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The T^2 chart with mixed samples to control bivariate autocorrelated processes

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In this paper, we propose the use of the T^2 chart with the mixed sampling strategy (MS) to monitor the mean vector of bivariate processes with observations that fit to a first-order vector autoregressive model. With the MS, rational subgroups of size n are taken from the process and the selected units are regrouped to form the mixed samples. The units of the mixed samples are units selected from the last two rational subgroups. The aim of the proposed sampling strategy is to reduce the negative effect of the autocorrelation on the performance of the T^2 chart. When the two variables are autocorrelated, the MS always enhances the T^2 chart performance, however, the mixed samples are not recommended for bivariate processes with only one autocorrelated variable which is rarely affected by the assignable cause.

Keywords: T^2 control chart; autocorrelation; mixed samples; sampling strategies

1. Introduction

The observations from multivariate processes are in general cross-correlated and, depending on the production rate, they are also autocorrelated (Pan and Jarrett 2007, 2011). The monitoring of multivariate process with autocorrelated variables is a growing area of research (Kalgonda and Kulkarni 2004; Niaki and Davoodi 2009; Hwang and Wang 2010; Kim, Jitpitaklert, and Sukchotrat 2010; Huang, Bisgaard, and Xu 2013; Huang, Xu, and Bisgaard 2013; Leoni et al. 2015; Leoni, Costa, and Machado 2015).

Previous studies with single variables have suggested the adoption of innovative sampling strategies to counteract the negative effect of the autocorrelation on the performance of the \bar{X} chart. Costa and Castagliola (2011) considered a sampling strategy where the samples are formed by collecting one item from the production line and then skipping one, two or more items before selecting the next one. They showed that the undesired effect of the autocorrelation might be reduced by building up the samples with the non-neighbouring items according to the time they were produced. The effect of the autocorrelation is minimised by just skipping two items, in the case of moderate autocorrelation, or three, in the case of high autocorrelation. Alternatively, Franco et al. (2013) introduced the mixed sampling strategy (MS) where the samples are composed with units selected from the last two rational subgroups. A numerical analysis shows that the mixed sampling outperforms the skip sampling strategy for high levels of autocorrelation. Recently, Franco et al. (2014) investigated the economic-statistical design of the \bar{X} chart with the skip sampling strategy. In the presence of the autocorrelation, the sampling strategy based on the rational subgroup concept never outperforms the skip sampling strategies. The choice between the skip sampling strategy and the MS is guided by the autocorrelation level. When the autocorrelation is low, the skip sampling strategy is more economically convenient than the MS. Conversely, when the autocorrelation is high, the MS is the best option. In all these studies, the observations of the quality characteristic X are described by a first-order autoregressive model AR (1) and the process mean was assumed to switch between two values, the target one and the off-target value resulting of the assignable cause occurrence.

In other studies with single variables, the AR (1) model has been used to describe a natural movement of the process mean (Reynolds, Arnold, and Baik 1996; Lu and Reynolds 1999, 2001; Lin and Chou 2008; Zou, Wang, and Tsung 2008; Lin 2009; Costa and Machado 2011; Franco, Costa, and Machado 2012). When the process mean wanders neither the skip nor the mixed sampling strategies reduce the effect of the autocorrelation on the \bar{X} chart's performance.

In this paper, we consider the use of the T^2 chart to control the mean vector of bivariate processes with observations that are modelled by a first-order autoregressive model – VAR (1). To reduce the negative effect of the autocorrelation on the performance of the T^2 chart, we propose the use of the MS. The next Section brings the VAR (1) model and the

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cross-covariance matrix of the sample mean vector when the sample items are collected according to the rational subgroup concept. In Section 3, we obtain the cross-covariance matrix of the mixed sample mean vector. Section 4 brings the statistical properties of the T^2 control chart and, in Section 5, we investigate the effect of the autocorrelation and the cross-correlation on the performance of the T^2 chart. Section 6 brings an example, and in Section 7 are the concluding remarks.

2. The VAR (1) model and the cross-covariance matrix

The multivariate autoregressive model for cross-correlated and serially correlated data has been adopted in studies dealing with control charts (Mastrangelo and Forrest 2002; Biller and Nelson 2003; Kalgonda and Kulkarni 2004; Arkat, Niaki, and Abbasi 2007; Jarrett and Pan 2007; Issam and Mohamed 2008; Niaki and Davoodi 2009; Kim, Jitpitaklert, and Sukchotrat 2010; Hwang and Wang 2010):

$$\mathbf{X}_t - \boldsymbol{\mu} = \boldsymbol{\Phi}(\mathbf{X}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\mathbf{X}_t \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Gamma})$ is the $(p \times 1)$ vector of observations at time t (p is the number of variables), $\boldsymbol{\mu}$ is the mean vector, $\boldsymbol{\varepsilon}_t$ is an independent multivariate normal random vector with a mean vector of zeros and covariance matrix $\boldsymbol{\Sigma}_e$ and $\boldsymbol{\Phi}$ is a $(p \times p)$ matrix of autocorrelation parameters. According to Kalgonda and Kulkarni (2004), the cross-covariance matrix of \mathbf{X}_t has the following property: $\boldsymbol{\Gamma} = \boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\Phi}' + \boldsymbol{\Sigma}_e$. After some algebra, we obtain:

$$\text{Vec } \boldsymbol{\Gamma} = (\mathbf{I}_{p^2} - \boldsymbol{\Phi} \otimes \boldsymbol{\Phi})^{-1} \text{Vec } \boldsymbol{\Sigma}_e \tag{2}$$

where \otimes is the Kronecker product and Vec is the operator that transforms a matrix into a vector by stacking its columns.

To study the effects of the autocorrelation and cross-correlation on the performance of the T^2 chart, we considered the bivariate case ($p = 2$) with:

$$\boldsymbol{\Phi} = \begin{pmatrix} a & c \\ d & b \end{pmatrix} \tag{3}$$

$$\boldsymbol{\Sigma}_e = \begin{pmatrix} \sigma_{e_X}^2 & \sigma_{e_{XY}} \\ \sigma_{e_{XY}} & \sigma_{e_Y}^2 \end{pmatrix} = \begin{pmatrix} \sigma_{e_X}^2 & \rho\sigma_{e_X}\sigma_{e_Y} \\ \rho\sigma_{e_X}\sigma_{e_Y} & \sigma_{e_Y}^2 \end{pmatrix} \tag{4}$$

where ρ is the correlation of X and Y.

when $\boldsymbol{\Phi} = \text{diag}(a, b)$, follows from (2) and (4):

$$\boldsymbol{\Gamma} = \begin{pmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{pmatrix} = \begin{pmatrix} (1 - a^2)^{-1}\sigma_{e_X}^2 & (1 - ab)^{-1}\sigma_{e_{XY}} \\ (1 - ab)^{-1}\sigma_{e_{XY}} & (1 - b^2)^{-1}\sigma_{e_Y}^2 \end{pmatrix} \tag{5}$$

Leoni, Machado, and Costa (2014) obtained the cross-covariance matrix $\boldsymbol{\Gamma}_{\bar{X}}$ of the sample mean vector \bar{X} when the sample items are collected according to the rational subgroup concept:

$$\boldsymbol{\Gamma}_{\bar{X}} = \begin{pmatrix} \zeta_X^2 & \text{cov}(\zeta_{XY}) \\ \zeta_{XY} & \zeta_Y^2 \end{pmatrix} = \begin{pmatrix} \frac{\sigma_X^2}{n} \left[1 + \frac{2}{n} \sum_{j=1}^{n-1} (n-j)a^j \right] & \frac{\sigma_{XY}}{n} \left[1 + \frac{1}{n} \sum_{j=1}^{n-1} (n-j)a^j + \frac{1}{n} \sum_{j=1}^{n-1} (n-j)b^j \right] \\ \frac{\sigma_{XY}}{n} \left[1 + \frac{1}{n} \sum_{j=1}^{n-1} (n-j)a^j + \frac{1}{n} \sum_{j=1}^{n-1} (n-j)b^j \right] & \frac{\sigma_Y^2}{n} \left[1 + \frac{2}{n} \sum_{j=1}^{n-1} (n-j)b^j \right] \end{pmatrix} \tag{6}$$

where n is the size of the samples.

The general cross-covariance matrix $\boldsymbol{\Gamma}_{\bar{X}}$ is a function of $\boldsymbol{\Phi}$, n and $\boldsymbol{\Sigma}_e$.

$$\boldsymbol{\Gamma}_{\bar{X}} = \begin{pmatrix} \zeta_X^2 & \zeta_{XY} \\ \zeta_{XY} & \zeta_Y^2 \end{pmatrix} = \frac{1}{n^2} \left\{ \left(\sum_{i=0}^{n-1} \boldsymbol{\Phi}^i \right) \boldsymbol{\Gamma} \left(\sum_{i=0}^{n-1} \boldsymbol{\Phi}^i \right)' + \sum_{k=1}^{n-1} \left[\left(\sum_{i=1}^k \boldsymbol{\Phi}^{i-1} \right) \boldsymbol{\Sigma}_e \left(\sum_{i=1}^k \boldsymbol{\Phi}^{i-1} \right)' \right] \right\} \tag{7}$$

It is well known that the autocorrelation has a negative impact on the performance of the \bar{X} chart; Leoni, Machado, and Costa (2014) proved that the autocorrelations also reduce the ability of the T^2 chart to signal.

3. The mixed samples and the corresponding cross-covariance matrix

When the MS is in use, the samples are composed with units selected from the last two rational subgroups. To illustrate, Figure 1 shows two rational subgroups ($n = 5$) and the mixed sample. The mixed sample i is composed with units of the current rational subgroup i (the first, third and the fifth ones according to the instant they were selected from the production line) and with units of the previous rational subgroup $i - 1$ (the second and fourth ones). The remaining units of the current rational subgroup, that is, the second and fourth units, are saved to compose the next mixed sample $i + 1$ and so on.

