbf{r}' are [24]

$$V_1(\mathbf{R}) = \frac{4\pi\hbar^2 a_1 \delta(\mathbf{R})}{m_1}, \quad V_{12}(\mathbf{R}) = \frac{2\pi\hbar^2 a_{12} \delta(\mathbf{R})}{m_R}, \quad (7)$$

$$V_2(\mathbf{R}) = \frac{3a_{dd}\hbar^2 V_{dd}(\mathbf{R})}{m_2} + \frac{4\pi\hbar^2 a_2 \delta(\mathbf{R})}{m_2}, \quad (8)$$

where $\mathbf{R} = (r - r')$, $a_{\text{dd}} = \mu_0 \mu^2 m_2 / (12\pi \hbar^2)$, $V_{\text{dd}}(\mathbf{R}) = (1 - 3 \cos^2 \theta) / \mathbf{R}^3$, the reduced mass $m_R = m_1 m_2 / (m_1 + m_2)$, a_{12} is the inter-species scattering length, a_{dd} is a dipolar length to measure the strength of dipolar interaction, μ_0 is the permeability of free space, μ is the magnetic moment of each atom in the dipolar soliton, θ is the angle made by the vector \mathbf{R} with the polarization z direction. The dimensionless mean-field Gross-Pitaevskii (GP) equations for the *trap-less* binary mixture are [24]

$$i \frac{\partial \phi_1(\mathbf{r}, t)}{\partial t} = \left[-\frac{\nabla^2}{2} + g_1 |\phi_1|^2 + g_{12} |\phi_2|^2 \right] \phi_1(\mathbf{r}, t), \quad (9)$$

$$i \frac{\partial \phi_2(\mathbf{r}, t)}{\partial t} = \left[-m_{12} \frac{\nabla^2}{2} + g_2 |\phi_2|^2 + g_{21} |\phi_1|^2 + g_{\text{dd}} \int V_{\text{dd}}(\mathbf{R}) |\phi_2(\mathbf{r}', t)|^2 d\mathbf{r}' \right] \phi_2(\mathbf{r}, t), \quad (10)$$

where $m_{12} = m_1/m_2$, $g_1 = 4\pi a_1 N_1$, $g_2 = 4\pi a_2 N_2 m_{12}$, $g_{12} = 2\pi m_1 a_{12} N_2 / m_R$, $g_{21} = 2\pi m_1 a_{12} N_1 / m_R$, $g_{\text{dd}} = 3N_2 m_{12} a_{\text{dd}} / m_R$. In (9) and (10), length is expressed in units of a scale l , probability density $|\phi_j|^2$ in units of l^{-3} , energy in units of $\hbar^2 / (2m_1 l^2)$ and time in units of $t_0 = 2m_1 l^2 / \hbar$.

To find a stationary quantized vortex of angular momentum \mathcal{L} along z axis in component 1, also called a dark soliton with circular symmetry, we look for axially-symmetric solution $\Phi_1(\mathbf{r}, t)$ in $x - y$ plane: $\phi_1(\mathbf{r}, t) \equiv \Phi_1(\mathbf{r}, t) e^{i\mathcal{L}\varphi}$, where φ is the azimuthal angle and $\Phi_1(\mathbf{r}, t)$ satisfies [4]

$$i \frac{\partial \Phi_1(\mathbf{r}, t)}{\partial t} = \left[-\frac{1}{2} \left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \mathcal{L}^2 / (x^2 + y^2) + g_1 |\Phi_1|^2 + g_{12} |\phi_2|^2 \right] \Phi_1(\mathbf{r}, t), \quad (11)$$

with the boundary conditions $\Phi_1(x = 0, y = 0, z) = 0$, $\Phi_1(x \rightarrow \infty, y \rightarrow \infty, z) = \text{constant}$. The same on the 1D dark soliton (3) are very similar: $\psi_1(x = 0) = 0$, $\psi_1(x \rightarrow \infty) = \text{constant}$. For a matter-wave soliton of component 2 in the vortex core of component 1 we solve the axially-symmetric equations (11) and (10). We consider a vortex of unit circulation $\mathcal{L} = 1$.

