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**Abstract.** Creation of Cold Dark Matter (CCDM), in the context of Einstein Field Equations, produces a negative pressure term which can be used to explain the accelerated expansion of the Universe. In this work we tested six different spatially flat models for matter creation using statistical criteria, in light of SNe Ia data: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Bayesian Evidence (BE). These criteria allow to compare models considering goodness of fit and number of free parameters, penalizing excess of complexity. We find that JO model is slightly favoured over LJO/$\Lambda$CDM model, however, neither of these, nor $\Gamma = 3\alpha H_0$ model can be discarded from the current analysis. Three other scenarios are discarded either because poor fitting or because of the excess of free parameters. A method of increasing Bayesian evidence through reparameterization in order to reducing parameter degeneracy is also developed.

**Keywords:** dark energy theory, dark matter theory, supernova type Ia - standard candles

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## Contents





# 1 Introduction

Since evidences for the accelerated cosmic expansion have been found [1–3], a large number of possible explanations for this unexpected behaviour have been proposed.

The most accepted proposal, the ΛCDM model, is successful at explaining many observational data, e. g., the Cosmic Microwave Background Radiation (CMB), Baryon Acoustic Oscillations (BAO) and $H(z)$ [4–7]. However, the standard model has important conceptual questions to be answered, namely, the Cosmological Constant Problem [8], Cosmic Coincidence Problem [9] and some small scale problems [10, 11].

In order to test other possibilities and trying to overcome the above difficulties, many alternatives to ΛCDM have been proposed. Among them, some models involve different kinds of dark energy with negative pressure in order to provide acceleration inside the Einstein Field Equations (EFE) while still assuming the Cosmological Principle (CP) [12]. Other proposals *to be considered are* modified gravitation theories [13] and the breaking of the CP [14].

On the other hand, the early investigations of the quantum particle creation from dynamical gravitational potentials [15–17] has shown that this process may result in a positive acceleration due to its negative creation pressure in the level of EFE.

This gave birth to the so-called Creation of Cold Dark Matter (CCDM) cosmologies, in which the creation rate of particles can influence the cosmic expansion rate [18–23]. In this scenario, many of the different models were phenomenologically proposed through dimensional arguments about the particle creation rate. In a different approach, recently, it was proposed a CCDM model which was equivalent to ΛCDM with respect to the background equations, the so called LJO model [21]. Evolution of density perturbations were calculated in the context of LJO, in a Neo-Newtonian framework, and it was shown that this model can be distinguished from ΛCDM in the linear order, but it can be compatible with observations if some amount of entropy perturbations are considered [24]. Later, it was shown that, even with no entropy perturbations, if one separates the obtained Cold Dark Matter (CDM) density as a clustered and a smooth components, this model is equivalent to ΛCDM even at higher orders of density perturbation theory [25, 26].

Meanwhile, more fundamental treatments of CCDM models were developed, as a particle creation rate calculated from quantum particle creation in a curved spacetime [27], and as the development of a kinetic theory of particle creation [28].

The most reliable way to compare the mentioned models and to determine which mechanism has been driving the late stage of cosmic accelerated expansion is by using Bayesian criteria to differentiate among them in light of current observational data [29].

In the present work, we focus on SNe Ia observations as it is the most straightforward evidence of cosmic acceleration with currently large amount of data, enabling the best model selection in what concerns the background evolution equations.

Through the use of different model selection criteria, namely, Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC) and Bayesian Evidence (BE), we ranked some models of interest, including some CCDM models and the ΛCDM model. Our conclusions are drawn by considering one of the currently largest Supernovae Ia data sample, the Union 2.1 [3].

In section 2, we discuss the dynamics of the universe with negative pressure due to matter creation. In section 3, we discuss the model selection methods used here. In section 4, we find the observational constraints from SNe Ia over some CCDM models, apply the model selection methods to distinguish among CCDM models and compare with other results obtained in literature. Finally, we summarize the main results in conclusion.

## 2 Creation of Cold Dark Matter (CCDM) models

We start by regarding the homogeneous and isotropic FRW line element (with $c = 1$):

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \tag{2.1}$$

where $k$ can assume values $-1$, $+1$ or 0.

In this background, the Einstein Field Equations are given by

$$8\pi G(\rho_{\rm rad} + \rho_{\rm b} + \rho_{\rm dm}) = 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2}, \tag{2.2}$$

and

$$8\pi G(p_{\rm rad} + p_{\rm c}) = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2}, \tag{2.3}$$

where $\rho_{\rm rad}$, $\rho_{\rm b}$ and $\rho_{\rm dm}$ are the radiation, baryons and CDM density, respectively, $p_{\rm rad} = \rho_{\rm rad}/3$ is the radiation pressure and $p_{\rm c}$ is the creation pressure.

The solutions of the EFE above are obtained considering an Energy-Momentum Tensor (EMT) in the form [15, 28]:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}, \tag{2.4}$$

where $T^{\mu\nu}_{\rm eq}$ characterizes thermodynamic equilibrium in the fluid and the creation of matter and entropy in universe are incorporated to the EFE through the correction term $\Delta T^{\mu\nu} = -p_{\rm c}(g^{\mu\nu} - u^\mu u^\nu)$ [15–17, 28].

