

Semiclassical estimate of the Coulomb excitation of the giant dipole resonance in ^{208}Pb

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(Received 15 January 1993)

A semiclassical approach to study pure Coulomb excitation of ^{208}Pb giant dipole isovector resonance is examined. We consider medium energy projectiles and assume the target excitation to be described by a simple Goldhaber-Teller model. It is shown that the main features concerning the angular distribution are obtained in the angular range described by the model and an estimate is made of the pure Coulomb dipole contribution to the measured cross sections.

PACS number(s): 24.30.Cz, 21.60.Ev, 25.10.+s, 27.80.+w

The use of medium energy heavy ions to excite giant resonances in nuclei has provided new perspectives for the study of that sort of nuclear collective motions. In particular, there have been extensive discussions of the effect of the Coulomb excitation by medium energy projectiles on giant resonance cross sections [1].

The result of the analysis of the Coulomb excitation process in terms of the equivalent virtual photons have pointed out to the very interesting fact that at incident energies around or higher than 50 MeV/nucleon, the giant resonance cross section with excitation energy between 10 and 20 MeV is dominated by the Coulomb interaction. Thus, within that range of energy the large enhancement of the observed giant resonance cross section is mainly due to the strong increase of the Coulomb excitation.

In this paper we intend to show how one could calculate the giant dipole resonance angular distribution using a semiclassical approach for the pure Coulomb excitation of ^{208}Pb induced by deuterons or alpha particles with incident energies in the region of interest and how one could estimate the contribution of that sort of giant resonance to the measured cross sections for forward angles. The method [2] makes use of the quantum phase space Wigner function [3] associated to a simple model describing an isovector Goldhaber-Teller dipole vibration degree of freedom for the target nuclei [4], while the projectile is treated as a structureless particle moving under a pure Coulomb force.

Although the present method could be applied to any kind of reactions, for convenience we are considering a nonreactive collision represented as

$$A + BC(n) \longrightarrow A + BC(m), \quad (1)$$

where n and m refer to any quantum vibrational numbers to be specified later.

On what concerns the target nuclei, we assume that it is constituted of two masses, M_c (neutrons) and M_b (protons) vibrating along the x axis. Through an adaptation of a coordinate transformation and an appropriate scaling proposed by Secrest and Johnson [5], the two body

oscillator system plus projectile can be replaced by an equivalent one body vibrator plus projectile. Therefore, in the present model, the Coulomb interaction term is considered as the ordinary one associated to only two point charges, $cZ_t Z_p e^2 / (|\mathbf{Q}_t - \mathbf{r}_p|)$, being the whole system described by an appropriate kinematics where $c = M_{bc}/M_c = (M_b + M_c)/M_c$ [5].

If we assume that the vibrational motion can be described by a harmonic oscillator in an n state, the corresponding Wigner function is written as [6]

$$W_{\text{target}}^{(n)} = (-1)^n \left(\frac{1}{\pi \hbar} \right) \exp \left(\frac{-2E_{\text{class}}}{\hbar \omega} \right) L_n \left(\frac{4E_{\text{class}}}{\hbar \omega} \right), \quad (2)$$

where E_{class} and ω are the classical energy and frequency of the harmonic oscillator, respectively. The last term $L_n(x)$ is the Laguerre polynomial. Furthermore, since we want to consider the projectile as a structureless particle, its corresponding phase space distribution function will be taken as a classical distribution:

$$\rho(\mathbf{r}, \mathbf{p}) = \delta[\mathbf{r} - \mathbf{r}_0(t)] \delta[\mathbf{p} - \mathbf{p}_0(t)]. \quad (3)$$

This indeed means that the distribution function for the projectile does not introduce any quantum correlation between r and p , contrary to (2) where the Wigner function describing the target vibration embodies at least the Heisenberg uncertainty correlation. In this way, at $t = -\infty$, when the ^{208}Pb target is considered in its ground state, the distribution for the whole system is written as a direct product

$$\Omega(\mathbf{Q}, \mathbf{P}, \mathbf{r}, \mathbf{p}) = W_{\text{target}}^{(0)}(\mathbf{Q}, \mathbf{P}) \rho_{\text{proj}}(\mathbf{r}, \mathbf{p}). \quad (4)$$

As it is known, the quantum full time evolution of the Wigner function is governed by the Wigner representative of the quantum von Neumann–Liouville equation [7]. In this work, however, we shall restrict ourselves to a semiclassical version of that equation which is equivalent to keep only the lowest order term of the corresponding \hbar -power series for the Liouville operator while the

quantum character is preserved in the Wigner function. Hence, the semiclassical time evolution of the Wigner function is given as

$$\Omega_t(\mathbf{Q}, \mathbf{P}, \mathbf{r}, \mathbf{p}) = \exp[-(t-t_0)L_{\text{class}}]\Omega_{t_0}(\mathbf{Q}, \mathbf{P}, \mathbf{r}, \mathbf{p}). \quad (5)$$