3. Numerical results in three dimensions

In the 3D simulation of the binary vortex-soliton we consider the nondipolar ^{87}Rb - ^{85}Rb and the dipolar ^{87}Rb - ^{164}Dy mixtures. The ^{164}Dy atom has the magnetic moment $\mu = 10\mu_B$ [16] with μ_B the Bohr magneton so that the dipolar length $a_{\text{dd}}(^{164}\text{Dy}) \approx 132.7a_0$ with a_0 the Bohr radius. We use scattering lengths $a(^{87}\text{Rb}) = a(^{87}\text{Rb} - ^{85}\text{Rb}) = a(^{87}\text{Rb} - ^{164}\text{Dy}) = 120a_0$ and take $a(^{85}\text{Rb})$ and $a(^{164}\text{Dy})$ as variables. The experimental values of these scattering lengths are not known precisely. The exact values of the inter-species scattering lengths are not important for our analysis. These positive scattering lengths are used as they simulate the inter-species repulsion required for the formation of binary vortex-

soliton. Furthermore, if needed, the variation of the scattering lengths can be achieved by the Feshbach resonance technique [25]. We solve (10) and (11) by the split-time-step Crank-Nicolson method using both real- and imaginary-time propagations in Cartesian coordinates using a space step of $0.2 \sim 0.4$ and a time step of $0.0025 \sim 0.005$ [26]. The dipolar term is treated by a Fourier transformation in momentum space using a convolution theorem [27]. In all cases we take the length scale $l = 1 \mu\text{m}$ and time scale $t_0 = 2m(^{87}\text{Rb})l^2/\hbar = 2.74 \text{ ms}$. The numerical simulation is performed in a cubic box, limited by $x = y = z = \pm 50$, containing 400000 ^{87}Rb atoms of density $4 \times 10^{11} / \text{cc}$.

We consider a matter-wave soliton of 1000 nondipolar ^{85}Rb atoms in the ^{87}Rb vortex core and perform imaginary-time simulation. A large intra-species attraction with scattering length $a(^{85}\text{Rb}) = -80a_0$ was necessary to obtain a compact soliton of small size. In the nondipolar case, unlike in 1D equation (2), no 3D vortex-soliton can be obtained for repulsive inter- and intra-species interactions. An attractive intra-species interaction facilitates the formation of the soliton. For smaller values of intra-species attraction, the size of the soliton was larger resulting in computational difficulty. The 1D densities of the nondipolar soliton, defined by $\rho_{1D}(x) = \int dy \int dz |\phi(\mathbf{r})|^2$, etc are plotted in figure 2(a).

In the dipolar case, we consider solitons of 500, 1000 and 2000 ^{164}Dy atoms, in the ^{87}Rb vortex core, with intra-species scattering lengths $a(^{164}\text{Dy}) = -40a_0, 10a_0$ and $30a_0$, respectively. Again the exact values of the scattering lengths are not important. These three values of scattering lengths simulate three distinct interactions: attractive, weakly repulsive and moderately repulsive. If needed, the intra-species and inter-species interactions can be manipulated by independent optical and magnetic Feshbach resonances in a laboratory [25]. The corresponding 1D densities of these solitons are presented in figures 2(b), (c), and (d), respectively. The 3D density $\rho_{3D}(\mathbf{r}) = |\Phi_1(\mathbf{r}, t)|^2$ of the axially-symmetric vortex core is plotted in figure 2(e). We also show the variational healing-length estimate of this density for an isolated vortex without the soliton: $\rho(x, y, 0) = \rho_0(x^2 + y^2) / (\xi^2 + x^2 + y^2)$, where ρ_0 is the density away from the vortex core, and the healing length $\xi = 1 / \sqrt{8\pi\rho_0 a} \approx 4.072 \mu\text{m}$. The numerical solution of the GP equation for the isolated vortex agrees well with healing-length estimate. The deviation of the vortex core of the binary vortex-soliton from the healing-length estimate of an isolated vortex is due to the presence of the soliton which increases the size of the vortex core by inter-species repulsion. In figure 2(f) the integrated 2D densities $\rho_{2D}(x, y) \equiv \int dz |\phi(\mathbf{r})|^2$ for the vortex and the soliton corresponding to figure 2(d) are plotted. Figures 2(a)–(f) show that the soliton is localized in the vortex core given by the minimum of the vortex density plotted in figure 2(e).

