Muon transverse polarization in the $K_{l2\gamma}$ decay in the standard model

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The muon transverse polarization in the $K^+ \to \mu^+ \nu\gamma$ process induced by the electromagnetic final state interaction is calculated in the framework of the standard model. It is shown that one loop contributions lead to a nonvanishing muon transverse polarization. The value of the muon transverse polarization averaged over the kinematical region of $E_\gamma \geqslant 20$ MeV is equal to 5.63×10^{-4}.

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I. INTRODUCTION

The study of the radiative K-meson decays is extremely interesting in searching for new physics effects beyond the standard model (SM). One of the most appealing possibilities is to probe new interactions, which could lead to CP violation. Contrary to the SM, where CP violation is caused by the presence of the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, the CP violation in extended models (for instance, in models with three and more Higgs doublets) can naturally arise due to the complex couplings of new Higgs bosons to fermions [1]. Such effects can be detected by using experimental observables, which are essentially sensitive to T-odd contributions. These observables, for instance, are the T-odd correlation $\{T = (1/M_K^3)\vec{p}_\gamma\cdot[\vec{p}_\pi\times\vec{p}_l]\}$ in the $K^\pm\to\pi^0\mu^\pm\nu\gamma$ decay [2] and muon transverse polarization (P_T) in $K^\pm\to\mu^\pm\nu\gamma$. The search for new physics effects using the T-odd correlation analysis in the $K^\pm\to\pi^0\mu^\pm\nu\gamma$ decay will be done in the proposed OKA experiment [3], where an event sample of 7.0×10^5 for the $K^+\to\pi^0\mu^+\nu\gamma$ decay is expected to be accumulated.

At the moment the E246 experiment at KEK [4] performs the analysis of the data on the $K^\pm\to\mu^\pm\nu\gamma$ process to put bounds on the T-violating muon transverse polarization. It should be noted that the expected value of new physics contribution to P_T can be of the order of $\simeq 7.0\times 10^{-3}$–6.0×10^{-2} [5,6], depending on the type of model beyond the SM. Thus, when one searches for new physics contributions to P_T, it is extremely important to estimate the effects coming from so called "fake" polarization, which is caused by the SM electromagnetic final state interactions and which is a natural background for the new interaction contributions.

In this paper we calculate muon transverse polarization in the $K^\pm\to\mu^\pm\nu\gamma$ process, induced by the electromagnetic final state interaction in the one-loop approximation of the minimal quantum electrodynamics.

In next section we present the calculations of the muon transverse polarization taking into account one-loop diagrams with final state interactions within the SM. The last section summarizes the results and conclusions.

II. MUON TRANSVERSE POLARIZATION IN THE $K^+\to\mu^+\nu\gamma$ PROCESS IN SM

The $K^+\to\mu^+\nu\gamma$ decay at the tree level of SM is described by the diagrams shown in Fig. 1. The diagrams in Figs. 1(b) and 1(c) correspond to the muon and kaon bremsstrahlung, while the diagram in Fig. 1(a) corresponds to the structure radiation. This decay amplitude can be written as follows:

$$M = ie\frac{G_F}{\sqrt{2}}V_{us}^*\varepsilon_\mu^*\Bigg[f_K m_\mu \overline{u}(p_\nu)(1+\gamma_5)\Bigg(\frac{p_K^\mu}{(p_K q)} - \frac{(p_\mu)^\mu}{(p_\mu q)} - \frac{\hat{q}\gamma^\mu}{2(p_\mu q)}\Bigg)v(p_\mu) - G^{\mu\nu} l_\nu\Bigg], \quad (1)$$

where

$$l_\mu = \overline{u}(p_\nu)(1+\gamma_5)\gamma_\mu v(p_\mu),$$
$$G^{\mu\nu} = iF_v\varepsilon^{\mu\nu\alpha\beta}q_\alpha(p_K)_\beta - F_a(g^{\mu\nu}(p_K q) - p_K^\mu q^\nu), \quad (2)$$

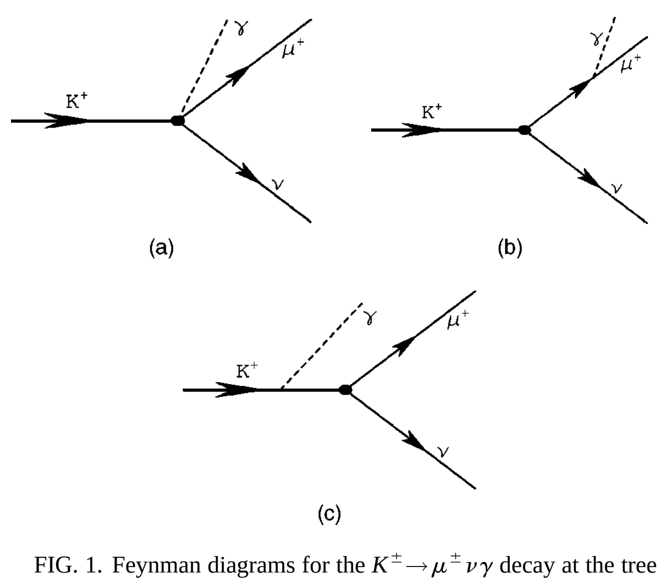

FIG. 1. Feynman diagrams for the $K^\pm\to\mu^\pm\nu\gamma$ decay at the tree level of SM.

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G_F is the Fermi constant, V_{us} is the corresponding CKM matrix element, f_K is the K-meson leptonic constant, p_K, p_μ, p_ν, q are the kaon, muon, neutrino, and photon four-momenta, correspondingly, and ε_μ is the photon polarization vector. F_v and F_a are the kaon vector and axial form factors. In Eq. (2) we use the following definition of Levi-Civita tensor: $\epsilon^{0123}=+1$.

