Chiral corrections to baryon properties with composite pions

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A calculational scheme is developed to evaluate chiral corrections to properties of composite baryons with composite pions. The composite baryons and pions are bound states derived from a microscopic chiral quark model. The model is amenable to standard many-body techniques such as the BCS and random phase approximation formalisms. An effective chiral model involving only hadronic degrees of freedom is derived from the macroscopic quark model by projection onto hadron states. Chiral loops are calculated using the effective hadronic Hamiltonian. A simple microscopic confining interaction is used to illustrate the derivation of the pion-nucleon form factor and the calculation of pionic self-energy corrections to the nucleon and $\Delta(1232)$ masses.

I. INTRODUCTION

The incorporation of chiral symmetry in quark models is an important issue in hadronic physics. The subject dates back to the early works [1–6] aimed at restoring chiral symmetry to the MIT bag model [7]. The early attempts were based on coupling elementary pion fields directly to quarks. A great variety of chiral quark-pion models have been constructed since then and the subject continues to be of interest in the recent literature [8–10]. Despite the long history, there are many important open questions in this field. In the present paper, we are concerned with one of such questions, namely the coupling of the pion as a quark-antiquark bound state to the baryons. Starting from a model chiral quark Hamiltonian, we construct an effective low-energy chiral pion-baryon Hamiltonian appropriate for calculating chiral loop corrections to hadron properties. The composite pion and baryon states are determined by the same underlying quark chiral dynamics.

The model we use belongs to a class of quark models inspired in the Coulomb gauge QCD Hamiltonian [11] and generalizes the Nambu–Jona-Lasinio model [12] to include confinement and asymptotic freedom. This class of models is amenable to standard many-body techniques such as the BCS formalism of superconductivity. The initial studies within these models were aimed at studying the interplay between confinement and dynamical chiral symmetry breaking (DχSB), and concentrated on critical couplings [13] for DχSB and light-meson spectroscopy [14]. The model has been extended to study the pion beyond BCS level and meson resonant decays in the context of a generalized resonating group method [15]. Since the model is formulated on the basis of a Hamiltonian, it provides a natural way to study finite temperature and chemical potential quark matter [16]. The model and the many-body techniques to solve it make direct contact with first-principle developments such as non-perturbative renormalization-group treatments of the QCD Hamiltonian [17] and Hamiltonian lattice QCD [18].

One important development of the model, in the context of the present paper, was its extension in Ref. [19] to baryon structure. In Ref. [19] a variational calculation was implemented for the masses of baryons and it was shown that a sizeable $\Delta(1232)$-N mass difference is obtained from the same underlying hyperfine interaction that gives a reasonable value for the π-ρ mass difference. This hyperfine interaction, along with other spin-dependent interactions such as tensor and spin-orbit, stem from Bogoliubov-Valatin rotated spinors that depend on the "chiral angle." The chiral angle gives the extent of the chiral condensation in the vacuum and determines the chiral condensate. The very same variational wave function was used later for studying S-wave kaon-nucleon [20] scattering and the repulsive core of the nucleon-nucleon force [21]. Both calculations obtain S-wave phase shifts that compare reasonably well with experimental data. A remarkable feature of all these results for the low-lying spectrum of mesons and baryons and S-wave scattering phase shifts is that they are obtained with a single free parameter, the strength of the confining potential.

In the present paper we go one step forward in the development of the model and set up a calculational scheme to treat chiral corrections in hadron spectroscopy. In a recent publication [22], two of us have calculated the pion-nucleon coupling constant in this model and obtained reasonable agreement with its experimental value. Here, we are interested in developing a scheme for calculating chiral corrections to hadron properties. We study the requirements to obtain in the context of the model the correct leading nonanalytic behavior (LNA) of chiral loops. Our scheme follows the standard practice [6,23] of constructing an effective baryon-pion Hamiltonian by projecting the quark Hamiltonian onto a Fock-space basis of single composite hadronic states. Chiral loop corrections are then calculated with the

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effective Hamiltonian in time-ordered perturbation theory. The difference here is that while in the previous works the pion is an elementary particle, in our approach the pions are composites described by a Salpeter amplitude.

A difficulty appears in the implementation of the projection of the microscopic quark Hamiltonian onto the composite hadron states, which is not present when the pion is treated as an elementary particle. The difficulty is related to the two-component nature of the Salpeter amplitude of the pion. The two components correspond to positive and negative energies (forward and backward moving, in the language of of time-ordered perturbation theory), they are 2×2 matrices in spin space and are called energy-spin (E-spin) wave functions. For the pion, the negative-energy component is as important as the positive-energy one—in the chiral limit they are equal—because of the Goldstone-boson nature of the pion. Because of this, the Fock-space representation of the pion state is not simple. We overcome the difficulty by rephrasing the formalism of the Salpeter equation in terms of the random-phase approximation (RPA) equations of many-body theory. The single-pion state is obtained in terms of a creation operator acting on the RPA vacuum. The pion creation operator is a linear combination of creation and annihilation operators of pairs of quark-antiquark operators; the positive-energy Salpeter component comes with the creation operator of the quark-antiquark pair, and the negative-energy one comes with the annihilation operator of the quark-antiquark pair. In this way, the projection of the microscopic quark Hamiltonian onto the single hadron states becomes feasible and simple.

The paper is organized as follows. In the next section we review the basic equations of the model. We show the relationship of the formalisms of the Salpeter equation and of the many-body technique of the RPA. In Sec. III we derive the pion-baryon vertex function in terms of the bound-state Salpeter amplitudes for the pion and the baryon. We obtain an expression that is valid for a general microscopic quark interaction, not restricted to a specific form of the potential. Given the pion-baryon vertex function, we derive the expression for the baryon self-energy correction in Sec. IV. Although the derivation of the expression for the self-energy is well-known in the literature, we repeat it here to make the paper easier to read. In Sec. V we obtain numerical results for the pion-baryon form factor and coupling constants. Numerical results and the discussion of the LNA contributions to the baryon masses are presented in Sec. VI. Section VII presents our conclusions and future directions.

II. THE MODEL

The Hamiltonian of the model is of the general form

$$H = H_0 + H_I, \tag{1}$$

where H_0 is the Dirac Hamiltonian

$$H_0 = \int d\boldsymbol{x}\, \psi^\dagger(\boldsymbol{x})(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m_q)\psi(\boldsymbol{x}), \tag{2}$$

with $\psi(\boldsymbol{x})$ the Dirac field operator, and H_I a chirally symmetric four-fermion interaction

$$H_I = \frac{1}{2}\int d\boldsymbol{x}\int d\boldsymbol{y}\, \psi^\dagger(\boldsymbol{x})T^a\Gamma\psi(\boldsymbol{x})V_\Gamma(\boldsymbol{x}-\boldsymbol{y})\psi^\dagger(\boldsymbol{y})T^a\Gamma\psi(\boldsymbol{y}). \tag{3}$$

Here, $T^a = 1/2\lambda^a, a = 1, \ldots, 8$ are the generators of the color SU(3) group, Γ is one, or a combination of Dirac matrices, and V_Γ contains a confining interaction and other spin-dependent interactions. One example of V_Γ will be presented in Sec. V, when we make a numerical application of the formalism.