A qualitative understanding of the formation of the matter-wave soliton can be obtained if we note that the densities corresponding to the vortex wave functions as presented in figure 2(e) for the four cases studied above are practically

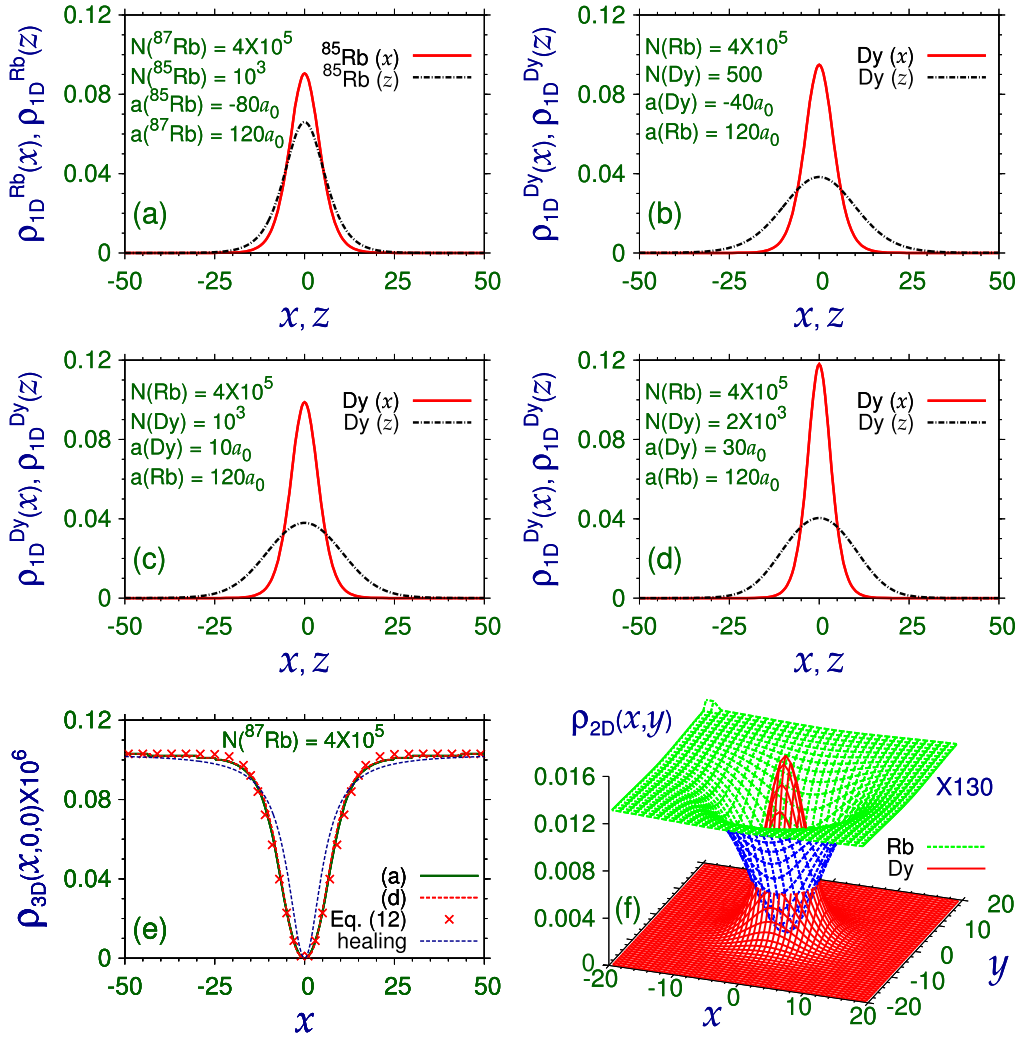


Figure 2. (a) Integrated 1D density $\rho_{1D}(x)$ and $\rho_{1D}(z)$ of a matter-wave soliton of 1000 ^{85}Rb atoms in the vortex core of a uniform ^{87}Rb BEC of 4×10^5 atoms in a box of volume $100^3 \mu\text{m}^3$. The same of (b) 500, (c) 1000, and (d) 2000 ^{164}Dy atoms. (e) 3D density $\rho_{3D}(x, 0, 0)$ of the vortex core for (a) and (d) together with analytic fit (12) and healing length estimate (healing) $\rho(x, 0, 0) = 0.00000103x^2/(x^2 + 2\xi^2)$ with healing length $\xi \approx 4.072 \mu\text{m}$. (f) Integrated 2D density $\rho_{2D}(x, y)$ of the ^{87}Rb vortex and the ^{164}Dy soliton of (d). The parameters are $a(^{87}\text{Rb} - ^{85}\text{Rb}) = a(\text{Rb} - \text{Dy}) = 120a_0$. Variables in all figures are dimensionless.