The mean vector of the mixed sample i is given by:

$$\bar{\mathbf{M}}_i = \frac{n_e}{n} \cdot \underbrace{\frac{1}{n_e} \sum_{t=1}^{n_e} \mathbf{X}_{i-1,2t}}_{\bar{\mathbf{Y}}_{i-1}} + \frac{n_o}{n} \cdot \underbrace{\frac{1}{n_o} \sum_{t=1}^{n_o} \mathbf{X}_{i,2t-1}}_{\bar{\mathbf{Z}}_i} \tag{8}$$

where $\mathbf{X}_{k,l}$ is the l -esime observation vector of the k -esime rational subgroup; $\bar{\mathbf{Y}}_{i,1}$ is the mean vector obtained with the even units of the $(i - 1)$ -esime rational subgroup, and $\bar{\mathbf{Z}}_i$ is the mean vectors obtained with the odd units of the i -esime rational subgroup. When n is even, $n_e = n_o = n/2$, and when n is odd, $n_e = (n - 1)/2$ and $n_o = (n + 1)/2$.

As the samples are taken from the process according to the rational subgroup concept, $\bar{\mathbf{Y}}_{i,1}$ and $\bar{\mathbf{Z}}_i$ are independent, consequently, the cross-covariance matrix $\Gamma_{\bar{\mathbf{M}}}$ of the mixed sample mean vector $\bar{\mathbf{M}}$ reduces to:

$$\Gamma_{\bar{\mathbf{M}}} = \begin{pmatrix} \zeta_X^2 & \zeta_{XY} \\ \zeta_{XY} & \zeta_Y^2 \end{pmatrix} = \left(\frac{n_e}{n}\right)^2 \cdot \Gamma_{\bar{\mathbf{Y}}} + \left(\frac{n_o}{n}\right)^2 \cdot \Gamma_{\bar{\mathbf{Z}}} \tag{9}$$

where $\Gamma_{\bar{\mathbf{Y}}}$ and $\Gamma_{\bar{\mathbf{Z}}}$ are, respectively, the cross-covariance matrix of the mean vector of the samples with the even and with the odd units of the rational subgroups. According to the VAR (1) model:

$$\Gamma_{\bar{\mathbf{Z}}} = \frac{1}{n_o^2} \left\{ \left(\sum_{l=0}^{n_o-1} \Phi^{2l} \right) \Gamma \left(\sum_{l=0}^{n_o-1} \Phi^{2l} \right)' + \sum_{k=1}^{n_o-1} \left\{ \sum_{j=1}^2 \left[\left(\sum_{l=1}^k \Phi^{2l-j} \right) \Sigma_e \left(\sum_{l=1}^k \Phi^{2l-j} \right)' \right] \right\} \right\} \tag{10}$$

and

$$\Gamma_{\bar{\mathbf{Y}}} = \frac{1}{n_e^2} \left\{ \left(\sum_{l=0}^{n_e-1} \Phi^{2l+1} \right) \Gamma \left(\sum_{l=0}^{n_e-1} \Phi^{2l+1} \right)' + \left(\sum_{l=0}^{n_e-1} \Phi^{2l} \right) \Sigma_e \left(\sum_{l=0}^{n_e-1} \Phi^{2l} \right)' + \sum_{k=1}^{n_e-1} \left\{ \sum_{j=1}^2 \left[\left(\sum_{l=1}^k \Phi^{2l-j} \right) \Sigma_e \left(\sum_{l=1}^k \Phi^{2l-j} \right)' \right] \right\} \right\} \tag{11}$$

To illustrate the computation of the cross-covariance matrix $\Gamma_{\bar{\mathbf{M}}}$, let be $n = 5$, $\Phi = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.5 \end{pmatrix}$ and $\Sigma_e = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. According to expressions (8), (10) and (11):

$$\bar{\mathbf{M}}_i = \frac{2}{5} \times \underbrace{\frac{1}{2} \sum_{t=1}^2 \mathbf{X}_{i-1,2t}}_{\bar{\mathbf{Y}}_{i-1}} + \frac{3}{5} \times \underbrace{\frac{1}{3} \sum_{t=1}^3 \mathbf{X}_{i,2t-1}}_{\bar{\mathbf{Z}}_i}$$

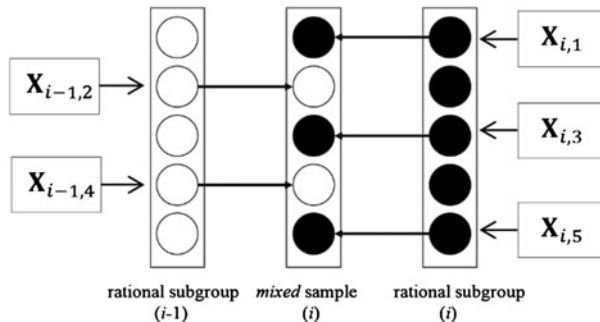


Figure 1. The mixed sample.

$$\Gamma_{\bar{Y}} = \frac{1}{4} \left[(\Phi + \Phi^3) \Gamma (\Phi + \Phi^3)' + (I + \Phi^2) \Sigma_e (I + \Phi^2)' + \Phi \Sigma_e \Phi' + \Sigma_e \right]$$

$$\Gamma_{\bar{Y}} = \begin{pmatrix} 0.5989 & 0.3441 \\ 0.3441 & 0.8333 \end{pmatrix}$$

and

$$\Gamma_{\bar{Z}} = \frac{1}{9} \left[(I + \Phi^2 + \Phi^4) \Gamma (I + \Phi^2 + \Phi^4)' + (\Phi + \Phi^3) \Sigma_e (\Phi + \Phi^3)' + (I + \Phi^2) \Sigma_e (I + \Phi^2)' + \Phi \Sigma_e \Phi' + \Sigma_e \right]$$

$$\Gamma_{\bar{Z}} = \begin{pmatrix} 0.4122 & 0.2451 \\ 0.2451 & 0.6111 \end{pmatrix}$$

Finally, with expression (9):

$$\Gamma_{\bar{M}} = \left(\frac{2}{5}\right)^2 \Gamma_{\bar{Y}} + \left(\frac{3}{5}\right)^2 \Gamma_{\bar{Z}}$$

$$\Gamma_{\bar{M}} = \begin{pmatrix} 0.2442 & 0.1433 \\ 0.1433 & 0.3533 \end{pmatrix}$$

The inverse of the cross-covariance matrix $\Gamma_{\bar{M}}$ is used to compute the T^2 values.

4. The statistical properties of the T^2 chart

The Hotelling T^2 chart is used to monitor the mean vector of multivariate processes (Hotelling 1947). With the MS, the T^2 statistic for bivariate processes, with the in-control mean vector $\mu_0 = (\mu_{0X}, \mu_{0Y})'$, is given by:

$$T^2 = (\bar{M} - \mu_0)' \Gamma_{\bar{M}}^{-1} (\bar{M} - \mu_0) \tag{12}$$

When the matrices Φ and Σ_e are known, the T^2 statistic follows a chi-squared distribution with $p = 2$ degrees of freedom (χ_p^2). After the assignable cause occurrence, the mean vector changes to $\mu_1 = (\mu_{1X}, \mu_{1Y})'$ and the distribution of the T^2 statistic changes to a non-central chi-squared distribution ($\chi_{(p,\lambda)}^2$) with non-centrality parameter $\lambda_{(p,\lambda)}^2$, where δ is the standardised mean vector shift $\delta = (\delta_X, \delta_Y)' = \left(\frac{\mu_{1X} - \mu_{0X}}{\sigma_{eX}}, \frac{\mu_{1Y} - \mu_{0Y}}{\sigma_{eY}}\right)'$, see Wu and Makis (2008) and Franco, Costa, and Machado (2012). The type I error of the T^2 chart is given by:

$$\alpha = 1 - \Pr(\chi_p^2 < \text{UCL}) \tag{13}$$

where UCL is the upper control limit of the chart. The UCL of the chart is chosen to be the $(1 - \alpha)$ th quantile of the chi-squared distribution to achieve a desired in-control average run length (ARL) of $1/\alpha$ (Champ, Jones-Farmer, and Rigdon 2005).

As the assignable cause always occurs between rational subgroups, i.e. $i - 1$ and i , it follows that $\bar{Y}_{i-1} \sim N_2[\mu_0; \Gamma_{\bar{Y}}]$ and $\bar{Z}_i \sim N_2[\mu_1; \Gamma_{\bar{Z}}]$, consequently, the type II error of the mixed sample i is:

$$\beta_1 = \Pr(\chi_{(p,\lambda_1)}^2 < \text{UCL}) \tag{14}$$

where $\lambda_1 = \delta' \Gamma_{\bar{M}}^{-1} \delta$ with $\delta = \frac{n_2}{n} (\delta_X, \delta_Y)'$. The type II error of the subsequent mixed samples $i + 1, i + 2 \dots$ is:

$$\beta_2 = \Pr(\chi_{(p,\lambda_2)}^2 < \text{UCL}) \tag{15}$$

where $\lambda_2 = \delta' \Gamma_{\bar{M}}^{-1} \delta$ with $\delta = (\delta_X, \delta_Y)'$.

According to Franco et al. (2013), the out of control ARL and the non-central moment of order 2 of the run length (SDRL) is given by:

Table 1. The ARL values for the T^2 control chart; $n = 3$.

