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Eduardo Machado Silva

The One-dimensional Multi-Period Cutting Stock Problem with Setup Cost

São José do Rio Preto
2023

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Tese apresentada como parte dos requisitos para obtenção do título de Doutor em Matemática, junto ao Programa de Pós-Graduação em Matemática, do Instituto de Biociências, Letras e Ciências Exatas da Universidade Estadual Paulista "Júlio de Mesquita Filho", Câmpus de São José do Rio Preto.

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RESUMO

Nesta tese o problema de corte de estoque unidimensional multiperíodo com minimização dos custos de preparação nos padrões de corte é estudado. Primeiro propomos um extenso conjunto de formulações pseudo-polinomiais e baseadas em padrões de corte, principalmente adaptando formulações conhecidas para problemas de corte de estoque da literatura. Reformulações baseadas no problema de localização de facilidades são discutidas para melhorar os limitantes inferiores dos modelos propostos. Em seguida, apresentamos uma análise teórica comparando as várias formulações propostas em relação ao seu limitante inferior. Apresentamos uma análise computacional a fim de complementar a análise teórica e apresentar mais *insights* com relação à complexidade e qualidade das formulações na prática. Os experimentos computacionais foram realizados em dois conjuntos de instâncias sendo o segundo mais difícil de ser resolvido. Ambos os conjuntos de instâncias mostraram que as reformulações baseadas no problema de localização de facilidade propostas melhoram a qualidade dos limitantes inferiores. Contudo, testes adicionais mostram que a melhoria do limitante é afetada quando maiores custos do objeto em estoque são considerados. Em uma abordagem diferente, um algoritmo genético de chaves aleatórias viciadas em que o controle dos parâmetros é adaptativo é proposto para resolver o problema. Para a inicialização da metaheurística, um procedimento de decodificação das chaves aleatórias em termos da solução (*decoder*) do problema é necessário. Dois (*decoders*) são propostos e avaliados com base nos resultados de uma heurística de geração de colunas integrada a um *software* de solução. Uma combinação da metaheurística e da heurística de geração de colunas também é apresentada. Os resultados computacionais mostram que ambos os processos de decodificação obtém um melhor desempenho que a heurística de geração de colunas para instâncias com items pequenos cujo custo de preparação nos padrões de corte é maior em relação ao custo do objeto em estoque. Por fim, são discutidas as conclusões finais e propostas de pesquisa futura.

Palavras chave: Problema de Corte de Estoque Multiperíodo. Setup nos Padrões de Corte. Formulações Fortes. Algoritmo Genético Adaptativo com Chaves Aleatórias Viciadas (*ABRKGA*).

ABSTRACT

In this thesis, we study the multi-period one dimensional cutting stock problem minimizing setup costs on cutting patterns. We first propose an extensive range of pattern-based and pseudo-polynomial formulations for the problem, primarily adapting known formulations for cutting stock problems from the literature. Facility location based reformulations are also discussed to improve the lower bounds of the proposed models. We then present a thorough theoretical analysis to establish the strength of the various proposed formulations in comparison to each other. A computational analysis is presented to complement the theoretical analysis and present further insights with respect to the complexity and strength of the formulations in practice. The computational experiments were performed over two sets of instances where the second set is more difficult to solve. Both sets of instances have shown that the proposed facility location reformulations significantly improve the quality of the lower bounds. However, additional computational tests show that the lower bound improvement is directly affected by the object cost. Then, a random key genetic algorithm with adaptive parameter control is proposed to solve the problem. The metaheuristic initialization requires a decoder process which maps the random keys to feasible solutions of the problem. Two different decoder processes are proposed and evaluated according to the performance of a column generation heuristic solved by an optimization software. An experiment combining both the metaheuristic and the column generation is also presented. Computational experiments show that both decoders outperform the column generation based heuristic for instances with small items length and setup cost greater than the object cost. Finally, some final conclusions and future research are discussed.