The part of the amplitude which corresponds to the structure radiation and kaon bremsstrahlung and which will be used further in one-loop calculations, has the form

$$M_K=ie\frac{G_F}{\sqrt{2}}V_{us}^*\varepsilon_\mu^*\bigg[f_K m_\mu \overline{u}(p_\nu)(1+\gamma_5)\times\left(\frac{p_K^\mu}{(p_K q)}-\frac{\gamma^\mu}{m_\mu}\right)v(p_\mu)-G^{\mu\nu}l_\nu\bigg]. \tag{3}$$

The partial width of the $K^+\to\mu^+\nu\gamma$ decay in the K-meson rest frame can be expressed as

$$d\Gamma=\frac{\sum|M|^2}{2m_K}(2\pi)^4\delta(p_K-p_\mu-q-p_\nu)\times\frac{d^3q}{(2\pi)^3 2E_q}\frac{d^3p_\mu}{(2\pi)^3 2E_\mu}\frac{d^3p_\nu}{(2\pi)^3 2E_\nu}, \tag{4}$$

where summation over muon and photon spin states is performed.

Introducing the unit vector along the muon spin direction in muon rest frame $\vec{s}$ where $\vec{e}_i$ $(i=L,N,T)$ are the unit vectors along the longitudinal, normal and transverse components of muon polarization, one can write down the matrix element squared for the transition into the particular muon polarization state in the following form:

$$|M|^2=\rho_0[1+(P_L\vec{e}_L+P_N\vec{e}_N+P_T\vec{e}_T)\cdot\vec{s}], \tag{5}$$

where ρ_0 is the Dalitz plot probability density averaged over polarization states. The unit vectors $\vec{e}_i$ can be expressed in terms of the three-momenta of final particles

$$\vec{e}_L=\frac{\vec{p}_\mu}{|\vec{p}_\mu|},\quad \vec{e}_N=\frac{\vec{p}_\mu\times(\vec{q}\times\vec{p}_\mu)}{|\vec{p}_\mu\times(\vec{q}\times\vec{p}_\mu)|},\quad \vec{e}_T=\frac{\vec{q}\times\vec{p}_\mu}{|\vec{q}\times\vec{p}_\mu|}. \tag{6}$$

With such definition of $\vec{e}_i$ vectors, P_T, P_L, and P_N denote transverse, longitudinal, and normal components of the muon polarization, correspondingly. It is convenient to use the following variables:

$$x=\frac{2E_\gamma}{m_K},\quad y=\frac{2E_\mu}{m_K},\quad \lambda=\frac{x+y-1-r_\mu}{x},\quad r_\mu=\frac{m_\mu^2}{m_K^2}, \tag{7}$$

where E_γ and E_μ are the photon and muon energies in the kaon rest frame.

The Dalitz plot probability density, as a function of the x and y variables, has the form

$$\rho_0=\frac{1}{2}e^2G_F^2|V_{us}|^2\Bigg\{\frac{4m_\mu^2|f_K|^2}{\lambda x^2}(1-\lambda)\bigg[x^2+2(1-r_\mu)\left(1-x-\frac{r_\mu}{\lambda}\right)\bigg]+m_K^6x^2(|F_a|^2+|F_v|^2)(y-2\lambda y-\lambda x+2\lambda^2)$$
$$+4\,\mathrm{Re}(f_KF_v^*)m_K^4r_\mu\frac{x}{\lambda}(\lambda-1)$$
$$+4\,\mathrm{Re}(f_KF_a^*)m_K^4r_\mu\left(-2y+x+2\frac{r_\mu}{\lambda}-\frac{x}{\lambda}+2\lambda\right)$$
$$+2\,\mathrm{Re}(F_aF_v^*)m_K^6x^2(y-2\lambda+x\lambda)\Bigg\}. \tag{8}$$

Calculating the muon transverse polarization P_T we follow the original paper [7] and assume that the decay amplitude is CP invariant, and form factors f_K, F_v, and F_a are real. In this case the tree level muon polarization $P_T=0$. When one-loop contributions are incorporated, the nonvanishing muon transverse polarization can arise due to the interference of tree-level diagrams and imaginary parts of one-loop diagrams, induced by the electromagnetic final state interaction.

To calculate the imaginary parts of formfactors one can use the S-matrix unitarity

$$S^+S=1 \tag{9}$$

and, using $S=1+iT$, one gets

$$T_{fi}-T_{if}^*=i\sum_n T_{nf}^*T_{ni}, \tag{10}$$

where i,f,n indices correspond to the initial, final, and intermediate states of the particle system. Further, using the T invariance of the matrix element one has

$$\mathrm{Im}\,T_{fi}=\frac{1}{2}\sum_n T_{nf}^*T_{ni}, \tag{11}$$

$$T_{fi}=(2\pi)^4\delta(P_f-P_i)M_{fi}. \tag{12}$$

One-loop diagrams of the SM, which contribute to the muon transverse polarization in the $K^+\to\mu^+\nu\gamma$ decay, are shown in Fig. 2. Using Eq. (3) one can write down the imaginary parts of these diagrams. For the diagrams in Figs. 2(a), 2(c) one has

$$\mathrm{Im}\,M_1=\frac{ie\alpha}{2\pi}\frac{G_F}{\sqrt{2}}V_{us}^*\overline{u}(p_\nu)(1+\gamma_5)\times\int\frac{d^3k_\gamma}{2\omega_\gamma}\frac{d^3k_\mu}{2\omega_\mu}\delta(k_\gamma+k_\mu-P)R_\mu(\hat{k}_\mu-m_\mu)\gamma^\mu\times\frac{\hat{q}+\hat{p}_\mu-m_\mu}{(q+p_\mu)^2-m_\mu^2}\gamma^\delta\varepsilon_\delta^*v(p_\mu). \tag{13}$$

