Lepton transverse polarization in the $B \to D l \nu_l$ decay due to the electromagnetic final state interaction

V. V. Braguta and A. E. Chalov
Institute for High Energy Physics, 142280, Protvino, Moscow Region, Russia

A. A. Likhoded*
Instituto de Física Teórica—UNESP, Rua Pamplona, 145, 01405-900 São Paulo, SP, Brazil
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The effect of lepton transverse polarization in $B^0 \to D^- l^+ \nu_l$, $B^+ \to \bar{D}^0 l^+ \nu_l$ decays ($l = \tau, \mu$) is analyzed within the framework of the standard model in the leading order of heavy quark effective theory. It is shown that a nonzero transverse polarization appears due to the electromagnetic final state interaction. The diagrams with intermediate D, D^* mesons contributing to the nonvanishing P_T are considered. Regarding only the contribution of these mesons, the values of the τ-lepton transverse polarization averaged over the physical region in the $B^0 \to D^- \tau^+ \nu_l$ and $B^+ \to \bar{D}^0 \tau^+ \nu_l$ decays are equal to 2.60×10^{-3} and -1.59×10^{-3}, respectively. In the case of muon decay modes the values of $\langle P_T \rangle$ are equal to 2.97×10^{-4} and -6.79×10^{-4}.

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I. INTRODUCTION

In spite of the remarkable phenomenological success of the standard model (SM) the problem of the CP-violation mechanism still remains unexplained. In the SM CP violation appears due to the complexity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix; however, there is a set of models offering other mechanisms of CP violation. For instance, the Weinberg three Higgs boson doublets model is one of the simplest SM extensions, where CP violation appears due to the complex Higgs boson couplings to fermions [1]. Investigation of the CP-violation phenomenon will help us to understand this mechanism and, hence, to clarify one of the fundamental problems of elementary particle physics.

The experimental observables sensitive to CP-violating effects are, for example, transverse lepton polarization in weak decays and the T-odd correlation. Muon transverse polarization in the $K^+ \to \mu^+ \nu \pi^0$, $K^+ \to \mu^+ \nu \gamma$ processes is object of intensive study by many theoretical and experimental groups. In some SM extensions a nonzero transverse muon polarization appears already at the tree level [2,3]. The SM contribution to lepton transverse polarization is equal to zero in the leading order, and this fact explains the smallness of the SM background. The final state interaction gives rise to a nonzero CP-conserving contribution to P_T. In $K^+ \to \mu^+ \nu \gamma$ decay the lepton transverse polarization appears at the one-loop level and varies in the range of $(0.0--1.1) \times 10^{-3}$ on a Dalitz plot. The P_T value averaged over the physical region with the cut on photon energy $E_\gamma \geq 20$ MeV is equal to 4.76×10^{-4} [2]. In $K^+ \to \mu^+ \nu \pi^0$ decay the muon transverse polarization is of order $\sim 10^{-6}$ [4,5], and therefore this decay is rather effective in searching for new physics effects. A measurement of muon transverse polarization in this process was carried out by the KEK-E246 experiment, where the following result was obtained [6]:

$$P_T = -0.0042 + 0.0049(\text{stat}) + 0.0009(\text{syst}). \tag{1}$$

This experimental result does not allow us to state that the value of P_T is stipulated by new physics effects. However, an increase of experimental accuracy is planned in the nearest future, which seems very promising from the point of CP-violation research.

Another experimental observable, suppressed in the SM, is the T-odd correlation in charged kaon decays [7] (the distribution of the decay width over the kinematical variable, which is the mixing product of the final particle momenta, for instance, $\vec{p}_\pi \cdot [\vec{p}_\mu \times \vec{q}]$ in $K^+ \to \pi^0 \mu^+ \nu \gamma$ decay). The small SM contribution to this observable can be explained in the same way, as the SM contribution to the lepton transverse polarization in $K_{l2\gamma}$ decay. Here, new perspectives to search for T-odd contributions from new physiscs are connected with the OKA experiment [8], where it is planned to achieve $\sim 7 \times 10^5$ events for $K_{l3\gamma}$ decay.

Aside from K-meson decays, it is possible to study the lepton transverse polarization in similar B-meson decays. It should be noted that the value of P_T is especially sensitive to CP-violating Higgs boson Yukawa couplings in these decays. Obviously, in the case of $B \to D^{(*)} \tau \nu_\tau$, the value of the transverse polarization, due to the complexity of these couplings, is $(m_b m_\tau)/(m_s m_\mu) \sim 800$ times greater then P_T in the analogous $K \to \pi \mu \nu_\mu$ decay.

In [9–11] the effects of CP-violating transverse polarization of leptons in the decays $B \to D(*) l \nu$ in various SM extensions were analyzed. From these studies it follows that the τ-lepton transverse polarization can have values $P_T \leq 1$ in models with CP-violation in the Higgs sector [9,10], and $P_T \leq 0.26$ in the leptoquark models [11]. Thus, one can expect that the value of the transverse polarization in various extensions of the SM is rather large. However, to estimate the impact of new physics and perform a dedicated study of the polarization phenomenon it is necessary to carry out the

*On leave of absence from Institute for High Energy Physics, Protvino, 142284 Russia. Email address: andre@ift.unesp.br

calculation of the SM contribution to this observable. In this paper we calculate the CP-conserving SM contribution to P_T in $B^+ \to \bar{D}^0 l^+ \nu$ decays ($l = \tau, \mu$). For simplicity, the calculations are carried out in the framework of heavy quark effective theory (HQET) the leading approximation of $1/m_Q$ expansion. It is shown below that the value of the transverse polarization is not equal to zero if and only if there is a nonzero phase shift of decay form factors. The electromagnetic final state interaction induces a nonzero phase shift at the one-loop level and that, in turn, results in a nonzero value of P_T. In our calculations we take into account only the (D, D^*) doublet contribution to the transverse polarization.