Therefore, the complete EMT (2.4) in the presence of matter creation has the explicit form:

$$T^{\mu\nu} = (\rho_{\rm rad} + \rho_{\rm b} + \rho_{\rm dm} + p_{\rm rad} + p_{\rm c})u^\mu u^\nu - (p_{\rm rad} + p_{\rm c})g^{\mu\nu}, \tag{2.5}$$

satisfying the conservation law $T^{\mu\nu}{}_{;\nu} = 0$.



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Assuming solely the creation of dark matter component, the densities of radiation and baryon components satisfy their respective usual conservation laws, namely:

$$\dot{\rho}_{\rm rad} + 4\frac{\dot{a}}{a}\rho_{\rm rad} = 0, \tag{2.6}$$

and

$$\dot{\rho}_{\rm b} + 3\frac{\dot{a}}{a}\rho_{\rm b} = 0, \tag{2.7}$$

where each overdot means one time derivative and we have used that $p_{\rm rad} = \rho_{\rm rad}/3$ and $p_{\rm b} = 0$.

On the other hand, considering the creation process, we have a source term at the level of the EFE [16, 17]:

$$\frac{\dot{\rho}_{\rm dm}}{\rho_{\rm dm}} + 3\frac{\dot{a}}{a} = \Gamma, \tag{2.8}$$

where $\Gamma$ is the rate of dark matter creation in units of $(\text{time})^{-1}$.

As shown by [16, 17], the creation rate of cold dark matter may be related to the creation pressure $p_{\rm c}$ in eq. (2.3) by assuming an "adiabatic" creation, i.e., the scenario where the entropy per particle is constant. The so called "adiabatic" regime is a simplifying hypothesis in which the only source of entropy increase in the universe is the matter creation [15]. Mathematically, according to Calvão, Lima & Waga [16, 17]:

$$\dot{\sigma} = \frac{\Psi}{nT}\left(\beta - \frac{\rho + p}{n}\right), \tag{2.9}$$

where $\sigma$ is the entropy per particle, $\Psi$ is the numeric particle creation rate, $n$ is the particle density, $T$ is the temperature and $\beta$ comes from a phenomenological assumption on the creation pressure:

$$p_{\rm c} = -\frac{\beta\Psi}{\Theta}, \tag{2.10}$$

where $\Theta = 3H$ is the bulk expansion rate and $H \equiv \dot{a}/a$. So, in case $\dot{\sigma} = 0$, as we assume, we find $\beta = \frac{\rho+p}{n}$, then creation pressure is given by

$$p_{\rm c} = -\frac{\rho + p}{\Theta}\frac{\Psi}{n} = -(\rho + p)\frac{\Gamma}{3H}. \tag{2.11}$$

On the other hand, if $\dot{\sigma} \neq 0$, $\beta$ remains as an unknown parameter, which can not be constrained by thermodynamics alone, as the second law of thermodynamics demands only $\Psi \geq -\frac{n\dot{\sigma}}{\sigma}$. By assuming creation of CDM only, we have $p = 0$ in (2.11) and:

$$p_{\rm c} = -\frac{\rho_{\rm dm}\Gamma}{3H}. \tag{2.12}$$

As a consequence of eq. (2.12), one can see that the dynamics of the universe is directly affected by the rate of creation of cold dark matter, $\Gamma$. In particular, in the case $\Gamma > 0$ (creation of particles) we have a negative pressure creation and in the case $\Gamma \to 0$ we recover the well known dynamics when the universe is lately dominated by pressureless matter (baryons plus dark matter).

Since we are interested only on the late phase of the dynamics of the universe, we can neglect the radiation terms from now on. Thus, by combining eqs. (2.2) and (2.3), we find

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\rm b} + \rho_{\rm dm} + 3p_{\rm c}). \tag{2.13}$$

Replacing $p_{\rm c}$ from eq. (2.12), we may write

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_{\rm b} + \rho_{\rm dm}\left(1 - \frac{\Gamma}{H}\right)\right]. \tag{2.14}$$

Using that $\frac{\ddot{a}}{a} = \dot{H} + H^2$ and changing variables from cosmological time $t$ to cosmological redshift $z$, we find

$$\frac{dH}{dz} = \frac{H}{1+z} + \frac{H_0^2\Omega_{\rm b}(1+z)^2\Gamma}{2H^2} + \frac{H^2 - H_0^2\Omega_k(1+z)^2}{2H(1+z)}\left(1 - \frac{\Gamma}{H}\right), \tag{2.15}$$

where we have used the solution of (2.7) to baryon density, $\rho_{\rm b} = \rho_{b0}(1+z)^3$, $\Omega_{\rm b} = \frac{\rho_{b0}}{\rho_{c0}}$ is the present baryon density parameter, and $\Omega_k = -\frac{k}{H_0^2}$ is the present curvature density parameter. Changing to dimensionless variable $\mathcal{H}(z) \equiv \frac{H(z)}{H_0}$, we find

$$\frac{d\mathcal{H}}{dz} = \frac{\mathcal{H}}{1+z} + \frac{\Omega_{\rm b}(1+z)^2}{2\mathcal{H}^2}\frac{\Gamma}{H_0} + \frac{\mathcal{H}^2 - \Omega_k(1+z)^2}{2\mathcal{H}(1+z)}\left(1 - \frac{\Gamma}{H_0\mathcal{H}}\right). \tag{2.16}$$

If the Universe is spatially flat, it can be further simplified:

$$\frac{d\mathcal{H}}{dz} = \frac{3\mathcal{H}}{2(1+z)}\left(1 - \frac{\Gamma}{3H_0\mathcal{H}}\right) + \frac{\Omega_{\rm b}(1+z)^2}{2\mathcal{H}^2}\frac{\Gamma}{H_0}. \tag{2.17}$$

By defining the dimensionless quantity $\Delta \equiv \frac{\Gamma}{3H_0}$, it can be written:

$$\frac{d\mathcal{H}}{dz} = \frac{3}{2}\left[\frac{\mathcal{H} - \Delta}{1+z} + \frac{\Omega_{\rm b}(1+z)^2}{\mathcal{H}^2}\Delta\right]. \tag{2.18}$$

So, if a CCDM model is defined with an expression $\Gamma = \Gamma(H)$, we can find a dependence $\Delta(\mathcal{H})$, replace it at eq. (2.18) and solve it for $\mathcal{H}(z)$.