			<i>a</i>		0.3		0.5		0.7		0.9		0.3		0	
			<i>b</i>		0.3		0.5		0.7		0.9		0.9		0.5	
			<i>c</i>		0		0		0		0		0		0	
			<i>d</i>		0		0		0		0		0		0	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	370.40	370.40	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	127.1	96.6	171.0	124.3	231.4	177.9	315.57	281.64	318.46	286.05	172.9	126.8		
	0	1	26.7	17.1	47.2	26.1	91.6	51.6	211.3	151.57	217.42	158.26	48.3	27.0		
	0	1.5	7.4	5.1	14.7	7.8	35.6	16.9	126.93	74.68	133.21	79.82	15.1	8.0		
	0.5	0	127.1	96.6	171.0	124.3	231.4	177.9	315.57	281.64	133.34	102.45	84.2	85.3		
	0.5	0.5	94.7	68.6	135.7	92.4	198.1	142.5	297.29	255.51	134.21	101.35	79.2	68.9		
	0.5	1	28.7	18.5	50.3	28.1	96.2	54.9	216.4	156.97	114.55	78.7	38.7	24.8		
	0.5	1.5	8.6	5.8	17.1	8.9	40.3	19.4	136.45	82.26	86.01	51.93	15.4	8.6		
	1	0	26.7	17.1	47.2	26.1	91.6	51.6	211.3	151.57	29.13	18.82	13.3	14.1		
	1	0.5	28.7	18.5	50.3	28.1	96.2	54.9	216.4	156.97	30.51	19.69	14.9	15.2		
	1	1	16.0	10.3	30.1	15.9	64.3	33.5	176.53	117.18	29.49	18.49	12.1	10.3		
	1	1.5	6.9	4.8	13.7	7.3	33.5	15.9	122.55	71.28	26.4	15.76	7.7	5.7		
	1.5	0	7.4	5.1	14.7	7.8	35.6	16.9	126.93	74.68	8.2	5.58	3.5	4.3		
	1.5	0.5	8.6	5.8	17.1	8.9	40.3	19.4	136.45	82.26	8.59	5.81	4.0	4.7		
	1.5	1	6.9	4.8	13.7	7.3	33.5	15.9	122.55	71.28	8.63	5.78	3.8	4.3		
	1.5	1.5	4.3	3.3	8.5	4.8	22.0	10.2	94.54	50.9	8.32	5.49	3.2	3.4		
0.6	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	93.3	67.4	134.1	91.0	196.5	140.9	296.34	254.2	313.44	279.17	144.1	102.7		
	0	1	15.6	10.0	29.5	15.6	63.2	32.8	174.89	115.65	206.91	147.95	33.7	18.9		
	0	1.5	4.2	3.3	8.3	4.7	21.5	9.9	93.15	49.94	122.54	71.97	9.7	5.6		
	0.5	0	93.3	67.4	134.1	91.0	196.5	140.9	296.34	254.2	122.67	93.48	62.7	65.7		
	0.5	0.5	114.2	85.3	157.4	111.6	219.0	164.3	309.08	272.19	136.63	105.22	89.8	82.5		
	0.5	1	29.9	19.3	52.1	29.2	98.8	56.7	219.11	159.87	126.12	89.38	48.2	29.3		
	0.5	1.5	7.3	5.0	14.5	7.6	35.0	16.7	125.81	73.81	98.63	60.92	17.1	8.7		
	1	0	15.6	10.0	29.5	15.6	63.2	32.8	174.89	115.65	25.03	16.25	8.5	9.7		
	1	0.5	29.9	19.3	52.1	29.2	98.8	56.7	219.11	159.87	28.77	18.89	13.4	15.2		
	1	1	22.1	14.1	40.0	21.7	80.5	44.0	198.21	138.15	30.48	19.66	14.7	13.4		
	1	1.5	8.6	5.8	16.9	8.9	40.0	19.3	135.86	81.79	29.55	18.18	10.5	7.4		
	1.5	0	4.2	3.3	8.3	4.7	21.5	9.9	93.15	49.94	6.89	4.87	2.4	3.2		
	1.5	0.5	7.3	5.0	14.5	7.6	35.0	16.7	125.81	73.81	7.73	5.4	3.2	4.2		
	1.5	1	8.6	5.8	16.9	8.9	40.0	19.3	135.86	81.79	8.36	5.75	3.9	4.7		
	1.5	1.5	6.0	4.3	12.0	6.4	29.7	13.9	114.1	64.89	8.64	5.82	3.9	4.1		
0.9	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	20.4	13.0	37.3	20.1	76.3	41.2	192.9	132.88	302.53	264.82	74.1	53.0		
	0	1	2.3	2.2	4.3	2.9	11.4	5.5	60.74	29.43	185.85	128.49	10.9	7.3		
	0	1.5	1.1	1.4	1.4	1.6	3.1	2.2	20.36	8.81	102.7	58.25	2.9	2.6		
	0.5	0	20.4	13.0	37.3	20.1	76.3	41.2	192.9	132.88	102.82	77.4	23.5	29.8		
	0.5	0.5	131.4	100.5	175.6	128.7	235.4	182.4	317.6	284.64	130.82	102.75	83.3	91.3		
	0.5	1	12.0	7.8	23.2	12.1	52.0	26.1	157.5	99.98	134.33	98.44	52.8	27.4		
	0.5	1.5	1.9	2.0	3.4	2.5	9.0	4.5	51.42	24.14	110.33	69.44	11.8	5.8		
	1	0	2.3	2.2	4.3	2.9	11.4	5.5	60.74	29.43	18.39	12.2	2.6	4.0		
	1	0.5	12.0	7.8	23.2	12.1	52.0	26.1	157.5	99.98	23.6	15.95	6.1	9.7		
	1	1	28.4	18.3	49.8	27.8	95.5	54.3	215.58	156.1	28.13	18.91	13.0	15.7		
	1	1.5	5.7	4.1	11.4	6.2	28.6	13.3	111.29	62.81	30.43	19.61	13.6	8.9		
	1.5	0	1.1	1.4	1.4	1.6	3.1	2.2	20.36	8.81	4.92	3.81	1.1	1.9		
	1.5	0.5	1.9	2.0	3.4	2.5	9.0	4.5	51.42	24.14	5.93	4.46	1.5	2.6		
	1.5	1	5.7	4.1	11.4	6.2	28.6	13.3	111.29	62.81	6.96	5.11	2.3	3.9		
	1.5	1.5	8.0	5.4	15.8	8.3	37.7	18.1	131.3	78.13	7.87	5.61	3.5	4.7		

Table 2. The ARL values for the T^2 control chart; $n = 3$.

			0.3		0.1		0.7		0.2		0.3	
			<i>a</i>		<i>b</i>		<i>c</i>		<i>d</i>			
			0.3		0.1		0.7		0.2		0.3	
			0.1		0.3		0.2		0.7		0.3	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	121.8	95.6	76.34	88.99	216.3	169.5	49.12	114.91	105.8	96.9
	0	1	24.7	16.8	11.37	15.05	78.3	46.8	6.04	22.8	19.3	17.2
	0	1.5	6.8	5.0	3.05	4.55	28.6	15.0	1.8	6.74	5.2	5.1
	0.5	0	121.8	95.6	76.34	88.99	216.3	169.5	49.12	114.91	105.8	96.9
	0.5	0.5	109.4	74.9	109.43	74.86	284.8	235.5	284.8	235.45	151.2	100.4
	0.5	1	33.0	20.1	25.51	19.46	150.0	97.6	41.16	72.6	43.0	27.2
	0.5	1.5	9.4	6.1	5.72	5.79	56.7	30.3	5.53	16.79	10.4	7.5
	1	0	24.7	16.8	11.37	15.05	78.3	46.8	6.04	22.8	19.3	17.2
	1	0.5	33.0	20.1	25.51	19.46	150.0	97.6	41.16	72.6	43.0	27.2
	1	1	20.5	11.6	20.46	11.62	156.2	95.8	156.19	95.78	37.0	18.2
	1	1.5	8.6	5.3	7.54	5.24	85.4	45.0	30.09	36.77	14.4	7.7
	1.5	0	6.8	5.0	3.05	4.55	28.6	15.0	1.8	6.74	5.2	5.1
	1.5	0.5	9.4	6.1	5.72	5.79	56.7	30.3	5.53	16.79	10.4	7.5
	1.5	1	8.6	5.3	7.54	5.24	85.4	45.0	30.09	36.77	14.4	7.7
	1.5	1.5	5.5	3.7	5.51	3.67	78.0	38.2	77.97	38.21	10.9	5.4
0.6	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	85.0	65.1	44.42	58.81	164.8	119.1	23.13	69.53	65.8	62.6
	0	1	13.5	9.6	5.3	8.36	43.8	24.3	2.55	10.47	9.1	9.1
	0	1.5	3.6	3.2	1.65	2.87	13.4	7.2	1.13	3.38	2.5	3.0
	0.5	0	85.0	65.1	44.42	58.81	164.8	119.1	23.13	69.53	65.8	62.6
	0.5	0.5	130.1	92.3	130.05	92.34	298.2	253.9	298.21	253.94	173.2	120.3
	0.5	1	32.2	20.3	21.02	19.22	126.7	80.8	20.91	51.54	35.4	25.1
	0.5	1.5	7.2	5.1	3.63	4.71	35.8	18.7	2.45	8.92	6.4	5.6
	1	0	13.5	9.6	5.3	8.36	43.8	24.3	2.55	10.47	9.1	9.1
	1	0.5	32.2	20.3	21.02	19.22	126.7	80.8	20.91	51.54	35.4	25.1
	1	1	27.8	15.9	27.82	15.94	178.1	115.4	178.13	115.36	48.4	24.7
	1	1.5	10.3	6.3	8.01	6.15	82.1	44.5	17.32	31.65	15.0	8.8
	1.5	0	3.6	3.2	1.65	2.87	13.4	7.2	1.13	3.38	2.5	3.0
	1.5	0.5	7.2	5.1	3.63	4.71	35.8	18.7	2.45	8.92	6.4	5.6
	1.5	1	10.3	6.3	8.01	6.15	82.1	44.5	17.32	31.65	15.0	8.8
	1.5	1.5	7.8	4.8	7.77	4.79	95.9	49.8	95.91	49.76	15.2	7.3
0.9	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	16.6	12.0	5.97	10.31	46.3	26.1	2.57	10.95	10.2	10.7
	0	1	1.9	2.1	1.11	1.98	5.6	3.5	1	2.03	1.4	2.0
	0	1.5	1.0	1.3	1	1.25	1.7	1.8	1	1.28	1.0	1.3
	0.5	0	16.6	12.0	5.97	10.31	46.3	26.1	2.57	10.95	10.2	10.7
	0.5	0.5	147.8	108.2	147.84	108.19	308.0	268.0	307.98	267.99	191.2	137.7
	0.5	1	10.9	7.6	4.68	6.82	41.1	22.4	2.52	9.96	8.1	7.7
	0.5	1.5	1.7	2.0	1.09	1.84	5.3	3.4	1	2	1.3	1.9
	1	0	1.9	2.1	1.11	1.98	5.6	3.5	1	2.03	1.4	2.0
	1	0.5	10.9	7.6	4.68	6.82	41.1	22.4	2.52	9.96	8.1	7.7
	1	1	35.4	20.6	35.39	20.59	196.1	132.6	196.08	132.57	59.6	31.4
	1	1.5	5.8	4.2	3.19	3.97	33.0	16.9	2.42	8.38	5.6	4.8
	1.5	0	1.0	1.3	1	1.25	1.7	1.8	1	1.28	1.0	1.3
	1.5	0.5	1.7	2.0	1.09	1.84	5.3	3.4	1	2	1.3	1.9
	1.5	1	5.8	4.2	3.19	3.97	33.0	16.9	2.42	8.38	5.6	4.8
	1.5	1.5	10.3	6.1	10.32	6.09	112.1	61.0	112.09	61.01	19.9	9.5

