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Self-consistent solution of the Schwinger-Dyson equations for the nucleon and meson propagators

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The Schwinger-Dyson equations for the nucleon and meson propagators are solved self-consistently in an approximation that goes beyond the Hartree-Fock approximation. The traditional approach consists in solving the nucleon Schwinger-Dyson equation with bare meson propagators and bare meson-nucleon vertices; the corrections to the meson propagators are calculated using the bare nucleon propagator and bare nucleon-meson vertices. It is known that such an approximation scheme produces the appearance of ghost poles in the propagators. In this paper the coupled system of Schwinger-Dyson equations for the nucleon and the meson propagators are solved self-consistently including vertex corrections. The interplay of self-consistency and vertex corrections on the ghosts problem is investigated. It is found that the self-consistency does not affect significantly the spectral properties of the propagators. In particular, it does not affect the appearance of the ghost poles in the propagators.

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I. INTRODUCTION

The development of relativistic many-body theories for the nucleus is one of the most important goals of contemporary nuclear theory. Models based on the methods of relativistic quantum field theory have been developed for more than two decades.

The starting point for understanding the many-nucleon problem is a description of the elementary processes in vacuum: the nucleon propagator, meson-nucleon scattering, and the N-N interaction. Successes and difficulties with relativistic meson-nucleon field theory have been the subject of papers for more than half a century. We will certainly not detail the history here, but note that a nagging inconsistency in (almost) all calculations has been the appearance of ghost poles.

Brown, Puff, and Wilets [1], for example, calculated the nucleon propagator by summing all planar meson diagrams with one nucleon line using π, ρ, and ω mesons. No cutoffs were introduced. The renormalized nucleon propagator was well defined and self-consistent, but contained a pair of complex conjugate poles located approximately 1 GeV off the real and imaginary axes. The full propagator, including these unphysical poles, was used with some success to describe the isovector nucleon magnetic moment, π-nucleon scattering [2], and nucleon-nucleon scattering [3]. (The last did require cutoffs in the N-N interaction, but yielded better chi-squared fits to scattering data with fewer parameters than the then current Paris potential). The inclusion of the complex poles was essential. Nevertheless, the occurrence of the complex poles remained an enigma.

Several interpretations of the appearance of the poles have been proffered, including the statement that it is a signal of the inconsistency of any local, relativistic field theory, and that a field theory with asymptotic freedom (e.g., QCD) is required.

The program of the previous section was driven by the interpretation that the appearance of the ghosts is an artifact of the approximations, and that progressively better calculations should lead to the receding or elimination of the ghosts, but that for consistency one must keep the ghosts as they emerge from the calculations at each stage.

Another interpretation is that it is an effective theory, and that one should be prepared to introduce further parameters to ensure physical quantities.

In a recent paper [4], the problem of ghosts poles in the nucleon propagator was investigated. The appearance of the ghost poles is related to the short distance behavior of the model interactions [1]; asymptotically free theories appear to be free of ghost poles [5]. An interesting possibility to eliminate the complex poles is the regularization of the theory by means of vector meson dressing of nucleon-meson vertices. It is known that in a theory with neutral vector mesons there are vertex corrections that generate a strongly damped vertex function in the ultraviolet [6]. In quantum electrodynamics (QED), such corrections give rise to the Sudakov form factor [7]. When the Sudakov form factor, generated by massive vector mesons, is included in the Hartree-Fock approximation to the Schwinger-Dyson equation (SDE) for the nucleon propagator, the ghost poles disappear. A similar result was obtained by Allendes and Serot [8] earlier in the study of the ghost pole in the meson propagator. Those authors concluded that the Sudakov corrected propagator is free of ghost poles.

It is the purpose of the present paper to solve self-consistently the coupled system of Schwinger-Dyson equations for the nucleon and meson propagators and investigate the role of self-consistency on the appearance of ghost poles in the propagators. Vertex corrections are introduced by means of form factors.

There is an extensive literature on calculations of nuclear matter and finite nuclei properties based on the Walecka scalar-vector model [9]. In general, the applications have been performed using Hartree-Fock (HF) type of approximations. In a relativistic HF approximation, the single-nucleon propagator is calculated by solving self-consistently the SDE using bare meson propagators and bare meson-nucleon vertices. An additional approximation has been the neglect of the quantum vacuum of the nucleon propagator. The corrections to the meson propagators are usually calculated considering the vacuum polarization correction using nucleon propagators with an effective mass. Although the nucleon propagator is solved self-consistently by means of the SDE, the self-consistency is only partial, since the meson propagators used are the bare ones. The meson propagators satisfy their own SDE's, which require for their solution the nucleon propagator. A self-consistent solution requires the consideration of the coupled system of nucleon and meson SDE's.

Besides the lack of self-consistency, the neglect of the quantum vacuum in the nucleon sector is a major limitation. It is exactly the nontrivial nature of the vacuum of a relativistic quantum field theory that motivates the introduction of models which go beyond the usual nonrelativistic approach. However, severe difficulties arise in including the vacuum effects beyond the one-loop Hartree approximation. The inclusion of these vacuum corrections leads to catastrophic results due to the presence of the ghost poles in the propagators. Among other things, the ghosts lead to a large imaginary part to the nuclear matter energy.