### 2.1 Models

We may regard $\Gamma(H)$ as a natural dependence for the creation rate, as it represents a relation between a creation rate and the expansion rate. Most of the CCDM models studied here follow this dependence. Furthermore, almost every model studied here can be written in a form $\Delta = \beta\mathcal{H} + \alpha\mathcal{H}^{-n}$, which corresponds to $\Gamma = 3\beta H + 3\alpha H_0\left(\frac{H_0}{H}\right)^n$. Another model we are interested in analysing is the so called LJO model [21], with a dependence $\Gamma = 3\alpha\frac{\rho_{c0}}{\rho_{\rm dm}}H$, which has been shown to have the same background dynamics as the $\Lambda$CDM model. That is, in this model, the cosmological constant is exactly mimicked by the creation of particles. The models we have analysed are shown on table 1.

In all models analysed here we have taken into account the contribution of baryons. The baryon density was assumed to be a fixed parameter, given by Planck as $\Omega_{\rm b} = 0.049$. For simplicity, we choose to work with a spatially flat Universe, as indicated from CMB and preferred by inflation, i.e., $\Omega_k \equiv 0$ in our analysis. Using the general expression for the creation rate, namely $\Gamma = 3\beta H + 3\alpha H_0\left(\frac{H_0}{H}\right)^n$, eq. (2.18) reads:

$$\frac{d\mathcal{H}}{dz} = \frac{3}{2}\left[\frac{(1-\beta)\mathcal{H} - \alpha\mathcal{H}^{-n}}{1+z} + \Omega_{\rm b}(1+z)^2\left(\frac{\alpha}{\mathcal{H}^{n+2}} + \frac{\beta}{\mathcal{H}}\right)\right]. \tag{2.19}$$

| Model | Creation rate | Reference | Priors/Fixed Parameters |
|---|---|---|---|
| $M_0$ | $\Gamma = \frac{3\alpha H_0^2}{H}$ | [20] (JO) | $\alpha \in [0,1]$, $\beta = 0$, $n = 1$ |
| $M_1$ | $\Gamma = 3\alpha \frac{\rho_{c0}}{\rho_{\rm dm}} H$ | [21] (LJO) | $\alpha \in [0,1]$ |
| $M_2$ | $\Gamma = 3\alpha H_0$ | [30] | $\alpha \in [0,1]$, $\beta = 0$, $n = 0$ |
| $M_3$ | $\Gamma = 3\beta H$ | — | $\alpha = 0$, $\beta \in [0,1]$ |
| $M_4$ | $\Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$ | — | $\alpha \in [0,1]$, $\beta = 0$, $n \in [-10,10]$ |
| $M_5$ | $\Gamma = 3\alpha \frac{H_0^2}{H} + 3\beta H$ | [30] | $\alpha \in [-2,3]$, $\beta \in [-2,2]$, $n = 1$ |

**Table 1**. Model parameters and priors.

This equation covers all models studied here, except LJO ($M_1$). However, even neglecting spatial curvature, these models in general have no analytical expression for $H(z)$, due to inclusion of baryons. One exception is LJO model, which recovers the $\Lambda$CDM dependence [21]:

$$\mathcal{H}(z) = \frac{H(z)}{H_0} = \left[\alpha + (1-\alpha)(1+z)^3\right]^{1/2}. \tag{2.20}$$

Another case in which $\mathcal{H}(z)$ can be analytically obtained, even with the presence of baryons, is model $M_3$, where the creation rate is proportional to expansion rate. In this case:

$$\mathcal{H}(z) = \left[(1-\Omega_{\rm b})(1+z)^{3-3\beta} + \Omega_{\rm b}(1+z)^3\right]^{1/2}. \tag{2.21}$$

If baryons parameter density $\Omega_{\rm b}$ could be neglected, eq. (2.19) would yield:

$$\mathcal{H}(z) = \left[\frac{\alpha + (1-\alpha-\beta)(1+z)^{\frac{3}{2}(n+1)(1-\beta)}}{1-\beta}\right]^{\frac{1}{n+1}}. \tag{2.22}$$

However, baryon density brings an important contribution and can not be neglected. So, one has to resort to numerical or semi-analytical methods. Throughout our analysis, we solve eq. (2.19) numerically.