In the next section we discuss the matrix elements contributing to the polarization value. In Sec. III the procedure for the transverse polarization calculation is given. The last section contains results and discussion.

II. MATRIX ELEMENTS

The general form for the $\langle D(D^*)|V^\mu(A^\mu)|B\rangle$ matrix elements is as follows:

$$\begin{aligned}
\langle D(k)|V^\mu|B(p)\rangle &= f_+(p^\mu + k^\mu) + f_-(p^\mu - k^\mu), \\
\langle D(k)|A^\mu|B(p)\rangle &= 0, \\
\langle D^*(k,\epsilon)|V^\mu|B(p)\rangle &= -iv e^{\mu\nu\alpha\beta}\epsilon^*_\nu k_\alpha p_\beta, \\
\langle D^*(k,\epsilon)|A^\mu|B(p)\rangle &= a_1(\epsilon^*)^\mu + a_2(\epsilon^* p)p^\mu \\
&\quad + a_3(\epsilon^* p)k^\mu,
\end{aligned} \tag{2}$$

where $V^\mu = \bar{b}\gamma^\mu c$ and $A^\mu = \bar{b}\gamma^\mu\gamma_5 c$.

The $\langle D(k)|A^\mu|B(p)\rangle$ matrix element is equal to zero, since it is impossible to construct an axial vector composed of the two momenta available. In our calculations we use the following definition of the Levi-Civitá tensor: $\epsilon^{0123} = 1$.

Estimates of the transverse polarization are carried out in the leading order of HQET, i.e., under the assumption of $m_b, m_c \to \infty$. In this approximation the form factors of the process are expressed in terms of the Isgur-Wise function $\xi(vv')$ [12,13] and, accordingly, expressions (2) can be rewritten as

$$\begin{aligned}
\langle D(k)|V^\mu|B(p)\rangle &= \frac{\xi(\omega)}{\sqrt{m_D m_B}}(m_D p^\mu + m_B k^\mu), \\
\langle D(k)|A^\mu|B(p)\rangle &= 0, \\
\langle D^*(k,\epsilon)|V^\mu|B(p)\rangle &= -i\frac{\xi(\omega)}{\sqrt{m_D m_B}} e^{\mu\nu\alpha\beta}\epsilon^*_\nu k_\alpha p_\beta, \\
\langle D^*(k,\epsilon)|A^\mu|B(p)\rangle &= \frac{\xi(\omega)}{\sqrt{m_D m_B}}[(m_B m_D + pk)\epsilon^{*\mu} \\
&\quad - (\epsilon^* p)k^\mu],
\end{aligned} \tag{3}$$

where $\omega = (pk)/(m_D m_B)$. Except for the given matrix elements, it is necessary to take into account the matrix elements of the vector current between D and D (D^* and D) states. In the framework of HQET the form factors of these matrix elements are also expressed through the Isgur-Wise function:

$$\begin{aligned}
\langle D(p')|\bar{c}\gamma^\mu c|D(k)\rangle &= \xi(\omega')(p'^\mu + k^\mu), \\
\langle D(p')|\bar{c}\gamma^\mu c|D^*(k,\epsilon)\rangle &= -i\frac{\xi(\omega')}{m_D} e^{\mu\nu\alpha\beta}\epsilon_\nu p'_\alpha k_\beta,
\end{aligned} \tag{4}$$

where $\omega' = (kp')/(m_D m_B)$. The D and D^* mass difference is neglected in Eqs. (3) and (4) as it does not contribute to these matrix elements in the leading order of HQET.

The function ξ is conventionally parametrized as

$$\xi(\omega) = 1 - \rho^2(\omega - 1). \tag{5}$$

In numerical estimates we use the value $\rho^2 = 0.94$ obtained in [14] within the framework of the potential quark model. This result is in good agreement with the experimental data [13]. Different values cannot drastically change the numerical results, since the kinematical area of the decay is quite narrow. In the $B \to D\tau\nu_\tau$ decay, ω varies from 1 to $(m_B^2 + m_D^2 - m_\tau^2)/2m_B m_D = 1.43$, and in the $B \to D\mu\nu_\mu$ decay this value varies in the range of 1–1.59.

