

3D calculation of Tucson-Melbourne 3NF effect in triton binding energy

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Abstract. As an application of the new realistic three-dimensional (3D) formalism reported recently for three-nucleon (3N) bound states, an attempt is made to study the effect of three-nucleon forces (3NFs) in triton binding energy in a non partial wave (PW) approach. The spin-isospin dependent 3N Faddeev integral equations with the inclusion of 3NFs, which are formulated as function of vector Jacobi momenta, specifically the magnitudes of the momenta and the angle between them, are solved with Bonn-B and Tucson-Melbourne NN and 3N forces in operator forms which can be incorporated in our 3D formalism. The comparison with numerical results in both, novel 3D and standard PW schemes, shows that non PW calculations avoid the very involved angular momentum algebra occurring for the permutations and transformations and it is more efficient and less cumbersome for considering the 3NF.

Keywords: triton binding energy, three dimensional approach, Bonn-B, Tucson-Melbourne, three-nucleon force

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INTRODUCTION

As already known, the non-relativistic calculations of few-nucleon bound and scattering states are not able to reproduce experimental results for relevant observables such as the triton binding, even when considering various available realistic nucleon-nucleon (NN) interactions. Deviations can be attributed to wrong off-shell behavior of such potentials, relativistic corrections and, probably more important, 3NFs effect. In order to incorporate the 3NF corrections in a spin-isospin dependent 3D approach [1] for the triton, we have recently formulated the corresponding Faddeev equations in terms of the vector Jacobi momenta, specifically the magnitudes of the momenta and the angle between them, as well as the spin-isospin quantum numbers.

We have shown that, for the full solution of the 3N bound system, the Tucson-Melbourne two-pion exchange 3NF can be included in a very simple manner in comparison to the PW representation. As indicated in Ref. [2], according to the number of spin-isospin states that one takes into account, the formalism for both ${}^3\text{H}$ and ${}^3\text{He}$ bound states leads to only strictly finite number of coupled three dimensional integral equations, which at most for fully charge dependent case leads to 24 coupled equations. In this communication we present our numerical results for triton binding energy, obtained by solving spin-isospin dependent three-dimensional Faddeev integral equations, when considering the Bonn-B and Tucson-Melbourne NN and 3N forces. We would like to mention that our next task, which is currently underway, is to incorporate relativistic

effects in 3N bound state calculations using the same 3D approach.

FADDEEV EQUATIONS IN 3D REPRESENTATION WITH 3NFs

In this section we briefly review the formalism of three-dimensional Faddeev integral equations in the realistic 3D approach. By considering the 3NF, the 3N bound state is described by the Faddeev equations

$$|\psi\rangle = G_0 t P |\psi\rangle + (1 + G_0 t) G_0 V_{123}^{(3)} |\Psi\rangle, \quad (1)$$

where $G_0 = (E - H_0)^{-1}$ is the free 3N propagator, the operator $t = v + v G_0 t$ is the NN transition matrix, $P = P_{12} P_{23} + P_{13} P_{23}$ is permutation operator, the quantity $V_{123}^{(3)}$ defines the 3NF and $|\Psi\rangle = (1 + P)|\psi\rangle$ is the total wave function. The representation of Eq. (1) in momentum space and in a non-PW scheme needs the following states in the 3D basis:

$$|\mathbf{p} \mathbf{q} \alpha\rangle \equiv |\mathbf{p} \mathbf{q} \alpha_S \alpha_T\rangle \equiv \left| \mathbf{p} \mathbf{q} \left(s_{12} \frac{1}{2} \right) S M_S \left(t_{12} \frac{1}{2} \right) T M_T \right\rangle. \quad (2)$$

As shown in Fig. (1) the states of the 3D basis involve two standard Jacobi momentum vectors \mathbf{p} and \mathbf{q} . A comparison of basis states in both 3D and PW schemes show that in a standard PW representation the angular dependence leads to two orbital angular momentum quantum numbers, i.e., l_{12} and l_3 :

$$|p q \alpha\rangle_{PW} \equiv |p q \alpha_J \alpha_T\rangle \equiv \left| p q \left((l_{12} s_{12}) j_{12} (l_3 \frac{1}{2}) j_3 \right) J M_J (t_{12} \frac{1}{2}) T M_T \right\rangle, \quad (3)$$

whereas in 3D representation the angular dependence explicitly appears in the Jacobi vector variables. So it is clear that in 3D formalism there is not any coupling between the orbital angular momenta and corresponding spin quantum numbers. Therefore the spin quantum number of two-nucleon subsystem s_{12} and the third nucleon $s_3 = \frac{1}{2}$ couple to the total spin S and its third component M_S . For the isospin quantum numbers, a similar coupling scheme leads to the total isospin T with its third component M_T .

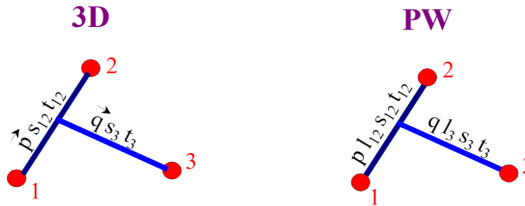


FIGURE 1. 3N basis states in both 3D and PW representation.