Table 3. The SDRL values for the T^2 control chart; $n = 3$.

			<i>a</i>		0.3		0.5		0.7		0.9		0.3		0	
			<i>b</i>		0.3		0.5		0.7		0.9		0.9		0.5	
			<i>c</i>		0		0		0		0		0		0	
			<i>d</i>		0		0		0		0		0		0	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	369.90	369.90	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	126.6	95.6	170.5	123.4	230.9	177.1	315.07	281	317.96	285.42	172.4	125.9		
	0	1	26.2	15.9	46.7	25.0	91.1	50.5	210.8	150.72	216.92	157.42	47.8	25.9		
	0	1.5	6.9	3.9	14.2	6.5	35.1	15.7	126.43	73.67	132.71	78.82	14.6	6.8		
	0.5	0	126.6	95.6	170.5	123.4	230.9	177.1	315.07	281	312.84	101.5	83.7	84.3		
	0.5	0.5	94.2	67.5	135.2	91.4	197.6	141.7	296.78	254.84	133.71	100.4	78.7	67.9		
	0.5	1	28.2	17.3	49.8	27.0	95.7	53.8	215.9	156.12	114.05	77.7	38.2	23.6		
	0.5	1.5	8.1	4.6	16.5	7.7	39.8	18.3	135.95	81.27	85.51	50.87	14.9	7.4		
	1	0	26.2	15.9	46.7	25.0	91.1	50.5	210.8	150.72	28.63	17.65	12.8	12.9		
	1	0.5	28.2	17.3	49.8	27.0	95.7	53.8	215.9	156.12	30	18.52	14.4	14.0		
	1	1	15.5	9.1	29.6	14.8	63.8	32.3	176.03	116.26	28.98	17.32	11.5	9.1		
	1	1.5	6.4	3.6	13.2	6.0	33.0	14.7	122.04	70.26	25.9	14.58	7.1	4.5		
	1.5	0	6.9	3.9	14.2	6.5	35.1	15.7	126.43	73.67	7.68	4.34	3.0	3.1		
	1.5	0.5	8.1	4.6	16.5	7.7	39.8	18.3	135.95	81.27	8.07	4.57	3.4	3.5		
	1.5	1	6.4	3.6	13.2	6.0	33.0	14.7	122.04	70.26	8.11	4.54	3.3	3.0		
	1.5	1.5	3.7	2.1	8.0	3.6	21.5	8.9	94.04	49.83	7.8	4.25	2.7	2.1		
0.6	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	92.8	66.4	133.6	90.0	196.0	140.0	295.84	253.52	312.95	278.53	143.6	101.8		
	0	1	15.1	8.8	28.9	14.4	62.7	31.6	174.39	114.73	206.41	147.09	33.2	17.7		
	0	1.5	3.6	2.0	7.8	3.5	21.0	8.7	92.64	48.87	122.04	70.95	9.2	4.4		
	0.5	0	92.8	66.4	133.6	90.0	196.0	140.0	295.84	253.52	122.17	92.51	62.2	64.7		
	0.5	0.5	113.7	84.3	156.9	110.7	218.5	163.5	308.58	271.53	136.13	104.28	89.3	81.5		
	0.5	1	29.4	18.1	51.6	28.1	98.3	55.6	218.61	159.03	125.62	88.4	47.7	28.1		
	0.5	1.5	6.8	3.8	14.0	6.4	34.5	15.5	125.31	72.8	98.13	59.88	16.6	7.5		
	1	0	15.1	8.8	28.9	14.4	62.7	31.6	174.39	114.73	24.52	15.06	8.0	8.5		
	1	0.5	29.4	18.1	51.6	28.1	98.3	55.6	218.61	159.03	28.27	17.72	12.9	14.0		
	1	1	21.5	12.9	39.5	20.5	80.0	42.9	197.7	137.27	29.98	18.49	14.2	12.2		
	1	1.5	8.0	4.5	16.4	7.6	39.5	18.1	135.36	80.79	29.05	17	10.0	6.2		
	1.5	0	3.6	2.0	7.8	3.5	21.0	8.7	92.64	48.87	6.37	3.63	1.8	2.0		
	1.5	0.5	6.8	3.8	14.0	6.4	34.5	15.5	125.31	72.8	7.22	4.16	2.6	3.0		
	1.5	1	8.0	4.5	16.4	7.6	39.5	18.1	135.36	80.79	7.84	4.51	3.3	3.5		
	1.5	1.5	5.5	3.1	11.5	5.2	29.2	12.7	113.59	63.85	8.13	4.58	3.4	2.9		
0.9	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	19.9	11.8	36.8	19.0	75.8	40.1	192.4	131.99	302.03	264.16	73.6	51.9		
	0	1	1.7	1.0	3.7	1.7	10.9	4.2	60.23	28.29	185.35	127.59	10.4	6.1		
	0	1.5	0.3	0.5	0.8	0.6	2.5	1.0	19.86	7.59	102.2	57.2	2.4	1.4		
	0.5	0	19.9	11.8	36.8	19.0	75.8	40.1	192.4	131.99	102.32	76.4	23.0	28.7		
	0.5	0.5	130.9	99.6	175.1	127.8	234.9	181.6	317.1	284.01	130.32	101.8	82.8	90.3		
	0.5	1	11.5	6.6	22.7	10.9	51.5	24.9	157	99.03	133.83	97.48	52.3	26.2		
	0.5	1.5	1.3	0.8	2.9	1.3	8.5	3.3	50.91	22.99	109.83	68.42	11.3	4.5		
	1	0	1.7	1.0	3.7	1.7	10.9	4.2	60.23	28.29	17.89	11	2.0	2.7		
	1	0.5	11.5	6.6	22.7	10.9	51.5	24.9	157	99.03	23.1	14.77	5.6	8.5		
	1	1	27.9	17.1	49.3	26.6	95.0	53.3	215.08	155.25	27.62	17.74	12.5	14.5		
	1	1.5	5.2	2.9	10.9	4.9	28.0	12.1	110.79	61.77	29.92	18.44	13.1	7.6		
	1.5	0	0.3	0.5	0.8	0.6	2.5	1.0	19.86	7.59	4.39	2.57	0.4	0.7		
	1.5	0.5	1.3	0.8	2.9	1.3	8.5	3.3	50.91	22.99	5.41	3.22	0.9	1.4		
	1.5	1	5.2	2.9	10.9	4.9	28.0	12.1	110.79	61.77	6.44	3.86	1.7	2.7		
	1.5	1.5	7.4	4.2	15.3	7.1	37.2	16.9	130.8	77.13	7.36	4.37	2.9	3.5		

Table 4. The SDRL values for the T^2 control chart; $n = 3$.

