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Universality in the Neutron– ^{19}C Scattering Using Finite-Range Separable Interactions

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Abstract We report a study on the low-energy properties of the elastic s -wave scattering of a neutron (n) in the carbon isotope ^{19}C near the critical condition for the occurrence of an excited Efimov state in the three-body n – n – ^{18}C system. For the separation energy of the two halo neutrons in ^{20}C we use the available experimental data. We also investigate to which extent the universal scaling laws, strictly valid in the zero-range limit, will survive when using finite-range interactions. By allowing to vary the n – ^{18}C binding energy, a scaling behavior for the real and imaginary parts of the s -wave phase-shift δ_0 is verified, emerging some universal characteristics given by the pole-position of $k \cot(\delta_0^R)$ and effective-range parameters.

1 Introduction

In the present communication we are reporting some results obtained when studying the elastic s -wave scattering of a neutron from ^{19}C , in the low-energy region, by considering a three-body n – n –core system, where the core is given by ^{18}C , with the binding energy of ^{20}C fixed to 3.5 MeV. Our aim is to investigate possible universal characteristics [1] obeyed by scattering observables of a three-body system which is close to form an excited Efimov state [2]. With this purpose, we notice that the structure of exotic halo nuclei has been shown quite fascinating to study when looking for universal characteristics in mass imbalanced systems [1, 3]. This motivated us to extend the corresponding studies from negative energy to the scattering low energy sector, in order to evidence universal effects in the continuum which could be associated to the Efimov physics. Therefore, we are moving from the bound n – n – c system to the case that a single neutron is scattered by the bound n – c subsystem. As an appropriate interesting example, with enough experimental data to be considered in theoretical models, we can mention the ^{20}C nucleus within the three-body n – n – ^{18}C model [4]. This neutron-rich exotic nucleus, with the n – ^{18}C subsystem forming an s -wave bound state, is close to produce

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an Efimov excited state [5]. Next, to verify possible universal properties which can be attributed to scattering observables, we allow the two-body n - ^{18}C bound-state energy to vary.

Within our present approach, we found convenient to remind some original studies considering the three-nucleon elastic scattering, given by neutron–deuteron (n - d) system. In the study of the spin-doublet s -wave n - d elastic scattering, a relevant quantity that was considered is $k \cot \delta_0$, where k is the on-energy shell relative momentum (incoming and final) and δ_0 is the s -wave phase shift. It was first pointed out in Ref. [6] that this quantity presents a pole when analyzing experimental data for the n - d doublet s -wave phase shift, which have also motivated some other pioneering related works [7–9].

The zero-range results reported in [10] for the n - ^{19}C elastic s -wave phase-shifts support the existence of a pole in the effective range expansion in a good qualitative agreement with what is found for the n - d low-energy scattering, as well as for the atom-dimer discussed above. It remains the question how much such universal features are preserved when using finite-range potentials, considering the range of the n - c and n - n interactions, when the two-neutron separation energy in ^{20}C is kept fixed. Therefore, in a recent work [11], we have performed a deeper investigation on the behavior of the low-energy pole $k \cot \delta_0$ for the n - ^{19}C scattering, beyond the zero-range approach used in Ref. [12], by considering a finite-range potential, which was chosen separable with Yamaguchi form. The effect of mass asymmetry associated with finite-range interactions was also verified in this study.

By reporting the main results given in Ref. [11], where the formalism is detailed, we found appropriate in this communication to introduce an extended analysis of our results on the n - ^{19}C elastic cross sections computations. The analysis is done with the assumption that the required separation energy for two neutrons in ^{20}C is fixed to an accepted experimental value, $E_{nnc} \equiv E_{20\text{C}} = -3.5$ MeV. Then, the less known two-body bound-state energy $E_{nc} \equiv E_{19\text{C}}$ is varied within our numerical investigation, looking for some characteristic universal behavior of an observable related to the s -wave scattering phase-shift δ_0 . Motivated by the well-known studies related to n - d , for such observable we choose the energy position of the poles verified in $k \cot \delta_0^R$ (where δ_0^R is the real part of δ_0). As this observable is expected to be quite sensible to the range, we found also appropriate to further discuss here some relevant results related to range effects and corresponding scaling reported in Ref. [11]. We have noticed that the scaling of the position of the pole in $k \cot \delta_0^R$ and the effective range parameters reproduces universal characteristics already found within the zero-range model under the realm of the Efimov physics.

