

Stabilization of the electroweak scale in 3-3-1 models

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One way of avoiding the destabilization of the electroweak scale through a strong coupled regime naturally occurs in models with a Landau-like pole at the TeV scale. Hence, the quadratic divergence contributions to the scalar masses are not considered as a problem anymore since a new nonperturbative dynamic emerges at the TeV scale. This scale should be an intrinsic feature of the models, and there is no need to invoke any other sort of protection for the electroweak scale. In some models based on the $SU(3)_C \otimes SU(3)_W \otimes U(1)_X$ gauge symmetry, a nonperturbative dynamics arise and it stabilizes the electroweak scale.

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Understanding the standard model (SM) of strong and electroweak interactions as an effective description of a more fundamental theory implies that, at an energy scale denoted by Λ , new physics must arise. Without any additional hypotheses, it is natural that $v_W < \Lambda \leq M_{\text{Pl}}$, where $v_W = (\sqrt{2}G_F/2)^{1/2} \simeq 246$ GeV is the electroweak scale and $M_{\text{Pl}} \sim 10^{19}$ GeV is the Planck scale, which is linked to the gravitational interactions according to semiclassical arguments. However, since there is no symmetry at the quantum level protecting the tree-level SM Higgs particle squared mass, $M_H^2 = -2\mu^2$, it receives quantum corrections so that $m_H^2 \rightarrow \bar{M}_H^2 = -2(\mu^2 + \delta\mu^2) = 2(\lambda + \delta\lambda)v_W^2$. Here μ^2 and λ are the usual tree-level parameters of the quadratic and the quartic terms in the renormalizable scalar potential at tree level, $V_0 = \mu^2 H^\dagger H + \lambda(H^\dagger H)^2$, with H the usual scalar doublet of the standard model; $\delta\mu^2$ and $\delta\lambda$ stand for the corresponding quantum corrections to the tree-level parameters. It so happens that $\delta\mu^2$ is proportional to Λ^2 , and $\delta\lambda$ is proportional to $\ln\Lambda$ [1]. Hence, the electroweak scale does not seem to be stable against quantum corrections unless some mechanism protects it. In the context of the SM, the Higgs, Z^0 and W bosons, and the top quark give the dominant contributions for the one-loop effective potential. Taking into account that Λ is the cutoff appearing in the momentum integrals, we have, with g , g' , and y_T , the $SU(2)$, $U(1)_Y$, and the top quark coupling constants of the SM, respectively,

$$\begin{aligned} \delta\mu^2 &\approx 3[2M_W^2 + M_Z^2 + M_H^2 - 4M_T^2] \left(\frac{\Lambda}{4\pi v_W} \right)^2 \\ &= 3 \left[\frac{3}{4}g^2 + \frac{1}{4}g'^2 + 2\lambda - 2y_T^2 \right] \left(\frac{\Lambda}{4\pi} \right)^2, \end{aligned} \quad (1)$$

plus terms proportional to $\ln\Lambda$ times loop factors. Independently of the value of Λ , a Higgs boson with a tree-level mass $M_H \approx 310$ GeV (i.e. $\lambda \approx 0.8$) could make $\delta\mu^2$ in Eq. (1) near zero [2]. However, according to the

electroweak precision data, such a heavy Higgs seems not to be favored by the global fit of the SM [3,4].

Ideas for stabilizing the electroweak scale have been put forward in the past. We assume that supersymmetry (SUSY) at the TeV scale was probably the first of such ideas[5]. More recently, solutions such as the little Higgs (LH) [6,7] and extra dimension (ED) models [8] were suggested. The LH models are constructions where the SM is contained in the low energy spectrum with the Higgs boson as a pseudo-Nambu-Goldstone boson of some particular global symmetry. This global symmetry protects one-loop quadratic divergences in $\delta\mu^2$. In LH models the common point resides in the fact that they are nonrenormalizable, defining, in principle, an energy cutoff which is around $\Lambda = 10$ TeV for the internal momentum integrals. It is interesting that this mechanism can be implemented if global and local $SU(3)$ symmetries are considered. In fact, Ref. [9] worked out a LH model which includes right-handed neutrinos transforming nontrivially under $SU(3)_W \otimes U(1)_X$ gauge symmetry. This was considered previously as an extension of the electroweak interactions in Ref. [10].