Once the model Hamiltonian is specified, the next step consists in constructing an explicit but approximate vacuum state of the Hamiltonian in the form of a pairing ansatz. This is most easily implemented in the form of a Bogoliubov-Valatin transformation (BVT). The transformation depends on a pairing function, or chiral angle φ that determines the strength of the pairing in the vacuum. The quark field operator is expanded as

$$\psi(\boldsymbol{x}) = \int \frac{d\boldsymbol{q}}{(2\pi)^{3/2}}[u_s(\boldsymbol{q})b(\boldsymbol{q}) + v_s(\boldsymbol{q})d^\dagger(-\boldsymbol{q})]e^{i\boldsymbol{q}\cdot\boldsymbol{x}}, \tag{4}$$

where the quark and antiquark annihilation operators b and d annihilate the paired vacuum, or BCS state $|0_{\text{BCS}}\rangle$. Here the spinors $u_s(\boldsymbol{q})$ and $v_s(\boldsymbol{q})$ depend upon the chiral angle φ as

$$u_s(\boldsymbol{q}) = \frac{1}{\sqrt{2}}\{[1+\sin\varphi(q)]^{1/2} + [1-\sin\varphi(q)]^{1/2}\boldsymbol{\alpha}\cdot\hat{\boldsymbol{q}}\}u_s^0, \tag{5}$$

$$v_s(\boldsymbol{q}) = \frac{1}{\sqrt{2}}\{[1+\sin\varphi(q)]^{1/2} - [1-\sin\varphi(q)]^{1/2}\boldsymbol{\alpha}\cdot\hat{\boldsymbol{q}}\}v_s^0, \tag{6}$$

where u_s^0 and v_s^0 are the spinor eigenvectors of Dirac matrix $\gamma^0 = \beta$ with eigenvalues ± 1, respectively.

The chiral angle can be determined from the minimization of the vacuum energy density

$$\frac{E_{\text{vac}}}{V} = \int \frac{d\boldsymbol{q}}{(2\pi)^3}\text{Tr}[\boldsymbol{\alpha}\cdot\boldsymbol{q}\,\Lambda^-(\boldsymbol{q})] + \frac{1}{2}\int \frac{d\boldsymbol{q}}{(2\pi)^3}\frac{d\boldsymbol{q}'}{(2\pi)^3}$$
$$\times \tilde{V}(\boldsymbol{q}-\boldsymbol{q}')\text{Tr}[T^a\Gamma\Lambda^+(\boldsymbol{q})T^a\Gamma\Lambda^-(\boldsymbol{q}')], \tag{7}$$

where Tr is the trace over color, flavor, and Dirac indices, and

$$\Lambda^+(\boldsymbol{q}) = \sum_s u_s(\boldsymbol{q})u_s^\dagger(\boldsymbol{q}), \quad \Lambda^-(\boldsymbol{q}) = \sum_s v_s(\boldsymbol{q})v_s^\dagger(\boldsymbol{q}). \tag{8}$$

The minimization of the vacuum energy leads to the gap equation

$$A(q)\cos\varphi(q) - B(q)\sin\varphi(q) = 0, \tag{9}$$

with

$$A(q) = m + \frac{1}{2} \int \frac{d\boldsymbol{q}'}{(2\pi)^3} \tilde{V}_\Gamma(\boldsymbol{q}-\boldsymbol{q}') \mathrm{Tr}[\beta T^a \Gamma (\Lambda^+(\boldsymbol{q}')$$
$$- \Lambda^-(\boldsymbol{q})) T^a \Gamma], \tag{10}$$

$$B(q) = q + \frac{1}{2} \int \frac{d\boldsymbol{q}'}{(2\pi)^3} \tilde{V}_\Gamma(\boldsymbol{q}-\boldsymbol{q}') \mathrm{Tr}[\boldsymbol{\alpha} \cdot \hat{\boldsymbol{q}} T^a \Gamma (\Lambda^+(\boldsymbol{q}')$$
$$- \Lambda^-(\boldsymbol{q})) T^a \Gamma], \tag{11}$$

where $\tilde{V}_\Gamma(\boldsymbol{q})$ is the Fourier transform of $V_\Gamma(\boldsymbol{x})$,

$$\tilde{V}_\Gamma(\boldsymbol{q}) = \int d\boldsymbol{x} \, e^{i\boldsymbol{q}\cdot\boldsymbol{x}} V_\Gamma(\boldsymbol{x}). \tag{12}$$

The pion bound-state equation is given by the field-theoretic Salpeter equation. The wave function has two components ϕ^+ and ϕ^-, the positive- and negative-energy components. Each of the ϕ's is a 2×2 matrix in spin space $\phi^+_{s_1,s_2}$ and $\phi^-_{s_1,s_2}$. For this reason, the ϕ's are also called energy-spin (E-spin) wave functions. The ϕ's satisfy the following coupled integral equations:

$$[M(\boldsymbol{q}) - E(\boldsymbol{q}_+) - E(\boldsymbol{q}_-)]\phi^+_k(\boldsymbol{q}) = u^\dagger(\boldsymbol{q}_+) K_\phi(\boldsymbol{k},\boldsymbol{q}) v(\boldsymbol{q}_-), \tag{13}$$

$$[M(\boldsymbol{q}) + E(\boldsymbol{q}_+) + E(\boldsymbol{q}_-)]\phi^{-T}_k(\boldsymbol{q}) = v^\dagger(\boldsymbol{q}_+) K_\phi(\boldsymbol{k},\boldsymbol{q}) u(\boldsymbol{q}_-), \tag{14}$$

where $\boldsymbol{q}_\pm = \boldsymbol{q} \pm \boldsymbol{k}/2$, and the kernel $K_\phi(\boldsymbol{k},\boldsymbol{q})$ is given by

$$K_\phi(\boldsymbol{k},\boldsymbol{q}) = \int \frac{d\boldsymbol{q}'}{(2\pi)^3} \tilde{V}_\Gamma(\boldsymbol{q}-\boldsymbol{q}') T^a \Gamma[u(\boldsymbol{q}'_+)\phi^+_k(\boldsymbol{q}')v^\dagger(\boldsymbol{k}'_-)$$
$$+ v(\boldsymbol{k}'_+)\phi^{-T}_k(\boldsymbol{q}')u^\dagger(\boldsymbol{q}'_-)]T^a \Gamma. \tag{15}$$

Here, the superscript T on ϕ^{-T} means spin transpose of ϕ^-, $\phi^{-T}_{s_1,s_2} = \phi^-_{s_2,s_1}$. The amplitudes are normalized as

$$\int \frac{d\boldsymbol{q}}{(2\pi)^3}[\phi^{+*}_k(\boldsymbol{q})\phi^+_{k'}(\boldsymbol{q}) - \phi^{-*}_k(\boldsymbol{q})\phi^-_{k'}(\boldsymbol{q})] = \delta(\boldsymbol{k}-\boldsymbol{k}'). \tag{16}$$