		a	0.3		0.1		0.7		0.2		0.3	
		b	0.3		0.1		0.7		0.2		0.3	
		c	0.1		0.3		0.2		0.7		0.3	
		d	0.1		0.3		0.2		0.7		0.3	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	121.3	94.6	75.84	88.02	215.8	168.7	48.62	113.99	105.3	95.9
	0	1	24.2	15.6	10.86	13.86	77.8	45.7	5.52	21.64	18.8	16.0
	0	1.5	6.3	3.8	2.5	3.31	28.1	13.8	1.2	5.51	4.7	3.9
	0.5	0	121.3	94.6	75.84	88.02	215.8	168.7	48.62	113.99	105.3	95.9
	0.5	0.5	108.9	73.9	108.93	73.85	284.3	234.7	284.3	234.74	150.7	99.4
	0.5	1	32.5	18.9	25	18.29	149.5	96.6	40.66	71.59	42.5	26.0
	0.5	1.5	8.9	4.9	5.2	4.56	56.2	29.2	5	15.61	9.9	6.3
	1	0	24.2	15.6	10.86	13.86	77.8	45.7	5.52	21.64	18.8	16.0
	1	0.5	32.5	18.9	25	18.29	149.5	96.6	40.66	71.59	42.5	26.0
	1	1	20.0	10.4	19.96	10.42	155.7	94.8	155.69	94.82	36.5	17.0
	1	1.5	8.1	4.1	7.02	4	84.8	44.0	29.59	35.66	13.9	6.5
	1.5	0	6.3	3.8	2.5	3.31	28.1	13.8	1.2	5.51	4.7	3.9
	1.5	0.5	8.9	4.9	5.2	4.56	56.2	29.2	5	15.61	9.9	6.3
	1.5	1	8.1	4.1	7.02	4	84.8	44.0	29.59	35.66	13.9	6.5
	1.5	1.5	5.0	2.4	4.99	2.43	77.5	37.1	77.47	37.11	10.4	4.2
0.6	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	84.5	64.0	43.92	57.76	164.3	118.2	22.62	68.51	65.3	61.5
	0	1	12.9	8.3	4.78	7.13	43.3	23.1	1.98	9.26	8.6	7.8
	0	1.5	3.0	1.9	1.03	1.64	12.9	6.0	0.38	2.14	1.9	1.8
	0.5	0	84.5	64.0	43.92	57.76	164.3	118.2	22.62	68.51	65.3	61.5
	0.5	0.5	129.6	91.4	129.55	91.37	297.7	253.3	297.7	253.26	172.7	119.4
	0.5	1	31.7	19.1	20.52	18.05	126.2	79.8	20.4	50.47	34.9	24.0
	0.5	1.5	6.7	3.9	3.09	3.46	35.3	17.5	1.89	7.7	5.8	4.4
	1	0	12.9	8.3	4.78	7.13	43.3	23.1	1.98	9.26	8.6	7.8
	1	0.5	31.7	19.1	20.52	18.05	126.2	79.8	20.4	50.47	34.9	24.0
	1	1	27.3	14.8	27.32	14.75	177.6	114.4	177.63	114.43	47.9	23.5
	1	1.5	9.7	5.1	7.5	4.91	81.6	43.4	16.81	30.52	14.5	7.6
	1.5	0	3.0	1.9	1.03	1.64	12.9	6.0	0.38	2.14	1.9	1.8
	1.5	0.5	6.7	3.9	3.09	3.46	35.3	17.5	1.89	7.7	5.8	4.4
	1.5	1	9.7	5.1	7.5	4.91	81.6	43.4	16.81	30.52	14.5	7.6
	1.5	1.5	7.3	3.6	7.26	3.55	95.4	48.7	95.41	48.68	14.7	6.1
0.9	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	16.0	10.8	5.45	9.09	45.8	25.0	2.01	9.74	9.6	9.5
	0	1	1.3	1.0	0.35	0.82	5.1	2.3	0.06	0.87	0.7	0.9
	0	1.5	0.2	0.5	0.01	0.44	1.1	0.7	0	0.45	0.1	0.4
	0.5	0	16.0	10.8	5.45	9.09	45.8	25.0	2.01	9.74	9.6	9.5
	0.5	0.5	147.3	107.3	147.34	107.25	307.5	267.3	307.49	267.34	190.7	136.9
	0.5	1	10.4	6.4	4.15	5.59	40.6	21.2	1.96	8.75	7.6	6.5
	0.5	1.5	1.1	0.8	0.31	0.72	4.8	2.1	0.06	0.84	0.6	0.8
	1	0	1.3	1.0	0.35	0.82	5.1	2.3	0.06	0.87	0.7	0.9
	1	0.5	10.4	6.4	4.15	5.59	40.6	21.2	1.96	8.75	7.6	6.5
	1	1	34.9	19.4	34.89	19.42	195.6	131.7	195.58	131.68	59.1	30.3
	1	1.5	5.3	3.0	2.64	2.73	32.5	15.7	1.85	7.16	5.1	3.6
	1.5	0	0.2	0.5	0.01	0.44	1.1	0.7	0	0.45	0.1	0.4
	1.5	0.5	1.1	0.8	0.31	0.72	4.8	2.1	0.06	0.84	0.6	0.8
	1.5	1	5.3	3.0	2.64	2.73	32.5	15.7	1.85	7.16	5.1	3.6
	1.5	1.5	9.8	4.9	9.81	4.85	111.6	60.0	111.58	59.97	19.4	8.2

Table 5. The ARL values for the T^2 control chart; $n = 5$.

			a		b		c		d		0.3		0		
			0.3	0.5	0.3	0.5	0	0	0	0	0.3	0.9	0.3	0.5	
			0	0	0	0	0	0	0	0	0	0.9	0	0	
			0	0	0	0	0	0	0	0	0	0	0	0	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	
0.3	0	0	370.40	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	88.02	59.47	137.69	88.28	211.75	149.26	312.11	271.46	314.94	276.04	138.95	90.29	
	0	1	14.23	8.54	30.93	14.92	74.6	36.54	204.22	137.2	209.98	143.5	31.46	15.44	
	0	1.5	3.78	2.98	8.79	4.57	26.79	11.26	119.89	64.24	125.6	68.73	8.97	4.71	
	0.5	0	88.02	59.47	137.69	88.28	211.75	149.26	312.11	271.46	93.05	63.75	46.84	48.13	
	0.5	0.5	61.37	39.29	104.25	61.75	177.01	115.03	292.92	243.47	95.67	64.7	46.2	39.37	
	0.5	1	15.44	9.22	33.22	16.11	78.75	39.09	209.41	142.5	84.63	52.48	22.56	13.5	
	0.5	1.5	4.39	3.29	10.25	5.17	30.6	12.97	129.26	71.19	66.08	36.18	8.87	4.95	
	1	0	14.23	8.54	30.93	14.92	74.6	36.54	204.22	137.2	15.58	9.35	5.68	6.58	
	1	0.5	15.44	9.22	33.22	16.11	78.75	39.09	209.41	142.5	16.45	9.83	6.5	7.16	
	1	1	8.22	5.26	18.83	8.97	50.66	22.89	169.13	103.9	16.3	9.54	5.58	5.27	
	1	1.5	3.53	2.86	8.18	4.32	25.15	10.55	115.59	61.15	15.18	8.59	3.86	3.36	
	1.5	0	3.78	2.98	8.79	4.57	26.79	11.26	119.89	64.24	4.14	3.17	1.72	2.53	
	1.5	0.5	4.39	3.29	10.25	5.17	30.6	12.97	129.26	71.19	4.33	3.27	1.88	2.7	
	1.5	1	3.53	2.86	8.18	4.32	25.15	10.55	115.59	61.15	4.4	3.29	1.87	2.56	
	1.5	1.5	2.29	2.24	5.04	3.08	16.1	6.82	88.32	42.88	4.34	3.22	1.7	2.24	
0.6	0	0	370.4	370.40	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	60.28	38.5	102.79	60.67	175.37	113.51	291.92	242.07	308.79	267.84	109.17	69.71	
	0	1	8.01	5.15	18.39	8.76	49.7	22.38	167.49	102.45	197.65	132.39	20.38	10.56	
	0	1.5	2.24	2.21	4.92	3.03	15.71	6.66	86.97	42.03	113.57	60.91	5.49	3.47	
	0.5	0	60.28	38.5	102.79	60.67	175.37	113.51	291.92	242.07	82.52	56.3	31.54	34.82	
	0.5	0.5	77.17	51.04	124.53	77.48	198.67	135.89	305.29	261.27	95.33	66.04	51.84	48.26	
	0.5	1	16.12	9.61	34.5	16.79	81.02	40.52	212.17	145.37	93.07	59.94	29.7	16.51	
	0.5	1.5	3.72	2.95	8.63	4.51	26.37	11.08	118.79	63.45	77.24	43.56	10.27	5.12	
	1	0	8.01	5.15	18.39	8.76	49.7	22.38	167.49	102.45	12.84	7.96	3.52	4.66	
	1	0.5	16.12	9.61	34.5	16.79	81.02	40.52	212.17	145.37	14.87	9.18	5.49	6.99	
	1	1	11.56	7.06	25.71	12.28	64.74	30.69	190.95	124.09	16.21	9.81	6.5	6.6	
	1	1.5	4.35	3.27	10.15	5.13	30.36	12.86	128.68	70.75	16.47	9.59	5.24	4.17	
	1.5	0	2.24	2.21	4.92	3.03	15.71	6.66	86.97	42.03	3.42	2.85	1.29	2.1	
	1.5	0.5	3.72	2.95	8.63	4.51	26.37	11.08	118.79	63.45	3.79	3.05	1.55	2.48	
	1.5	1	4.35	3.27	10.15	5.13	30.36	12.86	128.68	70.75	4.1	3.2	1.8	2.69	
	1.5	1.5	3.1	2.64	7.1	3.89	22.16	9.27	107.32	55.36	4.31	3.28	1.9	2.54	
0.9	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	10.63	6.55	23.85	11.36	61.06	28.59	185.6	118.98	294.86	250.48	42.94	31.44	
	0	1	1.41	1.76	2.62	2.16	8.16	3.88	55.94	24.25	172.38	111.47	5.08	4.23	
	0	1.5	1.01	1.19	1.14	1.43	2.28	1.93	18.29	7.23	91.02	47.39	1.6	2.01	
	0.5	0	10.63	6.55	23.85	11.36	61.06	28.59	185.6	118.98	63.57	43.53	8.32	13.73	
	0.5	0.5	91.81	62.47	142.16	92.05	216.03	153.78	314.26	274.71	85.75	61.32	39.58	52.31	
	0.5	1	6.1	4.16	14.2	6.87	40.27	17.56	150.12	87.65	96.41	65.14	35.18	16.22	
	0.5	1.5	1.26	1.65	2.16	1.98	6.47	3.31	47.13	19.79	87.28	50.87	7.12	3.68	
	1	0	1.41	1.76	2.62	2.16	8.16	3.88	55.94	24.25	8.65	5.87	1.25	2.37	
	1	0.5	6.1	4.16	14.2	6.87	40.27	17.56	150.12	87.65	11.12	7.41	2.14	4.39	
	1	1	15.24	9.11	32.84	15.91	78.07	38.67	208.58	141.64	13.65	8.88	4.59	7.27	
	1	1.5	2.96	2.57	6.77	3.76	21.22	8.88	104.59	53.49	15.66	9.76	6.47	5.01	
	1.5	0	1.01	1.19	1.14	1.43	2.28	1.93	18.29	7.23	2.39	2.37	1	1.52	
	1.5	0.5	1.26	1.65	2.16	1.98	6.47	3.31	47.13	19.79	2.78	2.6	1.03	1.83	
	1.5	1	2.96	2.57	6.77	3.76	21.22	8.88	104.59	53.49	3.2	2.84	1.16	2.3	
	1.5	1.5	4.05	3.12	9.44	4.84	28.5	12.02	124.2	67.4	3.63	3.06	1.5	2.69	