Notwithstanding, there are other motivations for considering $SU(3)_W \otimes U(1)_X$ symmetry as being realized at the TeV scale, as in the so-called 3-3-1 models [10–12]. One important point is that in some of these models [11,12], depending on the representation content, there is an intrinsic cutoff: the $U(1)_X$ coupling constant gets a Landau-like pole of the order of a few TeV [13–16]. Thus, the theory becomes strongly coupled at the TeV scale inducing, in this way, a natural cutoff for the quantum corrections.

One of the reasons that Λ can be arbitrarily high in the SM is that none of its gauge coupling constants reaches a value which invalidates the perturbative expansion at a testable energy. Let us imagine that a new particle content, which arises from a symmetry structure already revealed at the TeV scale, drives the gauge coupling constants towards

a region of values where the theory is nonperturbative. This would indicate that the theory goes to a strongly coupled regime for higher energies. That is the case if the electroweak sector of the SM is embedded into an $SU(3)_W \otimes U(1)_X$ symmetry, as in a specific class of 3-3-1 models [11,12]. Their symmetry reduction, $SU(3)_W \otimes U(1)_X \rightarrow SU(2)_W \otimes U(1)_Y$, to the SM group is realized through a scalar field condensation by a vacuum expectation value $\langle \chi^0 \rangle = v_\chi$, related to the energy scale denoted μ_{331} , by $\sqrt{2}v_\chi \approx \mu_{331}$. Below μ_{331} we have an effective $SU(2)_W \otimes U(1)_Y$ symmetry with the SM fermions, gauge boson multiplets, and two scalar doublets composing the light degrees of freedom. These are the active degrees of freedom below μ_{331} . All other fields are presumed to be heavy, i.e., with masses around μ_{331} .

In these 3-3-1 models there is the relation

$$\alpha_X(\mu_{331}) = \frac{\alpha(\mu_{331})}{1 - 4\sin^2\theta_W(\mu_{331})}, \quad (2)$$

where $\alpha_X = g_X^2/4\pi$ is the gauge coupling constant of the $U(1)_X$ gauge factor, written in terms of the electroweak mixing angle θ_W , and the electromagnetic coupling α , both defined at the μ_{331} scale. This relation is used to determine the initial value of $\alpha_X(\mu_{331})$, making the evolution of $\sin^2\theta_W \equiv \sin^2\theta_W(M_Z) \approx 0.231$ and $\alpha \equiv \alpha(M_Z) \approx 1/128$ from the Z^0 pole to the μ_{331} scale. We see that a Landau-like pole will be developed as $\sin^2\theta_W(\mu)$ evolves to the value 0.25. This value is reached for $\mu \approx 4.2$ TeV, making the evolution only with the active degrees of freedom below μ_{331} . This means that even if μ_{331} is above this value, the cutoff in Eq. (1) must be such that $\Lambda < 4.2$ TeV. Before reaching this pole, α_X goes outside the perturbative regime, and we cannot draw any conclusion based on perturbation theory. From the running equation for α_X , with the initial point as in Eq. (2), the upper energy limit $\bar{\Lambda} < \Lambda$ where the perturbative treatment loses its validity, i.e., $\alpha_X(\bar{\Lambda}) \approx 1$, is

$$\bar{\Lambda} = \mu_{331} \left(\frac{M_Z}{\mu_{331}} \right)^{7/13} e^{(\pi/13)[(1/\alpha)[1-4\sin^2\theta_W]-1]}. \quad (3)$$

The model predicts a mass relation between the neutral Z' and the double charged M_U gauge bosons,

$$\frac{M_{Z'}^2}{M_U^2} \approx \frac{4\cos^2\theta_W}{3 - 12\sin^2\theta_W}, \quad (4)$$

with $M_U^2 \approx g^2\mu_{331}^2/8$. So, using the lower bound for the Z' mass obtained in Ref. [17] as $M_{Z'} \geq 620$ GeV, which implies the minimal value $\mu_{331}^{\min} \approx 750$ GeV in Eq. (3), we have $\bar{\Lambda} \approx 2$ TeV. On the other hand, using the lower bound of $M_U \geq 750$ GeV [18] for the mass of the double charged gauge boson in the model, $\mu_{331}^{\min} \approx 3.2$ TeV so that $\bar{\Lambda} = 4$ TeV. It was pointed out in Ref. [19] that even for $M_{Z'} \approx 1.4$, thousands of new single charged vector bosons presented in the model could be produced at the LHC,

pointing out the possibility of distinguishing this model from other models. For other aspects of gauge boson phenomenology, see also [20].