In the language of many-body theory, these equations can be identified with the RPA equations [24]. In the RPA formulation, one writes for the one-pion state (suppressing isospin quantum numbers)

$$|\pi(\boldsymbol{k})\rangle = M^\dagger_\pi(\boldsymbol{k})|0_{\mathrm{RPA}}\rangle, \tag{17}$$

where $|0_{\mathrm{RPA}}\rangle$ is the RPA vacuum, which contains correlations beyond the BCS pairing vacuum $|0_{\mathrm{BCS}}\rangle$, and $M^\dagger_\pi(\boldsymbol{k})$ is the pion creation operator

$$M^\dagger_\pi(\boldsymbol{k}) = \int \frac{d\boldsymbol{q}}{(2\pi)^{3/2}}[\phi^+_k(\boldsymbol{q})b^\dagger(\boldsymbol{q}_+)d^\dagger(\boldsymbol{q}_-)$$
$$- \phi^-_k(\boldsymbol{q})b(\boldsymbol{q}_+)d(\boldsymbol{q}_-)], \tag{18}$$

where the $\boldsymbol{q}_\pm$ were defined above. In this formulation, Eqs. (13) and (14) above are obtained from the RPA equation of motion

$$\langle\pi(\boldsymbol{k})|[H,M^\dagger_\pi]|0_{\mathrm{RPA}}\rangle = [E_\pi(\boldsymbol{k}) - E_{\mathrm{vac}}]\langle\pi(\boldsymbol{k})|M^\dagger|0_{\mathrm{RPA}}\rangle. \tag{19}$$

The normalization is such that

$$\langle\pi(\boldsymbol{k})|\pi(\boldsymbol{k}')\rangle = \langle 0_{\mathrm{RPA}}|[M_\pi(\boldsymbol{k}),M^\dagger_\pi(\boldsymbol{k}')]|0_{\mathrm{RPA}}\rangle = \delta(\boldsymbol{k}-\boldsymbol{k}'). \tag{20}$$

The verification of DχSB consists in finding nontrivial solutions to the gap equation, Eq. (9), and the existence of solutions for the pion wave function. References [11,15] show that for a confining force, there is always a nontrivial solution for the gap and pion Salpeter equations. Moreover, good numerical values are obtained for the chiral parameters such as the pion decay constant and the chiral condensate when an appropriate spin-dependent potential is used [25].

The inclusion of RPA correlations beyond BCS pairing was shown in Ref. [24] to have a dramatic effect on the mass spectrum of the pseudoscalar mesons (π and η), while it has almost no effect on the mass spectrum of vector mesons (ρ and ω). One can trace this effect to the fact that the pseudo scalar mesons have a sizeable "negative energy" component wave function, while the vector mesons have a very small negative energy component [15]. For baryons (such as nucleon and Δ), since they do not have a sizeable negative energy component [19], one expects that they can be reliably obtained from the BCS vacuum. We write therefore for the one-baryon state

$$|B^{(0)}(\boldsymbol{p})\rangle = B^{(0)\dagger}(\boldsymbol{p})|0_{\mathrm{BCS}}\rangle, \tag{21}$$

where the baryon creation operator $B^{(0)\dagger}(\boldsymbol{p})$ is given by

$$B^{(0)\dagger}(\boldsymbol{p}) = \int d\boldsymbol{q}_1 d\boldsymbol{q}_1 d\boldsymbol{q}_1 \delta(\boldsymbol{p}-\boldsymbol{q}_1-\boldsymbol{q}_2-\boldsymbol{q}_3)$$
$$\times \Psi_p(\boldsymbol{q}_1\boldsymbol{q}_2\boldsymbol{q}_3)\epsilon^{c_1c_2c_3}\chi^{f_1f_2f_3s_1s_2s_3}$$
$$\times b^\dagger_{c_1s_1f_1}(\boldsymbol{q}_1)b^\dagger_{c_2s_2f_2}(\boldsymbol{q}_2)b^\dagger_{c_3s_3f_3}(\boldsymbol{q}_3). \tag{22}$$

Here $\epsilon^{c_1c_2c_3}$ is the Levi-Civita tensor, which guarantees that the baryon is a color singlet and $\chi^{f_1f_2f_3s_1s_2s_3}$ are the spin-isospin coefficients. The wave function $\Psi_p(\boldsymbol{q}_1\boldsymbol{q}_2\boldsymbol{q}_3)$ is determined variationally [19]. The index (0) on the baryon operators indicates a bare baryon, i.e., a baryon without pion cloud corrections.

The important fact to notice here is that the baryon wave function depends on the chiral angle φ and as such, spin splittings and other properties are determined by the same physics that determines vacuum properties. The pion-baryon vertex, that we will discuss in the next section, will therefore depend on the chiral angle not only because of the pion, but

also because of the baryon wave function. Numerical results for the masses of the the nucleon and $\Delta(1232)$ have been obtained previously [19] and are of the right order of magnitude as compared with experimental values. Of course, fine-tuning with different spin-dependent interactions can improve the numerical values of the calculated quantities.

III. PION-BARYON VERTEX FUNCTION

We obtain an effective baryon-pion Hamiltonian by projecting the model quark Hamiltonian onto the one-pion and one-baryon states, Eqs. (17) and (21). We use a shorthand notation. For the bare baryons, i.e., baryons without pionic corrections, we use the indices $\alpha, \beta, \ldots$, to indicate all the quantum numbers necessary to specify the baryon state, such as spin, flavor, and center-of-mass momentum. For the pion, we use $j, k, \ldots$, to specify all the quantum numbers of the pion state. With this notation, the effective Hamiltonian can be obtained as

$$H = \sum_{\alpha\beta} |\alpha\rangle\langle\alpha|H|\beta\rangle\langle\beta| + \sum_{jk} |j\rangle\langle j|H|k\rangle\langle k|$$
$$+ \sum_{j\alpha\beta} (|\alpha\rangle|j\rangle\langle\alpha,j|H_I|\beta\rangle\langle\beta| + |\beta\rangle\langle\beta|H_I|j,\alpha\rangle\langle\alpha|\langle j|).$$
(23)

This leads to an effective Hamiltonian that can be written as the sum of the single-baryon and single-pion contributions, and the pion-baryon vertex

$$H = H_0 + W,$$
(24)

where $H_0 = H_B + H_\pi$ contains the single-baryon and single-pion contributions

$$H_B = \sum_\alpha E_\alpha^{(0)} B_\alpha^{(0)\dagger} B_\alpha^{(0)}, \quad H_\pi = \sum_j E_j M_j^\dagger M_j,$$
(25)

and W is the pion-baryon vertex

$$W = \sum_{j\alpha\beta} W_{\alpha\beta}^j B_\beta^{(0)\dagger} B_\alpha^{(0)} M_j + \text{H.c.}$$
(26)

Here, $B_\alpha^{(0)\dagger}$ and $B_\alpha^{(0)}$ ($M_j^\dagger$ and M_j) are the baryon (pion) creation and annihilation operators, discussed in the previous section. Note that we have assumed that states with different quantum numbers are orthogonal. Note also that in writing the state $|j,\alpha\rangle$ we have implicitly assumed that the negative-energy component of the baryon is negligible and the baryon creation operator acting on the RPA vacuum has the same effect as acting on the BCS vacuum.