The $\langle D|J^\mu_{\text{e.m.}}|D^*\rangle$ and $\langle D|J^\mu_{\text{e.m.}}|D\rangle$ matrix elements, where $J^\mu_{\text{e.m.}}$ is the electromagnetic current, are also required. It is possible to write down this operator as the sum of heavy and light quark components:

$$J^\mu_{\text{e.m.}} = J^\mu_h + J^\mu_l. \tag{6}$$

The matrix elements of the heavy component of this electromagnetic current are expressed through the Isgur-Wise function (4):

$$\begin{aligned}
\langle D(p')|J^\mu_h|D(k)\rangle &= -q_c\xi(\omega')(p'^\mu + k^\mu), \\
\langle D(p')|J^\mu_h|D^*(k,\epsilon)\rangle &= iq_c\frac{\xi(\omega')}{m_D} e^{\mu\nu\alpha\beta}\epsilon_\nu p'_\alpha k_\beta,
\end{aligned} \tag{7}$$

where q_c is the c-quark charge. The matrix elements of J^μ_l have the form

$$\begin{aligned}
\langle D(p')|J^\mu_l|D(k)\rangle &= q_l f^1_l(q^2)(p'^\mu + k^\mu), \\
\langle D(p')|J^\mu_l|D^*(k,\epsilon)\rangle &= iq_l\beta f^2_l(q^2) e^{\mu\nu\alpha\beta}\epsilon_\nu p'_\alpha k_\beta,
\end{aligned} \tag{8}$$

where q_l is the light quark charge and $f_l^{(1,2)}(0) = 1$. The constant β evaluated in [15] is equal to 1.9 GeV^{-1}. In our calculations we use a q^2 dependence of the $f^i_l(q^2)$ form factors $(i = 1,2)$, obtained under the assumption of the dominant contribution of the ω and ρ resonances to these form factors [16]. Neglecting the ω- and ρ-meson mass difference, the expressions for f^i_l take the form

$$f_l^i = \frac{1}{1-q^2/m_\rho^2}, \quad i=1,2, \tag{9}$$

where m_ρ is the ρ-meson mass.

III. LEPTON TRANSVERSE POLARIZATION

The amplitude of $B \to D(D^*) l^+ \nu_l$ decay can be written as follows:

$$M = \frac{G_F}{\sqrt{2}} V_{cb}^* \langle D | V^\mu - A^\mu | B \rangle \overline{u}(p_\nu)(1+\gamma_5)\gamma_\mu v(p_l), \tag{10}$$

where G_F is the Fermi constant and V_{cb}^* is the corresponding CKM matrix element. The matrix elements $\langle D | V^\mu - A^\mu | B \rangle$ are discussed in the previous section. For the case of the $B \to D l \nu_l$ process it is convenient to introduce the following parametrization of the amplitude:

$$M = \frac{G_F}{\sqrt{2}} V_{cb}^* \overline{u}(p_\nu)(1+\gamma_5)(C_1 \hat{p} + C_2) v(p_l). \tag{11}$$

It should be noted that Eq. (11) is the most general form of the decay amplitude. The expressions for C_1, C_2 in the leading order of HQET can be written as follows:

$$C_1 = \frac{\xi(\omega)}{\sqrt{m_B m_D}}(m_D + m_B),$$

$$C_2 = \frac{\xi(\omega)}{\sqrt{m_B m_D}}(m_B m_l). \tag{12}$$

The partial width of the $B \to D l^+ \nu_l$ decay in the B-meson rest frame can be expressed as

$$d\Gamma = \frac{\sum |M|^2}{2m_B}(2\pi)^4 \delta(p - p_D - p_l - p_\nu) \times \frac{d^3 p_D}{(2\pi)^3 2E_D} \frac{d^3 p_l}{(2\pi)^3 2E_l} \frac{d^3 p_{\nu_l}}{(2\pi)^3 2E_{\nu_l}}, \tag{13}$$

where summation over lepton and photon spin states is performed.

Introducing the unit vector along the muon spin direction in the lepton rest frame, $\vec{\mathbf{s}}$, where $\vec{\mathbf{e}}_i (i = L, N, T)$ are the unit vectors along the longitudinal, normal, and transverse components of the lepton polarization, one can write down the matrix element squared for the transition into the particular lepton polarization state in the following form:

$$|M|^2 = \rho_0 [1 + (P_L \vec{\mathbf{e}}_L + P_N \vec{\mathbf{e}}_N + P_T \vec{\mathbf{e}}_T) \cdot \vec{\mathbf{s}}], \tag{14}$$

where ρ_0 is the Dalitz plot probability density averaged over polarization states. The unit vectors $\vec{\mathbf{e}}_i$ can be expressed in terms of the three-momenta of the final particles:

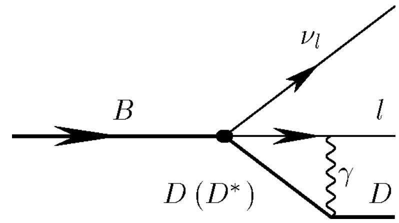

FIG. 1. Feynman diagrams contributing to lepton transverse polarization in $B^0 \to D^- l \nu_l$ decay at the one-loop level of the SM.

$$\vec{\mathbf{e}}_L = \frac{\vec{p}_l}{|\vec{p}_l|}, \quad \vec{\mathbf{e}}_N = \frac{\vec{p}_l \times (\vec{p}_D \times \vec{p}_l)}{|\vec{p}_l \times (\vec{p}_D \times \vec{p}_l)|},$$

$$\vec{\mathbf{e}}_T = \frac{\vec{p}_D \times \vec{p}_l}{|\vec{p}_D \times \vec{p}_l|}. \tag{15}$$

With this definition of the $\vec{\mathbf{e}}_i$ vectors, P_T, P_L, and P_N denote the transverse, longitudinal, and normal components of the muon polarization, respectively.