The subscripts in Ω refer to time at which the system is considered and L_{class} is the classical Liouville operator which is defined as

$$L_{\text{class}}(Q, P, \mathbf{r}, \mathbf{p}) = -\sum_n \left(\frac{\partial H}{\partial r_n} \frac{\partial}{\partial p_n} - \frac{\partial H}{\partial p_n} \frac{\partial}{\partial r_n} \right) + \left(\frac{\partial H}{\partial Q} \frac{\partial}{\partial P} - \frac{\partial H}{\partial P} \frac{\partial}{\partial Q} \right). \quad (6)$$

H is the model Hamiltonian written in terms of the variables defined by the proper kinematics [5],

$$H = \frac{P^2}{2\mu_{bc}} + \frac{p^2}{2m} + \frac{KQ^2}{2} + V_{\text{Coul}}(c|\mathbf{r} - \mathbf{Q}|), \quad (7)$$

where $\mu_{bc} = M_b M_c / (M_b + M_c)$ is the target reduced neutron-proton mass and m is the reduced projectile-target mass

$$m = \frac{m_a M_c^2}{(M_{bc} + m_a) M_{bc}}, \quad (8)$$

where m_a is the projectile mass. The spring constant K is our only free parameter, which was fitted to the 13.5-MeV isovector dipole giant resonance of ^{208}Pb .

In order to calculate the time evolution of the Wigner function, we could observe that the classical equations of motion lead to the following identity

$$\Omega_t(Q(t_0), P(t_0), \mathbf{r}(t_0), \mathbf{p}(t_0)) \equiv \Omega_{t_0}(Q(t), P(t), \mathbf{r}(t), \mathbf{p}(t)), \quad (9)$$

where $\{Q(t), P(t), \mathbf{r}(t), \mathbf{p}(t)\}$ are the phase space points at an arbitrary time obtained by solving classical equations of motion for the Hamiltonian (7) taking $\{Q(t_0), P(t_0), \mathbf{r}(t_0), \mathbf{p}(t_0)\}$ as the initial conditions. In fact, the value of the Wigner function at time t at corresponding phase space point $\{Q(t_0), P(t_0), \mathbf{r}(t_0), \mathbf{p}(t_0)\}$ is identical to the value of the Wigner function at time t_0 calculated at a phase point generated by the classical equations of motion taking $\{Q(t_0), P(t_0), \mathbf{r}(t_0), \mathbf{p}(t_0)\}$ as initial conditions. In this form, the calculations can be performed in the following way. First, we assume the projectile trajectory to be fixed on the x - y Cartesian plane and the time propagation is carried out by considering a mesh of points in the relevant domain of the target phase space. Each of these points (Q, P) is time evolved according to the six Hamilton differential equations for the previously chosen asymptotic initial conditions for the projectile, i.e., a set of parameters describing the laboratory energy E_{lab} , momenta p_{x_0}, p_{y_0} , impact parameter $y_0 = b$, asymptotic position (x_0) , and mass of the projectile m . In our calculations the set of 30×30 time evolved mesh points characterizing the Wigner function

at $t = -\infty$, allows us to calculate the target final Wigner function and the excitation probability as [9]

$$P_{if} = P_{\text{exc}} = 2\pi\hbar \int W_{t_i}(Q, P) W_{t_f}(Q, P) dQ dP, \quad (10)$$

where W_{t_i} is the target Wigner function corresponding to the initial state, while W_{t_f} corresponds to the final one. Such definition is exact in the present case since it is assumed as a one-dimensional vibrating target system. Besides, we have only considered the target prepared in its ground state at $t = -\infty$.

Calculations were performed for $E_{\text{lab}} = 218$ MeV and 172 MeV alpha particles and $E_{\text{lab}} = 108$ MeV deuterons. The first interesting result can be seen in Fig. 1, where in the intermediate range of impact parameters ($b \geq 8$ fm), the calculated points for the giant dipole resonance excitation probabilities are fitted by an exponential of the form

$$P_{\text{exc}} = A e^{-Db}, \quad (11)$$

where A and D are two adjustable parameters. The fitting correlation coefficient is 0.999 in all cases.