## 3 Model selection methods

In this section, we summarize the model selection methods used in this work. The likelihood function is the main ingredient of the analysis. It has to be built for each case and there is not a general recipe for it [31]. Here, we assume $N$ pairs of measurements $(x_i, y_i)$ for which we want to find the most likely relation between $x$ and $y$. As a maximum likelihood estimator, we can use the $\chi^2$ expression given by [32]:

$$\chi^2 = \sum_{i=1}^{N} \frac{[y_i - f(x_i,\theta)]^2}{\sigma_i^2} = -2\ln\frac{\mathcal{L}}{\mathcal{N}}, \tag{3.1}$$

where $f(x_i,\theta)$ represents the model with parameters $\theta$, $\mathcal{N}$ is a normalization constant and $\mathcal{L}$ is the likelihood function. The best fit values for the set of free parameters minimizes $\chi^2$ while maximizing the likelihood. The likelihood function indicates not only the most likely values for the relevant parameters of the statistical model, but also its distribution, in case we have no prior information over them.



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| $\Delta$AIC | Support |
|---|---|
| $\Delta\text{AIC} \leq 1$ | Not worth more than a bare mention |
| $4 \leq \Delta\text{AIC} \leq 7$ | Significant/Weak |
| $0 \leq \Delta\text{AIC} \leq 2$ | Strong to very strong/Significant |
| $\Delta\text{AIC} > 10$ | Decisive/Strong |

**Table 2**. Akaike Information Criterion.

### 3.1 Ockham's razor

Ockham's razor was proposed by William of Ockham (1285–1349) who was an English Franciscan friar and scholastic philosopher and theologian. The principle establishes: "if there are two models with some number of parameters, describing equally well a phenomenon, the simplest model will be better than the most complex model". This principle has two main reasons: aesthetic and empirical. First, if the model is simpler, it is more aesthetically beautiful because the simplest models would describe the Nature while excluding the least likely hypotheses. Second, the Nature is optimized i.e., it is economic and makes everything with parsimony. Ockham's razor is the main prerequisite for the construction of statistical models being used both in Frequentist and Bayesian analyses.

### 3.2 Akaike Information Criterion

The Akaike Information Criterion (AIC) provides a relative measure of the quality of models to describe a given data set. to each of the other models. AIC is a type of model selection that emerges from Information Theory, specifically an approximate minimization of the Kullback-Leibler (KL) information entropy which measures the distance between two probability distributions. Akaike [33] found this approximation to the KL quantity, which he called the Akaike Information Criterion (AIC), given by

$$\text{AIC} = -2\ln\mathcal{L}_{\text{max}} + 2p, \tag{3.2}$$

where $\mathcal{L}_{\text{max}}$ is the maximum likelihood and $p$ is the number of free parameters of the model. In our case, from eq. (3.1), we have $\mathcal{L} = \mathcal{N}\exp\left(-\frac{\chi^2}{2}\right)$, thus

$$\text{AIC} = \chi^2_{\text{min}} - 2\ln\mathcal{N} + 2p\,. \tag{3.3}$$

The normalization constant cancels out when we calculate the difference $\Delta$AIC between two different models:

$$\Delta\text{AIC} = \text{AIC}_j - \text{AIC}_i = \chi^2_{\text{min},j} - \chi^2_{\text{min},i} + 2(p_j - p_i)\,. \tag{3.4}$$

Table 2 shows how to interpret the outcomes of AIC. For two models, the ratio of likelihoods of one model against the other, with a correction given by the numbers of parameters, is quantified by the difference $\Delta$AIC. This approach is suitable for understanding the goodness of fit of one model against the other.



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| $\Delta$BIC | Support |
|---|---|
| $\Delta\text{BIC} \leq 1$ | No worth more than a bare mention |
| $1 \leq \Delta\text{BIC} \leq 3$ | Significant/Weak |
| $3 \leq \Delta\text{BIC} \leq 5$ | Strong to very strong/Significant |
| $\Delta\text{BIC} > 5$ | Decisive/Strong |

**Table 3**. Bayesian Information Criterion.

### 3.3 Bayesian Information Criterion

The Bayesian Evidence, in general, is given by multidimensional integrals over the parameters, so it is usually hard to evaluate. A way around this difficulty is by using its approximation, first obtained by Schwarz [34, 35], known as BIC. Differently from AIC, Bayesian Information Criterion (BIC) [31, 32, 36] heavily penalizes models with different number of free parameters. BIC incorporates Ockham's razor when it favours simple models against more complex models. BIC can be written as:

$$\text{BIC} = -2\ln\mathcal{L}_{\text{max}} + p\ln N, \tag{3.5}$$

where $N$ is the number of data, $\mathcal{L}_{\text{max}}$ is the maximum of likelihood and $p$ is the number of free parameters. Due to the term $p\ln N$, BIC drastically penalizes the excess of free parameters for big samples. In our case, BIC is given by

$$\text{BIC} = \chi^2_{\text{min}} - 2\ln\mathcal{N} + p\ln N \tag{3.6}$$

and the normalization constant $\mathcal{N}$ is cancelled out on $\Delta$BIC:

$$\Delta\text{BIC} = \text{BIC}_j - \text{BIC}_i = \chi^2_{\text{min},j} - \chi^2_{\text{min},i} + (p_j - p_i)\ln N\,. \tag{3.7}$$

The interpretation of $\Delta$BIC outcomes is described in table 3.