We would like to add the remark that in order to be able to evaluate the transition and the permutation operators we need the free 3N basis states $|\mathbf{p} \mathbf{q} \gamma\rangle \equiv |\mathbf{p} \mathbf{q} m_{s_1} m_{s_2} m_{s_3} m_{t_1} m_{t_2} m_{t_3}\rangle$. To this aim when we are changing the 3N basis states $|\alpha\rangle$ to the free 3N basis states $|\gamma\rangle$ we need to calculate the Clebsch-Gordan coefficients $\langle \gamma | \alpha \rangle \equiv g_{\gamma\alpha} = \langle m_{s_1} m_{s_2} m_{s_3} | (s_{12} \frac{1}{2}) S M_S \rangle \langle m_{t_1} m_{t_2} m_{t_3} | (t_{12} \frac{1}{2}) T M_T \rangle$.

By considering the symmetry property of the 3NF, the anti-symmetry property of the total wave function and the definition of anti-symmetrized NN t -matrix, i.e. ${}_a\langle |t| \rangle_a = \langle |t(1 - P_{12})| \rangle$, the representation of Eq. (1) in Eq. (2) leads to:

$$\begin{aligned}
\langle \mathbf{p} \mathbf{q} \alpha | \psi \rangle &= \frac{1}{E - \frac{p^2}{m} - \frac{3q^2}{4m}} \\
&\times \left[\int d^3 q' \sum_{\gamma', \gamma'', \alpha''} g_{\alpha \gamma'} g_{\gamma' \alpha''} \delta_{m_{s_3}''', m_{s_1}'} \delta_{m_{t_3}''', m_{t_1}'} \langle \mathbf{q} + \frac{1}{2} \mathbf{q}' \mathbf{q}' \alpha'' | \psi \rangle \right. \\
&\quad \times {}_a \langle \mathbf{p} m_{s_1}''' m_{s_2}''' m_{t_1}''' m_{t_2}''' | t(\epsilon) | \frac{-1}{2} \mathbf{q} - \mathbf{q}' m_{s_2}' m_{s_3}' m_{t_2}' m_{t_3}' \rangle_a \\
&\quad + \left\{ \langle \mathbf{p} \mathbf{q} \alpha | V_{123}^{(3)} | \Psi \rangle + \frac{1}{2} \sum_{\gamma', \gamma'', \alpha'''} g_{\alpha \gamma'} g_{\gamma' \alpha'''} \int d^3 p' \frac{\delta_{m_{s_3}'', m_{s_3}'} \delta_{m_{t_3}'', m_{t_3}'}}{E - \frac{p'^2}{m} - \frac{3q'^2}{4m}} \right. \\
&\quad \left. \left. \times {}_a \langle \mathbf{p} m_{s_1}' m_{s_2}' m_{t_1}' m_{t_2}' | t(\epsilon) | \mathbf{p}' m_{s_1}'' m_{s_2}'' m_{t_1}'' m_{t_2}'' \rangle_a \langle \mathbf{p}' \mathbf{q} \alpha''' | V_{123}^{(3)} | \Psi \rangle \right\} \right]. \quad (4)
\end{aligned}$$

As shown in Ref. [3], the evaluation of 3NF matrix elements $\langle \mathbf{p} \mathbf{q} \alpha | V_{123}^{(3)} | \Psi \rangle$ for 2π -exchange TM force avoids the cumbersome nature of the PW representation and leads to simple expressions which are more convenient for numerical calculations.

NUMERICAL RESULTS FOR ${}^3\text{H}$ BINDING ENERGY

In this section we present our numerical results for the triton binding energy, obtained by solving the three-dimensional Faddeev integral equations (4). These coupled integral equations have been solved before in Ref. [2] without 3NF term. By solving eight coupled Faddeev equations for $(\frac{1}{2} - \frac{1}{2})$ spin-isospin states and by using the operator form of Bonn-B NN potential [4], our calculation for the triton binding energy converges to a value of $E_t = -8.152$ MeV, whereas the PW calculations converges to $E_t = -8.14$ MeV for $j_{12}^{max} = 4$. In order to be able to compare our numerical results with available PW calculations, we have used TM force [5] (with cutoff mass $\Lambda_\pi = 5.828 m_\pi$) in an operator form [3] which is compatible with our 3D formalism. In table 1 we have shown the convergence of the triton binding energy in 3D approach as a function of the number of grid points. The corresponding PW results are also listed as function of total angular momentum j_{12}^{max} . As demonstrated in this table, by using the Bonn-B and TM combination in the 3D approach the calculation of triton binding energy converges to $E_t = -9.75$ MeV, whereas the corresponding result in a PW scheme for $j_{12}^{max} = 1$ yields $E_t = -9.80$ MeV. We should mention that this calculation is the first attempt toward the planned numerical investigations of ${}^3\text{H}$ binding energy with the most modern NN and 3N forces, i.e., AV18 and modified Tucson-Melbourne three-nucleon force (TM'), and also consistent NN and 3N chiral forces.

TABLE 1. The calculated triton binding energies E_t of the three-dimensional Faddeev integral equations as function of the number of grid points in Jacobi momenta N_{jac} and spherical angles N_{sph} , the number of grid points in polar angles is twenty. The corresponding PW results are listed for a comparison to our results. The used value for cutoff mass in TM 3NF is $\Lambda_\pi = 5.828 m_\pi$.

Potential	E_t [MeV]		
3D approach			
Bonn-B	N_{jac}	N_{sph}	
	40	24	-8.15 [2]
Bonn-B + TM			
	32	20	-9.73
	32	24	-9.73
	36	20	-9.74
	36	24	-9.74
	40	20	-9.75
	40	24	-9.75
	40	32	-9.75
PW approach			
Bonn-B	j_{12}^{max}		
	1		-8.17 [6]
	2		-8.10 [7]
	3		-8.14 [8]
	4		-8.14 [9]
Bonn-B + TM	1		-9.80 [6]

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