the same given by the following function

$$\Phi_1^2(\mathbf{r}) = A \left(1 - e^{-(x^2+y^2)\delta} \right) \left(1 + \nu - \nu e^{-z^2\delta} \right), \quad (12)$$

with $A = 0.00000103$, $\delta = 0.01$, $\nu = 0.25$. The function (12) with $\nu = 0$ simulates a vortex in a uniform BEC. A non-zero value of ν includes the effect of a small distortion of the vortex inside the soliton due to the presence of the soliton. The analytic density (12) is also plotted in figure 2(e) in good agreement with the numerical result. We perform an approximate variational calculation for the formation of a soliton with vortex density (12) substituted in (10) using the following Gaussian trial function for the soliton:

$$\phi_2 = \frac{\pi^{-3/4}}{w_\rho \sqrt{w_z}} \exp \left[-\frac{x^2 + y^2}{2w_\rho^2} - \frac{z^2}{2w_z^2} \right], \quad (13)$$

where w_ρ and w_z are the widths. The Lagrangian (10) can be written as

$$L = \frac{1}{2} \int d\mathbf{r} \left[m_{12} N_2 |\nabla \phi_2|^2 + N_2 g_2 \phi_2^4 + 2N_2 g_{21} |\Phi_1|^2 |\phi_2|^2 + \int d\mathbf{r}' \int d\mathbf{r}'' N_2 g_{dd} V_{dd}(\mathbf{R}) |\phi_2(\mathbf{r})|^2 |\phi_2(\mathbf{r}')|^2 \right], \quad (14)$$

$$= \frac{N_2 m_{12}}{2} \left[\frac{1}{w_\rho^2} + \frac{1}{2w_z^2} \right] + \frac{N_2^2 m_{12} [a_2 - a_{dd} f(\kappa)]}{\sqrt{2\pi} w_\rho^2 w_z} + \frac{N_2 g_{21} A \delta w_\rho^2}{e^2(w_\rho)} \left[1 + \nu - \frac{\nu}{e(w_z)} \right] \quad (15)$$

where $\kappa = w_\rho/w_z$, $f(\kappa) = [1 + 2\kappa^2 - 3\kappa^2 d(\kappa)]/(1 - \kappa^2)$, $d(\kappa) = \arctan(\sqrt{\kappa^2 - 1})/\sqrt{\kappa^2 - 1}$, $e(x) = \sqrt{1 + \delta x^2}$. Minimizing Lagrangian (15) we find the following conditions to