Here we will use the effective potential [21,22] in the formalism of Ref. [23]. Since Λ is an upper limit for evaluating the integrals, omitting constant terms proportional to Λ^4 , the one-loop contribution to the effective potential is

$$V_1(h) \approx \frac{1}{64\pi^2} \sum_i n_i \left[2\Lambda^2 \text{Tr}(\mathbf{M}_i^\dagger \mathbf{M}_i) + \text{Tr} \left\{ (\mathbf{M}_i^\dagger \mathbf{M}_i)^2 \left(\ln \frac{\mathbf{M}_i^\dagger \mathbf{M}_i}{\Lambda^2} - \frac{1}{2} \right) \right\} \right], \quad (5)$$

where n_i is the number of degrees of freedom of the field i , including a minus sign for fermions; $\mathbf{M}_i \equiv \mathbf{M}_i(h)$ are obtained from the tree-level mass matrices of the model using $v_W = h$. We will consider below only terms proportional to Λ^2 , since they are the most relevant contributions for our purposes.

The model we consider has an approximate global $SU(3)_L \otimes SU(3)_R$ symmetry [24]. In the scalar sector this global symmetry is supposed to be exact. Defining the tritriplet $\Phi = (\eta\rho\chi)$ transforming as $\Phi \rightarrow \Omega_L \Phi \Omega_R^\dagger$ under $SU(3)_L \otimes SU(3)_R$, the scalar potential is

$$V[\Phi] = \mu^2 \text{Tr}(\Phi^\dagger \Phi) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 + \frac{f}{6} \epsilon_{ijk} \epsilon_{mnl} (\Phi_{im} \Phi_{jn} \Phi_{kl} + \text{H.c.}). \quad (6)$$

The Yukawa interactions and the gauge interactions introduced by $U(1)_X$ symmetry explicitly break $SU(3)_L \otimes SU(3)_R$. For the Yukawa Lagrangian we have

$$\begin{aligned} -\mathcal{L}_Y = & \bar{Q}_{iL} [g_{i\alpha}^u u_{\alpha R} \rho^* + g_{i\alpha}^d d_{\alpha R} \eta^* + g_{ik}^j j_{kR} \chi^*] \\ & + \bar{Q}_{3L} [y_\alpha^u u_{\alpha R} \eta + y_\alpha^d d_{\alpha R} \rho + y^J J_R \chi] \\ & + \bar{\Psi}_{aL} [g_{ab}^\nu \nu_{bR} \eta^* + g_{ab}^l l_{bR} \rho^* + g_{ab}^E E_{bR} \chi^*] \\ & + \text{H.c.}, \end{aligned} \quad (7)$$

where repeated indices are to be summed accordingly with the fields. Terms like $y_{ab} \epsilon_{ijk} \bar{\Psi}_{iL} (\Psi_{jL})^c \eta_k$ and $m_{ab} \nu_{aR} (\nu_{bR})^c$ are not relevant for us here. We assume these terms are forbidden by some symmetry. The global and local symmetries are spontaneously broken with the vacuum expectation value for Φ ,

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \text{diag}(v_\eta, v_\rho, v_\chi), \quad (8)$$

leaving only the electromagnetic $U(1)$ factor as local symmetry.

In the Appendix the tree-level $\mathbf{M}_i(v_\eta, v_\rho, v_\chi)$ matrices needed to obtain the quadratic corrections for the present model are shown. Taking the first term in Eq. (5) and the trace of the matrices in the Appendix, we get the one-loop corrections for the bilinears in the potential

$$V_{\text{eff}} = (\mu^2 + \delta\mu_\eta^2)\eta^\dagger\eta + (\mu^2 + \delta\mu_\rho^2)\rho^\dagger\rho + (\mu^2 + \delta\mu_\chi^2)\chi^\dagger\chi + \dots \quad (9)$$

with

$$\begin{aligned} \delta\mu_\eta^2 &\approx \left[4g^2 + 20\lambda_1 + 12\lambda_2 \right. \\ &\quad \left. - 6 \sum_{\alpha=1}^3 (|y_\alpha^u|^2 + |g_{1\alpha}^d|^2 + |g_{2\alpha}^d|^2) \right] \left(\frac{\Lambda}{4\pi} \right)^2, \\ \delta\mu_\rho^2 &\approx \left[4g^2 + 3g_X^2 + 20\lambda_1 + 12\lambda_2 \right. \\ &\quad \left. - 6 \sum_{\alpha=1}^3 (|g_{1\alpha}^u|^2 + |g_{2\alpha}^u|^2 + |y_\alpha^d|^2) \right] \left(\frac{\Lambda}{4\pi} \right)^2, \\ \delta\mu_\chi^2 &\approx \left[4g^2 + 3g_X^2 + 20\lambda_1 + 12\lambda_2 \right. \\ &\quad \left. - 2 \sum_{a=1}^3 |g_{aa}^E|^2 - 6|y^J|^2 - 6 \sum_{i=1}^2 |g_{ii}^j|^2 \right] \left(\frac{\Lambda}{4\pi} \right)^2. \end{aligned} \quad (10)$$