Table 6. The ARL values for the T^2 control chart; $n = 5$.

			a		b		c		d			
			0.3	0.1	0.7	0.2	0.3	0.3	0.3	0.3		
			0.3	0.1	0.7	0.2	0.3	0.3	0.3	0.3		
			0.1	0.3	0.2	0.7	0.3	0.3	0.3	0.3		
			0.1	0.3	0.2	0.7	0.3	0.3	0.3	0.3		
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	83.08	58.53	36.03	49.82	201.74	144.03	18.06	68.84	63.83	57.1
	0	1	12.98	8.36	4.1	6.86	66.96	34.17	2.04	10.38	8.7	8.1
	0	1.5	3.46	2.94	1.4	2.59	23.17	10.44	1.06	3.42	2.4	2.88
	0.5	0	83.08	58.53	36.03	49.82	201.74	144.03	18.06	68.84	63.83	57.1
	0.5	0.5	73.01	43.46	73.01	43.46	253.8	191.73	253.8	191.73	108.06	61.78
	0.5	1	17.91	9.99	11.62	9.41	121.82	68.46	15.37	40.34	22.68	13.39
	0.5	1.5	4.73	3.4	2.41	3.15	43.43	19.69	1.94	7.79	4.74	3.91
	1	0	12.98	8.36	4.1	6.86	66.96	34.17	2.04	10.38	8.7	8.1
	1	0.5	17.91	9.99	11.62	9.41	121.82	68.46	15.37	40.34	22.68	13.39
	1	1	10.62	5.86	10.62	5.86	114.98	60.34	114.98	60.34	20.02	8.97
	1	1.5	4.35	3.06	3.57	3.01	59.1	26.48	11.53	18.91	6.99	4.05
	1.5	0	3.46	2.94	1.4	2.59	23.17	10.44	1.06	3.42	2.4	2.88
	1.5	0.5	4.73	3.4	2.41	3.15	43.43	19.69	1.94	7.79	4.74	3.91
	1.5	1	4.35	3.06	3.57	3.01	59.1	26.48	11.53	18.91	6.99	4.05
	1.5	1.5	2.86	2.37	2.86	2.37	49.21	20.66	49.21	20.66	5.39	3.08
0.6	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	53.49	36.82	17.85	29.61	151.49	97.59	7.39	36.55	35.14	33.29
	0	1	6.78	4.92	2.02	4	37.1	17.45	1.19	4.88	3.98	4.46
	0	1.5	1.96	2.16	1.06	1.96	10.93	5.26	1	2.15	1.38	2.06
	0.5	0	53.49	36.82	17.85	29.61	151.49	97.59	7.39	36.55	35.14	33.29
	0.5	0.5	90.37	56.06	90.37	56.06	270.65	212.89	270.65	212.89	128.6	77.5
	0.5	1	17.41	10.11	8.7	9.11	105.27	57.87	6.81	26.29	17.68	12.2
	0.5	1.5	3.65	2.97	1.61	2.68	28.21	12.6	1.18	4.31	2.91	3.09
	1	0	6.78	4.92	2.02	4	37.1	17.45	1.19	4.88	3.98	4.46
	1	0.5	17.41	10.11	8.7	9.11	105.27	57.87	6.81	26.29	17.68	12.2
	1	1	14.85	7.92	14.85	7.92	135.91	75.85	135.91	75.85	27.26	12.28
	1	1.5	5.17	3.48	3.59	3.36	59.63	27.39	5.84	15.61	7.14	4.5
	1.5	0	1.96	2.16	1.06	1.96	10.93	5.26	1	2.15	1.38	2.06
	1.5	0.5	3.65	2.97	1.61	2.68	28.21	12.6	1.18	4.31	2.91	3.09
	1.5	1	5.17	3.48	3.59	3.36	59.63	27.39	5.84	15.61	7.14	4.5
	1.5	1.5	3.95	2.84	3.95	2.84	63.05	27.83	63.05	27.83	7.59	3.89
0.9	0	0	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4	370.4
	0	0.5	47.39	6.07	2.18	4.71	40.13	19.24	1.19	5.07	4.39	5.11
	0	1	5.76	1.71	1	1.57	4.67	2.88	1	1.61	1.04	1.62
	0	1.5	1.74	1.16	1	1.08	1.51	1.68	1	1.1	1	1.1
	0.5	0	42.95	6.07	2.18	4.71	40.13	19.24	1.19	5.07	4.39	5.11
	0.5	0.5	92.86	68.19	106.14	68.19	283.26	229.61	283.26	229.61	146.35	92.09
	0.5	1	17.65	4.07	1.87	3.45	34.06	15.72	1.19	4.7	3.59	3.91
	0.5	1.5	3.47	1.62	1	1.5	4.32	2.74	1	1.6	1.03	1.56
	1	0	5.08	1.71	1	1.57	4.67	2.88	1	1.61	1.04	1.62
	1	0.5	14.78	4.07	1.87	3.45	34.06	15.72	1.19	4.7	3.59	3.91
	1	1	15.52	10.25	19.42	10.25	153.85	90.26	153.85	90.26	34.71	15.92
	1	1.5	5.51	2.61	1.49	2.41	25.4	11.18	1.17	4.11	2.61	2.78
	1.5	0	1.6	1.16	1	1.08	1.51	1.68	1	1.1	1	1.1
	1.5	0.5	2.93	1.62	1	1.5	4.32	2.74	1	1.6	1.03	1.56
	1.5	1	4.73	2.61	1.49	2.41	25.4	11.18	1.17	4.11	2.61	2.78
	1.5	1.5	4.13	3.39	5.21	3.39	76.17	35.23	76.17	35.23	10.08	4.84

Table 7. The SDRL values for the T^2 control chart; $n = 5$.

			0.3		0.5		0.7		0.9		0.3		0	
			0.3		0.5		0.7		0.9		0.9		0.5	
			0		0		0		0		0		0	
			0		0		0		0		0		0	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	369.90	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	87.52	58.36	137.19	87.24	211.25	148.36	311.62	270.79	314.43	275.37	138.45	89.25
	0	1	13.72	7.24	30.42	13.65	74.09	35.36	203.72	136.26	209.48	142.58	30.96	14.18
	0	1.5	3.24	1.65	8.28	3.25	26.29	9.98	119.39	63.14	125.1	67.64	8.46	3.39
	0.5	0	87.52	58.36	137.19	87.24	211.25	148.36	311.62	270.79	92.55	62.65	46.34	46.98
	0.5	0.5	60.87	38.12	103.75	60.65	176.51	114.05	292.42	242.74	95.17	63.6	45.7	38.2
	0.5	1	14.93	7.93	32.72	14.86	78.24	37.92	208.91	141.58	84.13	51.34	22.06	12.23
	0.5	1.5	3.86	1.96	9.74	3.85	30.1	11.7	128.76	70.11	65.58	35	8.35	3.63
	1	0	13.72	7.24	30.42	13.65	74.09	35.36	203.72	136.26	15.07	8.06	5.15	5.27
	1	0.5	14.93	7.93	32.72	14.86	78.24	37.92	208.91	141.58	15.94	8.54	5.98	5.85
	1	1	7.71	3.94	18.32	7.67	50.16	21.66	168.63	102.9	15.8	8.25	5.05	3.95
	1	1.5	2.99	1.53	7.67	3	24.64	9.26	115.09	60.04	14.67	7.29	3.32	2.03
	1.5	0	3.24	1.65	8.28	3.25	26.29	9.98	119.39	63.14	3.61	1.85	1.12	1.21
	1.5	0.5	3.86	1.96	9.74	3.85	30.1	11.7	128.76	70.11	3.8	1.94	1.29	1.37
	1.5	1	2.99	1.53	7.67	3	24.64	9.26	115.09	60.04	3.87	1.96	1.28	1.24
	1.5	1.5	1.72	0.93	4.52	1.75	15.59	5.51	87.82	41.72	3.81	1.89	1.09	0.94
0.6	0	0	369.9	369.90	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	59.78	37.33	102.29	59.56	174.87	112.53	291.42	241.34	308.3	267.16	108.67	68.63
	0	1	7.5	3.83	17.88	7.47	49.2	21.15	166.99	101.45	197.15	131.45	19.87	9.28
	0	1.5	1.67	0.91	4.39	1.71	15.21	5.36	86.47	40.87	113.07	59.81	4.97	2.14
	0.5	0	59.78	37.33	102.29	59.56	174.87	112.53	291.42	241.34	82.02	55.18	31.04	33.64
	0.5	0.5	76.67	49.91	124.03	76.41	198.17	134.95	304.79	260.58	94.83	64.95	51.34	47.12
	0.5	1	15.61	8.32	34	15.54	80.52	39.35	211.67	144.45	92.57	58.83	29.19	15.26
	0.5	1.5	3.18	1.62	8.12	3.18	25.86	9.79	118.29	62.34	76.74	42.4	9.75	3.8
	1	0	7.5	3.83	17.88	7.47	49.2	21.15	166.99	101.45	12.33	6.66	2.98	3.33
	1	0.5	15.61	8.32	34	15.54	80.52	39.35	211.67	144.45	14.36	7.89	4.97	5.68
	1	1	11.05	5.75	25.21	11	64.24	29.49	190.45	123.13	15.7	8.52	5.98	5.29
	1	1.5	3.82	1.94	9.64	3.81	29.85	11.59	128.18	69.67	15.96	8.3	4.72	2.85
	1.5	0	1.67	0.91	4.39	1.71	15.21	5.36	86.47	40.87	2.88	1.52	0.62	0.82
	1.5	0.5	3.18	1.62	8.12	3.18	25.86	9.79	118.29	62.34	3.25	1.72	0.92	1.16
	1.5	1	3.82	1.94	9.64	3.81	29.85	11.59	128.18	69.67	3.56	1.87	1.2	1.37
	1.5	1.5	2.55	1.32	6.59	2.56	21.65	7.98	106.82	54.24	3.78	1.95	1.31	1.22
0.9	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	10.12	5.24	23.34	10.07	60.55	27.38	185.1	118.01	294.35	249.77	42.44	30.24
	0	1	0.76	0.58	2.06	0.87	7.64	2.56	55.43	23.03	171.87	110.49	4.55	2.9
	0	1.5	0.08	0.39	0.4	0.5	1.7	0.68	17.78	5.92	90.52	46.25	0.98	0.74
	0.5	0	10.12	5.24	23.34	10.07	60.55	27.38	185.1	118.01	63.07	42.37	7.8	12.46
	0.5	0.5	91.31	61.37	141.66	91.02	215.53	152.88	313.76	274.04	85.25	60.21	39.08	51.17
	0.5	1	5.58	2.83	13.69	5.56	39.76	16.31	149.62	86.61	95.91	64.05	34.68	14.96
	0.5	1.5	0.57	0.54	1.58	0.72	5.95	1.98	46.62	18.55	86.77	49.73	6.6	2.35
	1	0	0.76	0.58	2.06	0.87	7.64	2.56	55.43	23.03	8.13	4.56	0.56	1.05
	1	0.5	5.58	2.83	13.69	5.56	39.76	16.31	149.62	86.61	10.61	6.11	1.56	3.07
	1	1	14.73	7.81	32.34	14.66	77.57	37.5	208.08	140.72	13.14	7.59	4.06	5.96
	1	1.5	2.41	1.25	6.25	2.43	20.71	7.58	104.09	52.37	15.15	8.47	5.95	3.69
	1.5	0	0.08	0.39	0.4	0.5	1.7	0.68	17.78	5.92	1.82	1.06	0.04	0.52
	1.5	0.5	0.57	0.54	1.58	0.72	5.95	1.98	46.62	18.55	2.22	1.28	0.17	0.62
	1.5	1	2.41	1.25	6.25	2.43	20.71	7.58	104.09	52.37	2.65	1.51	0.43	0.99
	1.5	1.5	3.51	1.79	8.93	3.52	28	10.75	123.69	66.31	3.09	1.73	0.86	1.36