In this semiclassical picture, the incident particle trajectories can be seen as classical so that we are allowed to use the standard relation for the impact parameter as a good approximation

$$b = \frac{Z_t Z_p e^2}{2E} \frac{1}{\tan(\frac{\theta}{2})}. \quad (12)$$

Hence, the excitation probability can be easily obtained as a function of the projectile scattering angle and the Coulomb excitation differential cross section is the direct product of the Rutherford differential cross section, calculated within the Secrest and Johnson kinematics, and the excitation probability, namely,

$$\frac{d\sigma}{d\Omega} = P \frac{d\sigma}{d\Omega} \Big|_{\text{Ruth}}. \quad (13)$$

We may observe that the location of the angular distribution peak, within the intermediate range of impact parameters ($b \geq 8$ fm), is given by

$$\theta_{\text{peak}}^{\text{lab}} = \arcsin \left[\left(\frac{M_{bc} Z_t e^2}{4M_c} \right) \frac{Z_p D}{E} \right]. \quad (14)$$

In particular, $a = (M_{bc} Z_t e^2) / (4M_c) = 48.7$ MeV fm for ^{208}Pb . Some results from our model are presented in Table I where they are compared to experimental data.

In spite of the simplicity of our model, Table I shows the good agreement of the data for the peak. Furthermore, the angular distributions presented in Fig. 2 confirm such agreement within the range for which Eq. (11) is applicable.

The calculated θ_{peak} for the $E = 218$ MeV α - ^{208}Pb is verified to lie at 8.7° but it is not presented in Table I since the experimental angle cannot be conclusively inferred from data. For large angles more accurate results

depend on small values for b and our model is not reliable in this region since, besides the vibration degree of freedom, no other structure was assigned to the target, no nuclear force is present in our Hamiltonian, and our projectile is a classical punctual object. Near the calculated peak the Coulomb contribution is expected to be dominant and this may explain the obtained good agreement in the present calculation even though nuclear effects can contribute in that region [10].

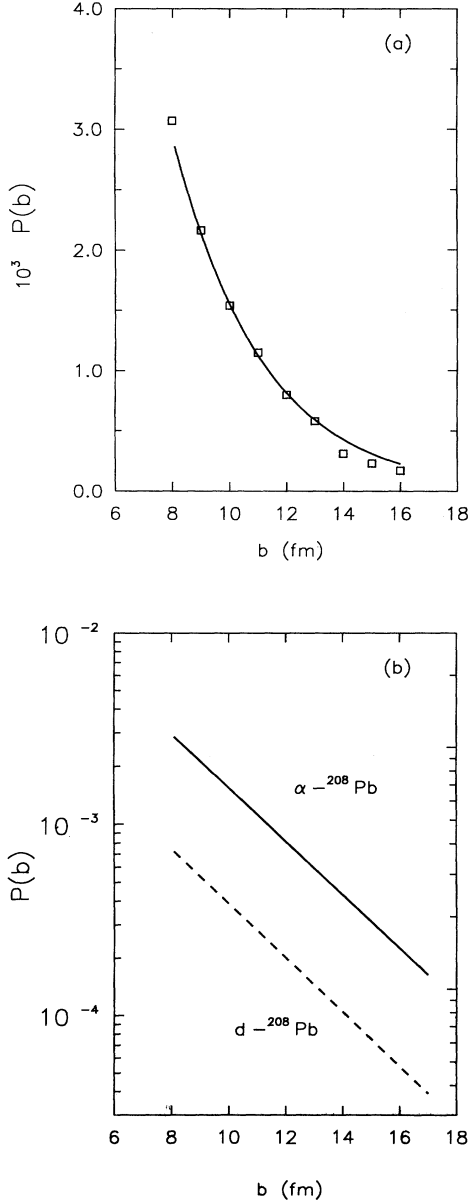


FIG. 1. (a) The results for excitation probabilities $P(b)_{0-1}$ and the fitting obtained according to Eq. (9) for the $E_{\text{lab}} = 218$ MeV $\alpha - {}^{208}\text{Pb}$ case. (b) The fitted excitation probabilities $P(b)_{0-1}$ for: (i) $E_{\text{lab}} = 218$ MeV $\alpha - {}^{208}\text{Pb}$; $A = 0.039$ and $D = 0.322 \text{ fm}^{-1}$; (ii) $E_{\text{lab}} = 108$ MeV $d - {}^{208}\text{Pb}$; $A = 0.010$ and $D = 0.328 \text{ fm}^{-1}$.

TABLE I. The values for the experimental and calculated angles refer to the center-of-mass system. The data for $E = 108$ MeV $d - {}^{208}\text{Pb}$ is found in Ref. [8], while for $E = 172$ MeV $\alpha - {}^{208}\text{Pb}$ was extracted from Ref. [10].