For the diagrams in Figs. 2(b), 2(d) one has

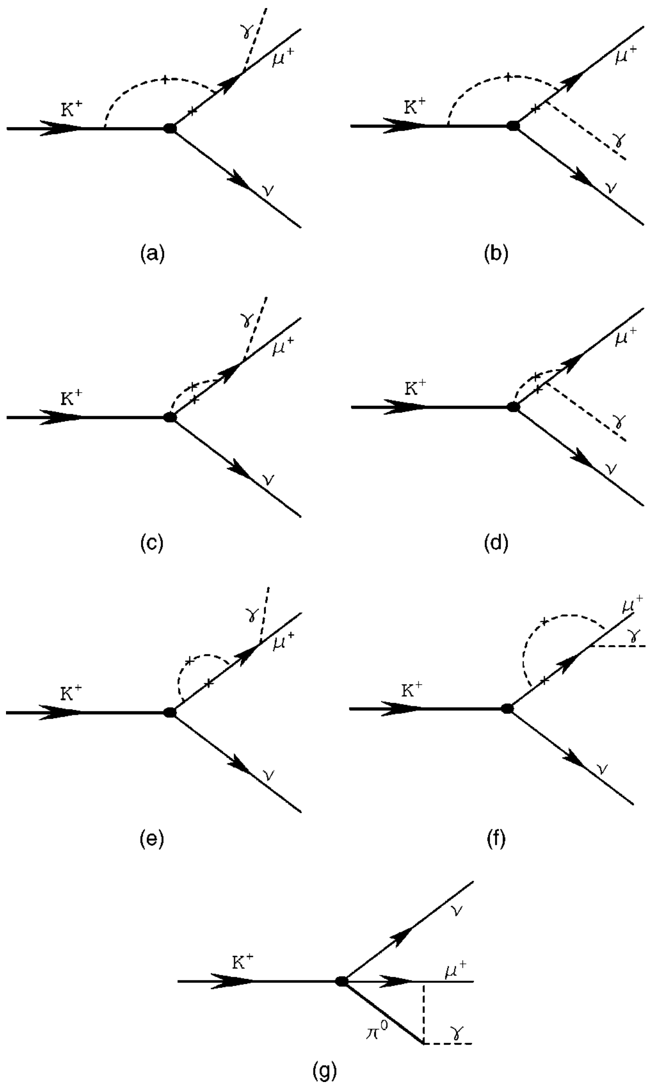

FIG. 2. Feynman diagrams contributing to the muon transverse polarization at the one-loop level of SM.

$$\mathrm{Im} M_2 = \frac{ie\alpha}{2\pi}\frac{G_F}{\sqrt{2}} V_{us}^* \overline{u}(p_\nu)(1+\gamma_5)\int \frac{d^3k_\gamma}{2\omega_\gamma}\frac{d^3k_\mu}{2\omega_\mu} \times \delta(k_\gamma+k_\mu-P)R_\mu(\hat{k}_\mu-m_\mu)\gamma^\delta \varepsilon_\delta^* \frac{\hat{k}_\mu-\hat{q}-m_\mu}{(k_\mu-q)^2-m_\mu^2} \times \gamma^\mu v(p_\mu), \tag{14}$$

where

$$R_\mu = f_K m_\mu\left(\frac{(p_K)_\mu}{(p_K k_\gamma)}-\frac{\gamma_\mu}{m_\mu}\right) - iF_v \varepsilon_{\mu\nu\alpha\beta}(k_\gamma)^\alpha (p_K)^\beta \gamma^\nu + F_a[\gamma_\mu(p_K k_\gamma)-(p_K)_\mu \hat{k}_\gamma]. \tag{15}$$

To write down the contributions of diagrams shown in Figs. 2(e), 2(f), one should substitute R_μ by

$$R_\mu = f_K m_\mu\left(\frac{\gamma_\mu}{m_\mu}-\frac{(k_\mu)_\mu}{(k_\mu k_\gamma)}-\frac{\hat{k}_\gamma\gamma_\mu}{2(k_\mu k_\gamma)}\right) \tag{16}$$

in expressions (13),(14).

Using the χ PT Lagrangian [8], one can derive decay amplitudes for the $K^+\to\pi^0\mu^+\nu$ and $\pi^0\to\gamma\gamma$ processes, which contribute to the imaginary part of the diagram in Fig. 2(g):

$$T(K^+\to\pi^0\mu^+\nu) = -\frac{G_F}{2}\overline{u}(p_\nu)(1+\gamma_5)(\hat{p}_K+\hat{p}_\pi)v(p_\mu),$$

$$T(\pi^0\to\gamma\gamma) = \frac{\alpha\sqrt{2}}{\pi F}\epsilon_{\mu\nu\lambda\sigma}k_1^\mu e_1^\nu k_2^\alpha e_2^\sigma, \tag{17}$$

where $F=132$ MeV. It should be noted that $T(\pi^0\to\gamma\gamma)$ is written at $O(p^2)$ level. In addition, the amplitude differs from the one in Ref. [8] by the sign, since we used the opposite sign of pseudoscalar octet of mesons. From Eq. (17) one can write down the imaginary part of the diagram shown in Fig. 2(g):

$$\mathrm{Im} M_3 = \frac{G_F\alpha}{8\sqrt{2}\pi^3 F} e\int \frac{d^3k_\pi}{2\omega_\pi}\frac{d^3k_\mu}{2\omega_\mu}\delta(k_\pi+k_\mu-P) \times \frac{\epsilon^{\rho\sigma\alpha\beta}q_\alpha e_\beta^* k_\rho^\pi}{k_\gamma^2}\overline{u}(p_\nu)(1+\gamma_5)(\hat{p}_K+\hat{k}_\pi) \times(\hat{k}_\mu-m_\mu)\gamma_\sigma v(p_\mu). \tag{18}$$

The details of the calculations of integrals entering Eqs. (13), (14), (18), and their dependence on kinematical parameters are given in Appendix A.

The expression for the amplitude including the imaginary one-loop contributions can be written as

$$M = ie\frac{G_F}{\sqrt{2}}V_{us}^*\varepsilon_\mu^*\left[\tilde{f}_K m_\mu \overline{u}(p_\nu)(1+\gamma_5)\left(\frac{p_K^\mu}{(p_K q)}-\frac{(p_\mu)^\mu}{(p_\mu q)}\right) \times v(p_\mu) + \tilde{F}_n \overline{u}(p_\nu)(1+\gamma_5)\hat{q}\gamma^\mu v(p_\mu) - \tilde{G}^{\mu\nu} l_\nu\right], \tag{19}$$

where

$$\tilde{G}^{\mu\nu} = i\tilde{F}_v \varepsilon^{\mu\nu\alpha\beta}q_\alpha(p_K)_\beta - \tilde{F}_a[g^{\mu\nu}(p_K q)-p_K^\mu q^\nu]. \tag{20}$$

The $\tilde{f}_K$, $\tilde{F}_v$, $\tilde{F}_a$, and $\tilde{F}_n$ form factors include one-loop contributions from diagrams shown in Figs. 2(a)–2(f). The choice of the form factors is determined by the matrix element expansion into set of gauge-invariant structures.