Although the primary aim of our studies is the construction of an intrinsically consistent relativistic quantum field theory for the nuclear many-body problem, these studies have connections to other fields that use the SDE's to study nonperturbative effects in field theory. Such fields include the problems of dynamical chiral symmetry breaking and color confinement in QCD, technicolor models and QED in four and lower dimensions. For a recent review on the subject of the SDE's in this context see Ref. [10].

It is common practice in QCD and QED to study the solutions of the fermion SDE in Euclidean space, instead of Minkowski space as we do in our studies. In principle, the formulation of the problem either in Minkowski space or in Euclidean space is entirely equivalent; both formulations are connected by an analytic continuation. However, this equivalence holds provided there are no singularities in the complex plane. Since the pioneering works of Fukuda and Kugo [11] and Atkinson and Blatt [12], it is known that the solution of the SDE for the electron propagator in Euclidean space treated in the Hartree-Fock approximation has pairs of complex conjugate branch points. The same feature was found in recent studies of the SDE in a variety of models of QCD [13]. The existence of the ghost poles in Minkowski space and of the complex branch points in Euclidean space spoils the equivalence of the Minkowski and Euclidean formulations. It would be interesting to investigate the possibility that the branch points have their origin in the ultraviolet behavior of the interaction as in the case of the ghost poles. This could be done by using a Sudakov corrected fermion–vector-boson vertex [4]. This would be particularly interesting for the case of QCD, were the running of the coupling constant provides extra logarithms in the Sudakov form factor.

The paper is organized as follows. In Sec. II we present the model for the interacting nucleon-meson system. We briefly review the spectral representation of the propagators and their inverses and discuss the renormalization procedure. In Sec. III we discuss the coupled system of Schwinger-Dyson equations for the nucleon and meson propagators in terms of their spectral representations. Section IV presents the method of solution of the equations and presents our numerical results. Conclusions are presented in Sec. V.

II. THE MODEL

In this paper we consider a model field theory with nucleons (ψ), pions ($\boldsymbol{\pi}$), and vector isoscalar mesons (ω). The model Lagrangian density is

$$\begin{aligned}\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - ig_{0\pi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} - g_{0\omega}\gamma_\mu\omega^\mu)\psi \\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} \\ -\frac{1}{2}m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\ ,\end{aligned} \quad (1)$$

where $F^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$.

This Lagrangian density is not compatible with the requirements of the partial conservation of the axial current (PCAC). Although it is true that a consistent hadronic model must be compatible with PCAC, in this paper we are mostly interested in the interplay of self-consistency of the nucleon and meson propagators and the problem of the ghost poles. Chiral symmetry allows the presence of self-interacting meson terms in the Lagrangian, as for instance in the linear sigma model. Such terms will alter the low momentum structure of the meson spectral functions, in comparison to those obtained in this paper. However, it is unlikely that the appearance or disappearance of the ghost poles, which are an ultraviolet phenomenon, will be altered by this. The implementation of chiral symmetry in a renormalizable hadronic model for practical uses in nuclear physics has difficulties due to the many-body forces implied by the self-interacting terms [14]. In this sense, our model is probably an appropriate starting point for studies towards a consistent relativistic many-body theory for the nucleus.

As usual, the nucleon propagator is defined by

$$G_{\alpha\beta}(x'-x) = -i\langle 0|T[\psi_\alpha(x')\bar{\psi}_\beta(x)]|0\rangle\ , \quad (2)$$

where $|0\rangle$ represents the physical vacuum state. The π- and ω-meson propagators are defined respectively by

$$D_\pi^{ij}(x'-x) = -i\langle 0|T[\pi^i(x')\pi^j(x)]|0\rangle \tag{3}$$

and

$$D_\omega^{\mu\nu}(x'-x) = -i\langle 0|T[\omega^\mu(x')\omega^\nu(x)]|0\rangle . \tag{4}$$

The Schwinger-Dyson equations for the nucleon and meson propagators in momentum space are given by the following expressions, Fig. 1:
(a) nucleon,

$$G(p) = G^{(0)}(p) + G^{(0)}(p)\Sigma(p)G(p) , \tag{5}$$

$$\Sigma(p) = -ig_{0\pi}^2 \int \frac{d^4q}{(2\pi)^4}\gamma_5\tau^i D_\pi(q^2)G(p-q)\Gamma_5^i(p-q,p;q) + ig_{0\omega}^2 \int \frac{d^4q}{(2\pi)^4}\gamma_\mu D_\omega^{\mu\nu}(q^2)G(p-q)\Gamma_\nu(p-q,p;q) ; \tag{6}$$

(b) pion,

$$D_\pi^{ij}(q^2) = D_\pi^{(0)^{ij}}(q^2) + D_\pi^{(0)^{ik}}(q^2)\Pi_\pi^{kl}(q^2)D_\pi^{lj}(q^2) , \tag{7}$$

$$\Pi_\pi^{ij}(q^2) = ig_{0\pi}^2 \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\gamma_5\tau^i G(p)\Gamma_5^j(p,p+q;q)G(p+q)] ; \tag{8}$$