2 Formalism and Notation

For the formalism, we follow the standard one for the elastic scattering amplitude of a neutron in the bound neutron-core subsystem, which is detailed in Refs. [11, 12]. For the two-body n - n and n - c interactions, we consider separable expressions, with strengths and ranges adjusted by the n - n virtual-state energy ($E_{nn} = -143$ keV) and n - c bound-state energy ($E_{nc} = E_{19\text{C}}$), respectively, such that the three-body ground-state binding energy remains fixed to ($E_{20\text{C}} = -3.5$ MeV). The neutron separation energy in ^{19}C has a sizable error, with given values ranging from -160 ± 110 keV [13] to -530 ± 130 keV [14]. Therefore, in our present study, a wide variation for E_{nc} will be allowed, from -400 up to -800 keV, such that all possible experimental values can be included. For the n - n and n - c interactions, considering range-parameters given by β_{nn} and β_{nc} , respectively, we assume they have a one-term separable form, given by

$$V_{ij}(p, p') = \lambda_{ij} \left(\frac{1}{p^2 + \beta_{ij}^2} \right) \left(\frac{1}{p'^2 + \beta_{ij}^2} \right) \quad (ij = nn, nc). \quad (1)$$

For negative two-body energies (bound or virtual), E_{ij} , defining $\kappa_{ij} \equiv \sqrt{2\mu_{ij}|E_{ij}|}$ with μ_{ij} being the reduced masses for n - n and n - c subsystems, $\mu_{nn} \equiv m_n/2$ and $\mu_{nc} \equiv m_n A/(A+1)$, where $A \equiv m_c/m_n$, the relations for the strengths and ranges are, respectively, given by

$$\lambda_{ij}^{-1} = \frac{-2\pi\mu_{ij}}{\beta_{ij}(\beta_{ij} \pm \kappa_{ij})^2}, \quad r_{ij} = \frac{1}{\beta_{ij}} + \frac{2\beta_{ij}}{(\beta_{ij} \pm \kappa_{ij})^2}, \quad (+/-) \text{ for bound/virtual}, \quad (2)$$

Our units are such that $\hbar = 1$, with momentum variables in fm^{-1} and the unit conversion given by $\hbar^2/m_n = 41.47 \text{ MeV fm}^2$.

The values of β_{nc} were obtained by fitting the two-neutron separation energy in ²⁰C (3.5 MeV [13]), with the n - n interactions fixed by the virtual state energy, $E_{nn} = -143$ keV. The fixed β_{nn} values for high and low ranges, respectively, are 1.34 fm^{-1} ($r_{nn} = 2.372 \text{ fm}$) and 24.50 fm^{-1} ($r_{nn} = 0.1228 \text{ fm}$).

The on-energy-shell incoming and final relative momentum are related to the three-body energy $E \equiv E(k_i)$ by $k \equiv k_i \equiv |\mathbf{k}_i| = |\mathbf{k}_f| = \sqrt{2\mu_{n(nc)}(E - E_{nc})}$, where $\mu_{n(nc)} = m_n(A + 1)/(A + 2)$. The coupled s -wave scattering equations can be cast in a single channel Lippmann-Schwinger-type equation for the relevant amplitude h_n , with the following form (for details, see Ref. [11]):

$$h_n(q, E) = \mathcal{V}(q, k, E) + \frac{2}{\pi} \int_0^\infty dq' q'^2 \frac{\mathcal{V}(q, q', E) h_n(q', E)}{q'^2 - k^2 - i\epsilon}, \quad (3)$$

where the kernel $\mathcal{V}(q, q', E)$ for the n -(nc) channel amplitude contains the contribution of the coupled (nn)- c channel.

3 Results and Discussion

The neutron-core interaction-range parameter, β_{nc} , of the one-term separable potential, is varied for different E_{19C} , by keeping fixed the low-energy observables S_{2n} and the virtual state energy E_{nn} . For that, we study situations with two distinct regions, with low and high values for the range parameters β_{nn} and β_{nc} , which will correspond to larger and smaller effective ranges of the n - n and n - c interactions, respectively. Here we include some representative results, which are relevant for our following discussed on the scaling characterization of the low-energy neutron scattering in ¹⁹C. The results are resumed in Table 1 and Fig. 1.

In Table 1, by keeping fixed the three-body bound-state energy, $E_{20C} = -3.5$ MeV, as well as the n - n parameters, such that $E_{nn} = -143$ keV, we report a few values for the one-neutron separation energies in ¹⁹C,

Table 1 One-neutron separation energy in ¹⁹C (left column), obtained by the given range parameters of the separable Yamaguchi potential, β_{nc} (second and fourth columns), with the corresponding effective ranges (third and fifth columns)

$ E_{19C} $ (keV)	β_{nc} (fm ⁻¹)	r_{nc} (fm)	β_{nc} (fm ⁻¹)	r_{nc} (fm)
200	0.971	2.736	18.970	0.157
400	0.754	3.233	17.036	0.174
600	0.598	3.720	15.592	0.190
800	0.477	4.255	14.395	0.205