Keywords: Multi-Period Cutting Stock Problem. Cutting Pattern Setup. Strong Formulations. Adaptive Biased Random Keys Genetic Algorithm (*ABRKGA*).

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LIST OF ABBREVIATIONS

OR	Operational Research;
CSP	Cutting Stock Problem;
PMP	Pattern Minimization Problem;
CSPs	Cutting Stock Problem with setups on cutting patterns;
$\mathcal{NP}$-hard	Non-deterministic polynomial-time hard;
MPCSP	Multi-Period Cutting Stock Problem;
MPCSPs	Multi-Period Cutting Stock Problem with Setup Costs;
VRP	Vehicle Routing Problem;
AGG	Adapted Gilmore and Gomory;
AJS	Adapted Johnston and Sadinlija;
ARE	Adapted Reflect;
AVR	Adapted Vehicle Routing;
AGGFL	Adapted Gilmore and Gomory Facility Location;
AJSFL	Adapted Johnston and Sadinlija Facility Location;
AREFL	Adapted Reflect Facility Location;
AVRFL	Adapted Vehicle Routing Facility Location;
EA	Evolutionary Algorithm;
GA	Genetic Algorithm;
RKGA	Random Key Genetic Algorithm;
BRKGA	Biased Random Key Genetic Algorithm;
ABRKGA	Adaptive Biased Random Key Genetic Algorithm;

CONTENTS

CHAPTER 1 INTRODUCTION

Operational Research (*OR*) is one of the managerial decision science tools used by profit and non-profit organizations. As the global environment becomes fiercely competitive, OR has gained significance in applications such as green logistic, quality management, benchmarking and decision making techniques. The growth of global markets and the resulting increase in competition have highlighted the need for *OR*. In order to be competitive, business must meet the challenges present in a global market by offering products and services with good value to their costumers (AGRAWAL et al. (2010)). *OR* leads to a more efficient use of resources, which is not only cost attractive, but lead to environment friendly decisions as well, being then an essential management tool to gain competitiveness in a industrial environment (DEKKER et al. (2012)).

Among the very first ideas to emerge from *OR* to be applied in practice, the cutting stock problem (*CSP*) was one of the problems identified by Kantorovich in his 1939 paper entitled "Mathematical methods of organizing and planning production" (later published in KANTOROVICH (1960)) (BEN AMOR and VALÉRIO DE CARVALHO (2005)). The *CSP* is concerned with determining the best way of cutting a set of objects into smaller items, in order to satisfy a given demand of the items and optimizing an objective function, often with a large potential of economic savings, such as the minimization of waste and the minimization of used objects. A strong characteristic of *CSP* is that the research direction has largely been motivated by taking inspiration, or directly solving, problems from industry. The literature describes a large range of problems that often include specific operational constraints and objectives, as describes the EURO Special Interest Group on Cutting and Packing (ESICUP). One of the Group main purposes is to improve communication among individuals working in this field. The *CSP* wide variety of industrial applications includes paper, steel, wood and glass industries.

The wide range of cutting stock applications motivated Dyckhoff (1990) to introduce a typology for the cutting problem. His typology organizes the problem according to four characteristics, which are the geometric dimensions $(1, 2, 3, n > 3)$, type of assignment (i.e selection of objects and items), assortment of large objects and the characterization of the assortment of the items. Later, Wäscher et al. (2007) proposed a revised typology that provided a consistent system of problem types which allows for a complete categorization of all known cutting problems and the corresponding current literature.