### 3.4 Bayesian Evidence

Bayesian Evidence (BE) emerges from Bayes' Theorem and it is a product of two probability distributions: likelihood and prior distribution. The posterior probability function is defined by [37]:

$$P(\theta_i, M_i|D) = \frac{P(D|\theta_i, M_i)P(\theta_i, M_i)}{P(D)}, \tag{3.8}$$

where $P(\theta_i, M_i)$ is a prior probability for the model $M_i$ with parameters $\theta_i$ and $D$ denoting the data. The $P(D)$ term is just a normalization term, defined by:

$$P(D) = \int P(D|\theta, M_i)P(\theta_i, M_i)d\theta. \tag{3.9}$$

$P(D)$ is calculated over all parameter space. One of the essential features of Bayesian framework is the marginalization over all parameters, also called the Bayesian Evidence (BE):

$$E(M_i) = \int P(D|\theta, M_i)P(\theta_i|M_i)d\theta. \tag{3.10}$$



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| $\ln B_{ij}$ | Support |
|---|---|
| $\ln B_{ij} \leq 1$ | Not worth more than a bare mention |
| $1 \leq \ln B_{ij} \leq 2.5$ | Significant/Weak |
| $2.5 \leq \ln B_{ij} \leq 5$ | Strong to very strong/Significant |
| $5 < \ln B_{ij}$ | Decisive/Strong |

**Table 4**. Bayesian Evidence.

BE conveys the principle of Ockham's razor and allows one to compare different models, through the Bayes factor [37–39]:

$$B_{ij} = \frac{E(M_j)}{E(M_i)}. \tag{3.11}$$

Note that in this definition we follow the convention of ref. [38] in such a way that if $E(M_j) > E(M_i)$ then $\ln B_{ij}$ is positive. The interpretation of BE through the Bayes factor is the so called Jeffreys scale [40], which is shown on table 4 as modified by [29, 41].

As discussed on [38], the Bayesian evidence can be written as

$$E = \int \mathcal{L}(p)\pi(p)dp, \tag{3.12}$$

where $\pi(p)$ is the prior probability distribution for the parameters. Assuming flat priors, we may write:

$$E = \frac{1}{V_\pi} \int_{V_\pi} \mathcal{L}(p)dp, \tag{3.13}$$

where $V_\pi$ is the volume in the parameter space spanned by the prior. As one may see, the Bayesian evidence can be quite dependent over the prior choice, even if it is flat. However, as discussed on [39], this dependence is weaker if one chooses large prior intervals. In fact, if the prior volume is large enough to encompass all the region that the likelihood is non-negligible, one can ensure that the logarithmic Bayesian evidence grows linearly with the logarithmic prior volume, as expected. Based on this, we choose conservative priors for the model parameters, ensuring that the $3\sigma$ likelihood constraints are quite inside of the prior volume region. In some cases, we had to limit the priors with physical considerations. For instance, to avoid big bangless models, we must impose $\alpha \in [0, 1]$. However, these limits were always inside the $3\sigma$ likelihood constraints, as one may see on figure 1.

## 4 Observational constraints

In this section, we used the 580 Supernovae Type Ia (SNe Ia) data set of Union 2.1 [3] in order to obtain constraints over the free parameters of the models listed on table 1.

### 4.1 Supernovae type Ia constraints

The parameter-dependent distance modulus for a supernova at the redshift $z$ can be computed through the expression

$$\mu(z|\mathbf{s}) = m - M = 5 \log d_L + 25, \tag{4.1}$$



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where $m$ and $M$ are respectively the apparent and absolute magnitudes, $\mathbf{s} \equiv (H_0, \alpha, \beta, n)$ is the set of free parameters of the model and $d_L$ is the luminosity distance in units of Megaparsecs.

Since in the general case, $H(z)$ has no analytical expression, we must define $d_L$ through a differential equation. The luminosity distance $d_L$ can be written in terms of a dimensionless comoving distance $D$ by:

$$d_L = (1+z)\frac{c}{H_0}D\,. \tag{4.2}$$

The comoving distance can be related to $H(z)$, neglecting spatial curvature, by the following relation [42]:

$$D'(z) = \frac{1}{\mathcal{H}(z)}, \tag{4.3}$$

where the prime denotes derivation with respect to redshift $z$. This equation, together with eq. (2.19) can be seen as a system of differential equations over the variables $(\mathcal{H}(z), D(z))$. The initial conditions are, naturally, $(\mathcal{H}(z=0)=1, D(z=0)=0)$.

This system may now be solved numerically. In order to constrain the free parameters of the models, we considered the Union 2.1 SNe Ia data set from Suzuki et al. [3]. The best fit set of parameters $\mathbf{s}$ was estimated from a $\chi^2$ statistics with

$$\chi^2_{\rm SN} = \sum_{i=1}^{N} \frac{\left[\mu^i(z|\mathbf{s}) - \mu^i_o(z)\right]^2}{\sigma_i^2}, \tag{4.4}$$

where $\mu^i(z|\mathbf{s})$ is given by (4.1), $\mu^i_o(z)$ is the corrected distance modulus for a given SN Ia at $z_i$ being $\sigma_i$ its corresponding individual uncertainty and $N = 580$ for the Union 2.1 data compilation.