The Dalitz plot probability density has the following form:

$$\rho_0 = G_F^2 |V_{cb}|^2 [4|C_1|^2 (p p_\nu)(p p_l) + 2|C_2|^2 (p_\nu p_l) - 2|C_1|^2 (p_\nu p_l) m_B^2 - 4 m_l \,\mathrm{Re}(C_2 C_1^*)(p p_\nu)]. \tag{16}$$

The expression for transverse polarization can be written as follows:

$$P_T = \frac{\rho_T}{\rho_0}, \tag{17}$$

where ρ_T has the form

$$\rho_T = 4 G_F^2 |V_{cb}|^2 m_B \,\mathrm{Im}(C_1 C_2^*) |\vec{p}_D \times \vec{p}_l|. \tag{18}$$

Obviously, the lepton transverse polarization arises only in the case of nonzero phase shift between C_1 and C_2 form factors. At the tree level of the SM these form factors are real and thus the lepton transverse polarization in this case is equal to zero. The nonvanishing P_T arises due to the effect of the final state interaction. To calculate the imaginary parts of the form factors one can use the S-matrix unitarity as was done in [17] for the case of the $K^0 \to \pi \mu \nu$ decay.

The diagrams inducing the transverse lepton polarization are shown in Fig. 1. As was mentioned earlier, in our calculations we take into account only the diagrams with intermediate D, D^* mesons. The contribution of these diagrams to the imaginary part of the decay amplitude can be written as follows:

$$\mathrm{Im}\, M = \frac{G_F}{\sqrt{2}} V_{cb}^* \frac{\alpha}{2\pi} \int \frac{d\rho}{k_\gamma^2} \overline{u}(p_\nu)(1+\gamma_5)\gamma_\sigma(\hat{k}_l - m_l)\gamma_\lambda v(p_l) \times \sum_{n=D,D^*} \langle D | J_{em}^\lambda | n \rangle \langle n | V^\sigma - A^\sigma | B \rangle, \tag{19}$$

where k_l, k_D denote the four-momenta of the intermediate D, D^* mesons, respectively, $k_\gamma^2 = (k_D - p_D)^2$ is the squared transferred momentum, and $d\rho$ is the two-particle phase space. Expressions for the $\langle D | J^\lambda_{\rm e.m.} | n \rangle, \langle n | V^\sigma - A^\sigma | B \rangle$ matrix elements are given in the previous section.

It should be noted that in the case of $B^0 \to D^- l \nu_l$ decay the D^--meson contribution to the imaginary part comprises an infrared divergence, but the latter does not affect the value of transverse polarization. One may explain this fact by the factorization of the soft photon contribution, which, in turn, does not lead to the nonzero phase difference of form factors required for a nonvanishing transverse polarization. Really, in the limit of soft photon exchange $k_\gamma \to 0$, one may disregard the difference between k_D, k_l and p_D, p_l in the numerator of Eq. (19). Regarding the divergent part only and applying the Dirac equation, the formula (19) may be rewritten as follows:

$$\operatorname{Im} M = -\frac{G_F}{\sqrt{2}} V_{cb}^* \frac{\alpha}{\pi} \langle D | (V-A)^\sigma | B \rangle \overline{u(p_\nu)} (1+\gamma_5) \times \gamma_\sigma v(p_l) (2 p_l p_D) \int \frac{d\rho}{k_\gamma^2}.$$

So we see that the above expression repeats formula (10) up to the divergent coefficient that leads to equal phase shifts in C_1, C_2. As for the D^*-meson contribution, it is free from infrared divergence, since the matrix element $\langle D | J_{\rm e.m.} | D^* \rangle$ is proportional to k_γ.

In contrast to the $B^0 \to D^- l \nu_l$ decay, formula (19) for $B^+ \to \overline{D}{}^0 l \nu_l$ is free from infrared divergence since the matrix elements for $\overline{D}{}^*$-$\overline{D}$ and $\overline{D}{}^0$-$\overline{D}{}^0$ electromagnetic transitions are proportional to k_γ and k_γ^2, respectively.

Evidently, there are contributions to transverse polarization coming from excited D-meson states. But we assume that this contribution cannot change the result dramatically, since the Isgur-Wise function defined in Eq. (5) is greater than that in the case of B-meson decay into an excited D meson. The Isgur-Wise function for transition into the $(0^+, 1^+)$ doublet, $\tau_{1/2}(\omega)$, was calculated in [18]. The value of $\tau_{1/2}(1)$ obtained in this paper is equal to 0.24. One can see that the heavy quark contribution to P_T is proportional to the second power of $\tau_{1/2}(1)$ and the light quark contribution is linear in $\tau_{1/2}(1)$. Furthermore, the physical region, where the value of transverse polarization is not equal to zero is bounded by inequality $(p_l + p_D)^2 \geqslant (m_D + m_l)^2$. In the case of the intermediate D^* meson this restriction does not cut the physical region significantly. Contrary to the case of the intermediate state with a D^* meson, the physical region for the $(0^+, 1^+)$ case is strongly confined, which results in the reduction of the average transverse polarization.

The integrals entering the imaginary part of the one-loop decay amplitude are given in Appendix A, and Appendix B contains expressions for the imaginary parts of the form factors.

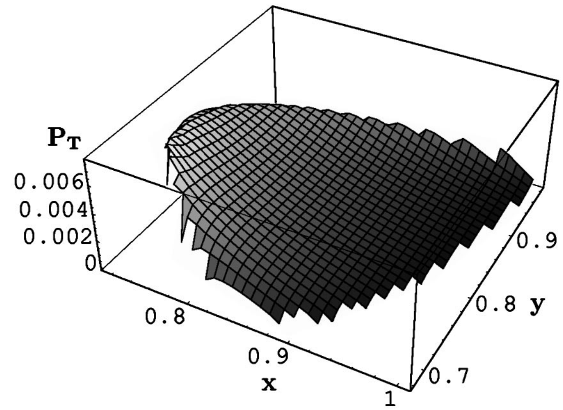

FIG. 2. The 3D plot for τ-lepton transverse polarization for the case of $B^0 \to D^- \tau \nu_\tau$ decay.