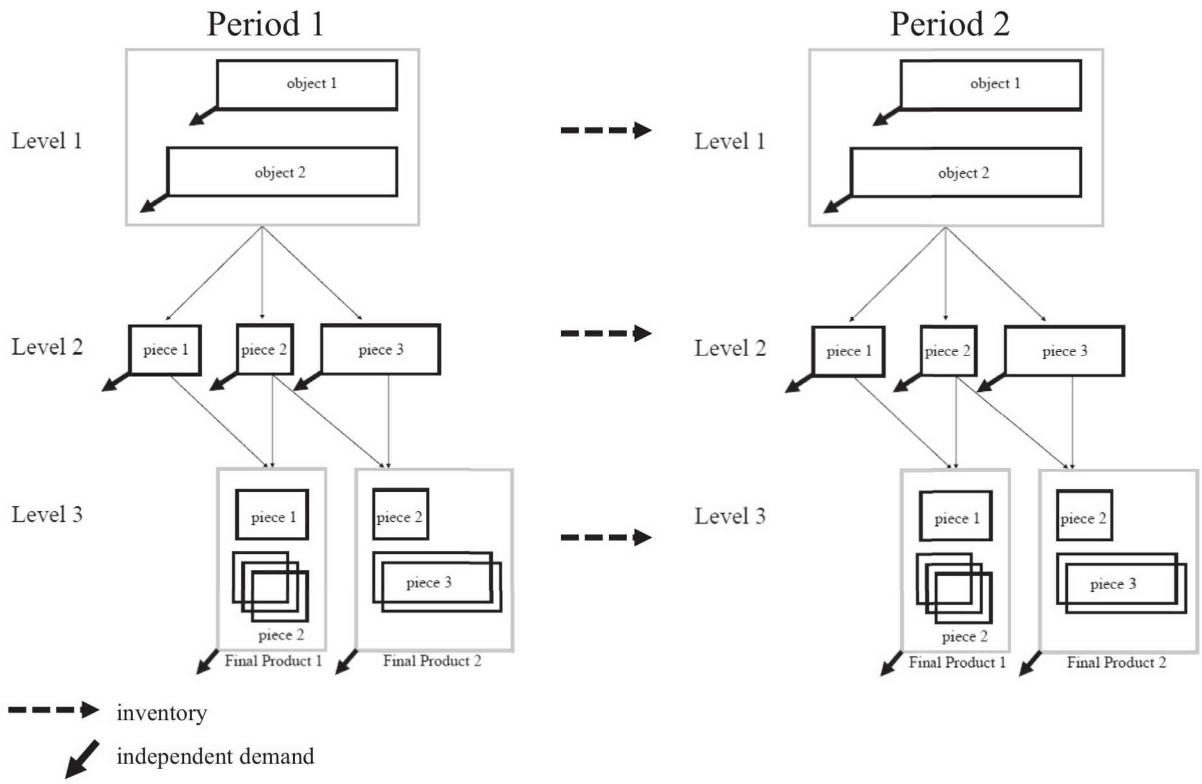
The cutting process in *CSP* may be affected by various factors, particularly by the number of times one has to switch between different cutting patterns, e.g., changing the positions of the cutting knives (Wuttke and Heese (2018)) or the position of lasers. Such adjustments often interrupt production and/or may impose a setup cost every time a different cutting pattern is used, often leading to impractical applications due the use of many different cutting patterns. Therefore, it is desirable to have a cutting plan composed of fewer cutting patterns. The problem that focuses only on minimizing of the number of different cutting patterns while satisfying demands is known in the literature as the Pattern Minimization Problem (*PMP*) (Vanderbeck (2000)). In the remainder of this thesis, *PMP* will be refereed to as the single period problem with setup costs on the cutting patterns where only the minimization of different cutting patterns is considered as objective. When additional costs are considered in the objective function (trim loss or object costs), the problem will be denoted as *CSPs*. We note that it is important to have bi-criteria decision problems when setup costs are significant when compared to material costs (Yanasse and Limeira (2006)). We also remark that even the single period version of this problem is known to be $\mathcal{NP}$-hard (McDiarmid (1999)).