We note that the projection of the microscopic quark Hamiltonian to an effective hadronic Hamiltonian can be obtained in a systematic and controlled way using a mapping procedure [26]. We do not follow such a procedure here because we are mainly concerned with tree-level pion-baryon coupling and the projection we are using is enough to obtain the desired effective coupling. For processes that involve
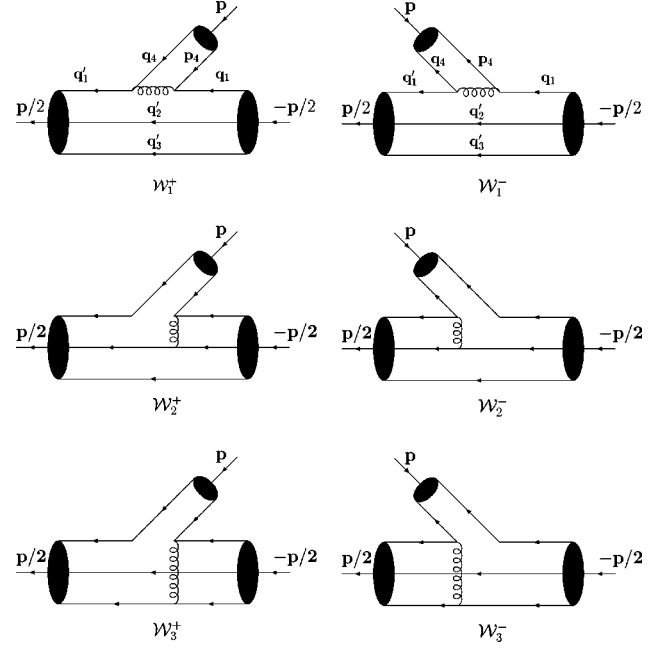

FIG. 1. Graphical representation of the functions $\mathcal{W}_i^\pm$, $i = 1,2,3$.

quark exchange, such as baryon-meson or baryon-baryon interactions, a mapping procedure would be useful. In a future publication, we intend to address such processes in the context of the present model.

The single-hadron Hamiltonians H_0 and H_π give

$$H_0|B_\alpha^{(0)}\rangle = E_B^{(0)}|B^{(0)}\rangle, \quad H_\pi|\pi\rangle = E_\pi^{(0)}|\pi\rangle,$$
(27)

with $E_B^{(0)} = M_B^{(0)}$ and $E_\pi^{(0)} = m_\pi^{(0)}$ in the rest frame. The vertex W gives the coupling of the pion to the baryon and, as we will show later, leads to loop corrections to the baryon self-energy.

The pion-nucleon vertex function can be written generally as

$$W = \sum_{i=1}^3 (W_i^+ + W_i^-),$$
(28)

where the $W_i^\pm$'s are of the general form (for simplicity we suppress the color and spin-flavor wave functions in the following)

$$W_i^\pm(\boldsymbol{p},\boldsymbol{p}';k) = \int \frac{d\boldsymbol{q}\,d\boldsymbol{q}'\,d\boldsymbol{q}''}{(2\pi)^9} V_\Gamma(\boldsymbol{q}-\boldsymbol{q}') \Psi_{\boldsymbol{p}'}^*(\boldsymbol{q}_1'\boldsymbol{q}_2'\boldsymbol{q}_3')$$
$$\times \mathcal{W}_i^\pm[\Gamma,\phi_{\mathbf{k}}]\Psi_{\boldsymbol{p}}(\boldsymbol{q}_1\boldsymbol{q}_2\boldsymbol{q}_3),$$
(29)

where the $\mathcal{W}_i^\pm$'s involve the Dirac spinors and the pion wave functions. In Fig. 1 we present a pictorial representation of the different contributions to the vertex function. Explicitly, the $\mathcal{W}_i^\pm$'s are given by

$$\mathcal{W}_1^+ = [u^\dagger(\boldsymbol{q}_1')T^a\Gamma u(\boldsymbol{q}_4)]\phi_{\mathbf{k}}^+(\boldsymbol{p}_4)[v^\dagger(-\boldsymbol{p}_4)T^a\Gamma u(\boldsymbol{q}_1)],$$
(30)

$$\mathcal{W}_2^+ = \phi_k^+(\boldsymbol{p}_4)[v^\dagger(-\boldsymbol{p}_4)T^a\Gamma u(\boldsymbol{q}_1)][u^\dagger(\boldsymbol{q}_2')T^a\Gamma u(\boldsymbol{q}_2)], \quad (31)$$

$$\mathcal{W}_3^+ = \mathcal{W}_2^+, \quad (32)$$

$$\mathcal{W}_1^- = [u^\dagger(\boldsymbol{q}_1')T^a\Gamma v(-\boldsymbol{q}_4)]\phi_k^{-T}(\boldsymbol{p}_4)[u^\dagger(\boldsymbol{p}_4)T^a\Gamma u(\boldsymbol{q}_1)], \quad (33)$$

$$\mathcal{W}_2^- = [u^\dagger(\boldsymbol{q}_2')T^a\Gamma u(\boldsymbol{q}_2)][u^\dagger(\boldsymbol{q}_1')T^a\Gamma v(-\boldsymbol{q}_4)]\phi_k^{-T}(\boldsymbol{p}_4), \quad (34)$$

$$\mathcal{W}_3^- = \mathcal{W}_2^-. \quad (35)$$

In these formulas, the quark momenta in the initial (final) nucleon $\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3$ ($\boldsymbol{q}_1',\boldsymbol{q}_2',\boldsymbol{q}_3'$) and the momenta of the quark and antiquark in the pion, $\boldsymbol{p}_4$ and $\boldsymbol{q}_4$, are expressed in terms of the loop momenta $\boldsymbol{q},\boldsymbol{q}',\boldsymbol{q}''$ by momentum conservation (see Fig. 1).

Once the effective baryon-pion Hamiltonian is obtained, one can calculate the pionic corrections to baryon properties as in the CBM and in the traditional Chew-Low model. This will be done in the next section.

Before leaving this section, we recall that the use of the Breit frame is essential in calculations of form factors (vertex functions) in static models [27], like the present one. This is true for composite models for which approximate solutions that maintain relativistic covariance are very difficult to implement. This was the case for all old, static source, pion-nucleon models such as the Chew-Low model [28]. In particular, as explained in Ref. [27], electromagnetic gauge invariance is respected in this frame. Therefore, in calculating loop corrections to baryon properties, we employ the Breit-frame vertex functions. In the Breit frame, we denote the incoming pion and nucleon momenta by $\boldsymbol{p}$ and $-\boldsymbol{p}/2$, respectively, and the outgoing nucleon momentum by $\boldsymbol{p}/2$. In this frame, the internal momenta of quarks and antiquarks $\boldsymbol{q}_1',\boldsymbol{q}_2',\ldots$ are given in terms of the loop momenta $\boldsymbol{q}$, $\boldsymbol{q}'$, and $\boldsymbol{q}''$ as

$$\boldsymbol{q}_1 = \boldsymbol{p}/2 + \boldsymbol{q} + \boldsymbol{q}'', \quad \boldsymbol{q}_1' = \boldsymbol{p}/2 + \boldsymbol{q}' + \boldsymbol{q}'',$$

$$\boldsymbol{q}_2 = \boldsymbol{q}_2' = -\boldsymbol{q}' + \boldsymbol{q}'', \quad \boldsymbol{q}_3 = \boldsymbol{q}_3' = -2\boldsymbol{q}'',$$

$$\boldsymbol{p}_4 = \boldsymbol{p}/2 - \boldsymbol{q} - \boldsymbol{q}'', \quad \boldsymbol{q}_4 = \boldsymbol{p}/2 - \boldsymbol{q} - \boldsymbol{q}'', \quad (36)$$

for the vertex $\mathcal{W}^+$ and

$$\boldsymbol{q}_1 = -\boldsymbol{p}/2 + \boldsymbol{q}' + \boldsymbol{q}'', \quad \boldsymbol{q}_1' = \boldsymbol{p}/2 + \boldsymbol{q}' + \boldsymbol{q}'',$$

$$\boldsymbol{q}_2 = \boldsymbol{q}_2' = -\boldsymbol{q}' + \boldsymbol{q}'', \quad \boldsymbol{q}_3 = \boldsymbol{q}_3' = -2\boldsymbol{q}'',$$

$$\boldsymbol{p}_4 = -\boldsymbol{p}/2 + \boldsymbol{q} + \boldsymbol{q}'', \quad \boldsymbol{q}_4 = -\boldsymbol{p}/2 - \boldsymbol{q} - \boldsymbol{q}'', \quad (37)$$

for the vertex $\mathcal{W}^-$. In following equations, we also denote the vertex function as $W(-\boldsymbol{p}/2,\boldsymbol{p}/2;\boldsymbol{p}) \equiv W(\boldsymbol{p})$.