As usual on SNe Ia analyses, we rewrite the distance modulus:

$$\mu = 5\log(D_L) + M_*, \tag{4.5}$$

where $D_L = (1+z)D$ is dimensionless luminosity distance and $M_* \equiv 25 + 5\log\frac{c}{H_0}$ comprises all the dependence over $H_0$. Then, we marginalize the likelihood over $M_*$:

$$\tilde{\mathcal{L}}(\alpha,\beta,n) = \mathcal{N}\int_{-\infty}^{+\infty} \exp\left[-\frac{1}{2}\chi^2(M_*,\alpha,\beta,n)\right] dM_*, \tag{4.6}$$

where $\mathcal{N}$ is a normalization constant. The corresponding $\tilde{\chi}^2 = -2\ln\left(\frac{\tilde{\mathcal{L}}}{\mathcal{N}}\right)$ is given by:

$$\tilde{\chi}^2 = C - \frac{B^2}{A}, \tag{4.7}$$

where

$$A = \sum_{i=1}^{N}\frac{1}{\sigma_i^2}, \qquad B = \sum_{i=1}^{N}\frac{5\log[D_L(z_i)] - \mu_{o,i}}{\sigma_i^2}, \qquad C = \sum_{i=1}^{N}\left\{\frac{5\log[D_L(z_i)] - \mu_{o,i}}{\sigma_i}\right\}^2. \tag{4.8}$$

The result of this analysis can be seen on figure 1.

As one may see on figure 1, the models are well constrained by SNe Ia Union 2.1 data. For panels 1a–1d we may see the likelihood $\mathcal{L}$ for parameters $\alpha$ and $\beta$ of models $M_0$ to $M_3$, respectively. In panel 1e, we see the likelihood contours for model $M_4$, with free parameters $\alpha$ and $n$, corresponding to 68.3%, 95.4% and 99.7% c.l. The same corresponding contours we may see on panel 1f for model $M_5$, with free parameters $\alpha$ and $\beta$. The detailed results for each model can be seen on table 5.

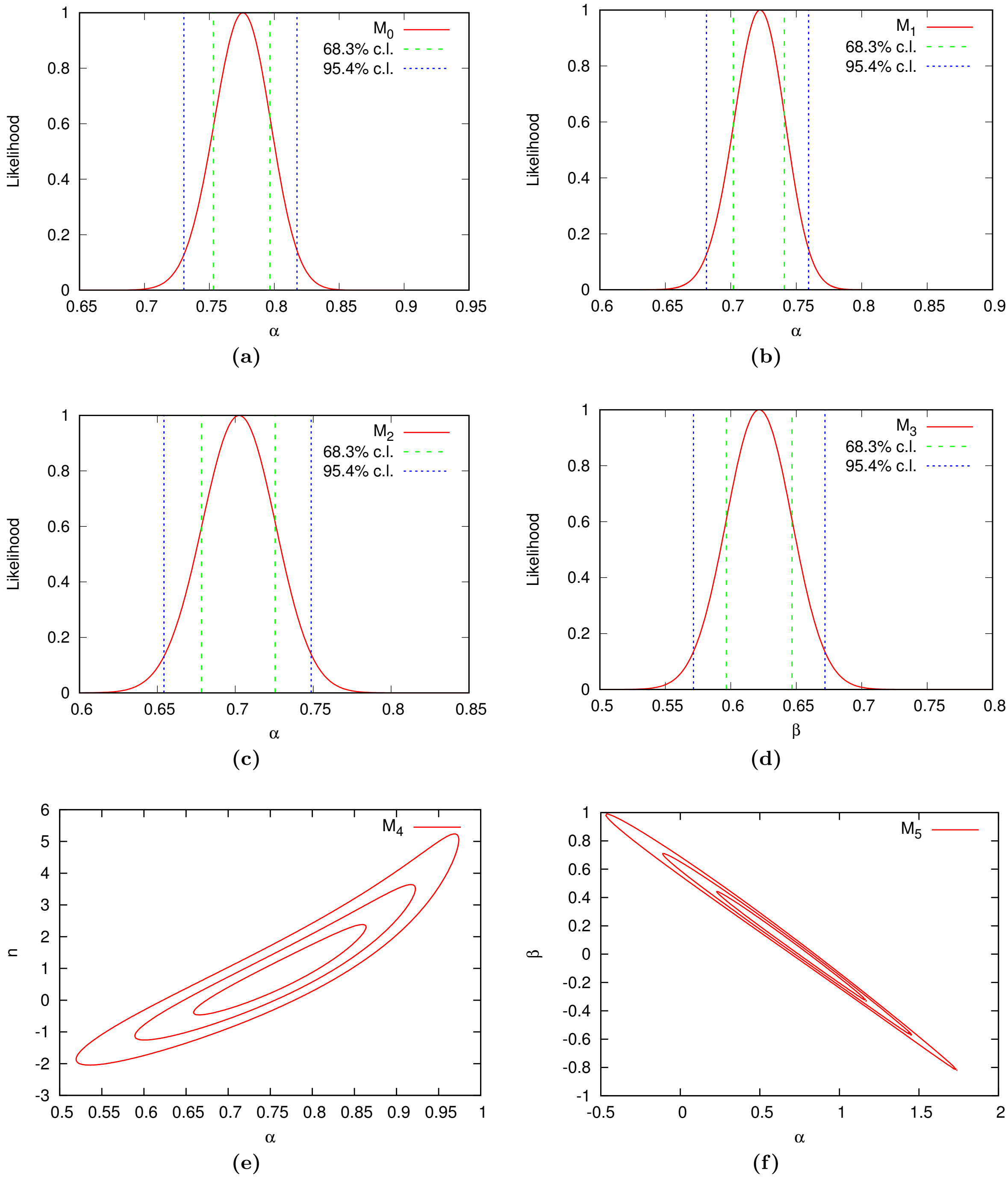


**Figure 1**. The results of our statistical analysis, with constraints from SNe Union 2.1 data. *Panels (a)–(d)*: likelihoods for the parameters on each indicated model, $M_0$–$M_3$, including 68.3% and 95.4% confidence levels. *Panels (e)–(f)*: contours for 68.3%, 95.4% and 99.7% confidence intervals for each indicated model, $M_4$ and $M_5$. Explanation of each model is on text and table 1.