As long as we are interested in the contributions of imaginary parts of one-loop diagrams only (since they lead to a nonvanishing value of the transverse polarization), we neglect the real parts of these diagrams and assume that $\mathrm{Re}\tilde{f}_K, \mathrm{Re}\tilde{F}_v, \mathrm{Re}\tilde{F}_a$ coincide with their tree-level values f_K, F_v, F_a, correspondingly, and $\mathrm{Re}\tilde{F}_n = -f_K m_\mu/2(p_\mu q)$. Explicit expressions for imaginary parts of the form factors are given in Appendix B.

The muon transverse polarization can be written as

$$P_T = \frac{\rho_T}{\rho_0}, \tag{21}$$

where

$$\rho_T = 2m_K^3 e^2 G_F^2 |V_{us}|^2 x\sqrt{\lambda y - \lambda^2 - r_\mu}\Bigg[m_\mu \mathrm{Im}(\tilde{f}_K \tilde{F}_a^*) \times\left(1-\frac{2}{x}+\frac{y}{\lambda x}\right) + m_\mu \mathrm{Im}(\tilde{f}_K \tilde{F}_v^*)\left(\frac{y}{\lambda x}-1-2\frac{r_\mu}{\lambda x}\right) + 2\frac{r_\mu}{\lambda x}\mathrm{Im}(\tilde{f}_K \tilde{F}_n^*)(1-\lambda) + m_K^2 x\,\mathrm{Im}(\tilde{F}_n \tilde{F}_a^*)(\lambda-1) + m_K^2 x \mathrm{Im}(\tilde{F}_n \tilde{F}_v^*)(\lambda-1)\Bigg]. \tag{22}$$

It should be noted that Eq. (20) disagrees with the expression for ρ_T in Ref. [9]. In particular, the terms containing $\mathrm{Im}\,F_n$ are missing in the ρ_T expression given in Ref. [9]. Moreover, calculating the muon transverse polarization we took into account the diagrams shown in Figs. 2(e)–2(g), which have been neglected in Ref. [9], and which give the contribution comparable with the contribution from other diagrams in Fig. 2.

III. RESULTS AND DISCUSSION

For the numerical calculations we use the following form factor values:

$$f_K = 0.16 \text{ GeV}, \quad F_v = \frac{0.095}{m_K}, \quad F_a = -\frac{0.043}{m_K}.$$

The f_K form factor is determined from experimental data on kaon decays [10], and the F_v, F_a ones are calculated at the one loop-level in the chiral perturbation theory [11]. It should be noted that our definition for F_v differs by a sign from that in Ref. [11]. With this choice of form factor values the decay branching ratio $\mathrm{Br}(K^\pm \to \mu^\pm \nu\gamma)$, with the cut on photon energy $E_\gamma \geqslant 20$ MeV, is equal to $= 3.3\times 10^{-3}$, which is in good agreement with the PDG data.

The three-dimensional distribution of muon transverse polarization, calculated in the one-loop approximation of SM is shown in Figs. 3 and 4. P_T, as function of the x and y parameters, is characterized by the sum of individual contributions of diagrams in Figs. 2(a)–2(f), while the contributions from diagrams 2(a)–2(d) [12] and 2(e)–2(f) are comparable in absolute value, but they are opposite in sign, so that the total $P_T(x,y)$ distribution is the difference of these group contributions and in absolute value it is about one order of magnitude less than each individual one of those.

It should be noted that the value of muon transverse polarization is positive in the whole Dalitz plot region. Averaged value of transverse polarization can be obtained by integrating the function $2\rho_T/\Gamma(K^+ \to \mu^+ \nu\gamma)$ over the physical region, and with the cut on photon energy $E_\gamma > 20$ MeV it is equal to

$$\langle P_T^{SM} \rangle = 5.63\times 10^{-4}. \tag{23}$$

Let us note that the obtained numerical value of the averaged transverse polarization and $P_T(x,y)$ kinematical dependence in Dalitz plot differ from those given in Refs. [9,13].

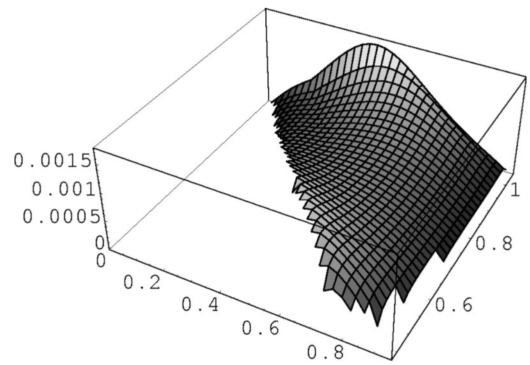

FIG. 3. The 3D Dalitz plot for the muon transverse polarization as a function of $x = 2E_\gamma/m_K$ and $y = 2E_\mu/m_K$ in the one-loop approximation of SM.

Note that in Ref. [13] only the diagram shown in Fig. 2(g) was calculated and the result for that diagram does not coincide with ours.

As it was calculated in Ref. [9], the P_T value varies in the range of $(-0.1–4.0)\times 10^{-3}$ for cuts on the muon and photon energies $200 < E_\mu < 254.5$ MeV, $20 < E_\gamma < 200$ MeV. We have already mentioned above that (1) the authors of Ref. [9] did not take into account terms containing the imaginary part of the F_n form factor (contributing to ρ_T), which, in general, are not small being compared with others, and (2) the authors of Ref. [9] omitted the diagrams, shown in Figs. 2(e)–2(f), though, as was mentioned above, their contribution to P_T is comparable with the one of diagrams in Figs. 2(a)–2(d), and (3) the authors of Ref. [9] did not take into account the diagram shown in Fig. 2(g).

All these points lead to serious disagreement between our results and results obtained in Ref. [9]. In particular, our calculations show that the value of the muon transverse polarization has positive sign in the whole Dalitz plot region and its absolute value varies in the range of (0.0–1.5) $\times 10^{-3}$, and the P_T dependence on the x,y parameters is different from that in Ref. [9].