IV. SELF-ENERGY CORRECTION TO BARYON MASSES

For completeness we review the derivation of the expression for the self-energy correction from the effective baryon-pion Hamiltonian of Eq. (24) in the "one-pion-in-the air"

approximation [28,6]. The baryon self-energy is defined as the difference of bare- and dressed-baryon energies

$$\Sigma(E_B) = E_B - E_B^{(0)}. \quad (38)$$

The physical baryon mass M_B is given by the (in general, nonlinear) equation

$$M_B = M_B^{(0)} + \Sigma(M_B), \quad (39)$$

where $M_B^{(0)}$ is the bare baryon mass (i.e., without pionic corrections) and $\Sigma(E_B)$ is the self-energy function.

Let $|B\rangle$ denote the physical baryon state, and $|B_0\rangle$ the "bare" undressed state. Let Z_2^B be the probability of finding $|B_0\rangle$ in $|B\rangle$. Then one can write

$$|B\rangle = \sqrt{Z_2^B}|B_0\rangle + \Lambda|B\rangle, \quad (40)$$

where Λ is a projection operator that projects out the component $|B_0\rangle$ from $|B\rangle$,

$$\Lambda = 1 - |B_0\rangle\langle B_0|. \quad (41)$$

We have that

$$\langle B|W|B_0\rangle = \langle B|(H-H_0)|B_0\rangle = (E_B - E_B^{(0)})\langle B|B_0\rangle$$
$$= \sqrt{Z_2^B}(E_B - E_B^{(0)}) = \sqrt{Z_2^B}\Sigma(E_B). \quad (42)$$

We can now express $|B\rangle$ in terms of $|B_0\rangle$ and the pion-baryon interaction Hamiltonian W as

$$|B\rangle = \sqrt{Z_2^B}\left[1 - \frac{1}{E_B - H_0 - \Lambda W\Lambda}W\right]|B_0\rangle. \quad (43)$$

On the other hand, since

$$\langle B|W|B_0\rangle = \sqrt{Z_2^B}\langle B_0|W\frac{1}{E_B - H_0 - \Lambda W\Lambda}W|B_0\rangle, \quad (44)$$

we have that the self-energy is given by

$$\Sigma(E_B) = \frac{1}{\sqrt{Z_2^B}}\langle B|W|B_0\rangle = \langle B_0|W\frac{1}{E_B - H_0 - \Lambda W\Lambda}W|B_0\rangle. \quad (45)$$

This expression can be further approximated so as to avoid solving complicated integral equations for the self-energy. We can manipulate the expression for Σ to obtain (for details, see Ref. [6])

$$\Sigma(E_B) = \langle B_0|W\frac{1}{E_B - H_0 - \Sigma_0(E_B)}W|B_0\rangle, \quad (46)$$

with

$$\Sigma_0(E_B) = W\Lambda\frac{1}{E_B - H_0}\Lambda W. \quad (47)$$

The approximation consists in absorbing $\Sigma_0(E_B)$ into H_0 such that

$$H_0 + \Sigma_0(E_B) \equiv \tilde{H}_0, \tag{48}$$

with

$$\tilde{H}_0 = \sum_\alpha E_\alpha B_\alpha^{(0)\dagger} B_\alpha^{(0)} + \sum_j E_\pi M_j^\dagger M_j, \tag{49}$$

where E_α and E_π are the *physical* energies. Therefore, the baryon self-energy can be written as

$$\Sigma(E_B) = \langle B_0 | W \frac{1}{E_B - \tilde{H}_0} W | B_0 \rangle. \tag{50}$$

Finally, insertion of a sum over intermediate baryon-pion states in Eq. (50) leads to

$$\Sigma(E_B) = \sum_n \langle B_0 | W | n \rangle \frac{1}{E_B - E_n} \langle n | W | B_0 \rangle. \tag{51}$$

The structure vertex-propagator-vertex $W(E - \tilde{H}_0)^{-1} W$ in Eq. (50) is an effective baryon-pion interaction. The main difference here with the hybrid approaches [1–6] is that we do not have a pointlike pion coupling to pointlike quarks and antiquarks. The pion-baryon vertex arises through the "Z graphs" in which the antiquark of the pion is annihilated with a quark of the "initial" baryon and the quark of the pion appears in the "final" baryon. Therefore, the vertex function incorporates not only the extension of the baryons, but also the extension of the pion.

We truncate the sum over the intermediate states in Eq. (51) to the lowest mass states, namely, the nucleon and the $\Delta(1232)$. In this case, we obtain for the on-shell N and $\Delta(1232)$ self-energies the coupled set of equations

$$\Sigma_N(M_N) = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{|W_{NN}(\mathbf{k})|^2}{M_N - [M_N + E_\pi(\mathbf{k})]} \right.$$
$$\left. + \frac{W_{N\Delta}(\mathbf{q}) W_{\Delta N}(\mathbf{k})}{M_N - [M_\Delta + E_\pi(\mathbf{k})]} \right], \tag{52}$$

$$\Sigma_\Delta(M_\Delta) = \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{W_{\Delta N}(\mathbf{k}) W_{N\Delta}(\mathbf{k})}{M_\Delta - [M_N + E_\pi(\mathbf{k})]} \right.$$
$$\left. + \frac{|W_{\Delta\Delta}(\mathbf{k})|^2}{M_\Delta - [M_\Delta + E_\pi(\mathbf{k})]} \right]. \tag{53}$$

This is the final result for the pion loop correction for the nucleon and $\Delta(1232)$.

One important consequence of projecting the microscopic quark interaction onto hadronic states is that the leading nonanalytic (LNA) contributions in the pion mass as predicted by chiral perturbation theory are correctly obtained [9]. In particular, as we discuss in the next section, Eq. (52) leads to an LNA contribution to the nucleon mass as predicted by QCD [29], namely,

$$M_N^{LNA} = -\frac{3}{16\pi^2 f_\pi^2} g_A^2 m_\pi^2. \tag{54}$$

In models where the pion is treated as a pointlike particle, this result follows trivially [9] from Eq. (52). In the context of the present model, where the pion is not treated covariantly, such a result does not follow in general for an arbitrary interaction. The difficulty is related to the fact the the pion dispersion relation $E_\pi = \sqrt{\mathbf{k}^2 + m_\pi^2}$ is not obtained in general in a noncovariant model. In the CBM for example, the pion is point like and the normalization is correct from the very beginning. However, the microscopic quark interaction can be chosen such that the pion dispersion relation is correctly obtained [14,25]. These issues will be discussed in Sec. VI.