| Model | $\alpha$ | $\beta$ | $n$ | $\chi^2_\nu$ |
|---|---|---|---|---|
| $M_0 : \Gamma = \frac{3\alpha H_0^2}{H}$ | $0.776^{+0.021}_{-0.023}$ | 0 | 1 | 0.97107 |
| $M_1 : \Gamma = 3\alpha \frac{\rho_{c0}}{\rho_{\rm dm}} H$ | $0.722^{+0.019}_{-0.020}$ | — | — | 0.97103 |
| $M_2 : \Gamma = 3\alpha H_0$ | $0.702 \pm 0.024$ | 0 | 0 | 0.97259 |
| $M_3 : \Gamma = 3\beta H$ | 0 | $0.622 \pm 0.025$ | — | 0.97916 |
| $M_4 : \Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$ | $0.766^{+0.098}_{-0.11}$ | 0 | $0.8^{+1.5}_{-1.3}$ | 0.97270 |
| $M_5 : \Gamma = 3\alpha \frac{H_0^2}{H} + 3\beta H$ | $0.74^{+0.28}_{-0.34}$ | $0.03^{+0.27}_{-0.23}$ | 1 | 0.97270 |

**Table 5**. Results of the analysis for the different models. Limits on the parameters correspond to 68.3% c.l. as explained on text.

| Model | $\chi^2_{\rm min}$ | $\chi^2_\nu$ | $\nu$ | $\Delta$AIC | $\Delta$BIC | $V_P$ | $\ln B_{i0}$ |
|---|---|---|---|---|---|---|---|
| $M_0 : \Gamma = \frac{3\alpha H_0^2}{H}$ | 562.251 | 0.97107 | 1 | 0 | 0 | 1 | 0 |
| $M_1 : \Gamma = 3\alpha \frac{\rho_{c0}}{\rho_{\rm dm}} H$ | 562.227 | 0.97103 | 1 | $-0.024$ | $-0.024$ | 1 | 0.043 |
| $M_2 : \Gamma = 3\alpha H_0$ | 563.131 | 0.97259 | 1 | 0.880 | 0.880 | 1 | 0.155 |
| $M_3 : \Gamma = 3\beta H$ | 566.936 | 0.97916 | 1 | 4.685 | 4.685 | 1 | 0.955 |
| $M_4 : \Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$ | 562.220 | 0.97270 | 2 | 1.969 | 6.332 | 20 | 0.921 |
| $M_5 : \Gamma = 3\alpha \frac{H_0^2}{H} + 3\beta H$ | 562.213 | 0.97269 | 2 | 1.962 | 6.325 | 20 | 1.463 |

**Table 6**. Results of the model selection analysis for the different models.

### 4.2 Model selection of matter creation models

Next, we have calculated AIC, BIC and Bayesian Evidence for all models studied here. AIC and BIC are relatively easy to compute, as they are directly obtained from $\chi^2_{\rm min}$ (3.3), (3.6). The results are shown on table 6.

The $\Delta$AIC values for the six models studied here are shown on fifth column of table 6. In the second column, we have the values of reduced chi-square, $\chi^2_\nu = \chi^2_{\rm min}/\nu$. The values of $\chi^2_\nu$ vary little for all the models studied here, favouring slightly the model $M_1$, the so called LJO model, which gives the same background dynamics as $\Lambda$CDM. $\Delta$AIC goes in the same direction and indicates an slight preference for LJO. The values of $\Delta$AIC in this column are relative to $M_0$, the CCDM model where $\Gamma \propto \frac{1}{H}$. Let us call it JO, for short. The models that are excluded by the Akaike criterion are $M_3$, $M_4$ and $M_5$, but mainly $M_3$, due to its high $\chi^2$ value. Because AIC penalizes too weakly the number of free parameters, it favours $M_4$ and $M_5$ over $M_3$ because they provide a lower $\chi^2_{\rm min}$, although $M_4$ and $M_5$ have more free parameters than $M_3$. A Bayesian criterion, one which drastically penalizes the excess of free parameters is thus necessary.

The values of $\Delta$BIC for CCDM are shown on sixth column of table 6. As one may see, BIC excludes model $M_3$ due to bad fitting ($\Delta\chi^2 = 4.685$ relatively to $M_0$) and it excludes $M_4$ and $M_5$ due to excess of parameters. In fact, although the $\chi^2_{\rm min}$ is slightly lower for