Observe that the global $SU(3)_L \otimes SU(3)_R$ is recovered when g_X , and the Yukawa couplings in Eq. (7) are made equal to zero, resulting in $\delta\mu_\eta^2 = \delta\mu_\rho^2 = \delta\mu_\chi^2$, as it should be.

For the potential in Eq. (6) we have to have $v_\eta = v_\rho = v_\chi$. The quantum corrections bring into the constraint equations, Eqs. (11) below, the effects of the explicit breakdown of $SU(3)_L \otimes SU(3)_R$, making it possible to have $v_\eta \neq v_\rho \neq v_\chi$. Considering the dominant contributions in Eqs. (10) and disregarding the corrections for the couplings in the self-interaction terms, the constraint equations for minimizing the potential are

$$\begin{aligned} -(\mu^2 + \delta\mu_\eta^2) &= \lambda_2 v_\eta^2 + \lambda_1 v^2 + \frac{\sqrt{2}}{4} \frac{f}{v_\eta} v_\rho v_\chi, \\ -(\mu^2 + \delta\mu_\rho^2) &= \lambda_2 v_\rho^2 + \lambda_1 v^2 + \frac{\sqrt{2}}{4} \frac{f}{v_\rho} v_\eta v_\chi, \\ -(\mu^2 + \delta\mu_\chi^2) &= \lambda_2 v_\chi^2 + \lambda_1 v^2 + \frac{\sqrt{2}}{4} \frac{f}{v_\chi} v_\rho v_\eta, \end{aligned} \quad (11)$$

where $v^2 = v_\eta^2 + v_\rho^2 + v_\chi^2$. Since $\delta\mu_\eta^2 \neq \delta\mu_\rho^2 \neq \delta\mu_\chi^2$ it is possible to have $v_\eta \neq v_\rho \neq v_\chi$. Once the scale Λ is really limited in the model, we do not expect that any severe fine-tuning is needed in Eqs. (11).

Next we show the masses, at leading order, for some of the scalar fields. There are two single charged scalars with masses given by

$$\begin{aligned} M_{1+}^2 &= \left(\lambda_2 - \frac{\sqrt{2}}{4} \frac{f v_\chi}{v_\eta v_\rho} \right) (v_\eta^2 + v_\rho^2), \\ M_{2+}^2 &= \left(\lambda_2 - \frac{\sqrt{2}}{4} \frac{f v_\rho}{v_\eta v_\chi} \right) (v_\eta^2 + v_\chi^2); \end{aligned} \quad (12)$$

a double charged scalar with mass given by

$$M_{++}^2 = \left(\lambda_2 - \frac{\sqrt{2}}{4} \frac{f v_\eta}{v_\chi v_\rho} \right) (v_\rho^2 + v_\chi^2); \quad (13)$$

and a pseudoscalar (CP odd) with mass given by

$$M_A^2 = -\frac{\sqrt{2}}{4} f v_\eta v_\rho v_\chi \left(\frac{1}{v_\chi^2} + \frac{1}{v_\rho^2} + \frac{1}{v_\eta^2} \right). \quad (14)$$

Assuming v_η , v_ρ , and v_χ are real and positive, we have that $f < 0$. This condition, along with $\lambda_2 > 0$, guarantees positive squared masses also for the charged scalars, as we see from Eqs. (12) and (13). We have not displayed the expressions for the last three CP even scalars because they do not have a simple closed form.

The existence of an ultraviolet singularity in one of the running coupling constants, through a Landau-like pole, may indicate an energy scale at which new phenomena could intervene. It is not clear at all what new phenomena would arise at energies near or above the Landau-like pole in these 3-3-1 models, but it could modify the running of the low energy coupling constant. One possibility is the appearance of new particles from fields forming representations affecting the running of the coupling constants such that the pole is avoided at reachable energies [16]. An investigation with this hypothesis is needed in order to see how the electroweak scale would then be stabilized in this case.