V. THE PION-NUCLEON AND PION-Δ(1232) FORM FACTORS

Our aim is to obtain an estimate for the numerical values of the pionic self-energies. It happens that nature has produced a sort of low energy filter (chiral symmetry) for the details of strong interactions. Indeed it is remarkable that although intermediate theoretical concepts such as gluon propagators, quark effective masses and so on, might vary (in fact they are not gauge invariant and hence they are not physical observables), chiral symmetry contrives for the final physical results, e.g., hadronic masses and scattering lengths, to be largely insensitive to the above mentioned theoretical uncertainties. The pion mass furnishes the ultimate example: In the case of massless quarks, the pion mass is bound to be zero, regardless of the form of the effective quark interaction *provided* it supports the mechanism of spontaneous breakdown of chiral symmetry. The other example is provided by the π-π scattering lengths [30] which are equally independent of the form of the quark kernel [31]. Furthermore it has become more and more evident through the accumulation of theoretical calculations on low-energy hadronic phenomena, ranging from calculations on Euclidean space to instantaneous approximations and from harmonic kernels to linear confinement, that low-energy hadronic phenomenology only seems to depend mildly on the details of the quark kernels used. To this extent, we will use for the quark-quark interaction a kernel of the form

$$\Gamma = \gamma^0, \tag{55}$$

$$V(\mathbf{k}) = \frac{3}{4} (2\pi)^3 K_0^3 \Delta_k \delta(\mathbf{k}), \tag{56}$$

where K_0 is a free parameter. This potential has been widely used in the context of chiral symmetry breaking because it allows a great deal of simple analytic calculations (which is not the case for the linear potential). The harmonic potential basically differs from the linear potential in domains of the baryon-pion-baryon overlap kernel which contribute little to the total geometrical overlap so that, at least for results proportional to these overlaps, they should not differ too much.

The momentum-dependent part of the Salpeter amplitude for the baryon $\Psi_p(q_1 q_2 q_3)$ in Eq. (22), is taken to be of a Gaussian form

$$\Psi_p(q_1 q_2 q_3) = \frac{e^{-(\rho^2 + \lambda^2)/2\alpha_B^2}}{\mathcal{N}_B(p)}; \quad \rho = \frac{\mathbf{p}_1 - \mathbf{p}_2}{\sqrt{2}};$$

$$\lambda = \frac{\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3}{\sqrt{6}}, \tag{57}$$

where α_B^2 is the variational parameter and $\mathcal{N}_B(p)$ is the normalization. Notice that since the integrations of the quark momenta in the functions $W_i^{\pm}$ in Eq. (29) are made through a Monte Carlo integration, the Gaussian ansatz is not essential and does not simplify our calculations, but we still use it to make contact with previous calculations.

As in our previous calculation [22] for the pion-nucleon coupling constant, the Salpeter amplitudes $\phi_k^{\pm}(q)$ up to first order in k are given by

$$\phi_k^+(q) \simeq \mathcal{N}(k)^{-1}[+\sin\varphi(q) + E_1(k)f_1(q)$$
$$+ ig_1(q)k(\hat{q} \times \boldsymbol{\sigma})]\chi_\pi \mathcal{S}_{\text{color}}, \tag{58}$$

$$\phi_k^-(q) \simeq \mathcal{N}(k)^{-1}[-\sin\varphi(q) + E_1(k)f_1(q)$$
$$- ig_1(q)k(\hat{q} \times \boldsymbol{\sigma})]\chi_\pi \mathcal{S}_{\text{color}}, \tag{59}$$

where φ is the chiral angle and $E_1(k)$ is the first-order correction to the pion energy. The normalization $\mathcal{N}(k)$ is proportional to $E_1(k)$ and is given as

$$\mathcal{N}^2(k) = 4E_1(k)\int \frac{d\mathbf{q}}{(2\pi)^3}\sin\varphi(q)f_1(q) \equiv E_1 a^2. \tag{60}$$

The energy $E_1(k)$ is given in terms of the second derivatives of the diagonal components of the Salpeter kernel with respect to k and its explicit form is given in Eq. (24) of Ref. [22]. Note that the truncation up to first order in k of the Salpeter amplitude constitutes a reasonable approximation due to the fact that c.m. momenta-dependent distortions of the pion and nucleon wave functions are geometrically damped because of the geometric overlap kernel integrations for the functions $W_i^{\pm}$ in Eq. (29)—see Ref. [32]. Explicit numerical solutions were obtained in Ref. [22] for the functions $f_1(q)$ and $g_1(q)$.

For completeness, we initially repeat the results of Ref. [22] for the coupling constants $f_{\pi NN}$ and $f_{\pi N\Delta}$. In Ref. [22], they were obtained as

$$\frac{f_{\pi NN}}{m_\pi}\boldsymbol{\sigma}_N \cdot \mathbf{p} = \frac{5}{3\sqrt{3}}\frac{\mathcal{O}_{fs}(p)}{2a}\boldsymbol{\sigma}_N \cdot \hat{\mathbf{p}}, \tag{61}$$

$$\frac{f_{\pi N\Delta}}{m_\pi}\mathbf{S}\cdot\mathbf{p} = \left[\frac{2\sqrt{2}}{\sqrt{3}}\frac{\mathcal{O}_{fs}(p)}{2a} + \sqrt{2}\frac{\mathcal{O}'_{fs}(p)}{2a}\right]\mathbf{S}\cdot\hat{\mathbf{p}}, \tag{62}$$

where the isospin matrix is omitted and

$$\mathcal{O}'_{fs}(p) \simeq 0, \quad \mathcal{O}_{fs}(p) = -\frac{\int [d\mathbf{q}](a^+ + a^- + b^+ + b^-)}{\int [d\mathbf{q}]\Psi_{\text{in}}^* \Psi_{\text{out}}}, \tag{63}$$

where $[d\mathbf{q}]$ means integration over $\mathbf{q}$, $\mathbf{q}'$, and $\mathbf{q}''$ [see Eq. (29)] and the set of functions a^+, a^-, b^+, b^- is given by

$$a^+ = \phi^+ \left\{ \varphi'(q_1)\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\Psi_{\text{out}} \right.$$
$$\left. + \left[\varphi'(q_1) + \frac{\cos\varphi(q_1)}{q_1}\right]\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_1 \times (\hat{\mathbf{q}}_1 \times \boldsymbol{\nabla}\Psi_{\text{out}})si \right\}\Psi_{\text{in}}, \tag{64}$$

$$a^- = \Psi_{\text{out}} \left\{ \varphi'(q_1')\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\Psi_{\text{in}} \right.$$
$$\left. + \left[\varphi'(q_1') + \frac{\cos\varphi(q_1')}{q_1'}\right]\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_1' \times (\hat{\mathbf{q}}_1' \times \boldsymbol{\nabla}\Psi_{\text{in}}) \right\}\phi^-, \tag{65}$$

$$b^+ = \phi^+ \frac{1 - \sin\varphi(q_1')}{2q_1'}\left\{ 2\frac{\cos\varphi(p_1)}{p_1}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_1' \right.$$
$$\left. + \left[\varphi'(q_1) + \frac{\cos\varphi(q_1)}{q_1}\right]\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_1 \times (\hat{\mathbf{q}}_1 \times \hat{\mathbf{q}}_1') \right\}\Psi_{\text{out}}\Psi_{\text{in}}, \tag{66}$$

$$b^- = \frac{1 - \sin\varphi(q_1)}{2q_1}\left\{ 2\frac{\cos\varphi(q_1')}{q_1'}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_1 \right.$$
$$+ \left[\varphi'(q_1) + \frac{\cos\varphi(q_1')}{q_1'}\right]\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}_1'$$
$$\left. \times (\hat{\mathbf{q}}_1' \times \hat{\mathbf{q}}_1) \right\}\Psi_{\text{out}}\Psi_{\text{in}}\phi^-. \tag{67}$$

Here, $\Psi_{\text{in,out}}$ stand for the baryon in and out Salpeter amplitudes and $\phi^{+,-}$ represent the pion Salpeter amplitudes.