We would like to remark that the muon transverse polarization for the same process was calculated in Ref. [14], where the contributions from diagrams 2(e), 2(f), and 2(g) were taken into account. However, our result differs from the one obtained in Ref. [14]: P_T value has opposite sign in comparison to ours and in numerical calculation the author of Ref. [14] used constant f_π instead of f_K in Eq. (1). Since

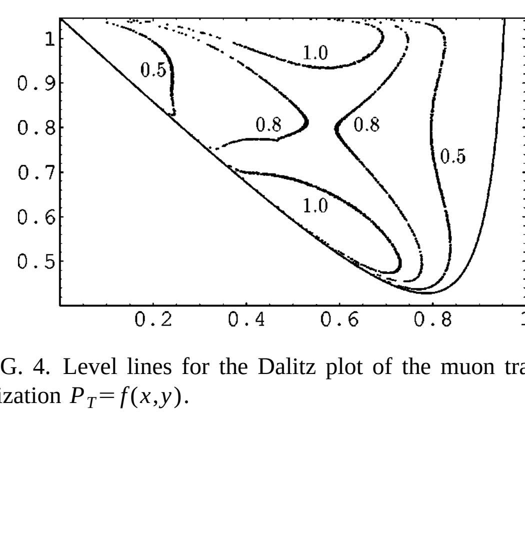

FIG. 4. Level lines for the Dalitz plot of the muon transverse polarization $P_T = f(x,y)$.

the calculation is produced at $O(p^4)$ level, one needs to use f_K, as have been done in our paper. The kinematical structures for diagrams Figs. 2(a)–2(g) in Ref. [14] coincide with ours.

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APPENDIX A

For the integrals, which contribute to Eqs. (14) and (15), we use the following notations:

$$P = p_\mu + q, \tag{A1}$$

$$d\rho = \frac{d^3k_\gamma}{2\omega_\gamma}\frac{d^3k_\mu}{2\omega_\mu}\delta(k_\gamma + k_\mu - P).$$

We present below either the explicit expressions for integrals, or the set of equations, which being solved, give the parameters, entering the integrals:

$$J_{11} = \int d\rho = \frac{\pi}{2}\frac{P^2 - m_\mu^2}{P^2},$$

$$J_{12} = \int d\rho \frac{1}{(p_K k_\gamma)} = \frac{\pi}{2I}\ln\left(\frac{(Pp_K)+I}{(Pp_K)-I}\right),$$

where

$$I^2 = (Pp_K)^2 - m_K^2 P^2,$$

$$\int d\rho \frac{k_\gamma^\alpha}{(p_K k_\gamma)} = a_{11}p_K^\alpha + b_{11}P^\alpha.$$

The a_{11} and b_{11} parameters are determined by the following equations:

$$a_{11} = -\frac{1}{(Pp_K)^2 - m_K^2 P^2}\left(P^2 J_{11} - \frac{J_{12}}{2}(Pp_K)(P^2 - m_\mu^2)\right),$$

$$b_{11} = \frac{1}{(Pp_K)^2 - m_K^2 P^2}\left((Pp_K)J_{11} - \frac{J_{12}}{2}m_K^2(P^2 - m_\mu^2)\right),$$

$$\int d\rho k_\gamma^\alpha = a_{12}P^\alpha,$$

$$\int d\rho k_\gamma^\alpha k_\gamma^\beta = a_{13}g^{\alpha\beta} + b_{13}P^\alpha P^\beta,$$

where

$$a_{12} = \frac{(P^2 - m_\mu^2)}{2P^2}J_{11},$$

$$a_{13} = -\frac{1}{12}\frac{(P^2 - m_\mu^2)^2}{P^2}J_{11},$$

$$b_{13} = \frac{1}{3}\left(\frac{P^2 - m_\mu^2}{P^2}\right)^2 J_{11}.$$

$$J_1 = \int d\rho \frac{1}{(p_K k_\gamma)[(p_\mu - k_\gamma)^2 - m_\mu^2]} = -\frac{\pi}{2I_1(P^2 - m_\mu^2)}\ln\left(\frac{(p_K p_\mu)+I_1}{(p_K p_\mu)-I_1}\right),$$

$$J_2 = \int d\rho \frac{1}{(p_\mu - k_\gamma)^2 - m_\mu^2} = -\frac{\pi}{4I_2}\ln\left(\frac{(Pp_\mu)+I_2}{(Pp_\mu)-I_2}\right),$$

where

$$I_1^2 = (p_K p_\mu)^2 - m_\mu^2 m_K^2,$$

$$I_2^2 = (Pp_\mu)^2 - m_\mu^2 P^2.$$

$$\int d\rho \frac{k_\gamma^\alpha}{(p_\mu - k_\gamma)^2 - m_\mu^2} = a_1 P^\alpha + b_1 p_\mu^\alpha,$$

$$a_1 = -\frac{m_\mu^2(P^2 - m_\mu^2)J_2 + (Pp_\mu)J_{11}}{2((Pp_\mu)^2 - m_\mu^2 P^2)},$$

$$b_1 = \frac{(Pp_\mu)(P^2 - m_\mu^2)J_2 + P^2 J_{11}}{2[(Pp_\mu)^2 - m_\mu^2 P^2]},$$

The integrals below are determined by the parameters, which can be obtained by solving the sets of equations:

$$\int d\rho \frac{k_\gamma^\alpha}{(p_K k_\gamma)[(p_\mu - k_\gamma)^2 - m_\mu^2]} = a_2 P^\alpha + b_2 p_K^\alpha + c_2 p_\mu^\alpha,$$

$$a_2(Pp_K)+b_2m_K^2+c_2(p_Kp_\mu)=J_2,\quad a_2(Pp_\mu)+b_2(p_Kp_\mu)+c_2m_\mu^2=-\frac{1}{2}J_{12},\quad a_2P^2+b_2(Pp_K)+c_2(Pp_\mu)=(p_\mu q)J_1,$$

$$\int d\rho\frac{k_\gamma^\alpha k_\gamma^\beta}{(p_Kk_\gamma)[(p_\mu-k_\gamma)^2-m_\mu^2]}=a_3g^{\alpha\beta}+b_3(P^\alpha p_K^\beta+P^\beta p_K^\alpha)+c_3(P^\alpha p_\mu^\beta+P^\beta p_\mu^\alpha)+d_3(p_K^\alpha p_\mu^\beta+p_K^\beta p_\mu^\alpha)$$
$$+e_3p_\mu^\alpha p_\mu^\beta+f_3P^\alpha P^\beta+g_3p_K^\alpha p_K^\beta,$$