IV. NUMERICAL RESULTS AND DISCUSSION

It is convenient to use the following x, y variables:

$$E_D = \frac{m_B}{2} x, \quad E_\gamma = \frac{m_B}{2} y, \tag{20}$$

where E_D and E_γ are the D-meson and photon energies; m_B is the B-meson mass.

The three-dimensional distributions of lepton transverse polarization in the kinematical region (x,y) for the processes $B^0 \to D^- \tau \nu_\mu$, $B^0 \to D^- \mu \nu_\tau$ are shown in Figs. 2 and 3, respectively. The contour lines for P_T in these decays are shown in Figs. 4 and 5.

In our calculation we have taken into account D and D^* contributions only. In the $B^0 \to D^- l \nu_l$ decay the D^- meson gives the dominant contribution to lepton polarization. Keeping in mind that electromagnetic form factors reach their maximum at $k_\gamma^2 \to 0$ one may expect that P_T reaches a maximum at

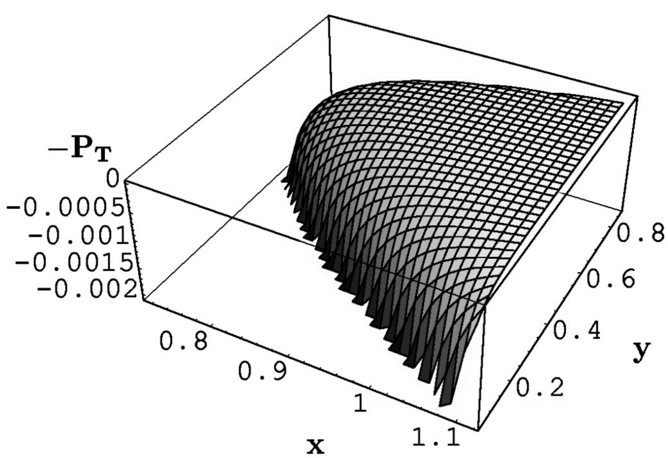

FIG. 3. The 3D plot for muon transverse polarization for the case of $B^0 \to D^- \mu \nu_\mu$ decay.

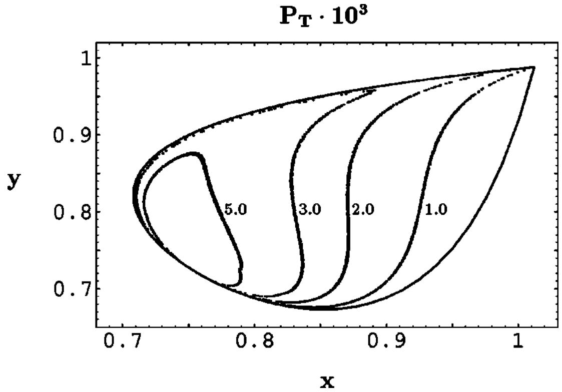

FIG. 4. The level lines for τ-lepton transverse polarization for the case of $B^0\to D^-\tau\nu_\tau$ decay.

$$x=\frac{m_D}{m_D+m_l}\frac{m_B^2+(m_D+m_l)^2}{m_B^2},$$

$$y=\frac{m_l}{m_D+m_l}\frac{m_B^2+(m_D+m_l)^2}{m_B^2}. \tag{21}$$

Indeed, the three-dimensional distribution of τ -lepton transverse polarization (Fig. 2). has a maximum at (0.76,0.72). The maximum of the muon transverse polarization is at (1.08,0.06).

The three-dimensional distributions of lepton transverse polarization in the kinematical region (x,y) for the processes $B^+\to\overline{D^0}\tau\nu_\mu$, and $B^+\to\overline{D^0}\mu\nu_\tau$ are shown in Figs. 6 and 7, respectively. The contour lines for P_T in these decays are shown in Figs. 8 and 9.

In contrast to $B^0\to D^-l\nu_l$ decay, the contribution of the intermediate $\overline{D^0}$ does not dominate in $B^+\to\overline{D^0}l\nu_l$ decay. This fact may be explained by the partial cancellation of the heavy and light components of the electromagnetic current in the physical region due to the neutrality of the $\overline{D^0}$ meson. If $k_\gamma^2\ll m_\rho^2$, the electromagnetic current of the $\overline{D^0}$-$\overline{D^0}$ transition

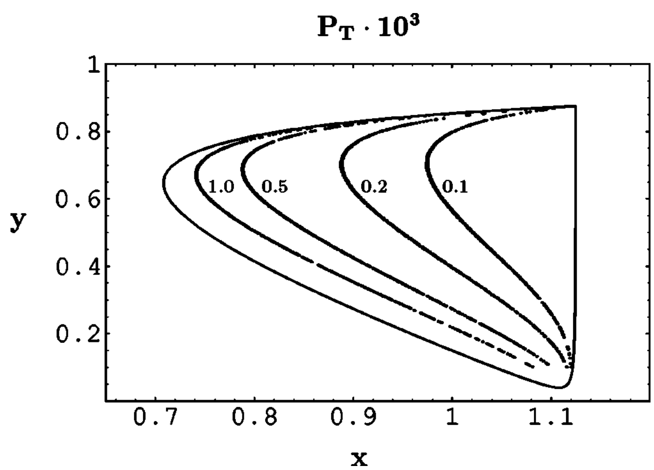

FIG. 5. The level lines for muon transverse polarization for the case of $B^0\to D^-\mu\nu_\mu$ decay.