The baryon-pion coupling constants are obtained as the zero limit of the nucleon (or Δ) momentum $\mathbf{p}\to 0$ of the above overlap functions. For simplicity, we are defining the couplings at zero momentum, and not at the physical pion mass. In order to facilitate the integration, in Ref. [22] a Gaussian parametrization for the $[\cos\varphi(k)]/k$ and $[1 - \sin\varphi(k)]/k^2$ was used. Here, since we need the vertex function for $\mathbf{p}\neq 0$, we use a Monte Carlo integration to perform the multidimensional integral that gives the overlap function and use the full numerical solution for the gap function (not the Gaussian parametrization). We first checked the correctness of our Monte Carlo integration with the result of Ref. [22] for the special case of $\mathbf{p}=0$ using the same Gaussian parametrization as was used there. This was done by calcu-
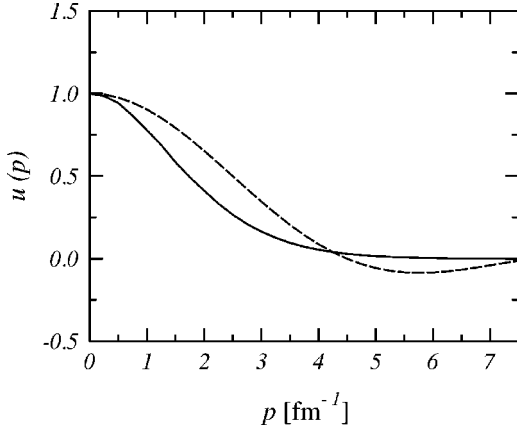
FIG. 2. The function $u(p)$. The solid line is the form factor obtained with the baryon amplitude of Eq. (57) and pion Salpeter amplitudes of Eqs. (58) and (59). The dashed line is the CBM form factor of Eq. (71) for $R=1$ fm.

lating $\mathcal{O}_{fs}(\boldsymbol{p})$ for $\boldsymbol{p}=(0,0,p_z)$ and finding the limit of $\mathcal{O} = \mathcal{O}_{fs}/p_z$ when $p_z \to 0$ to obtain $f_{\pi NN}$.

As in Ref. [22] we have used $K_0 = 247$ MeV for the strength of the potential. The variational determination of α of the baryon amplitude, Eq. (57), leads to $\alpha_N = 1.2 K_0$. For $\Delta(1232)$, the result is not much different and therefore we use $\alpha_N = \alpha_\Delta$.

Introducing the quantities

$$F_1 = \frac{5}{3\sqrt{12}}, \quad F_2 \sqrt{\frac{2}{3}}, \quad F_3 = \frac{1}{3\sqrt{12}}, \tag{68}$$

we can summarize the couplings of the pion to the nucleon and $\Delta(1232)$ as follows:

$$f_{\pi NN} = F_1 \mathcal{O}(0) \frac{m_\pi}{a},$$

$$f_{\pi N\Delta} = F_2 \mathcal{O}(0) \frac{m_\pi}{a}, \quad f_{\pi\Delta\Delta} = F_3 \mathcal{O}(0) \frac{m_\pi}{a}. \tag{69}$$

For the value of K_0 given above, we have $m_\pi/a = 3.47$. The numerical values for the couplings are then

$$f_{\pi NN} = 1.19, \quad f_{\pi N\Delta} = 2.02, \quad f_{\pi\Delta\Delta} = 0.24. \tag{70}$$

The effect of the Gaussian parametrization can be assessed by comparing with the corresponding numbers of Ref. [22]. For example, $f_{\pi NN} \simeq 1.0$ and $f_{\pi N\Delta} = 1.8$ in Ref. [22]; the effect of the parametrization is therefore of the order of 20%.

Next, we calculated the full overlap function for $\boldsymbol{p} \neq 0$. In Fig. 2 we plot the function $u(p) = \mathcal{O}(p)/\mathcal{O}(0)$ for the parameters given above. It is instructive to compare the momentum dependence of this form factor with the one given by the CBM [6,23]:

$$u(p) = 3 \frac{j_1(pR)}{pR}, \tag{71}$$

where j_1 is the spherical Bessel function and R is the radius of the underlying MIT bag. The solid line is our result and the dashed one is the CBM result for $R=1$ fm. The faster fall-off of our result is clearly a consequence of our Gaussian ansatz. As we will discuss soon, this rapid falloff will have the consequence of giving a smaller value of the self-energy correction to the nucleon mass, as compared to the corrections obtained with the CBM.

VI. SELF-ENERGY CORRECTIONS TO THE NUCLEON AND $\Delta(1232)$ MASSES

In this section we present numerical results for the pionic self-energy corrections to the nucleon and $\Delta(1232)$ masses and discuss the LNA contribution to the masses. We start by rewriting the vertex function in a manner to make clear the problem with the pion dispersion relation. The pion energy is given, for low $\boldsymbol{k}$, as [14,25]

$$E_1^2(\boldsymbol{k}) = m_\pi^2 + \boldsymbol{k}^2 \sqrt{\frac{f_\pi^{(s)}}{f_\pi^{(t)}}}, \tag{72}$$

where

$$m_\pi^2 = -\frac{2m_q \langle \bar{\psi}\psi \rangle}{(f_\pi^{(t)})^2}, \quad \langle \bar{\psi}\psi \rangle = -6 \int \frac{d\boldsymbol{q}}{(2\pi)^3} \sin \varphi(q). \tag{73}$$

The point is that for an arbitrary quark-quark interaction one obtains in general two different values for the pion decay constant $f_\pi^{(t)}$ and $f_\pi^{(s)}$ (the explicit calculations can be found in Refs. [14,25]), depending on how one defines the decay constant. When using the time component of the axial current, one obtains $f_\pi^{(t)}$, and when using the space component one obtains $f_\pi^{(s)}$. However, as suggested in Ref. [14], and explicitly demonstrated in Ref. [25], this problem can be cured by adding a transverse gluon interaction. Therefore, to illustrate the point of obtaining the correct LNA term from Eq. (52) with composite pions, we use the correct pion dispersion relation and assume $f_\pi^{(t)} = f_\pi^{(s)} \equiv f_\pi$ and denote $E_1(k) = \omega(k)$.