$$4a_3+2b_3(Pp_K)+2c_3(Pp_\mu)+2d_3(p_Kp_\mu)+g_3m_K^2+e_3m_\mu^2+f_3P^2=0,$$
$$c_3(p_Kp_\mu)+b_3m_K^2+f_3(Pp_K)-a_1=0,\quad c_3(Pp_K)+d_3m_K^2+e_3(p_Kp_\mu)-b_1=0,\quad a_3+b_3(Pp_K)+d_3(p_Kp_\mu)+g_3m_K^2=0,$$
$$b_3(p_Kp_\mu)+c_3m_\mu^2+f_3(Pp_\mu)=-\frac{1}{2}b_{11},\quad b_3(Pp_\mu)+d_3m_\mu^2+g_3(p_Kp_\mu)=-\frac{1}{2}a_{11},$$
$$a_3P^2+2b_3P^2(Pp_K)+2c_3P^2(Pp_\mu)+2d_3(Pp_\mu)(Pp_K)+e_3(Pp_\mu)^2+f_3(P^2)^2+g_3(Pp_K)^2=(p_\mu q)^2J_1,$$

$$\int d\rho\frac{k_\gamma^\alpha k_\gamma^\beta}{(p_\mu-k_\gamma)^2-m_\mu^2}=a_4g_{\alpha\beta}+b_4(P^\alpha p_\mu^\beta+P^\beta p_\mu^\alpha)+c_4P^\alpha P^\beta+d_4p_\mu^\alpha p_\mu^\beta,$$

$$a_4+d_4m_\mu^2+b_4(Pp_\mu)=0,\quad b_4m_\mu^2+c_4(Pp_\mu)=-\frac{1}{2}a_{12},\quad 4a_4+2b_4(Pp_\mu)+c_4P^2+d_4m_\mu^2=0,$$
$$a_4P^2+2b_4P^2(Pp_\mu)+c_4(P^2)^2+d_4(Pp_\mu)^2=\frac{(P^2-m_\mu^2)^2}{4}J_2,$$

$$\int d\rho\frac{k_\gamma^\alpha k_\gamma^\beta k_\gamma^\delta}{(p_\mu-k_\gamma)^2-m_\mu^2}=a_5(g^{\alpha\beta}p_\mu^\delta+g^{\delta\alpha}p_\mu^\beta+g^{\beta\delta}p_\mu^\alpha)+b_5(g^{\alpha\beta}P^\delta+g^{\delta\alpha}P^\beta+g^{\beta\delta}P^\alpha)+c_5p_\mu^\alpha p_\mu^\beta p_\mu^\delta+d_5P^\alpha P^\beta P^\delta$$
$$+e_5(P^\alpha p_\mu^\beta p_\mu^\delta+P^\delta p_\mu^\alpha p_\mu^\beta+P^\beta p_\mu^\delta p_\mu^\alpha)+f_5(P^\alpha P^\beta p_\mu^\delta+P^\delta P^\alpha p_\mu^\beta+P^\beta P^\delta p_\mu^\alpha),$$

$$2a_5+c_5m_\mu^2+e_5(Pp_\mu)=0,\quad a_5m_\mu^2+b_5(Pp_\mu)=-\frac{1}{2}a_{13},\quad b_5+e_5m_\mu^2+f_5(Pp_\mu)=0,$$
$$d_5(Pp_\mu)+f_5m_\mu^2=-\frac{1}{2}b_{13},\quad 6a_5+c_5m_\mu^2+2e_5(Pp_\mu)+f_5P^2=0,$$
$$3a_5P^2(Pp_\mu)+3b_5(P^2)^2+c_5(Pp_\mu)^3+d_5(P^2)^3+3e_5P^2(Pp_\mu)^2+3f_5(P^2)^2(Pp_\mu)=\frac{(P^2-m_\mu^2)^3}{8}J_2.$$

For the rest of the integrals the following notations are used:

$$Pk_\pi=\frac{1}{2}(P^2+m_\pi^2-m_\mu^2),\quad d\rho=\frac{d^3k_\pi}{2\omega_\pi}\frac{d^3k_\mu}{2\omega_\mu}\delta(k_\pi+k_\mu-P).$$

In terms of this notation the integrals can be rewritten as follows:

$$J_3=\int\frac{d\rho}{k_\gamma^2}=-\frac{\pi}{4Pq}\ \mathrm{Ln}\left|\frac{2(Pq)Pk_\pi+2(Pq)\sqrt{Pk_\pi^2-m_\pi^2P^2}-m_\pi^2P^2}{2(Pq)Pk_\pi-2(Pq)\sqrt{Pk_\pi^2-m_\pi^2P^2}-m_\pi^2P^2}\right|,\quad J_4=\int d\rho=\frac{\pi}{P^2}\sqrt{(Pk_\pi)^2-m_\pi^2P^2}.$$

APPENDIX B

Here we present the expressions for imaginary parts of form factors as the functions of parameters, calculated in Appendix A.