(c) omega,

$$D_\omega^{\mu\nu}(q^2) = D_\omega^{\mu\nu(0)}(q^2) + D_\omega^{\mu\rho(0)}(q^2)\Pi_\omega^{\rho\sigma}(q^2)D_\omega^{\sigma\nu}(q^2) , \tag{9}$$

$$\Pi_\omega^{\mu\nu}(q^2) = -ig_{0\omega}^2 \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu G(p)\Gamma^\nu(p,p+q;q)G(p+q)] . \tag{10}$$

In the above equations, $\Gamma_5^i(p,p+q;q)$ and $\Gamma^\mu(p,p+q;q)$ are the three-point π-nucleon and ω-nucleon vertex functions, respectively. They satisfy their own Schwinger-Dyson equations. These relate the three-point functions to four-point vertices and so on *ad infinitum.* In practice one has to truncate this infinite set. In this paper we truncate the SDE's by postulating a specific form for the three-point functions (see below).

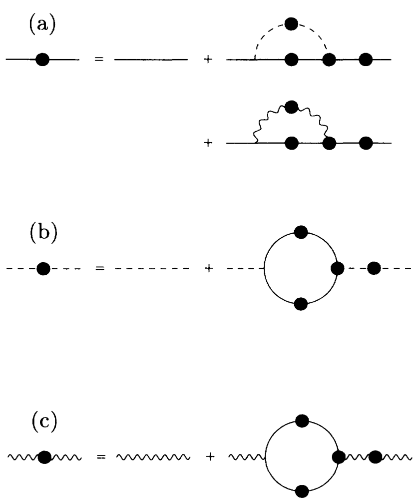

FIG. 1. Diagrammatic representation of the Schwinger-Dyson equations for the full (a) nucleon, (b) pion, and (c) omega propagators. The solid, wavy and dashed lines represent respectively the nucleon, the ω, and the π. The solid circles represent full propagators and vertices.

Next, we discuss the spectral representations of the propagators and their inverses. We do not intend to review the subject of spectral representations; we simply make use of the relevant equations for the purposes of the present paper. We refer the reader to Refs. [15–18] for an extensive discussion on the subject. Let us start with the nucleon propagator. The spectral representation of the nucleon propagator (in momentum space) can be written as

$$G(p) = \int_{-\infty}^{+\infty} d\kappa \, \frac{A(\kappa)}{\not p - \kappa + i\epsilon} . \tag{11}$$

$A(\kappa)$ is the spectral function. It represents the probability that a state of mass $|\kappa|$ is created by ψ or $\bar\psi$, and as such it must be non-negative. Negative κ corresponds to states with opposite parity to the nucleon.

Defining the projection operators

$$P_\pm(p) = \frac{1}{2}\left(1 \pm \frac{\not p}{w_p}\right) , \tag{12}$$

where

$$w_p = \sqrt{p^2} = \begin{cases} \sqrt{p^2} & \text{if } p^2 > 0, \\ i\sqrt{-p^2} & \text{if } p^2 < 0. \end{cases} \tag{13}$$

$G(p)$ can be rewritten conveniently as

$$G(p) = P_+(p)\tilde G(w_p + i\epsilon) + P_-(p)\tilde G(-w_p - i\epsilon) , \tag{14}$$

where $\tilde{G}(z)$, $z = \pm(w_p + i\epsilon)$, is given by the dispersion integral

$$\tilde{G}(z) = \int_{-\infty}^{+\infty} d\kappa \, \frac{A(\kappa)}{z - \kappa} \; . \tag{15}$$

The inverse of the propagator can also be written in terms of the projection operators $P_\pm(p)$ as

$$G^{-1}(p) = P_+(p)\tilde{G}^{-1}(w_p + i\epsilon) + P_-(p)\tilde{G}^{-1}(-w_p - i\epsilon) \; . \tag{16}$$

The spectral representation for $\tilde{G}^{-1}(z)$ is written as

$$\begin{aligned}\tilde{G}^{-1}(z) &= z - M_0 - \tilde{\Sigma}(z) \\ &= z - M_0 - \int_{-\infty}^{+\infty} d\kappa \frac{T(\kappa)}{z - \kappa} \; . \end{aligned} \tag{17}$$

The function $\tilde{\Sigma}(z)$ is related to the $\Sigma(q)$ of Eq. (5) by the projection operators $P_\pm(q)$ as in Eq. (16).