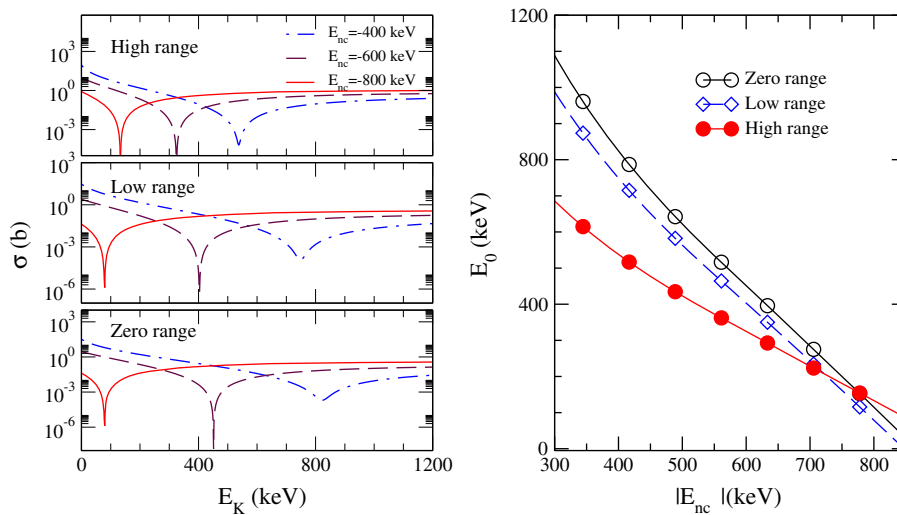


Fig. 1 Left panels Several plots for the s -wave n -¹⁹C elastic cross-section are shown as functions of the center-of-mass kinetic energy E_K . In each panel we have three plots corresponding to different E_{nc} binding energies, which are identified in the top panel. For the range parameters used in the Yamaguchi potential, with E_{20C} fixed, high range are in the top panel; low-range in the middle panel; and zero-range in the bottom panel. Right panel The energy positions of the $kcot\delta_0$ poles, given by E_0 , are shown as the n -¹⁸C binding energy (E_{nc}) is changed, considering results obtained with zero-, low- and high-range potential parameters

given by E_{19C} , with the respective values for the parameter β_{nc} and corresponding effective ranges, given by r_{nc} . For the n - n parameters, we have fixed $\beta_{nn} = 1.34 \text{ fm}^{-1}$ for the high-range cases, and $\beta_{nn} = 24.50 \text{ fm}^{-1}$ for the low-ranges ones. For lower values of β 's, the effective range of the n - c interaction is comparable to the ^{18}C root-mean-square matter radius, $r_m[^{18}\text{C}] = 2.82 \pm 0.04 \text{ fm}$ [15], while for high values of β 's, $r_m[^{18}\text{C}]$ is quite large compared to the effective-range, being not realistic. The case when both β_{nn} and β_{nc} are large, the corresponding effective ranges become much smaller than the core size and the n - n effective range, 2.75 fm [16].

The energy positions E_0 of the $k \cot \delta_0^R$ poles are plotted in the right-side panel of Fig. 1 as a function of the corresponding n - c binding energies, for the zero-range, high-beta (low-range) and low-beta (high-range) parameters. As shown, E_0 obtained with high-range parameters are less affected by the changes of E_{nc} than the other two cases. The quite different slope of the curves obtained with high-range parameters when compared with the corresponding ones with low- or zero-range parameters can also be observed in the results presented in the left-hand-side panels of Fig. 1.

In the panels at the left side of Fig. 1 we have the results for the s -wave scattering cross sections (in logarithmic scale) obtained from $d\sigma/d\Omega = |h_n(k; E)|^2$, given as functions of the center-of-mass kinetic energy E_K . There are three panels corresponding, respectively, to high-, low- and zero-range parameters. In each panel we show three representative values for the n - c binding energy ($E_{19C} = -400, -600$ and -800 keV). The poles of $k \cot \delta_0^R$ correspond to the observed minima appearing in the cross-section for $E_K = E_0$, clearly seen for E_0 below the breakup threshold or when the absorption to the breakup channel is not large. In this trend of moving the poles with the variation of E_{19C} we should notice that such behavior affects more the low-range cases, which reflects the slope changes verified in the right side of Fig. 1.

We finally should observe that the poles are correlating the s -wave quantities with the binding energy of the three-body halo system, namely the ^{20}C nucleus. The strong correlation, previously verified using zero-range interactions, survives when considering finite-range potential with effective n - n and n - c ranges compatible with physical values.

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