Considering the *CSPs* with multiple time periods (Tomat and Gradisar (2017) and Trkman and Gradisar (2007)), there is a strong relevance to the lot-sizing problem, which has been an area of very active research over the last six decades (Brahimi et al. (2017)), offering significant cost savings to the manufacturing sector by generating the least costly production plan over a planning horizon with multiple periods as well for the green manufacturing sector (e.g. when carbon emission constraints are considered (Absi et al. (2016))). The lot-sizing problem deals with key decisions such as when and how much to produce or stock, while respecting limitations such as satisfying demands on time. The lot-sizing problem can also be characterized by the number of levels in the production structure (single level or multi-level), time horizon (finite or infinite) and the presence of capacity constraints. The research devoted to the topic (and solution methodologies therein) is extensive, ranging from polyhedral methods such as extended formulations and valid inequalities (Doostmohammadi and Akartunali (2018), Gruson et al. (2019), and Zhao and Zhang (2020)) to decomposition and relaxations (Akartunali et al. (2016), de Araujo et al. (2015), and Van Vyve et al. (2014)), and heuristics designed for real-world problems (Absi and van den Heuvel (2019), Fiorotto et al. (2017), and Wu et al. (2018)), as well as stochastic and robust approaches to tackle uncertainty in a broad range of settings (Alem et al. (2020), Attila et al. (2021), and Quezada et al. (2020)). An extensive analysis of lot-sizing problems can be found in the book by Pochet and Wolsey (2006).

Before considering integrated decisions during the production process, the literature has dealt with the cutting stock and the lot-sizing problems separately and sequentially. Firstly, the lot-sizing is solved and subsequently, the *CSP* is solved. However, this approach may increase the total cost, especially if the cutting process is economically relevant (Gramani et al. (2009) and Poltroniere et al. (2008)). The integration of these problems opens an interesting area for further research. Over the last decades this tendency was observed for the cutting stock and the lot-sizing. The interest in this problem often originates from direct practical applications of the integrated environments in various industries. For example, in the paper industry, large reels are manufactured or purchased and a decision about the size of the lots must be taken. Next, the large reels are cut into smaller reels that might correspond to customer requests, and a decision related to the cutting stock problem is needed (Melega et al. (2018)). Another motivation for studying these kind of problems is to explore models that capture the interdependence between both decisions in order to obtain better solutions. In general, the integrated lot-sizing and cutting stock problem consists of determining the cutting patterns and multiplicities of the corresponding cutting patterns (i.e., occurrence frequency) in each period of the planning horizon to satisfy customer demands while optimizing a given objective function, such as minimizing the costs associated with cutting pattern setup, inventory holding or objects consumed.

The integrated lot-sizing and cutting stock problems were reviewed in the extensive research of Melega et al. (2018), where a deterministic mathematical model, that considers multiple dimensions of integration and comprises several aspects found in practice, is proposed. This model is used as a framework to classify the current literature in this field. The authors essentially identify three levels of production, with the first level associated to the purchase/manufacture of object(s), the second level to the cutting process of objects into pieces, and third level to production of final products from pieces. An example of the generalized three level integration for two periods is presented in Figure 1. In their classification scheme, the case that considers exclusively the second level, with multiple time periods in a planning horizon, the inventory of cut pieces providing the link between different periods and cutting of objects planned for each period is called the Multi-Period Cutting Stock Problem (*MPCSP*). It is also worth noticing that when more than one level is considered, with multiple time periods in a planning horizon, Melega et al. (2018) classify the problem as the Integrated Lot-sizing and Cutting Stock Problem.

Figure 1: Integration between three production levels and two time periods. Source (Melega et al. (2018))



Although integrated decision problems have gained more attention over the last decades, there are only a small number of papers dealing with setups on cutting patterns with multiple periods in the literature. Moreover, most of this literature uses the multi-period adaptation of the well-known *CSP* formulations of Gilmore and Gomory (1961, 1963) and Kantorovich (1960). The cutting stock formulations based on the model of Gilmore and Gomory (1961, 1963) are classified as pattern-based formulations. The generalized model of Melega et al. (2018) is a pattern-based model and it address most of their literature review papers considering the *MPCSP* with setups. Even though this type of formulation avoids linearity issues, the number of variables in pattern-based models is exponential in the number of items.