Since $A(\kappa)$ is supposed to be non-negative, $\tilde{G}(z)$ can have no poles or zeros off the real axis. This is known as the Herglotz property. The absence of zeros can be demonstrated as follows [1]: $\tilde{G}(z)$ can be written as

$$\tilde{G}(x + iy) = \int_{-\infty}^{+\infty} d\kappa \, \frac{(x - \kappa - iy)A(\kappa)}{(x-\kappa)^2 + y^2} \; ; \tag{18}$$

from this one has that the imaginary part of $\tilde{G}(z)$ is given by

$$\mathrm{Im}\tilde{G}(x + iy) = -y \int_{-\infty}^{+\infty} d\kappa \, \frac{A(\kappa)}{(x-\kappa)^2 + y^2} \; , \tag{19}$$

which is nonzero for $y \neq 0$. This is a necessary condition for writing a spectral representation for $\tilde{G}^{-1}(z)$; the absence of poles off the real axis for $\tilde{G}(z)$ follows then from the absence of zeros for $\tilde{G}^{-1}(z)$. This last property can be demonstrated from Eq. (17) for $T(\kappa)$ non-negative.

In general, the integral in Eq. (17) needs renormalization. The usual mass and wave-function renormalizations are performed by imposing the condition that the renormalized propagator have a pole at the physical nucleon mass M, with unit residue. This implies that the renormalized propagator $\tilde{G}_R(z)$, defined as

$$\tilde{G}_R(z) \equiv \tilde{G}(z)/Z_2 \; , \tag{20}$$

is given by the following expression:

$$\tilde{G}_R(z) = \int_{-\infty}^{+\infty} d\kappa \, \frac{A_R(\kappa)}{z - \kappa} \; . \tag{21}$$

The renormalized inverse is given by

$$\tilde{G}_R^{-1}(z) = (z - M)\left[1 - (z - M)\int_{-\infty}^{+\infty} d\kappa \, \frac{T_R(\kappa)}{(\kappa - M)^2(z - \kappa)}\right] \; . \tag{22}$$

In the above expressions, $A_R(\kappa) = A(\kappa)/Z_2$ and $T_R(\kappa) = Z_2 T(\kappa)$. In terms of renormalized quantities, Z_2 can be written as

$$Z_2 = 1 - \int_{-\infty}^{+\infty} d\kappa \frac{T_R(\kappa)}{(\kappa - M)^2} \tag{23}$$

$$= \left[\int_{-\infty}^{+\infty} d\kappa A_R(\kappa)\right]^{-1} \; . \tag{24}$$

The spectral functions $A_R(\kappa)$ and $T_R(\kappa)$ are related by

$$A_R(\kappa) = \delta(\kappa - M) + |\tilde{G}_R^{-1}(\kappa(1 + i\epsilon))|^{-2} T_R(\kappa) \tag{25}$$

$$\equiv \delta(\kappa - M) + \bar{A}_R(\kappa) \; . \tag{26}$$

The possibility of writing such an expression, relating the spectral function of the propagator to the spectral function of its inverse, is of course only permissible if the Herglotz property is valid. In the presence of ghosts, this expression is not valid.

Let us now consider the spectral representations of the meson propagators. The isospin structure of the π-meson propagator is such that $D_\pi^{ij}(q^2) = \delta^{ij} D_\pi(q^2)$. For $D_\pi(q^2)$ one can write the spectral representation

$$D_\pi(z) = \int_0^\infty d\sigma^2 \, \frac{\rho_\pi(\sigma^2)}{z - \sigma^2} \; , \tag{27}$$

where $\rho_\pi(\sigma^2)$ is the pion spectral function. It represents the probability that a state of mass $\sqrt{\sigma^2}$ is created by the pion field and as such it must be non-negative. The meaning of the complex variable z is that the physical propagator $D_\pi(q^2)$ is the limit of $D_\pi(z)$ when $z \to q^2 + i\epsilon$.

Using the SDE for the pion, Eq. (7), the inverse of $D_\pi(z)$ can be written in terms of the pion self-energy $\Pi_\pi(z)$ as

$$D_\pi^{-1}(z) = z - {m_\pi^0}^2 - \Pi_\pi(z) \; . \tag{28}$$

Similarly to the case of the nucleon, one can write a spectral representation for $D_\pi^{-1}(z)$,

$$D_\pi^{-1}(z) = z - {m_\pi^0}^2 - \int_0^\infty d\sigma^2 \, \frac{S_\pi(\sigma^2)}{z - \sigma^2} \; . \tag{29}$$

The renormalized propagator is again obtained by fixing the pole position at the physical mass, and the residue at the pole equal to 1. The renormalized propagator $D_{\pi R}(z)$, defined as

$$D_{\pi R}(z) \equiv D_\pi(z)/Z_{3\pi} \; , \tag{30}$$

is then

$$D_{\pi R}(z) = \int_0^\infty d\sigma^2 \, \frac{\rho_{\pi R}(\sigma^2)}{z - \sigma^2} \; . \tag{31}$$

Its inverse is given by

$$D^{-1}_{\pi R}(z) = (z - m_\pi^2)\left[1 - (z - m_\pi^2)\int_0^\infty d\sigma^2 \, \frac{S_{\pi R}(\sigma^2)}{(\sigma^2 - m_\pi^2)^2(z - \sigma^2)}\right] . \tag{32}$$

The renormalized spectral functions are defined as $\rho_{\pi R}(\sigma^2) = \rho_\pi(\sigma^2)/Z_{3\pi}$ and $S_{\pi R}(\sigma^2) = Z_{3\pi} S_\pi(\sigma^2)$.