Table 8. The SDRL values for the T^2 control chart; $n = 5$.

			0.3		0.1		0.7		0.2		0.3	
			0.3		0.1		0.7		0.2		0.3	
			0.1		0.3		0.2		0.7		0.3	
			0.1		0.3		0.2		0.7		0.3	
ρ	δx	δy	STD	MS	STD	MS	STD	MS	STD	MS	STD	MS
0.3	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	82.58	57.41	35.52	48.68	201.24	143.12	17.56	67.75	63.33	55.98
	0	1	12.47	7.07	3.56	5.55	66.45	32.98	1.46	9.1	8.18	6.8
	0	1.5	2.91	1.61	0.75	1.27	22.66	9.15	0.25	2.09	1.83	1.55
	0.5	0	82.58	57.41	35.52	48.68	201.24	143.12	17.56	67.75	63.33	55.98
	0.5	0.5	72.51	42.3	72.51	42.3	253.3	190.9	253.3	190.9	107.56	60.67
	0.5	1	17.4	8.7	11.11	8.12	121.32	67.37	14.86	39.17	22.17	12.12
	0.5	1.5	4.2	2.08	1.84	1.82	42.93	18.44	1.35	6.49	4.21	2.58
	1	0	12.47	7.07	3.56	5.55	66.45	32.98	1.46	9.1	8.18	6.8
	1	0.5	17.4	8.7	11.11	8.12	121.32	67.37	14.86	39.17	22.17	12.12
	1	1	10.11	4.55	10.11	4.55	114.48	59.23	114.48	59.23	19.52	7.68
	1	1.5	3.81	1.73	3.03	1.68	58.6	25.27	11.01	17.66	6.48	2.73
	1.5	0	2.91	1.61	0.75	1.27	22.66	9.15	0.25	2.09	1.83	1.55
	1.5	0.5	4.2	2.08	1.84	1.82	42.93	18.44	1.35	6.49	4.21	2.58
	1.5	1	3.81	1.73	3.03	1.68	58.6	25.27	11.01	17.66	6.48	2.73
	1.5	1.5	2.31	1.06	2.31	1.06	48.71	19.42	48.71	19.42	4.86	1.76
0.6	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	52.98	35.64	17.34	28.4	150.99	96.57	6.88	35.37	34.64	32.1
	0	1	6.26	3.6	1.43	2.68	36.59	16.2	0.48	3.56	3.44	3.13
	0	1.5	1.38	0.87	0.25	0.7	10.41	3.94	0.03	0.86	0.72	0.78
	0.5	0	52.98	35.64	17.34	28.4	150.99	96.57	6.88	35.37	34.64	32.1
	0.5	0.5	89.87	54.94	89.87	54.94	270.16	212.1	270.16	212.1	128.09	76.44
	0.5	1	16.9	8.82	8.19	7.81	104.77	56.75	6.29	25.07	17.17	10.92
	0.5	1.5	3.11	1.64	0.99	1.36	27.71	11.32	0.46	2.98	2.35	1.76
	1	0	6.26	3.6	1.43	2.68	36.59	16.2	0.48	3.56	3.44	3.13
	1	0.5	16.9	8.82	8.19	7.81	104.77	56.75	6.29	25.07	17.17	10.92
	1	1	14.34	6.62	14.34	6.62	135.41	74.78	135.41	74.78	26.75	11.01
	1	1.5	4.64	2.15	3.05	2.03	59.13	26.18	5.32	14.35	6.62	3.17
	1.5	0	1.38	0.87	0.25	0.7	10.41	3.94	0.03	0.86	0.72	0.78
	1.5	0.5	3.11	1.64	0.99	1.36	27.71	11.32	0.46	2.98	2.35	1.76
	1.5	1	4.64	2.15	3.05	2.03	59.13	26.18	5.32	14.35	6.62	3.17
	1.5	1.5	3.41	1.51	3.41	1.51	62.54	26.62	62.54	26.62	7.08	2.56
0.9	0	0	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9	369.9
	0	0.5	46.89	4.75	1.6	3.39	39.63	18	0.48	3.75	3.86	3.79
	0	1	5.24	0.56	0.03	0.53	4.14	1.55	0	0.53	0.21	0.54
	0	1.5	1.14	0.37	0	0.27	0.88	0.55	0	0.3	0	0.31
	0.5	0	42.44	4.75	1.6	3.39	39.63	18	0.48	3.75	3.86	3.79
	0.5	0.5	92.36	67.1	105.64	67.1	282.76	228.86	282.76	228.86	145.85	91.05
	0.5	1	17.14	2.74	1.27	2.12	33.55	14.47	0.47	3.37	3.05	2.58
	0.5	1.5	2.93	0.54	0.03	0.51	3.79	1.41	0	0.53	0.19	0.52
	1	0	4.55	0.56	0.03	0.53	4.14	1.55	0	0.53	0.21	0.54
	1	0.5	14.27	2.74	1.27	2.12	33.55	14.47	0.47	3.37	3.05	2.58
	1	1	15.01	8.96	18.91	8.96	153.35	89.23	153.35	89.23	34.21	14.67
	1	1.5	4.98	1.29	0.86	1.1	24.89	9.9	0.45	2.78	2.04	1.45
	1.5	0	0.98	0.37	0	0.27	0.88	0.55	0	0.3	0	0.31
	1.5	0.5	2.38	0.54	0.03	0.51	3.79	1.41	0	0.53	0.19	0.52
	1.5	1	4.2	1.29	0.86	1.1	24.89	9.9	0.45	2.78	2.04	1.45
	1.5	1.5	3.59	2.06	4.69	2.06	75.67	34.05	75.67	34.05	9.57	3.52

Table 9. The in-control ARL, $n = 5$.

The values of the estimated parameters		\hat{a}	\hat{b}	\widehat{ARL}	
				$\rho = 0.3$	$\rho = 0.9$
$\hat{a} = 0.80a$	$\hat{b} = 0.80b$	0.56	0.56	40.33	39.92
$\hat{a} = 0.90a$	$\hat{b} = 0.90b$	0.63	0.63	96.67	96.80
$\hat{a} = a$	$\hat{b} = 0.80b$	0.70	0.56	81.03	56.24
$\hat{a} = a$	$\hat{b} = 0.90b$	0.70	0.63	162.33	128.35
$\hat{a} = a$	$\hat{b} = 0.95b$	0.70	0.665	244.39	220.52
$\hat{a} = a$	$\hat{b} = b$	0.70	0.70	370.22	370.30
$\hat{a} = a$	$\hat{b} = 1.05b$	0.70	0.735	552.11	476.80
$\hat{a} = a$	$\hat{b} = 1.10b$	0.70	0.77	769.55	457.92
$\hat{a} = 1.05a$	$\hat{b} = 0.95b$	0.735	0.665	322.48	229.41
$\hat{a} = 1.10a$	$\hat{b} = 1.10b$	0.77	0.77	3470.27	2693.80
$\hat{a} = 1.30a$	$\hat{b} = 0.70b$	0.91	0.49	69.26	50.46

Table 10. Preliminary sample.