$$\mathrm{Im}\,\tilde{f}_K=\frac{\alpha}{2\pi}f_K[-4a_3(p_Kq)+4a_2m_\mu^2(p_Kq)-2b_3m_\mu^2(p_Kq)+4c_2m_\mu^2(p_Kq)-4c_3m_\mu^2(p_Kq)-2d_3m_\mu^2(p_Kq)-2e_3m_\mu^2(p_Kq)$$
$$-2f_3m_\mu^2(p_Kq)+4a_2(p_Kq)(p_\mu q)-4b_3(p_Kq)(p_\mu q)-4c_3(p_Kq)(p_\mu q)-4f_3(p_Kq)(p_\mu q)]+\frac{\alpha}{2\pi}F_a[8a_4(p_Kq)$$
$$-8a_5(p_Kq)-8b_5(p_Kq)+8b_4m_\mu^2(p_Kq)+4c_4m_\mu^2(p_Kq)-2c_5m_\mu^2(p_Kq)+4d_4m_\mu^2(p_Kq)-2d_5m_\mu^2(p_Kq)$$
$$-6e_5m_\mu^2(p_Kq)-6f_5m_\mu^2(p_Kq)+12b_4(p_Kq)(p_\mu q)+8c_4(p_Kq)(p_\mu q)+4d_4(p_Kq)(p_\mu q)-4d_5(p_Kq)(p_\mu q)$$
$$-4e_5(p_Kq)(p_\mu q)-8f_5(p_Kq)(p_\mu q)]+\frac{\alpha}{2\pi}F_v[8a_4(p_Kq)-8a_5(p_Kq)-8b_5(p_Kq)+8b_4m_\mu^2(p_Kq)+4c_4m_\mu^2(p_Kq)$$
$$-2c_5m_\mu^2(p_Kq)+4d_4m_\mu^2(p_Kq)-2d_5m_\mu^2(p_Kq)-6e_5m_\mu^2(p_Kq)-6f_5m_\mu^2(p_Kq)+12b_4(p_Kq)(p_\mu q)+8c_4(p_Kq)(p_\mu q)$$
$$+4d_4(p_Kq)(p_\mu q)-4d_5(p_Kq)(p_\mu q)-4e_5(p_Kq)(p_\mu q)-8f_5(p_Kq)(p_\mu q)],$$

$$\mathrm{Im}\,\tilde{F}_a=\frac{\alpha}{2\pi}f_K\left(a_2m_\mu^2+2c_2m_\mu^2-c_3m_\mu^2-2d_3m_\mu^2-e_3m_\mu^2-\frac{a_1m_\mu^2}{(p_\mu q)}-\frac{b_1m_\mu^2}{(p_\mu q)}+\frac{2b_4m_\mu^2}{(p_\mu q)}+\frac{c_4m_\mu^2}{(p_\mu q)}+\frac{d_4m_\mu^2}{(p_\mu q)}\right)+\frac{\alpha}{2\pi}F_v[8a_4-4a_5$$
$$-12b_5-2a_1m_\mu^2+4b_4m_\mu^2+5c_4m_\mu^2-c_5m_\mu^2-d_4m_\mu^2-3d_5m_\mu^2-5e_5m_\mu^2-7f_5m_\mu^2+2a_1(p_Kp_\mu)-4b_4(p_Kp_\mu)$$
$$-4c_4(p_Kp_\mu)+2d_5(p_Kp_\mu)+2e_5(p_Kp_\mu)+4f_5(p_Kp_\mu)+2a_1(p_Kq)-2b_4(p_Kq)-4c_4(p_Kq)+2d_5(p_Kq)+2f_5(p_Kq)$$
$$-4a_1(p_\mu q)+6b_4(p_\mu q)+10c_4(p_\mu q)-6d_5(p_\mu q)-2e_5(p_\mu q)-8f_5(p_\mu q)]+\frac{\alpha}{2\pi}F_a[-6a_4+2a_5+c_4m_\mu^2-d_4m_\mu^2$$
$$-d_5m_\mu^2-e_5m_\mu^2-2f_5m_\mu^2+2a_1(p_Kp_\mu)-4b_4(p_Kp_\mu)-4c_4(p_Kp_\mu)+2d_5(p_Kp_\mu)+2e_5(p_Kp_\mu)+4f_5(p_Kp_\mu)$$
$$+2a_1(p_Kq)-2b_4(p_Kq)-4c_4(p_Kq)+2d_5(p_Kq)+2f_5(p_Kq)+2c_4(p_\mu q)-2d_5(p_\mu q)-2f_5\ (p_\mu q)].$$

$$\mathrm{Im}\,\tilde{F}_n=\frac{\alpha}{2\pi}f_K\Bigg[4a_1m_\mu+2a_3m_\mu+2b_1m_\mu+b_{11}m_\mu-2b_4m_\mu-2c_4m_\mu-J_{12}m_\mu-2J_2m_\mu-b_2m_K^2m_\mu+g_3m_K^2m_\mu-2a_2m_\mu^3$$
$$-c_2m_\mu^3+c_3m_\mu^3+f_3m_\mu^3-2a_2m_\mu(p_Kp_\mu)-2b_2m_\mu(p_Kp_\mu)+2b_3m_\mu(p_Kp_\mu)-2c_2m_\mu(p_Kp_\mu)+2d_3m_\mu(p_Kp_\mu)$$
$$+2J_1m_\mu(p_Kp_\mu)+2b_3m_\mu(p_Kq)-\frac{a_{12}m_\mu^3}{(p_\mu q)^2}-\frac{J_{11}m_\mu^3}{(p_\mu q)^2}-\frac{a_{12}m_\mu}{(p_\mu q)}-\frac{2a_4m_\mu}{(p_\mu q)}+\frac{J_{11}m_\mu}{(p_\mu q)}-\frac{a_{11}m_K^2m_\mu}{2(p_\mu q)}+\frac{3a_1m_\mu^3}{(p_\mu q)}+\frac{3b_1m_\mu^3}{(p_\mu q)}$$
$$+\frac{b_{11}m_\mu^3}{2(p_\mu q)}-\frac{2J_2m_\mu^3}{(p_\mu q)}-\frac{b_{11}m_\mu(p_Kp_\mu)}{(p_\mu q)}+\frac{J_{12}m_\mu(p_Kp_\mu)}{(p_\mu q)}-\frac{b_{11}m_\mu(p_Kq)}{(p_\mu q)}+\frac{J_{12}m_\mu(p_Kq)}{(p_\mu q)}-2a_2m_\mu(p_\mu q)+2c_3m_\mu(p_\mu q)$$
$$+2f_3m_\mu(p_\mu q)\Bigg]+\frac{\alpha}{2\pi}F_v\Big(2a_4m_\mu-4a_5m_\mu+2b_{13}m_\mu-4b_5m_\mu-2a_1m_\mu^3+c_4m_\mu^3-c_5m_\mu^3-d_4m_\mu^3-d_5m_\mu^3-3e_5m_\mu^3$$
$$-3f_5m_\mu^3+2a_1m_\mu(p_Kp_\mu)-2c_4m_\mu(p_Kp_\mu)+2d_4m_\mu(p_Kp_\mu)+2d_5m_\mu(p_Kp_\mu)+2e_5m_\mu(p_Kp_\mu)+4f_5m_\mu(p_Kp_\mu)$$
$$-2c_4m_\mu(p_Kq)+2d_5m_\mu(p_Kq)+2f_5m_\mu(p_Kq)+\frac{3a_{13}m_\mu}{(p_\mu q)}+\frac{b_{13}m_\mu^3}{(p_\mu q)}-\frac{b_{13}m_\mu(p_Kp_\mu)}{(p_\mu q)}-\frac{b_{13}m_\mu(p_Kq)}{(p_\mu q)}-2a_1m_\mu(p_\mu q)$$
$$+2c_4m_\mu(p_\mu q)-2d_4m_\mu(p_\mu q)-2d_5m_\mu(p_\mu q)-2e_5m_\mu(p_\mu q)-4f_5m_\mu(p_\mu q)\Big)+\frac{\alpha}{2\pi}F_a\Big(-6a_4m_\mu+8a_5m_\mu-b_{13}m_\mu$$
$$+8b_5m_\mu-4b_4m_\mu^3-2c_4m_\mu^3+c_5m_\mu^3-2d_4m_\mu^3+d_5m_\mu^3+3e_5m_\mu^3+3f_5m_\mu^3+2a_1m_\mu(p_Kp_\mu)-2c_4m_\mu(p_Kp_\mu)$$
$$+2d_4m_\mu(p_Kp_\mu)+2d_5m_\mu(p_Kp_\mu)+2e_5m_\mu(p_Kp_\mu)+4f_5m_\mu(p_Kp_\mu)-2c_4m_\mu(p_Kq)+2d_5m_\mu(p_Kq)+2f_5m_\mu(p_Kq)$$
$$-\frac{3a_{13}m_\mu}{(p_\mu q)}-\frac{b_{13}m_\mu^3}{2(p_\mu q)}-\frac{b_{13}m_\mu(p_Kp_\mu)}{(p_\mu q)}-\frac{b_{13}m_\mu(p_Kq)}{(p_\mu q)}-6b_4m_\mu(p_\mu q)-4c_4m_\mu(p_\mu q)-2d_4m_\mu(p_\mu q)+2d_5m_\mu(p_\mu q)$$
$$+2e_5m_\mu(p_\mu q)+4f_5m_\mu(p_\mu q)\Big),$$