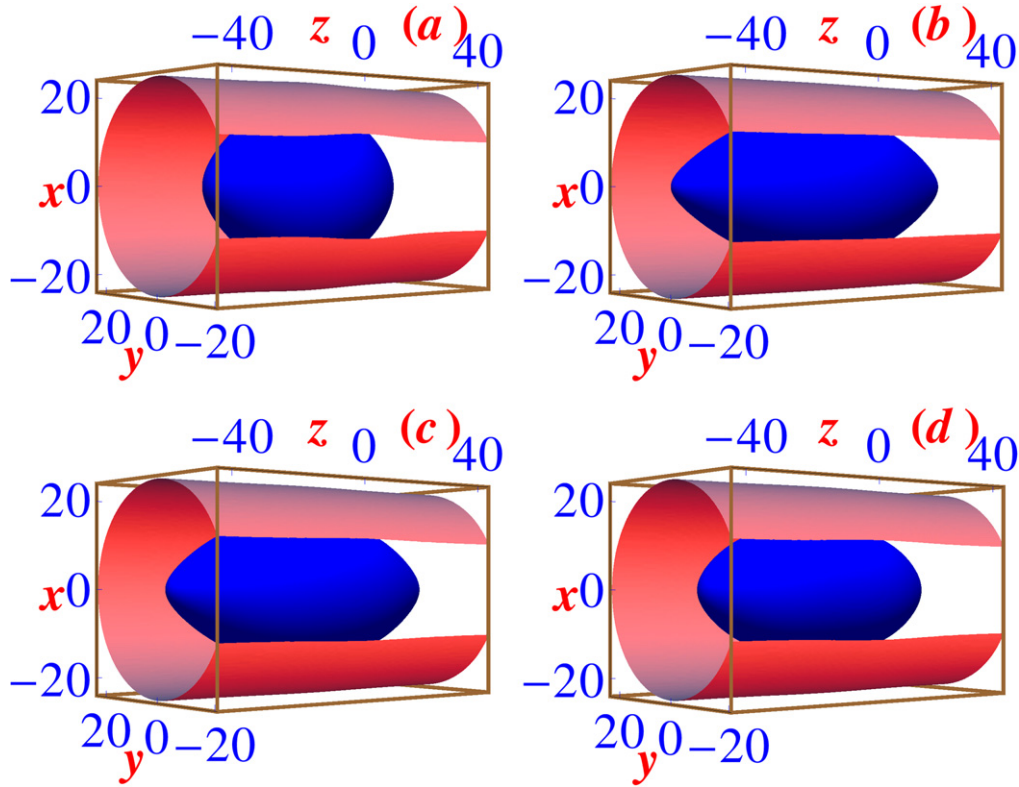


Figure 3. 3D isodensity contour of the binary vortex-soliton, showing the vortex core (grey, pink in color) and the soliton (black, blue in color) profiles, corresponding to (a) figure 2(a), (b) figure 2(b), (c) figure 2(c), and (d) figure 2(d). Density on the contour is 10^6 /cc.

Table 1. Numerical and analytic RMS sizes of the four solitons presented in figures 3(a)–(d).

	$\langle x, y \rangle_{num}$	$\langle x, y \rangle_{anal}$	$\langle z \rangle_{num}$	$\langle z \rangle_{anal}$	$\langle z/x \rangle_{num}$
(a)	4.89	4.762	6.97	7.519	1.43
(b)	4.66	4.341	11.05	10.557	2.37
(c)	4.47	4.226	10.90	11.711	2.44
(d)	3.79	3.870	9.72	11.884	2.56

determine the widths of the soliton [18, 19]

$$\frac{1}{w_\rho^3} + \frac{N_2 e(\kappa)}{\sqrt{2\pi} w_\rho^3 w_z} - \frac{2w_\rho g_{21} A \delta}{e^4(w_\rho) m_{12}} \left[1 + \nu - \frac{\nu}{e(w_z)} \right] = 0, \quad (16)$$

$$\frac{m_{12}}{w_z^3} + \frac{2N_2 m_{12} h(\kappa)}{\sqrt{2\pi} w_\rho^2 w_z^2} - \frac{2g_{21} A \nu \delta^2 w_\rho^2 w_z}{e^2(w_\rho) e^3(w_z)} = 0, \quad (17)$$

where $e(\kappa) = 2a_2 - a_{dd}[2 - 7\kappa^2 - 4\kappa^4 + 9\kappa^4 d(\kappa)] / (1 - \kappa^2)^2$, $h(\kappa) = a_2 - a_{dd}[1 + 10\kappa^2 - 2\kappa^4 - 9\kappa^2 d(\kappa)] / (1 - \kappa^2)^2$. A solution of (16) and (17) determines the widths, and hence the sizes of the soliton. For the solitons of figures 2 (a)–(d) the numerical and analytic root-mean-square (RMS) sizes are given in table 1. The analytic result presented is not a variational solution of the full dynamics given by (11) and

(10) and hence there are no variational bounds on the energy or sizes. Nevertheless, the approximate analytic results of table 1, in reasonable agreement with the numerical results, provide a qualitative understanding of the formation of the soliton.

The density for the dipolar matter-wave solitons is strongly anisotropic with distinct 1D densities $\rho_{1D}(x)$ and $\rho_{1D}(z)$ as can be found in figures 2(b)–(d), whereas in the nondipolar case, shown in figure 2(a), these two densities are nearly equal. The anisotropy in the shape of the soliton arises partly due to the anisotropy of the dipolar interaction and partly due to the anisotropy of the vortex core. The increase in total dipolar interaction makes the soliton more elongated (prolate) along the polarization z direction. This is reflected in the ratio of numerical RMS sizes $\langle z \rangle / \langle x \rangle$ shown in table 1.

The 3D isodensity contours of the four matter-wave solitons of figures 2(a)–(d) together with the respective vortex cores are shown in figures 3(a)–(d), respectively. The central prolate spheroid is the soliton and the outer shell is the vortex core. The net dipolar interaction and hence the anisotropy of the soliton increases with the number of dipolar ^{164}Dy atoms. The 3D profile of the soliton is more prolate, in figures 3(b)–(d), due to dipolar interaction, compared to that in figure 3(a) for the nondipolar case. The nondipolar soliton is also slightly prolate due to the axially-symmetric inter-species repulsion of the vortex.