In terms of the renormalized quantities, $Z_{3\pi}$ is given by

$$Z_{3\pi} = 1 - \int_0^\infty d\sigma^2 \, \frac{S_{\pi R}(\sigma^2)}{(\sigma^2 - m_\pi^2)^2} \tag{33}$$

$$= \left[\int_0^\infty d\sigma^2 \, \rho_{\pi R}(\sigma^2)\right]^{-1} . \tag{34}$$

The spectral functions $\rho_{\pi R}$ and $S_{\pi R}$ are related by

$$\rho_{\pi R}(q^2) = \delta(q^2 - m_\pi^2) + |D^{-1}_{\pi R}|^2 S_{\pi R}(q^2) \tag{35}$$

$$\equiv \delta(q^2 - m_\pi^2) + \bar{\rho}_{\pi R}(q^2) . \tag{36}$$

Let us now consider the ω-meson propagator. Since the baryon current is conserved, the ω-meson self-energy $\Pi_\omega^{\mu\nu}(q^2)$ must satisfy

$$q_\mu \Pi_\omega^{\mu\nu}(q^2) = q_\nu \Pi_\omega^{\mu\nu}(q^2) = 0 . \tag{37}$$

Therefore, the Lorentz structure of $\Pi_\omega^{\mu\nu}$ must be

$$\Pi_\omega^{\mu\nu}(q^2) = \left(g^{\mu\nu} - q^\mu q^\nu/q^2\right)\Pi_\omega(q^2) . \tag{38}$$

Substituting this in the SDE for the ω-meson propagator, Eq. (9), $D_\omega^{\mu\nu}(q^2)$ can be written as

$$D_\omega^{\mu\nu}(q^2) = -g^{\mu\nu} D_\omega(q^2) , \tag{39}$$

where

$$D_\omega(q^2) = \frac{1}{q^2 - {m_\omega^0}^2 - \Pi_\omega(q^2) + i\epsilon} . \tag{40}$$

Terms proportional to $q^\mu q^\nu$ in Eq. (39) can be neglected when using $D_\omega^{\mu\nu}$ in Eq. (6), because of current conservation.

The spectral representation of D_ω is

$$D_\omega(z) = \int_0^\infty d\sigma^2 \, \frac{\rho_\omega(\sigma^2)}{z - \sigma^2} . \tag{41}$$

As in the case of the pion, one can write the Cauchy representation for the inverse of the ω-meson propagator as

$$D_\omega^{-1}(z) = z - {m_\omega^0}^2 - \int_0^\infty d\sigma^2 \, \frac{S_\omega(\sigma^2)}{z - \sigma^2} . \tag{42}$$

Renormalization proceeds as for the pion. The renormalized quantities are given by expressions similar to the ones for the pion, Eqs. (31, 32, 34, 36), with the π index replaced by ω index.

III. SCHWINGER-DYSON EQUATIONS

We start with the nucleon SDE, Eq. (5). It can be written as

$$G^{-1}(p) = G^{-1}(p) - \Sigma(p) , \tag{43}$$

where $\Sigma(p)$ is given by Eq. (6). To proceed, we need to specify the form of the vertex functions $\Gamma_5^i(p, p+q; q)$ and $\Gamma^\mu(p, p+q; q)$. In the usual HF approximation, $\Gamma_5^i(p, p+q; q) = \tau^i\gamma_5$ and $\Gamma^\mu(p, p+q; q) = \gamma^\mu$. In this paper we consider vertex functions written as

$$\Gamma_5^i(p_1, p_2; q) = \tau^i \gamma_5 F_5(p_1, p_2; q), \tag{44}$$

$$\Gamma^\mu(p_1, p_2; q) = \gamma^\mu F_V(p_1, p_2; q) , \tag{45}$$

where $F_5(p_1, p_2; q)$ and $F_V(p_1, p_2; q)$ are scalar functions.

It is important to note the inconsistency of our ansatz for the $NN\omega$ vertex function with the (first) Ward-Takahashi identity. This identity is an exact statement for the conservation of the baryon current; it relates (the longitudinal part of) $\Gamma^\mu(p, p+q; q)$ to the nucleon self-energy. There are attempts [19] to incorporate vertices consistent with this identity in studies of model SDE's. It would be very interesting to pursue such an approach in hadronic models, mainly in connection with the ultraviolet behavior of the vertex function. For the purposes of the present paper, we use the above ansatz and reserve for a future publication the study of an ansatz consistent with the Ward-Takahashi identity.