	$d - {}^{208}\text{Pb}$	$\alpha - {}^{208}\text{Pb}$
θ_{exp}	8.0°	11.0°
θ_{calc}	8.6°	12.0°

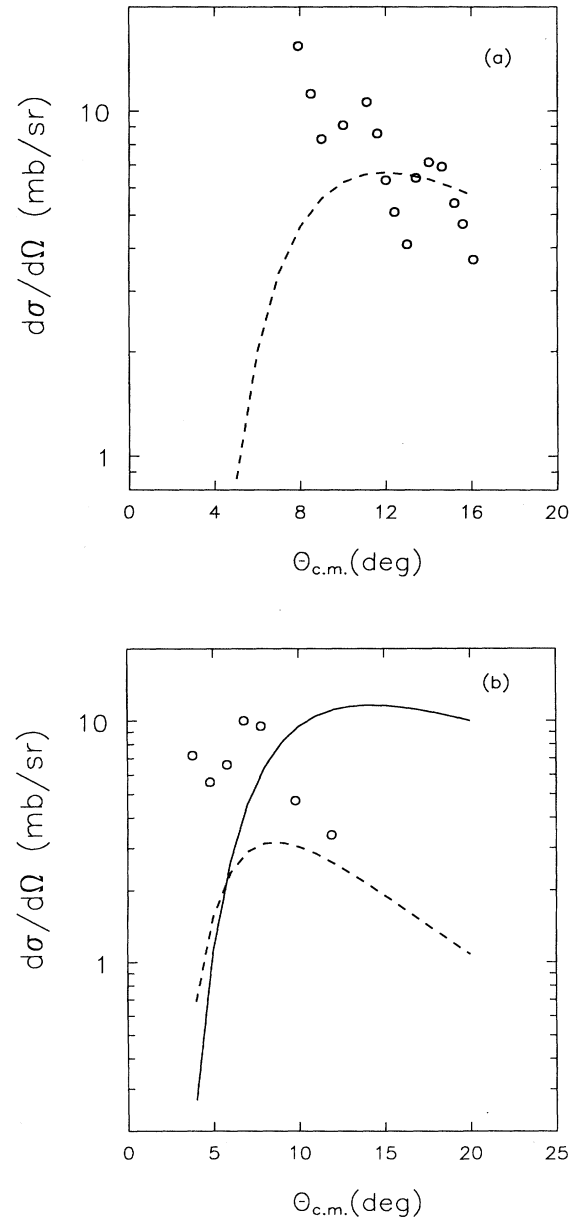


FIG. 2. (a) The angular distribution for the $E_{\text{lab}} = 172$ MeV $\alpha - {}^{208}\text{Pb}$ case ($A = 0.049$ and $D = 0.358 \text{ fm}^{-1}$) and (b) $E_{\text{lab}} = 108$ MeV $d - {}^{208}\text{Pb}$. Experimental data have been extracted from Refs. [8 and 10].

Our calculations estimate the pure Coulomb contribution – for the present strongly collective model – of the isovector dipole giant resonance to be of the order of 65% of the experimental cross section of ^{208}Pb at angles about the peak (Fig. 2). This supports the suggestion that the excitation of that sort of giant resonance is the dominant contribution to the forward angle spectra for large incident energies [8], in contrast to the monopole giant resonance whose excitation energy almost coincides with that of the former and also contributes in the same region. We want to mention that calculations using the well-established Alder and Winther approach [11] for the present model predict a cross section which is higher than ours in the relevant angular domain, Fig. 2(b), although the general trends are common to both, due to the same form assumed for the ansatz of the excitation cross section, Eq. (13), for which P has essentially the same features in both cases.

As a conclusion we want to note that, although assuming only an extremely simple collective nuclear Goldhaber-Teller model for a ^{208}Pb vibrating system, the

location of the peak in the intermediate range of the experimental angular distributions is well described and the above results besides being a relevant argument in favor of the present semiclassical scheme as a method for giving estimates of the pure Coulomb excitation probability for the giant dipole resonance predicts the dominance of this kind of collective motion in the small angle medium energy experiments. Finally we would like to point out the importance of the particular kinematics adapted from that of Ref. [5] and employed in our semiclassical approach. A direct comparison between the results derived from the Alder and Winther model and our semiclassical method stands for an appropriated way to investigate the kinematics role. We think that deeper discussions concerning the comparison are necessary, but such subject escapes the original scope of the present short note and will appear in a future paper.

D.G. was partially supported by CNPq-Brazil. A.E. was supported by CNPq-Brazil.

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