By carefully adjusting the parameters of the mean-field model equations a balance between attraction and repulsion

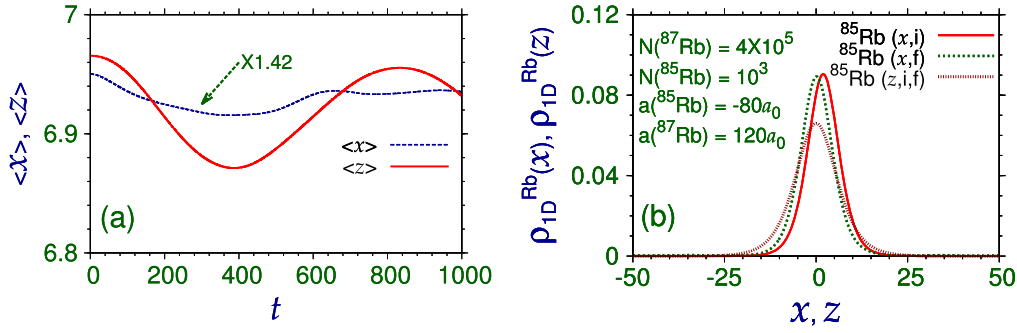


Figure 4. (a) RMS sizes $\langle x \rangle$, $\langle z \rangle$ during breathing oscillation of the nondipolar soliton of figures 2(a) and 3(a) initiated by a sudden change of $a(^{85}\text{Rb})$ from $-80a_0$ to $-80.5a_0$. (b) Initial (i) and final (f) 1D densities along x and z directions after real-time dynamics of the vortex-soliton of figure 3(a) during 1000 units of time. The dynamics is started after shifting the initial soliton along x direction through 2 units of length and maintaining the initial vortex position unchanged. In both (a) and (b) the initial state was the imaginary-time stationary profile of the vortex-soliton of figure 3(a).

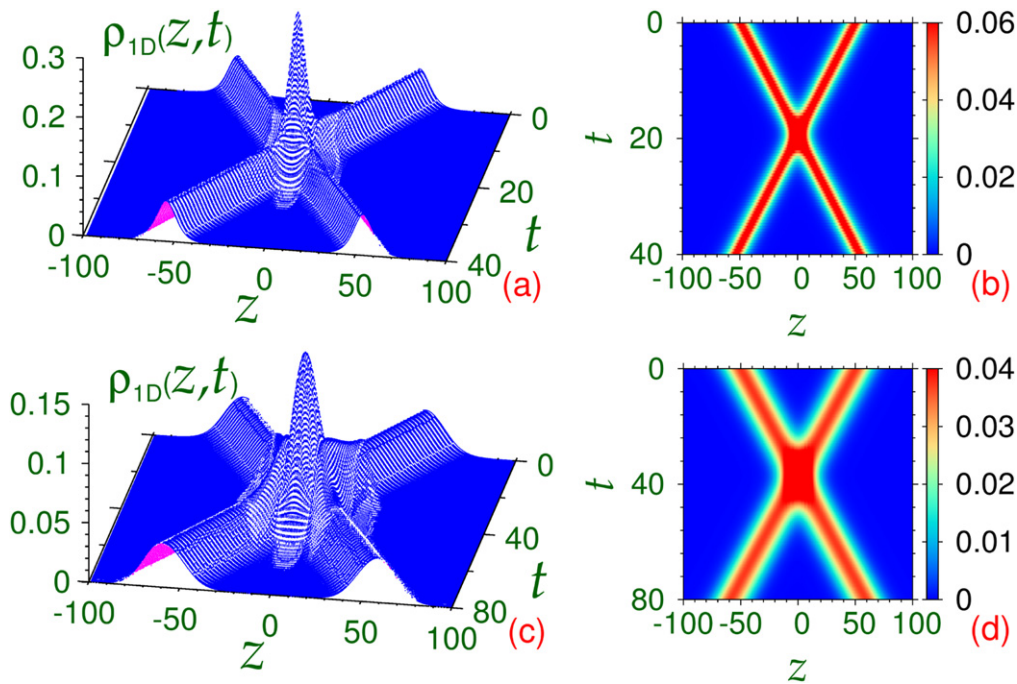


Figure 5. (a) The 1D density $\rho_{1D}(z, t)$ and (b) its contour plot during collision of two nondipolar solitons of $1000 \text{ }^{85}\text{Rb}$ atoms of figure 3 (a) initially placed at $z = \pm 50$, upon real-time propagation. The initial wave functions are multiplied by $\exp(\pm i 20 z)$ to set them in motion. The same for two dipolar solitons of $1000 \text{ }^{164}\text{Dy}$ atoms of figure 3(c) are shown in (c) and (d).