The normalization of the pion Salpeter amplitude, Eq. (60), can be rewritten as

$$\mathcal{N}^2(\boldsymbol{p}) = 4\omega(p) \int \frac{d\boldsymbol{k}}{(2\pi)^3} \sin \varphi(q) f_1(\boldsymbol{q}) = \frac{2}{3} \omega(p) f_\pi^2. \tag{74}$$

That is, a^2 from Eq. (60) is $2/3 f_\pi^2$. We next extract from the vertex function (we concentrate on the NN form factor) this normalization in the following way:

$$W_{NN}^i(\boldsymbol{p}) = \frac{1}{\sqrt{2\omega(p)}} \frac{G_A(p)}{2f_\pi} \tau_N^i \boldsymbol{\sigma}_N \cdot \boldsymbol{p}. \tag{75}$$

The relation of the function $G_A(k^2)$ to the overlap function $\mathcal{O}(p)$ can be trivially obtained by comparing with Eq. (61).
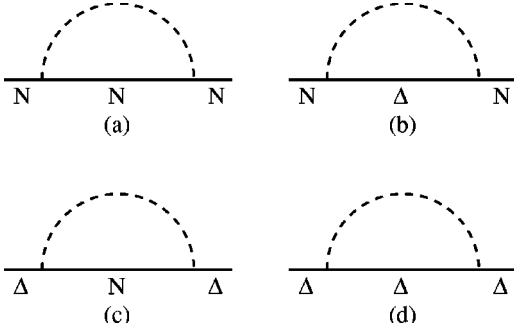
FIG. 3. Schematic representation of the pion self-energy corrections to the nucleon (N) and delta (Δ) masses.

Inserting Eq. (75) in the expression for the N and Δ self-energies, Eqs. (52) and (53), and after performing rather straightforward spin-isospin algebra one obtains

$$M_N = M_N^{(0)} - f_0^2 \int_0^\infty dp \frac{p^4 u^2(p)}{\omega^2(p)}$$

$$- \frac{32}{25} f_0^2 \int_0^\infty dp \frac{p^4 u^2(p)}{\omega(p)[\Delta M + \omega(p)]}, \qquad (76)$$

$$M_\Delta = M_\Delta^{(0)} + \frac{8}{25} f_0^2 \int_0^\infty dp \frac{p^4 u^2(p)}{\omega(p)[\Delta M - \omega(p)]}$$

$$- f_0^2 \int_0^\infty dp \frac{p^4 u^2(p)}{\omega^2(p)}, \qquad (77)$$

where

$$\Delta M = M_\Delta - M_N, \quad \omega(p) = \sqrt{p^2 + m_\pi^2} \qquad (78)$$

and

$$u(p) = \frac{\mathcal{O}(p)}{\mathcal{O}(0)}, \quad f_0^2 = \frac{108}{m_\pi^2} \frac{f_{\pi NN}^2}{4\pi}. \qquad (79)$$

Note that in principle we have different spatial dependencies for the NN, $N\Delta$, ..., vertices, but for simplicity we have written them here as being equal. A schematic representation of Eqs. (76) and (77) is presented in Fig. 3. It is important to note that these equations are not the ones one would obtain by simple perturbation theory; they are actually nonperturbative, because of the dependence on $\Delta M = M_\Delta - M_N$ on the right-hand side.

It is easy now to obtain the LNA contributions to the masses [9]. For the nucleon, the LNA contribution comes from the first term in Eq. (76) by performing the integral. The integral can be done by transforming it into a contour integral and making use of Cauchy's theorem. The result is Eq. (54). For the Δ, the LNA contribution follows in a similar way from the last term in Eq. (77).

To conclude, we discuss numerical results for the pionic corrections. Initially we solve variationally the bare nucleon case. As discussed above, using $K_0 = 247$ MeV, we obtain

for the variational size parameter the value $\alpha_N = 1.2K_0$. We also use here $\alpha_N = \alpha_\Delta$. This leads to the following values for the bare N and Δ masses:

$$M_N^{(0)} = 1174 \text{ MeV}, \quad M_\Delta^{(0)} = 1373 \text{ MeV}. \qquad (80)$$

The difference between the masses, of the order of 200 MeV, comes from the hyperfine splitting induced by the confining interaction. Given these values, we solve the two self-consistent equations given in Eqs. (76) and (77). They are solved by iteration. We obtain for the masses

$$M_N = 1125 \text{ MeV}, \quad M_\Delta = 1342 \text{ MeV}. \qquad (81)$$

Comparing with the values above, we see that the pionic effect is relatively small, as it should be, and of the order of 50 MeV for the N and 30 MeV for the Δ. The pionic effect is smaller for the Δ, as one expects from spin-isospin considerations [9]. The results obtained with the CBM for a $R = 1$ fm are a bit larger [23]. The difference can be traced to the rapid falloff of the form factor in our model.

We certainly do not expect these numbers to be definitive. Once more realistic microscopic quark interactions and ansatze for the baryon wave function are used, they might be improved. However, independently of the microscopic model, our scheme is general and able to incorporate such interactions and new baryon amplitudes. It would be of particular interest to have the numbers for a linear confining interaction with short range gluonic interactions that respect asymptotic freedom.

VII. CONCLUSIONS AND FUTURE PERSPECTIVES

We developed a calculational scheme to calculate chiral loop corrections to properties of composite baryons with composite pions. The composite baryons and pions are bound states derived from a microscopic chiral quark model inspired in Coulomb gauge QCD and provides a generalization of the Nambu–Jona-Lasinio model to include confinement and asymptotic freedom. An effective chiral hadronic model is constructed by projecting the microscopic quark Hamiltonian onto a Fock-space basis of single composite hadronic states. The composite pions and baryons are obtained from the same microscopic Hamiltonian that describes the chiral vacuum condensate. The projection of the quark Hamiltonian onto the pion states is nontrivial because of the two-component nature of the Salpeter amplitude of the pion. As explained before, the two components correspond to positive and negative energies which complicates the Fock-space representation of the pion state. The projection is made possible by rephrasing the formalism of the Salpeter equation in terms of the RPA equations.

The development of models and calculational methods of the sort described in the present paper are relevant in the context of a phenomenological understanding of nonperturbative phenomena of strong QCD-like confinement and dynamical chiral symmetry breaking. Eventually full lattice QCD simulations aimed at studying hadronic structure will be available and phenomenological models will play a central role in the interpretation of the data generated. The de-

velopments of the present paper are of particular interest for the first-principle developments based on the QCD Hamiltonian, such as the nonperturbative renormalization program for the QCD Hamiltonian [17] and Hamiltonian lattice QCD [18]. We intend to implement the technique developed here to such first-principle QCD calculations.

We illustrated the applicability of the formalism with a numerical calculation using a simple microscopic interaction, namely a confining harmonic potential, and a simple Gaussian ansatz for the baryon amplitude. This very same S-wave interaction has been used in a variety of earlier calculations, such as meson and baryon spectroscopy and S-wave nucleon-nucleon interaction. Numerical results were obtained here for the pion-nucleon form factor and for the pionic self-energy corrections to the nucleon and $\Delta(1232)$ masses in the nonperturbative one-loop approximation. Despite the simplicity of the interaction, the results obtained are very reasonable.

For the future, the most pressing development would be to use a microscopic interaction that is consistent with asymptotic freedom and describes confinement by a linear potential. The calculation of the pion wave function beyond lowest order in momentum must be implemented and the variational ansatz for the baryon amplitude must be improved. A more ambitious development would be to include explicit gluonic degrees of freedom. In this case renormalization issues will show up and the new techniques such as discussed in Ref. [17] will certainly be useful. Another very interesting direction would be to employ the techniques developed here in Hamiltonian lattice QCD.

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