Substituting Eqs. (44,45) and the spectral representations for $G(q)$, D_π, and D_ω, in the integral for $\Sigma(q)$, Eq. (6), and applying the projection operators $P_\pm(p)$ to Eq. (43), one obtains

$$T_R(\kappa) = \int_{-\infty}^{+\infty} d\kappa' K(\kappa, \kappa') A_R(\kappa') , \tag{46}$$

where $K(\kappa, \kappa')$ is given by

$$K(\kappa, \kappa') = K_\pi(\kappa, \kappa'; m_\pi^2) + 2K_\omega(\kappa, \kappa'; m_\omega^2) + \int_0^\infty d\sigma^2 \bar{\rho}_{\pi R}(\sigma^2) K_\pi(\kappa, \kappa'; \sigma^2) + 2\int_0^\infty d\sigma^2 \bar{\rho}_{\omega R}(\sigma^2) K_\omega(\kappa, \kappa'; \sigma^2) . \tag{47}$$

$K_\pi(\kappa, \kappa'; m^2)$ and $K_\omega(\kappa, \kappa'; m^2)$ are respectively the π-nucleon and ω-nucleon scattering kernels,

$$K_\pi(\kappa, \kappa'; m^2) = 3\left(\frac{g_\pi}{4\pi}\right)^2 \left[\kappa^4 - 2\kappa^2(\kappa'^2 + m^2) + (\kappa'^2 - m^2)^2\right]^{1/2} \times \frac{1}{2|\kappa|^3}\left[(\kappa - \kappa')^2 - m^2\right]\theta(\kappa^2 - (|\kappa'| + m)^2) F_5(\kappa, \kappa'; m) \tag{48}$$

and

$$K_\omega(\kappa,\kappa';m^2) = \left(\frac{g_\omega}{4\pi}\right)^2 \left[\kappa^4 - 2\kappa^2(\kappa'^2+m^2) + (\kappa'^2-m^2)^2\right]^{1/2} \times \frac{1}{2|\kappa|^3}\left[(\kappa-\kappa')^2 - 2\kappa\kappa' - m^2\right]\theta(\kappa^2 - (|\kappa'|+m)^2)F_V(\kappa,\kappa';m) \,. \tag{49}$$

$\bar{\rho}_{\pi R}(\sigma^2)$ is related to $S_{\pi R}$ as shown in Eqs. (35–36) [similarly for $\bar{\rho}_{\omega R}(\sigma^2)$].

The meson self-energies $S_{\pi R}(q^2)$ and $S_{\omega R}(q^2)$ are obtained using the spectral representation of the nucleon propagator in Eqs. (8, 10). $S_{\pi R}(q^2)$ is given by

$$S_{\pi R}(q^2) = S_\pi(M,M;q^2) + 2\int_{-\infty}^{\infty} d\kappa \bar{A}_R(\kappa) S_\pi(M,\kappa;q^2) + \int_{-\infty}^{\infty} d\kappa d\kappa' \bar{A}_R(\kappa)\bar{A}_R(\kappa')S_\pi(\kappa,\kappa';q^2) \,, \tag{50}$$

where $\bar{A}(\kappa)$ is defined in Eq. (26), and

$$S_\pi(\kappa,\kappa';q^2) = \left(\frac{g_\pi^2}{4\pi^2}\right)\left[1 - \frac{(\kappa-\kappa')^2}{q^2}\right][q^4 - 2q^2(\kappa^2+\kappa'^2) + (\kappa^2-\kappa'^2)^2]^{\frac{1}{2}}\Theta(q^2 - (|\kappa|+|\kappa'|)^2)F_5(\kappa,\kappa';q) \,, \tag{51}$$

with $g_\pi^2 = Z_2 g_{0\pi}$ and $g_\omega^2 = Z_2 g_{0\omega}$.

For the ω meson, we have the same expression as in Eq. (50), with the index π replaced by ω and

$$S_\omega(\kappa,\kappa';q^2) = \left(\frac{g_\omega^2}{8\pi^2}\right)\left\{1 - \frac{(\kappa-\kappa')^2}{q^2} + \frac{1}{3q^4}[q^4 - 2q^2(\kappa^2+\kappa'^2) + (\kappa^2-\kappa'^2)^2]\right\} \times [q^4 - 2q^2(\kappa^2+\kappa'^2) + (\kappa^2-\kappa'^2)^2]^{\frac{1}{2}}\Theta(q^2 - (|\kappa|+|\kappa'|)^2)F_V(\kappa,\kappa';q) \,. \tag{52}$$

IV. NUMERICAL RESULTS

The problem consists in solving for the spectral functions $A_R(\kappa)$, $\rho_{\pi R}(\sigma^2)$, and $\rho_{\omega R}(\sigma^2)$. The equations related to $A_R(\kappa)$ are Eqs. (46)–(49), (22), and (26). For $\rho_{\pi R}(\sigma^2)$, the relevant equations are Eqs. (50), (51), (32), and (36). For the ω meson, the equations are the equivalent ones of Eqs. (50), (51), and (32), with the index π replaced by ω, and Eq. (52). These represent a set of coupled nonlinear integral equations, which we solve by iteration.

We start solving for A_R with the bare π and ω propagators, for which the spectral functions are given by

$$\rho_{\pi R}(\sigma^2) = \delta(\sigma^2 - m_\pi^2) \,, \quad \rho_{\omega R}(\sigma^2) = \delta(\sigma^2 - m_\omega^2) \,. \tag{53}$$

This is the usual Hartree-Fock approximation for the nucleon propagator including vertex corrections by means of the form factors of Eqs. (44), (45). The solution for $A_R(\kappa)$ is obtained, as in Refs. [1] and [4], by iteration from the perturbative solution.