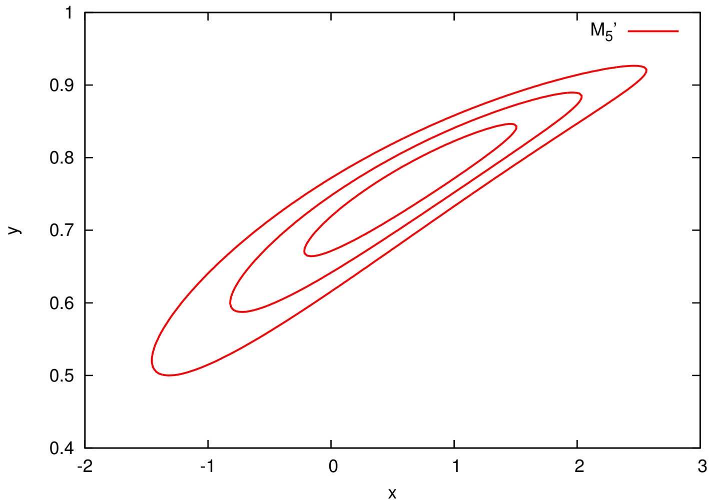


**Figure 2**. The results of our statistical analysis for model $M_5'$, with constraints from SNe Union 2.1 data.

these models, this small advantage is quite penalized due to the term $\ln 580 = 6.363$ in BIC equation (3.7). We may say that the big effort of adding one free parameter in $M_4$ or $M_5$ can not be justified by the small improvement of fitting obtained.

However, as we know, BIC is only an approximation of the Bayesian evidence. Bayesian evidence is the most reliable tool to perform model selection, from the Bayesian point of view. We have then calculated Bayesian evidences.

In order to calculate Bayesian evidences, we have used Romberg's integration method, written in FORTRAN, as explained in [43]. This method is interesting because it runs fast and provides full control of fractional error. As multiple integrals involved in computing BE in models $M_4$ and $M_5$ are only bidimensional, we choose to keep a deterministic integration method, by replicating the Romberg integration, as explained in [43]. In order to control convergence, we have integrated the posteriors with fractional errors $10^{-6}$ and $10^{-7}$. No significant deviation was found, indicating convergence was achieved.

We have compared the CCDM models by using the Bayes factor (3.11). As mentioned before, we use a convention where $\ln B_{ij}$ is positive in case that $E_j > E_i$. The results of this analysis for the models studied here is in the eighth column of table 6, *where we show the values of $B_{i0}$, the Bayes factors relatively to model $M_0$ (JO).*

As one may see on table 6, while models $M_3$ and $M_4$ were barely acceptable in comparison with model $M_0$, the only model that can surely be discarded by this analysis is model $M_5$.

As shown on figure 1f, in the case of model $M_5$, there is a strong degeneracy between parameters $\alpha$ and $\beta$, which prevents their independent determination. In order to alleviate the degeneracy, we reparameterize this model with $x = \alpha - \beta$, $y = \alpha + \beta$. Let us call this new parameterization model $M_5'$. The resulting constraints from SNe Ia can be seen on figure 2.

As can be seen on figure 2, the degeneracy is alleviated in the plane $x - y$. In fact, the correlation coefficient decreased from $r_{\alpha\beta} = -0.9971$ to $r_{xy} = 0.9392$. The best fit parameters were $x = 0.66^{+0.85}_{-0.88}$ and $y = 0.664^{+0.082}_{-0.10}$. The reduced $\chi^2_\nu = 562.213/578 = 0.97269$.

As we have just reparameterized the model $M_5$, we may calculate $E(M_5')$ from $E(M_5)$ by changing variables in the integral (3.13). As the Jacobian determinant $|J| = \left|\frac{\partial(x,y)}{\partial(\alpha,\beta)}\right| = 2$,

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the new Bayesian evidence will be given by $E(M_5') = \frac{2E(M_5)V_\pi}{V_\pi'}$. Choosing $V_\pi' = 15$, given that $V_\pi = 20$, the new Bayesian evidence is $E(M_5') = 2.67E(M_5) = 0.09891$, that is, the Bayesian evidence is increased in this case. It yields to a Bayes factor $\ln B_{05'} = 0.912$, which makes $M_5'$ barely acceptable and more competitive than $M_3$, $M_4$ and $M_5$. One must be aware, however, that this parameterization is not known *a priori*, nor it has any physical motivation. It just shows that $M_5$ can not be surely discarded if one allows for a reparameterization in order to break parameter degeneracy.

An obvious extension of the models studied here would be a model with creation rate $\Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n + 3\beta H$. However, in our preliminary study we have found that SNe Ia from Union 2.1 alone are not enough to constrain this model. In fact, even values of $n$ up to $n \sim 100$ are allowed by this analysis. Thus, we choose not to include this analysis, as more data would be needed to constrain this model, which is beyond the scope of the present work.

In order to compare our results with the current literature, the ref. [30] have obtained $\Delta$AIC for three models analysed here, $M_1$, $M_2$ and $M_5$ (CCDM1, CCDM2 and CCDM3 in their analysis, respectively). They have calculated $\Delta$AIC relatively to $M_1$. Comparing with their analysis, our result for $M_5$ is quite similar. However, we did not find such a large $\Delta$AIC $= 33.21$ as they have found. Our $\Delta$AIC for $M_5$, relatively to $M_1$, is $\Delta$AIC $= 1.993$.

## 5 Conclusion

We have compared 6 spatially flat CCDM models, *including one that is degenerate with* the $\Lambda$CDM model. The JO model is slightly favoured over $\Lambda$CDM in the Bayesian evidence, however, $\Lambda$CDM and $\Gamma = 3\alpha H_0$ can not be discarded from this analysis. Models $M_3$ and $M_4$ can be moderately weak and $M_5$ can certainly be discarded, unless a reparameterization is made in order to break the parameter degeneracy. At this point, it is important to mention that JO model is equivalent to the late phase of the model from ref. [23].

Further investigations of CCDM models may include spatial curvature, other background data and the evolution of density perturbations.

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