Let us compare this sort of 3-3-1 model with the SUSY, LH, and ED solutions for the stabilization of the electroweak scale. SUSY, at the electroweak scale, is a *renormalizable* theory; however, we need to assume that the scale related with SUSY and the masses of the supersymmetric partners are at the TeV scale. It is also useful for the unification of three of the fundamental forces, but it has trouble with the stabilization of the proton [25]. The LH models are nonlinear realizations of the spontaneous symmetry breaking of a global symmetry, thus they are *nonrenormalizable* theories and remain perturbative, by construction, until an energy scale of the order of 10 TeV; to have naturalness beyond this scale, some similar mechanism has to be invoked, i.e., a second LH model, and so on. The ED proposals are also *nonrenormalizable* theories, and the energy scale to solve the hierarchy problem is chosen by hand as well. The 3-3-1 models are different in the following sense: they are *renormalizable* theories and the energy scale at the TeV scale is an intrinsic property of the theories. We did not assume that they have an appropriate value for solving the problem of the electroweak scale. The LH and ED solutions need ultraviolet completion; the 3-3-1 models do not. LH and ED solutions to the stabilization of the electroweak scale are *ad hoc* since they are proposed *just* to solve the problem. The 3-3-1 models were proposed for other reasons and have interesting consequences and predictive power [24].

Finally, we stress that the argument that the running of $\sin^2\theta_W$ gives an energy bound defining the model symmetry structure, as in Eq. (2), must be seen as a real prediction of this minimal 3-3-1 model. It is not possible to arbitrarily raise the scale related to the new particles in this model, which become inconsistent for energies μ at the standard model level such that $\sin^2\theta_W(\mu) > 0.25$. This gives us a reason why $\sin^2\theta_W(M_Z) < 0.25$ and also why there are no threatening divergences for the mass of the Higgs boson.

We have discussed the stabilization of the electroweak scale in a specific 3-3-1 model [11]. The same arguments we present here are valid for the version of the model in Ref. [12] which does not have heavy charged leptons, and a scalar sextet is mandatory for generating mass to the known charged leptons. It may be that there is a whole

class of models where the arguments we have put forward here are valid.

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APPENDIX: MASS MATRICES

1. Scalar fields

Defining $X = (1/\sqrt{2})(v_x + X_R + iX_I)$, where $X = \eta, \rho, \chi$, we have for the mass matrix of the neutral real scalars, in the basis $(\eta_R \rho_R \chi_R)$,

$$\mathbf{M}_R^2 = \mu^2 \mathbf{1} + \begin{bmatrix} 3\lambda_1 v_\eta^2 + \lambda_2(2v_\eta^2 + v^2) & 2\lambda_1 v_\eta v_\rho + \frac{\sqrt{2}}{4} f v_\chi & 2\lambda_1 v_\eta v_\chi + \frac{\sqrt{2}}{4} f v_\rho \\ 2\lambda_1 v_\eta v_\rho + \frac{\sqrt{2}}{4} f v_\chi & 3\lambda_1 v_\rho^2 + \lambda_2(2v_\rho^2 + v^2) & 2\lambda_1 v_\rho v_\chi + \frac{\sqrt{2}}{4} f v_\eta \\ 2\lambda_1 v_\eta v_\chi + \frac{\sqrt{2}}{4} f v_\rho & 2\lambda_1 v_\rho v_\chi + \frac{\sqrt{2}}{4} f v_\eta & 3\lambda_1 v_\chi^2 + \lambda_2(2v_\chi^2 + v^2) \end{bmatrix}; \quad (\text{A1})$$

for the neutral pseudoscalars, in the basis $(\eta_I \rho_I \chi_I)$,

$$\mathbf{M}_I^2 = \mu^2 \mathbf{1} + \begin{bmatrix} \lambda_1 v^2 + \lambda_2 v_\eta^2 & -\frac{\sqrt{2}}{4} f v_\chi & -\frac{\sqrt{2}}{4} f v_\rho \\ -\frac{\sqrt{2}}{4} f v_\chi & \mu^2 + \lambda_1 v^2 + \lambda_2 v_\rho^2 & -\frac{\sqrt{2}}{4} f v_\eta \\ -\frac{\sqrt{2}}{4} f v_\rho & -\frac{\sqrt{2}}{4} f v_\eta & \lambda_1 v^2 + \lambda_2 v_\chi^2 \end{bmatrix}; \quad (\text{A2})$$