This $A_R(\kappa)$ is used to obtain the spectral function of the inverse of the pion propagator $S_{\pi R}(q^2)$, Eq. (50), and the equivalent one with $\pi \to \omega$. Using Eq. (36) one obtains $\bar{\rho}_{\pi R}$ and similarly $\bar{\rho}_{\omega R}$. This completes the first iteration.

The next iteration starts calculating the fermion kernel $K(\kappa,\kappa')$ of Eq. (47) using the spectral functions $\bar{\rho}_{\pi R}$ and $\bar{\rho}_{\omega R}$ obtained in the first iteration. With this $K(\kappa,\kappa')$, we solve for A_R by iteration starting from the $A_R(\kappa)$ obtained in the first iteration. The process is then repeated to convergence, for $A_R(\kappa)$, $\rho_{\pi R}(\sigma^2)$, and $\rho_{\omega R}(\sigma^2)$.

Initially, we considered bare vertices $F_5(p_1,p_2,q) = F_V(p_1,p_2,q) = 1$, and investigated the role of the self-consistency on the spectral functions. We used the following values for the coupling constants and masses:

$$\frac{g_\pi^2}{4\pi} = 14.6 \,, \quad m_\pi = 0.144M \,, \tag{54}$$

$$\frac{g_\omega^2}{4\pi} = 6.36 \,, \quad m_\omega = 0.833M \,, \tag{55}$$

where M is the nucleon mass.

The converged spectral functions A_R, $\rho_{\pi R}$, and $\rho_{\omega R}$ are shown (without the delta functions) in Figs. 2–4. The solid (dashed) lines represent the self-consistent (not self-consistent) solutions. The not self-consistent meson spectral functions are the ones obtained by calculating the nucleon loop in Figs. 1(b), 1(c) using the bare nucleon propagators ($\bar{A}_R = 0$); i.e., these are the first order perturbative spectral functions. As discussed in Ref. [4], the contribution of the ω meson to the kernel $K(\kappa,\kappa')$ in Eq. (47) has a finite jump at $\kappa = M + m_\omega$. This introduces a discontinuity in the integrand of Eq. (22). At the discontinuity, the real part (principal value integral) of Eq. (22) has a logarithmic singularity, implying that $A_R(\kappa)$ has a (sharp) zero at $\kappa = M + m_\omega$. This zero is represented in Figs. 2 and 5 by the vertical straight line which hits the κ axis at the discontinuity.

The self-consistency does not affect the fermion spectral function perceptively; therefore we have plotted only the self-consistent one. However, the self-consistency does affect the meson spectral functions, although not very importantly (see Figs. 3 and 4). It is interesting to note that the effect of the self-consistency is opposite in

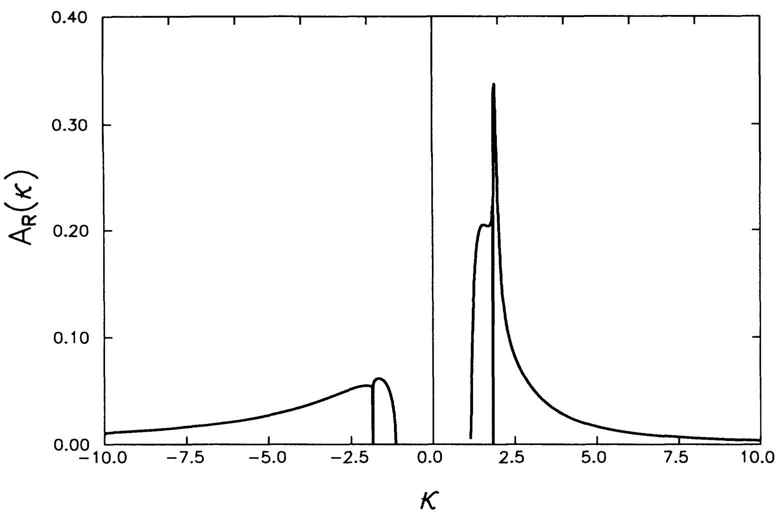

FIG. 2. Self-consistent (solid curve) and not self-consistent (dashed curve) nucleon spectral function $A_R(\kappa)$. κ is in units of the nucleon mass M and $A_R(\kappa)$ is in units of M^{-1}. The curves are multiplied by 5 for negative κ.

$\rho_{\pi R}$ and $\rho_{\omega R}$; it increases the former and decreases the last.

Next, we search for ghost poles. This is done by searching the zeros in the complex variable z of the term in square brackets in Eq. (22). The search is done using a Newton-Raphson method. Once complex zeros of $\tilde{G}_R^{-1}(z)$ are located, the residues of the corresponding poles in $\tilde{G}_R \equiv [\tilde{G}_R(z)]^{-1}$ are easily computed regarding this function as the ratio of two analytic functions. The role of the self-consistency on the appearance of ghost poles is shown in Table I. Clearly, the self-consistency does not change much the position of the poles and residues of the nucleon propagator, although it changes somewhat the ones of the meson propagators.