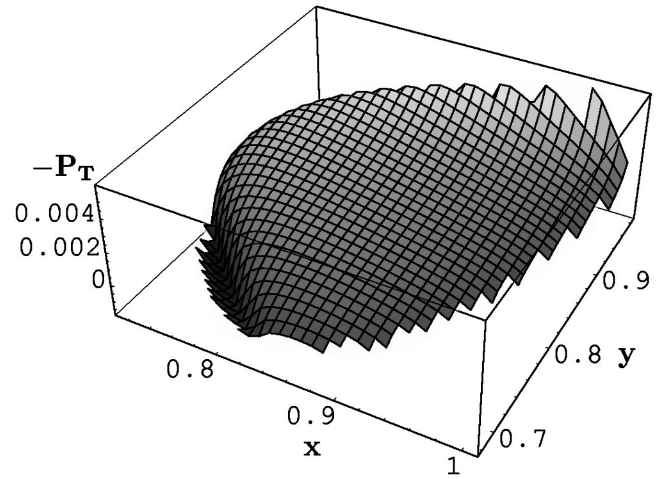

FIG. 6. The 3D plot for τ-lepton transverse polarization for the case of $B^+\to\overline{D^0}\tau\nu_\tau$ decay.

behaves as $\sim k_\gamma^2$ so one may suppose that the maximum of P_T is shifted from the point defined by Eq. (21) that corresponds to our result.

The values of transverse polarization averaged over the physical region are shown in Table I.

Finally, we would like to remark that the averaged values of P_T in the SM are quite small in comparison to the predictions of some models [9–11]. This allows one to conclude that B-meson decays provide an appealing possibility to search for new physics effects.

ACKNOWLEDGMENTS

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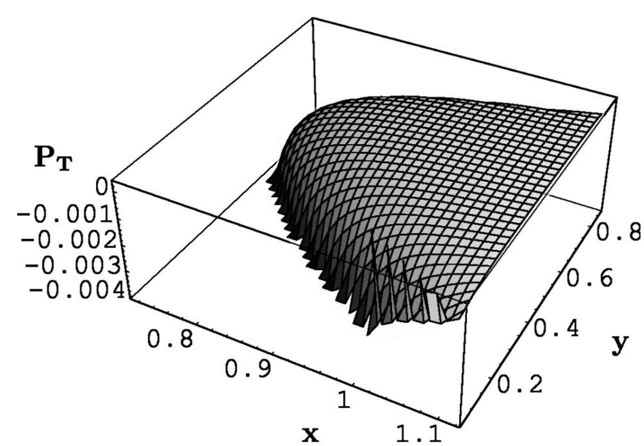

FIG. 7. The 3D plot for muon transverse polarization for the case of $B^+\to\overline{D^0}\mu\nu_\mu$ decay.

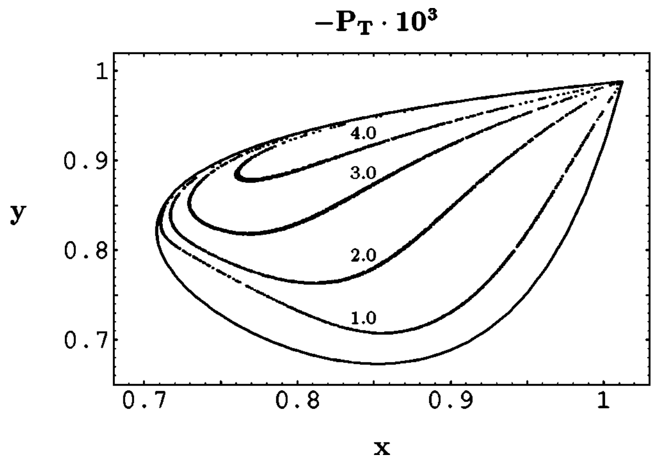

FIG. 8. The level lines for τ-lepton transverse polarization for the case of $B^+ \to \overline{D}^0 \tau \nu_\tau$ decay.

APPENDIX A

Let us introduce the following notation:

$$P^\mu = p_l^\mu + p_D^\mu,$$

$$l^\mu = p_D^\mu - \frac{(Pp_D)}{(P^2)} P^\mu,$$

$$Pk_D = \frac{1}{2}(P^2 + m_{D*}^2 - m_l^2),$$

$$l^2 = m_D^2 - \frac{(Pp_D)^2}{P^2},$$

$$l_1^2 = m_{D*}^2 - \frac{(Pk_D)^2}{P^2},$$

$$\eta^2 = m_D^2 + m_{D*}^2 - 2\frac{(Pk_D)(Pp_D)}{P^2}.$$

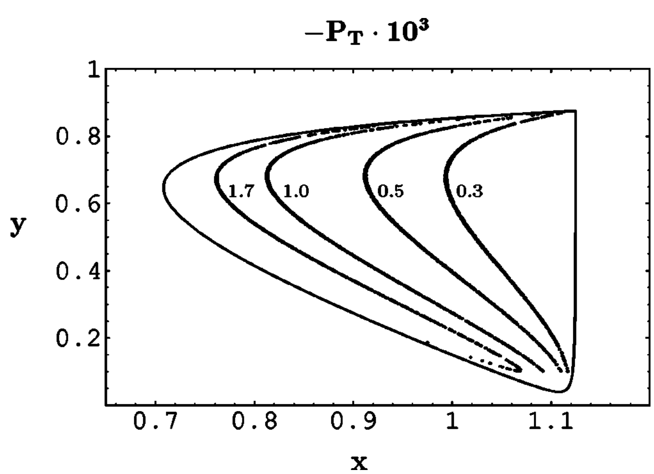

FIG. 9. The level lines for muon transverse polarization for the case of $B^+ \to \overline{D}^0 \mu \nu_\mu$ decay.

TABLE I. Values of averaged transverse polarization.

Decay	$\langle P_T \rangle$
$B^0 \to D^- \tau \nu_\tau$	2.60×10^{-3}
$B^0 \to D^- \mu \nu_\mu$	2.97×10^{-4}
$B^+ \to \overline{D}^0 \tau \nu_\tau$	-1.59×10^{-3}
$B^+ \to \overline{D}^0 \mu \nu_\mu$	-6.79×10^{-4}

So the integrals contributing the imaginary parts of the one-loop amplitude can be written as follows:

$$J_1 = \int \frac{d\rho}{k_\gamma^2} f_l(k_\gamma^2) = \frac{\pi}{4} \frac{1}{\sqrt{-l^2 P^2}} \left(\ln \left| \frac{-\eta^2 - 2\sqrt{l_1^2 l^2}}{-\eta^2 + 2\sqrt{l_1^2 l^2}} \right| - \ln \left| \frac{-\eta^2 - 2\sqrt{l_1^2 l^2} + m_\rho^2}{-\eta^2 + 2\sqrt{l_1^2 l^2} + m_\rho^2} \right| \right), \tag{A1}$$

$$J_2 = \int d\rho f_l(k_\gamma^2) = -\frac{\pi}{4} \frac{m_\rho^2}{\sqrt{-l^2 P^2}} \left(\ln \left| \frac{-\eta^2 - 2\sqrt{l_1^2 l^2} + m_\rho^2}{-\eta^2 + 2\sqrt{l_1^2 l^2} + m_\rho^2} \right| \right), \tag{A2}$$

$$J_3 = \int d\rho = \pi \frac{\sqrt{-l_1^2 P^2}}{P^2}; \tag{A3}$$

$$\int \frac{k_D^\alpha}{k_\gamma^2} f_l(k_\gamma^2) d\rho = a_1 P^\alpha + b_1 l^\alpha, \tag{A4}$$

where

$$a_1 = \frac{Pk_D}{P^2} J_1,$$

$$b_1 = \frac{\eta^2}{2l^2} J_1 - \frac{1}{2l^2} J_2; \tag{A5}$$

$$\int k_D^\alpha f_l(k_\gamma^2) d\rho = A_1 P^\alpha + B_1 l^\alpha, \tag{A6}$$

where

$$A_1 = \frac{Pk_D}{P^2} J_2,$$

$$B_1 = \frac{\eta^2}{2l^2} J_2 - \frac{m_\rho^2}{2l^2}(J_2 - J_3); \tag{A7}$$

$$\int \frac{k_D^\alpha k_D^\beta}{k_\gamma^2} f_l(k_\gamma^2) d\rho = a_2 g^{\alpha\beta} + b_2 \frac{P^\alpha P^\beta}{P^2} + c_2(l^\alpha P^\beta + l^\beta P^\alpha) + d_2 \frac{l^\alpha l^\beta}{l^2}, \tag{A8}$$

where

$$a_2 = \frac{l_1^2}{2} J_1 - \frac{\eta^2}{4} b_1 + \frac{B_1}{4},$$

$$b_2 = \frac{(Pk_D)^2}{P^2} J_1 - a_2,$$

$$c_2 = \frac{Pk_D}{P^2} b_1,$$

$$d_2 = \frac{\eta^2}{2} b_1 - \frac{B_1}{2} - a_2; \tag{A9}$$

$$\int k_D^\alpha k_D^\beta f_l(k_\gamma^2) d\rho = A_2 g^{\alpha\beta} + B_2 \frac{P^\alpha P^\beta}{P^2} + C_2(l^\alpha P^\beta + l^\beta P^\alpha) + D_2 \frac{l^\alpha l^\beta}{l^2}, \tag{A10}$$

where

$$A_2 = \frac{l_1^2}{2} J_2 - \frac{\eta^2}{4} B_1 + \frac{m_\rho^2 B_1}{4},$$

$$B_2 = \frac{(Pk_D)^2}{P^2} J_2 - A_2,$$

$$C_2 = \frac{Pk_D}{P^2} B_1,$$

$$D_2 = \frac{\eta^2}{2} B_1 - \frac{m_\rho^2 B_1}{2} - A_2; \tag{A11}$$

$$\begin{aligned}\int \frac{k_D^\alpha k_D^\beta k_D^\gamma}{k_\gamma^2} d\rho = {} & a_3(g_{\alpha\beta}P^\gamma + g_{\alpha\gamma}P^\beta + g_{\beta\gamma}P^\alpha) \\ & + b_3(g_{\alpha\beta}l^\gamma + g_{\alpha\gamma}l^\beta + g_{\beta\gamma}l^\alpha) \\ & + c_3(l^\alpha P^\beta P^\gamma + l^\beta P^\alpha P^\beta + l^\gamma P^\alpha P^\beta) \\ & + d_3(l^\alpha l^\beta P^\gamma + l^\beta l^\alpha P^\beta + l^\gamma l^\alpha P^\beta) \\ & + e_3 l^\alpha l^\beta l^\gamma + f_3 P^\alpha P^\beta P^\gamma,\end{aligned} \tag{A12}$$