for the single charged scalars, in the basis $(\eta_1^- \rho^-)$,

$$\mathbf{M}_{1+}^2 = \mu^2 \mathbf{1} + \begin{bmatrix} \lambda_1 v^2 + \lambda_2(v^2 - v_\chi^2) & \lambda_2 v_\eta v_\rho - \frac{\sqrt{2}}{4} f v_\chi \\ \lambda_2 v_\eta v_\rho - \frac{\sqrt{2}}{4} f v_\chi & \lambda_1 v^2 + \lambda_2(v^2 - v_\chi^2) \end{bmatrix}, \quad (\text{A3})$$

and, in the $(\eta_2^- \chi^-)$ basis,

$$\mathbf{M}_{2+}^2 = \mu^2 \mathbf{1} + \begin{bmatrix} \lambda_1 v^2 + \lambda_2(v^2 - v_\rho^2) & \lambda_2 v_\eta v_\chi - \frac{\sqrt{2}}{4} f v_\rho \\ \lambda_2 v_\eta v_\chi - \frac{\sqrt{2}}{4} f v_\rho & \lambda_1 v^2 + \lambda_2(v^2 - v_\rho^2) \end{bmatrix}; \quad (\text{A4})$$

and for the double charged scalars, in the basis $(\rho^{--} \chi^{--})$,

$$\mathbf{M}_{++}^2 = \mu^2 \mathbf{1} + \begin{bmatrix} \lambda_1 v^2 + \lambda_2(v^2 - v_\eta^2) & \lambda_2 v_\rho v_\chi - \frac{\sqrt{2}}{4} f v_\eta \\ \lambda_2 v_\rho v_\chi - \frac{\sqrt{2}}{4} f v_\eta & \lambda_1 v^2 + \lambda_2(v^2 - v_\eta^2) \end{bmatrix}. \quad (\text{A5})$$

2. Gauge bosons

For the real neutral gauge bosons in the $(W_\mu^3, W_\mu^8, B_\mu)$ basis, the mass matrix is

$$M_{\text{n.g.b.}}^2 = \begin{pmatrix} \frac{g^2}{4}(v_\eta^2 + v_\rho^2) & \frac{\sqrt{3}g^2}{12}(v_\eta^2 - v_\rho^2) & -\frac{g_X g}{2} v_\rho^2 \\ \frac{\sqrt{3}g^2}{12}(v_\eta^2 - v_\rho^2) & \frac{g^2}{12}(v^2 + 3v_\chi^2) & \frac{\sqrt{3}}{6} g_X g(v_\rho^2 + 2v_\chi^2) \\ \frac{g^2}{12}(v^2 + 3v_\chi^2) & \frac{\sqrt{3}}{6} g_X g(v_\rho^2 + 2v_\chi^2) & g_X^2(v_\rho^2 + v_\chi^2) \end{pmatrix}; \quad (\text{A6})$$

and for the non-Hermitian gauge bosons

$$M_W^2 = \frac{g^2}{4}(v_\eta^2 + v_\rho^2), \quad M_V^2 = \frac{g^2}{4}(v_\eta^2 + v_\chi^2), \quad M_U^2 = \frac{g^2}{4}(v_\rho^2 + v_\chi^2). \quad (\text{A7})$$

3. Fermion fields

For type u and d quarks

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} g_{11}^u v_\rho^* & g_{12}^u v_\rho^* & g_{13}^u v_\rho^* \\ g_{21}^u v_\rho^* & g_{22}^u v_\rho^* & g_{23}^u v_\rho^* \\ y_1^u v_\eta & y_2^u v_\eta & y_3^u v_\eta \end{pmatrix}, \quad M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} g_{11}^d v_\eta^* & g_{12}^d v_\eta^* & g_{13}^d v_\eta^* \\ g_{21}^d v_\eta^* & g_{22}^d v_\eta^* & g_{23}^d v_\eta^* \\ y_1^d v_\rho & y_2^d v_\rho & y_3^d v_\rho \end{pmatrix}; \quad (\text{A8})$$

for the j_i and J quarks, and exotic leptons

$$M_j = \begin{pmatrix} g_{11}^j & g_{12}^j \\ g_{21}^j & g_{22}^j \end{pmatrix} \frac{v_\chi}{\sqrt{2}}, \quad M_J = \frac{y^J}{\sqrt{2}} v_\chi; \quad M_E = \begin{pmatrix} g_{11}^E & g_{12}^E & g_{13}^E \\ g_{21}^E & g_{22}^E & g_{23}^E \\ g_{31}^E & g_{32}^E & g_{33}^E \end{pmatrix} \frac{v_\chi}{\sqrt{2}}. \quad (\text{A9})$$

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