As we note, different types of *CSP* models derive different types of *MPCSP* models. The arc-flow based models of Alves and Valério de Carvalho (2008a), Delorme (2017), and Valério de Carvalho (1999, 2002) are a different branch for modeling the *CSP*. The model of Valério de Carvalho (1999, 2002) was firstly proposed for the *MPCSP* by Poldi and de Araujo (2016) (through without cutting pattern setup minimization). The fist *MPCSP* model considering setups on cutting pattern was proposed, to the best of our knowledge, by Ma et al. (2019) and is based on the *PMP* arc-flow of Alves and Valério de Carvalho (2008a). Another types of *CSP* model formulations are the One-cut formulation of Dyckhoff (1981) and the knapsack-based formulation of Kantorovich (1960). A non-linear knapsack-based formulation was presented by Vanderbeck (2000).

In this thesis, we consider the *MPCSP* with setup costs on cutting patterns and an one-dimensional cutting process, which will hereafter refer to it as the *MPCSPs*. This research makes important contributions in this domain. First, we present four formulations to provide a rather complete picture of alternative formulations for the *MPCSPs*. To the best of our knowledge three of these *MPCSPs* formulations are proposed here for the first time in the literature, the ones inspired by the *CSP* models of Johnston and Sadinlija (2004), Delorme and Iori (2019) and Ben Amor and Valério de Carvalho (2005). Secondly, we consider strengthening the formulations by using extended reformulations. More specifically, we use the facility location reformulation of Krarup and Bilde (1977). Although this is an effectively used method in the lot-sizing domain, its application to the *MPCSPs* is not trivial due to cutting patterns. Thirdly, we present a thorough theoretical analysis investigating the strength of various formulations given in the thesis, providing a comparative ranking with respect to lower bounds to be expected from the formulations. To complement our theoretical analysis with an understanding of performance in practice, a computational analysis is provided. Moreover, in order to solve large instances of the problem, we also propose a solution method based on the Adaptive Biased Random Key Genetic Algorithm (*ABRKGA*) presented by Chaves et al. (2018).

We remark that although the concepts of the cutting-stock and lot-sizing formulations are used, the formulations proposed are not direct adaptations from the literature of these individual problems. Considering the point of view of the *CSP*, the inclusion of setups on the single period case is already an area of extensive research (see Section 2.2). In this thesis, some of the proposed formulations are presented for the first time in the literature, even considering their simplified single period version. From the lot sizing problem point of view, since we are considering the production of cutting patterns (which contains a set of items), the classical production variables of the Uncapacitated Lot Sizing (*ULS*) problem had to be modified and the theoretical results for the *ULS* are no longer valid. As the adaptations are not direct, an interesting problem arises, which is different from the classical *ULS* and has not been fully explored in the literature. It is important to highlight that, to the best of our knowledge, the facility location reformulation has never been applied to such a problem, and it presents high quality lower bounds.

It is worth remarking that in the *MPCSPs* addressed in this study, the setup cost only reflects the direct or indirect costs related to a setup, for example, when changeovers imply an unavoidable loss of material (ARBIB and MARINELLI (2007)), or when several workers are needed to perform the setup, which implies high labor cost (KOLEN and SPIEKSMA (2000)), or when a setup involves a costly craft-work (BONNEVAY et al. (2016)). The considered setup costs do not include penalty costs for lost production capacity, since this should be taken into account via the introduction of setup times, which may impact time-related parameters or indicators (such as due dates, throughput, production capacity). In general, when considering practical applications, production capacity and its consequent constraints must be considered when integrating cutting stock and lot sizing problems. Since the research developed in this thesis does not consider production capacity, it is limited to some exceptional practical applications, and it is also relevant as a relaxation of several real problems, where production capacity is apparent.

