Reply to ''Comment on 'Conditionally exactly soluble class of potentials' ''

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We show that the problems raised by Znojil [Phys. Rev. A **61**, 066101 (2000)] preceding paper, can be resolved and that, in fact, there exist the proposed singular conditionally, exactly solvable potentials [Phys. Rev. A **47**, R2435 (1993)].

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Regarding the criticisms raised in the paper by Znojil [1], I should divide my reply in two parts. In the first place, concerning the arguments to the problem of boundary conditions at the origin, I partially agree with the author [1]. Since it can be resolved, it should be necessary to use the general solution of the harmonic oscillator, the parabolic cylinder function instead of the usual Hermite polynomials, and then impose the boundary condition at the origin. For instance, in the first example, the energy can be obtained substituting the zeroes of

$$D_{(16B^2/\omega^3)-(4A/\omega)-(1/2)}\left(\frac{8B}{\omega^{3/2}}\right)=0,$$

in the expression for the energy $E=-\omega^2/16$. In fact this is exactly the same procedure used to solve problems like that of the double oscillator, appearing in standard textbooks [3], where the zeroes of the parabolic cylinder function at the origin are used in order to define the energy eigenvalues as a transcendental equation. Really this is quite analogous to the well-known solution of the finite square well, where once again the solution comes from a transcendental equation, which, despite of being nonelementary, still is an exact one.

On the other hand, based on the above arguments, I disagree with the author of [1] when he suggests that these

potentials are not exactly soluble. One could argue that it is not an elementary solution, but that it is an exact one. (Has anyone doubt about the exact solvability of the harmonic oscillator in a box?) This can be seen by looking for the roots of the wave function at the origin (as he correctly suggests, including by presenting an example of ''salvation'' by using the Hermite polynomials). In this process one finds the roots of a polynomial in the energy, and it is not necessary to use any kind of numerical solution of the Schrödinger equation or even perturbative techniques. The approximation will be of another sort; it will be of the same kind of any exact function like $\sin(x)$, where in the process of calculation one takes, for instance, a convergent series that has infinitely many terms, and truncate it, obtaining the value of the function with any desired accuracy. This happens precisely in the same way for the eigenenergies of these conditionally exactly soluble potentials. By the way, the case of the quasi-exactly-soluble potentials is also similar because, even when one has an arbitrary number of exact energy levels, such levels are obtained from the roots of a polynomial on the energy, which for polynomials of order higher than four need to be solved numerically.

In fact, Stillinger has discussed these potentials in Ref. [4], and gave the same discussion regarding the wave functions, with respect to the correct boundary conditions at the origin. In addition, in that paper Stillinger did an exhaustive analysis of the spectrum of the first conditionally, exactly soluble potential appearing in my paper [2].

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[1] M. Znojil, preceding paper, Phys. Rev. A **61**, 066101 (2000).
[2] A. de Souza Dutra, Phys. Rev. A **47**, R2435 (1993).
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[4] F. H. Stillinger, J. Math. Phys. **20**, 1891 (1979), and private communication.