can be achieved to obtain a vortex-soliton similar to a Townes soliton in 2D [28], which is weakly unstable and collapses upon a small perturbation [7]. So it is quite relevant to establish the stability of the vortex-soliton. To demonstrate the stability of the soliton, we consider the one in figure 3(a) and subject the stationary state(s) obtained by imaginary-time propagation to real-time propagation introducing a small perturbation, e.g. jumping intra-species scattering length $a(^{85}\text{Rb})$ from $-80a_0$ to $-80.5a_0$ at $t = 0$. Long-time stable oscillation of the resultant RMS sizes, illustrated in figure 4 (a), guarantees the stability of the soliton. To test the stability of the soliton after a small displacement along x direction, we performed real-time simulation of the vortex-soliton of figure 3 (a) after displacing it

through 2 units of length along x direction. The soliton is found to come back to its stable position on the z axis at the center of the vortex core without executing oscillatory motion along x axis. In figure 4 (b) the plot of initial and final 1D densities of the soliton after 1000 units of time confirms the stability. This transverse stability of the vortex-soliton is important for an experimental realization.

Next we study the head-on collision between two solitons moving along the vortex core in opposite directions. The imaginary-time profile of the vortex-soliton of figure 3(a) is used as the initial function in the real-time simulation of collision, with two identical solitons placed at $z = \pm 50$. To set the solitons in motion along the z axis in opposite directions the soliton wave functions are multiplied by

$\exp(\pm ivz)$, $v = 20$. To illustrate the dynamics upon real-time simulation, we plot the time evolution of 1D density $\rho_{1D}(z, t)$ in figure 5(a) and its contour plot in figure 5(b). The same for the collision of two solitons of figure 3(c) are shown in figures 5(c) and (d). In figures 5(a) and (b), the dimensionless velocity of a soliton is ~ 2.5 , whereas in figures 5(c) and (d) this velocity is ~ 1.25 . The quasi-elastic nature of collision is established at a relative velocity (two times the velocity of a single soliton) of about $2 \sim 5$ in dimensionless units.

4. Summary

Summarizing, we demonstrated the possibility of the creation of a nondipolar or dipolar matter-wave soliton in the vortex core of a uniform nondipolar BEC. The soliton is localized by a strong inter-species repulsion. This binary vortex-soliton is a stable stationary state. A dipolar soliton can be created for repulsive inter- and intra-species contact interactions. However, for the creation of a nondipolar soliton an attractive intra-species contact interaction in the soliton is necessary. The matter-wave soliton can move with a constant velocity along the vortex core without any deformation. The stability of the vortex-soliton is demonstrated by a stable oscillation of the soliton upon a small perturbation in real-time simulation using the initial states obtained in imaginary-time propagation, figure 4. At medium velocities, the collision between the two solitons is quasi elastic with no visible deformation, figure 5.

The techniques of generating a vortex in BECs [29] are well known, hence a binary vortex-soliton can be realized in experiments. Here, for the sake of computational simplicity, we considered the vortex in a uniform BEC. However, for an experimental observation, a small matter-wave soliton can be created in the core of a large low-density weakly-trapped BEC vortex. To achieve this the binary vortex-soliton should be realized under a weak harmonic trap on both components in a laboratory and eventually the weak trap on the soliton should be slowly removed to obtain the trapless soliton in the vortex core maintaining the weak trap on the vortex. The weak initial harmonic trap on both components will bring the soliton inside the vortex core. We tested the stability of the binary vortex-soliton under a small transverse perturbation. The transverse confining force on the soliton due to inter-species repulsion brings the soliton back to the center inside the vortex core. An interesting future work would be to study the possibility of creating a binary vortex-soliton in two components of a spin-orbit coupled BEC with one component supporting a coreless vortex [30] and the other supporting a soliton.

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