The remainder chapters of this thesis are organized as follows: a literature review with models and methods related to the *MPCSPs*, as well as a discussion regarding the impact of setup costs, setup times and production capacity, is presented in Chapter 2. The formulations and their descriptions are presented in Chapter 3, in the following order: 1) a multi-period adaptation of the classical *CSP* model of GILMORE and GOMORY (1961) (denoted by *AGG*), 2) an extension of the *CSP* model of JOHNSTON and SADINLIJA (2004) (denoted by *AJS*), 3) an arc-flow extension of the ALVES and VALÉRIO DE CARVALHO (2008a) model motivated by the reflect formulation of DELORME and IORI (2019) (denoted by *ARE*), and 4) an extension of the *CSP* vehicle routing problem formulation firstly proposed by BEN AMOR (1997) (denoted by *AVR*). Later, on the same chapter, we strength the formulations using the facility location reformulation and present theoretical results evaluating the strengths of the different formulations. In Chapter 4, the two heuristic procedures used in this thesis are presented, namely, the column generation technique and the *ABRKGA*. Two applications of the *ABRKGA* are proposed to solve the *MPCSPs*. Then, in Chapter 5 a discussion about the computational experiments regarding the models considering two sets of instances is presented, next, a sensitivity analysis of the lower bounds over the object cost is presented, followed by the computational results regarding the *ABRKGA*. The chapter is then concluded with an additional experiment combining the *ABRKGA* and the column generation procedure. Finally, in Chapter 6 we make our concluding remarks and discuss some potential directions for future research.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCHES

The multi-period cutting stock problem (*MPCSP*) is studied in this thesis. This problem addresses multiple periods in a finite planning horizon, the inventory of items which comprises the link between periods, and the cutting process of objects in each period. In this way, the decision-making of such processes might occur at different levels of the supply chain. For instance, the planning managers are usually responsible for the production planning of the items in order to meet the demand, whereas the machine manufacturers perform the optimization of the cuts in the cutting process. In this work, we proposed eight formulations for the *MPCSPs*, one formulation was adapted from the literature, while seven others were new formulations proposed for the problem, four of which were reformulations based on the facility location problem with stronger lower bounds. A thorough theoretical study regarding lower bound strength was conducted in order to establish a comparative understanding among these formulations. Except for the lower bound relationship between the *AGG* model and the knapsack based facility location reformulations, a dominance relation could be found for all other models. In addition, a computational study was performed based on randomly generated instances to evaluate the formulations in terms of computational performance and so that theoretical relationships could be better understood.

The computational experiments were performed over two sets of instances. In the first set, we fix the item length and vary the holding costs and the object length while in the second, we fix the holding costs and the object length and vary the item length, resulting in more difficult instances. For both sets of instances the integer solutions were obtained considering the setup cost as 100 times the object cost. Regarding the upper bound achievements of the proposed formulations when using the first data set, the *AGGFL* and *AREFL** models were able to find the smallest relative gaps on 22 out of 24 classes, with 1.77% and 2.11% on average, respectively. In addition, the *AGGFL* and *AREFL** models were in average always faster than their respective original formulations. As for the second set of instances, the pattern based formulations present slight differences on overall integer solution while *AGGFL* showed a better computational performance. The *AJS* model obtained the best average of relative gaps for classes with small items, however, as the instances size increase, the model could not find all feasible solutions, whereas *AVR* could find more feasible solutions than *AJS* for all classes (though with bigger relative gaps). The arc-flow formulations obtained, in general, the best relative gaps for instances with medium and high item lengths, whereas *AREFL** obtained the best gaps for classes where feasible solutions could be found. In general, as the item length becomes smaller in relation to the object length the facility location formulation presented more difficulties in finding feasible solution for the problem. This observation is clearer when the knapsack based models were considered, since their facility location reformulation had difficulties even for obtaining smaller relative gaps than the respective original formulations. In general, except for the pattern-based formulation, the arc-flow and knapsack-based models struggle to solve medium-size instances where 6 periods and 20 items are considered. As for the lower bounds, the facility location improvement is highly affected by the increase of the object cost value. However, all experiments showed that the facility location reformulations improve the quality of the lower bounds. Considering the second set of instances, the improvement ranges from 139% to 181%, when the object cost is considered as 1, down to 0.49% to 1.59% when the cost is considered as 1000. We highlight that Chapters 1, 2, 3 and 4 were partially published in Silva et al. (2023).