where

$$a_3 = \frac{Pk_D}{P^2} a_2,$$

$$b_3 = \frac{1}{2l^2}(\eta^2 a_2 - A_2),$$

$$c_3 = \frac{1}{2l^2P^2}(\eta^2 b_2 - B_2),$$

$$d_3 = \frac{Pk_D}{P^2 l^2} d_2,$$

$$e_3 = \frac{1}{2(l^2)^2}(\eta^2 d_2 - D_2) - \frac{2}{l^2} b_3,$$

$$f_3 = \frac{Pk_D}{(P^2)^2}(b_2 - 2a_2); \tag{A13}$$

$$\begin{aligned}\int k_D^\alpha k_D^\beta k_D^\gamma f_l(k_\gamma^2) d\rho = {} & A_3(g_{\alpha\beta}P^\gamma + g_{\alpha\gamma}P^\beta + g_{\beta\gamma}P^\alpha) \\ & + B_3(g_{\alpha\beta}l^\gamma + g_{\alpha\gamma}l^\beta + g_{\beta\gamma}l^\alpha) \\ & + C_3(l^\alpha P^\beta P^\gamma + l^\beta P^\alpha P^\beta + l^\gamma P^\alpha P^\beta) \\ & + D_3(l^\alpha l^\beta P^\gamma + l^\beta l^\alpha P^\beta + l^\gamma l^\alpha P^\beta) \\ & + E_3 l^\alpha l^\beta l^\gamma + F_3 P^\alpha P^\beta P^\gamma,\end{aligned} \tag{A14}$$

where

$$A_3 = \frac{Pk_D}{P^2} A_2,$$

$$B_3 = \frac{1}{2l^2}\left(\eta^2 A_2 - m_\rho^2 A_2 + \frac{1}{3} m_\rho^2 l_1^2 J_3\right),$$

$$C_3 = \frac{1}{2l^2P^2}\left[\eta^2 B_2 - m_\rho^2\left(B_2 - \frac{(Pk_D)^2}{P^2} J_3 + \frac{l_1^2}{3} J_3\right)\right],$$

$$D_3 = \frac{Pk_D}{P^2 l^2} D_2,$$

$$E_3 = \frac{1}{2(l^2)^2}(\eta^2 D_2 - m_\rho^2 D_2) - \frac{2}{l^2} B_3,$$

$$F_3 = \frac{Pk_D}{(P^2)^2}(B_2 - 2A_2). \tag{A15}$$

APPENDIX B

First, we take into account the contribution of the diagram with an intermediate D meson to lepton polarization. The expression for $\mathrm{Im}(C_1 C_2^*)$ entering Eq. (18) can be represented as the sum of heavy and light components:

$$\mathrm{Im}(C_1 C_2^*) = \frac{\alpha}{2\pi}\xi(\omega)\frac{m_l}{m_B m_D}\left[q_l\left((1+\rho^2)C_l - \frac{\rho^2}{m_B m_D}C_l'\right) - q_c\left((1+\rho^2)C_h - \frac{\rho^2}{m_B m_D}C_h'\right)\right], \tag{B1}$$

where $\xi(\omega)$ is the Isgur-Wise function; ρ^2 is the slope of this function; q_l, q_c are the light quark and c-quark charges. The coefficients C_l and C_l' entering Eq. (B1) have the form

$$C_l = 2b_1\frac{P p_D}{P^2}(m_B^2 m_l^2 - M^2 P^2), \tag{B2}$$

$$C_l' = a_2[2P^2(m_B^2 - m_D^2) + 4(Pp)M m_D - 2m_B^2 m_l^2] + 2b_2\frac{(Pp)}{P^2}(P^2 M^2 - m_B^2 m_l^2) + \frac{2}{P^2}c_2(m_B^2 m_l^2 - M^2 P^2)$$
$$\times[-(lp)P^2 + (Pp)(Pp_D)] + \frac{2(Pp_D)(lp)}{P^2 l^2}d_2(m_B^2 m_l^2 - M^2 P^2), \tag{B3}$$

where $M = m_B + m_D$. In Appendix A we present the coefficients b_1, a_2, b_2, c_2, d_2 which are valid for the case of an intermediate D^*-meson, i.e., the integration is performed assuming $m_{D*} \neq m_D$. To obtain these coefficients for the case of the intermediate D-meson one should substitute m_{D*} by m_D. After the substitution the integral J_1 becomes divergent, but, as mentioned above, it will not affect the value of the transverse polarization and can be omitted. To obtain the C_h, C_h' coefficients one has to expand C_l, C_l' in powers of $1/m_\rho^2$ up to linear terms and substitute $1/m_\rho^2$ by $\rho^2/(2m_D^2)$.

As for the contribution of the intermediate D^* meson to the imaginary parts of C_1, C_2, we do not give them here since the formulas are too bulky.

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