Observation	X	Y
1	988.47	986.72
2	989.63	987.28
3	986.40	990.88
4	989.77	991.93
5	987.81	990.12
:	:	:
496	990.87	989.81
497	989.72	990.68
498	991.76	990.93
499	991.05	990.00
500	992.41	991.35

$$ARL = \frac{\beta_1}{(1 - \beta_2)} + 1 \quad (16)$$

$$SDRL = \frac{\sqrt{\beta_1(1 + \beta_2 - \beta_1)}}{(1 - \beta_2)} \quad (17)$$

5. The effect of the autocorrelation and cross-correlation on the performance of the T^2 control chart

In this Section, we investigate the influence of the autocorrelation parameters (a, b, c, d), the cross-correlation (ρ) and the mean shifts (δ_X, δ_Y) on the statistical performance of the Hotelling T^2 control chart. The MS is compared with the standard sampling strategy (STD), where the sample units are selected according to the rational subgroup concept. We assume that Φ remains unchanged when the process shifts to an out-of-control condition. The in-control ARL is equal to 370.4 and the size of the samples is equal to 3 and 5.

Tables 1–8 present the ARL and SDRL values of the T^2 charts with the MS and with the STD. The SDRL is slightly lower than ARL. The ARL values of the best strategy are in bold. In these Tables, the cross-correlation (ρ) are stratified in low (0.3), medium (0.6) and high (0.9) levels. Tables 1–4 consider the cases in which $n = 3$ and Tables 5–8 consider the cases in which $n = 5$.

In general, the use of the MS reduces the negative effect of the autocorrelations on the T^2 chart's performance. For example, Table 1 shows that when the observations are medially autocorrelated ($a = b = 0.5$; $\rho = 0.3$)

Table 11. Mixed sample.

ith sample		Sample observations					\bar{M}	T_i^2
1	X	989.28	990.07	989.29	992.22	993.1	990.3	0.18
	Y	990.08	989.06	989.22	991.35	991.3		
2	X	990.31	989.09	989.87	990.53	989.03	990.112	1.02
	Y	989.8	990.14	990.35	991.67	990		
3	X	989.39	991.14	991.98	990.81	992.19	990.636	0.73
	Y	987.84	991.79	992.21	991.02	990.88		
4	X	989.76	987.82	987.78	989.46	991.31	990.16	1.67
	Y	989.97	988.85	988.22	991.07	991.87		
5	X	989.53	988.42	988.96	987.28	990.75	989.304	5.41
	Y	990.23	988.8	989.35	987.77	991.71		
6	X	988.03	989.45	988.33	988.64	989.69	988.35	7.67
	Y	989.34	986.5	989.44	989.69	990.67		
7	X	988.3	988.83	991.28	989.69	988.97	989.328	6.24
	Y	988.53	988.98	990.3	985.87	985.58		
8	X	988.33	990.83	989.79	990.22	989.62	989.252	1.18
	Y	989.4	991.03	989.55	990.39	987.69		
9	X	989.66	990.88	989.86	991.31	990.75	990.264	1.1
	Y	988.24	988.1	987.86	989.91	989.86		
10	X	988.84	991.19	991.89	991.34	989.81	990.546	2.17
	Y	990.28	991.09	991.7	990.37	988.89		
11	X	986.13	989.13	992.89	991.2	990.16	990.342	7.15
	Y	989.1	989.91	993.24	994.49	991.19		
12	X	991.32	991.25	989.97	990.46	990.74	990.472	0.15
	Y	992.22	991.25	991.24	991.79	991.1		
13	X	988.2	989.37	990.43	990.08	988.61	989.79	0.62
	Y	988.03	989.31	990.35	987.88	988.95		
14	X	990.63	988.04	987.14	989.74	991.49	989.742	6.78
	Y	988.3	990.54	990.41	990.37	991.42		
15	X	989.96	986.73	988.32	989.29	989.76	989.164	0.48
	Y	993.1	989.99	989.58	988.65	991.91		
	X	989.87	989.25	992.14	991.41	989.65	989.536	
	Y	989.97	990.98	990.07	991.08	991.11	989.958	

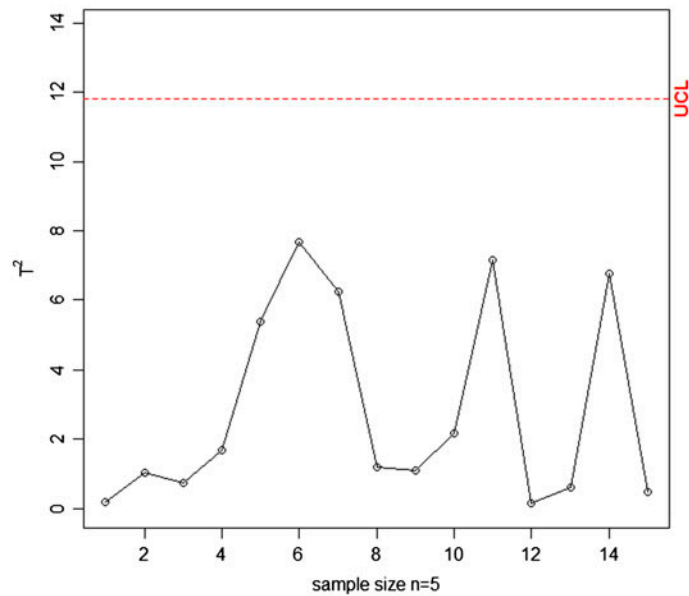


Figure 2. The T^2 chart with mixed sample.

Table 12. The ARL for the T^2 chart with the STD and with MS strategies.

δ_1	δ_2	STD	MS	ARL reduction (%)
		ARL		
0	0.5	142.35	103.91	27.00
1	0	38.84	21.98	43.41
0.5	1	42.62	25.49	40.19
0.5	0	155.09	112.32	27.58
0	1	32.93	19.32	41.33
0.5	0.5	133.16	94.34	29.15
1	1	29.06	16.54	43.08

and $\delta = (0.5, 1.0)'$, the T^2 chart with the STD requires, on average, 50.3 samples to signal; the ARL reduces to 28.1 (around 44% lower) with the MS.

When the autocorrelated variable is not affected by the assignable cause and the observations of the other variable are independent, that is, $(a = 0, b > 0, \delta_x > 0, \delta_y = 0)$ or $(a > 0, b = 0, \delta_x = 0, \delta_y > 0)$ the MS reduces the ability of the T^2 chart to detect the specified shift. These are the cases where $\beta_1 > \beta$ and β_2 is slightly lower or even equal to β , being β the type II error of the standard T^2 chart. For example, if $(a = 0, b = 0.5; \rho = 0.3)$ and $\delta = (1.0, 0)'$, the T^2 chart with the STD requires, on average, 5.7 samples to signal, see Table 5. With the MS, the ARL increases to 6.6.

In many applications, the magnitudes of the mean shifts are difficult to be predicted. Based on this fact, the MS is only recommended for bivariate processes with two autocorrelated variables.

With regard to the autocorrelation parameters, it is worth noting that the T^2 control chart with the MS has a more robust statistical performance than STD. In other words, fixing δ_x, δ_y, ρ , and n , the T^2 control chart with the MS is less sensitive to the negative effect of the autocorrelation. For instance, fixing $\delta_x = 1.0, \delta_y = 0.5, \rho = 0.6, n = 5$ and varying a and b as in Table 5, the range of the ARLs values is equal to 30.9 ($=40.5 - 9.6$) for the MS, and is equal to 64.9 ($=81.0 - 16.1$) for the STD. This robustness can lead quality practitioners to use the MS when there is some uncertainty with regard to the autocorrelation level.

In an additional study of robustness, it was investigated the a and b misestimation effect on the in-control ARL. According to Table 9, when a and/or b are underestimated, the risk of false alarms increase, and an opposite effect is observed when a and/or b are overestimated. Similar comments are also valid for other ρ values. Based on that, the MS should be considered with care if the phase (I) data are not enough to obtain precise estimates of the model's parameters.

The SPC literature has innumerable examples of autocorrelated processes; in almost all of them, not to say all of them, the variables observations at time (i) are correlated, however, the observation of one variable at time (i) is independent of the other variable observation at time ($i - 1$). Because of that the study of robustness was done with $c = d = 0$.

6. An example application

With the process in control, a preliminary investigation proved that the bivariate data from a two-valve filling milk machine fit to a VAR (1) model with $\hat{a} = 0.36, \hat{b} = 0.32$ and $\hat{\rho} = 0.42$ (see Table 10). The database was also used to estimate Γ and Σ_e . Leoni, Costa, and Machado (2015) describe in details how to estimate Γ and Σ_e .

The quality engineer has decided to use the T^2 chart with mixed samples of size five to control the filling process, being 990 mL the specified quantity inside the filled container. Based on the specifications, the in-control means vector $\mu_0 = (990, 990)'$. According to expression (9), the cross-covariance matrix $\Gamma_{\bar{M}} = \begin{pmatrix} 0.5074 & 0.2044 \\ 0.2044 & 0.4646 \end{pmatrix}$. The means vector μ_0 and the inverse of $\Gamma_{\bar{M}}$ are used to compute the monitoring statistic (see expression 12). Table 11 presents the bivariate data of 16 samples, the mean of the mixed samples and the values of the monitoring statistic. Figure 2 brings the mixed T^2 chart with the T_i^2 values from Table 11.

Table 12 was built considering $a = 0.36, b = 0.32, \rho = 0.42$ in order to present the ARL reduction when the MS is used to control the filling process rather than the STD. According to Table 12, the overall ARLs' reduction is around 36% due to the MS.

7. Conclusion

This paper considered the T^2 chart with the MS to monitor bivariate processes with autocorrelated variables. With the MS, the T^2 statistic depends on the cross-covariance matrix of the mixed sample mean vector. Numerical comparisons between standard and mixed sampling strategies showed that the mixed one is highly recommended to control the mean vector of bivariate processes with two autocorrelated variables. If the observations of one variable are autocorrelation free, the MS should be used with care; depending on the mean vector shift, the mixed strategy reduces the speed with which the chart signals. EWMA, CUSUM or synthetic schemes can be used with the MS to enhance the ability of the T^2 chart in signalling changes in the mean vector of multivariate processes with autocorrelated variables.

Based on recent studies with univariate processes and wandering means, the MS is not recommended to control wandering mean vectors.

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