$$
\begin{aligned}
\mathrm{Im}\,\tilde{F}_v=&\frac{\alpha}{2\pi}f_K\left(a_2m_\mu^2+c_3m_\mu^2+e_3m_\mu^2+\frac{a_1m_\mu^2}{(p_\mu q)}+\frac{b_1m_\mu^2}{(p_\mu q)}-\frac{2b_4m_\mu^2}{(p_\mu q)}-\frac{c_4m_\mu^2}{(p_\mu q)}-\frac{d_4m_\mu^2}{(p_\mu q)}\right)+\frac{\alpha}{2\pi}F_a[6a_4-2a_5-8b_5+c_4m_\mu^2\\
&-d_4m_\mu^2-d_5m_\mu^2-e_5m_\mu^2-2f_5m_\mu^2-2a_1(p_Kp_\mu)+4b_4(p_Kp_\mu)+4c_4(p_Kp_\mu)-2d_5(p_Kp_\mu)-2e_5(p_Kp_\mu)-4f_5(p_Kp_\mu)\\
&-2a_1(p_Kq)+2b_4(p_Kq)+4c_4(p_Kq)-2d_5(p_Kq)-2f_5(p_Kq)+2c_4(p_\mu q)-2d_5(p_\mu q)-2f_5(p_\mu q)]+\frac{\alpha}{2\pi}F_v[-8a_4\\
&+4a_5+4b_5+2a_1m_\mu^2-4b_4m_\mu^2-3c_4m_\mu^2+c_5m_\mu^2-d_4m_\mu^2+d_5m_\mu^2+3e_5m_\mu^2+3f_5m_\mu^2-2a_1(p_Kp_\mu)+4b_4(p_Kp_\mu)\\
&+4c_4(p_Kp_\mu)-2d_5(p_Kp_\mu)-2e_5(p_Kp_\mu)-4f_5(p_Kp_\mu)-2a_1(p_Kq)+2b_4(p_Kq)+4c_4(p_Kq)-2d_5(p_Kq)-2f_5(p_Kq)\\
&+4a_1(p_\mu q)-6b_4(p_\mu q)-6c_4(p_\mu q)+2d_5(p_\mu q)+2e_5(p_\mu q)+4f_5(p_\mu q)].
\end{aligned}
$$

The contribution to imaginary parts coming from the diagram shown in Fig. 2(g) may be written as follows:

$$\mathrm{Im}\ \tilde{f}_K=0,$$

$$\mathrm{Im}\,\tilde{F}_v=-\frac{\alpha}{8\pi^3F}\left(\frac{3J_4m_\mu^2}{4P^2}-\frac{J_4m_\mu^4m_\pi^2}{8(p_\mu q)^2P^2}+\frac{J_3m_\mu^4m_\pi^4}{8(p_\mu q)^2P^2}-\frac{3J_4m_\mu^2m_\pi^2}{8(p_\mu q)P^2}+\frac{J_3m_\mu^2m_\pi^4}{4(p_\mu q)P^2}+\frac{2J_4(p_\mu q)}{P^2}\right)\theta[P^2-(m_\mu+m_\pi)^2],$$

$$\mathrm{Im}\tilde{F}_a=-\mathrm{Im}\tilde{F}_v\,,$$

$$\mathrm{Im}\,\tilde{f}_n=-\frac{\alpha}{8\pi^3F}\left(\frac{-3J_4m_\mu m_\pi^2}{2P^2}+\frac{J_3m_\mu m_\pi^4}{P^2}-\frac{J_4m_\mu^5m_\pi^2}{4(p_\mu q)^2P^2}+\frac{J_3m_\mu^5m_\pi^4}{4(p_\mu q)^2P^2}-\frac{5J_4m_\mu^3m_\pi^2}{4(p_\mu q)P^2}+\frac{J_3m_\mu^3m_\pi^4}{(p_\mu q)P^2}\right)\theta[P^2-(m_\mu+m_\pi)^2].$$

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