A metaheuristic based on the Adaptive Biased Random Key Genetic Algorithm (*ABRKGA*) is also presented for the problem. A chromosome represented as random item production in different periods and rules to construct cutting patterns where a decoder procedure uses these information to obtain a feasible cutting plan is presented. Two different processes of encoding and decoding, D_1 and D_2, were proposed based on different ways of selecting the production and construct the cutting patterns for each period. A column generation approach based on the *AGG* model is also used for comparative purpose. The solution procedures where tested using the second set of instances. Three costs Scenarios such that the setup price is 10 times bigger, equal and 10 times smaller than the object cost, denoted here as Scenarios 1, 2 and 3, respectively, were considered for test evaluation.

The computational experiments show that D_1 and D_2 outperforms the column generation technique for all instances in the classes with small items with cost Scenarios 1 and 2. D_2 is the only decoder which obtained comparable results to the column generation procedure for the classes with small items when considering the third Scenario. Comparable results for the medium and high item length classes were also found for Scenarios 1 and 2 while the column generation outperforms both decoders in these classes when Scenario 3 is considered. D_1 is almost 3 times faster than D_2 and slight variation in computational time is noted from both decoders as the object cost varies. In general, D_1 works better than D_2 for smaller object costs while D_2 works better than D_1 when higher costs were considered. An additional experiment using D_1 and the column generation procedure is conducted for the costs Scenarios 2 and 3. The combined procedure obtained comparable results in all instances in better computational time for both costs configuration.

An interesting subject for future research is to extend the theoretical insights and reformulations proposed in this thesis to cutting stock problems considering different practical aspects, such as: several machines, capacity constraints, sequence-dependent cut setups (Arbib and Marinelli (2007) and Wuttke and Heese (2018)), among others. In addition, other strength strategies from the lot-sizing literature, such as shortest path reformulation and facet defining inequalities can be extended to enhance formulations based on cutting stock problems. The column-and-row method addressed by Sadykov and Vanderbeck (2013) may also be used to generate cutting patterns considering the facility location reformulation. Considering the capacitated case, applying Lagrangian relaxation to the capacity constraint will result in a sub-problem similar to the one studied in this thesis. A multi-objective approach can also be applied to analyze the complex trade-offs present in the objective function.

As for future research regarding the $ABRKGA$, sequential heuristics such as the one proposed by Haessler (1975) may be adapted in the decoder since such strategies have been proven effective to deal with items with high length. Other strategies such as implementing local search and adapting the decoder to consider capacity constraints are interesting topics of research. In addition, the work of Chaves and Lorena (2021) proposed a $BRKGA$ with Q-Learning algorithm ($BRKGA - QL$), where the $BRKGA$ parameters are set during the evolutionary process using Reinforcement Learning, indicating the best configuration at each stage. Therefore, testing decoders under the $BRKGA - QL$ framework is also a research topic.

We also would like to highlight that the feature considered in this thesis, where a cutting pattern produces several items, can be extended to several process industries, where the products are obtained by processes that can produce several types of products simultaneously. These processes can be a specific mode of configuration of a production system that can produce several different items simultaneously and in varied quantities. A recent general discussion about it can be found in Villas Boas et al. (2021). Examples of such process industries are: refineries (Göthe-Lundgren et al. (2002) and Shi et al. (2014)), molded pulp (Martínez et al. (2018, 2019)), electrofused grains (Luche et al. (2009)), foundry (de Araujo et al. (2008)), offset printing industry (Baumann et al. (2015)) industries, among others. In practice, industries define a list of alternative processes as input data, and the decision is related to the selection of the processes to be used in each period of the planning horizon. However, according to Martínez et al. (2019), for some production environments, the number of configurations might be large and hence the complete enumeration not possible while considering only a subset of them may lead to sub-optimal solutions. Hence, an integrated approach that considers the process configuration together with other decisions is also an interesting avenue for future research.

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