As discussed in Refs. [1] and [4], the signal for the presence of ghosts in the nucleon propagator is revealed by the fact that the renormalization constant Z_2 calculated via the spectral function of the nucleon self-energy, $T_R(\kappa)$, gives $Z_2 = -\infty$. The minus sign is the indication of a ghost. This happens because, for large κ or κ', one has

$$K_\pi(\kappa,\kappa') \longrightarrow \frac{1}{2|\kappa|^3}(\kappa^2-\kappa'^2)(\kappa-\kappa')^2\theta(\kappa^2-\kappa'^2)\ , \quad (56)$$

and since the integral of $A_R(\kappa)$ is finite, one has [1]

$$T_R(\kappa) \longrightarrow |\kappa|\ , \quad (57)$$

and the integral for Z_2 is therefore logarithmically divergent. On the other hand, the integral over $A_R(\kappa)$ is not zero, and therefore Z_2 calculated via A_R, Eq. (24), does not give $Z_2^{-1} = 0$. However, as shown in Ref. [1], consistency is recovered if one includes the pair of complex conjugate poles in $\tilde{G}_R$ [note that the real parts of the residues are negative (see Table I)].

For the case of the renormalization constants of the π and ω mesons, we obtain exactly the same result: The Z_3's calculated via the spectral function of the self-

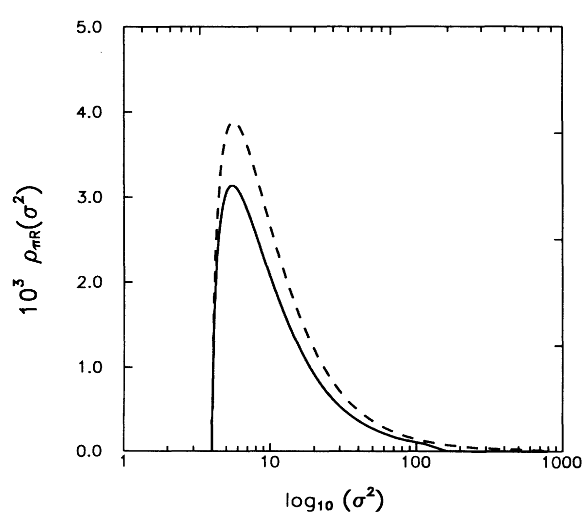

FIG. 3. Self-consistent (solid curve) and not self-consistent (dashed curve) π spectral function $\rho_{\pi R}(\sigma^2)$. σ^2 is in units of M^2 and $\rho_{\pi R}(\sigma^2)$ is in units of M^{-2}.

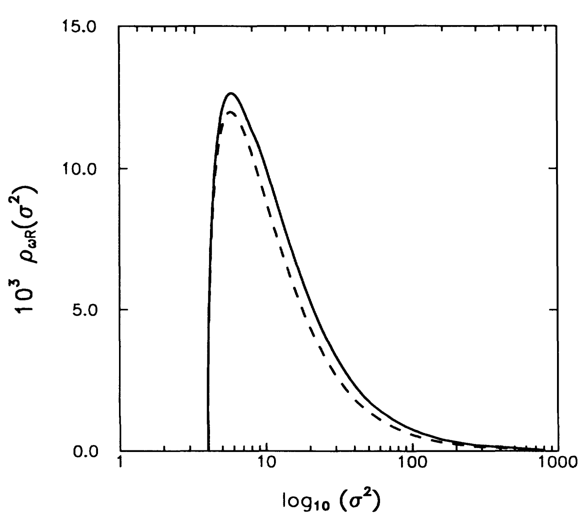

FIG. 4. Self-consistent (solid curve) and not self-consistent (dashed curve) ω spectral function $\rho_{\omega R}(\sigma^2)$. The units are the same as in Fig. 3.

TABLE I. Ghost poles. The first value is the pole position and the second is the residue at the pole. The nucleon poles are in units of M and of the mesons are in units of M^2.

	Self-consistent		Not self-consistent	
N	$1.06 \pm 1.25i$	$-0.77 \pm 0.20i$	$1.05 \pm 1.26i$	$-0.77 \pm 0.20i$
π	-1.04	-1.08	-1.44	-1.13
ω	-3.50	-1.30	-5.68	-1.49

energy, S_R, give $Z_3 \longrightarrow -\infty$. To obtain zero for the integral over the spectral function of the propagator, ρ_R, the residue of the ghost pole has to be included.

In Ref. [4], the problem of ghosts poles in the nucleon propagator was investigated using form factors at the nucleon-meson vertices. Two types of form factors were used: (a) a Sudakov form factor, which is generated by vector meson dressing of the vertices, and (b) a phenomenological form factor, of the monopole type. The conclusion there was that both types of form factors are able to kill the ghosts. However, as remarked in that reference, a proper extension of the Sudakov form factor to lower momenta is necessary for a better study of these issues. In this paper we use only the simple monopole form factor to investigate the interplay of self-consistency and vertex corrections on the spectral functions. As in Ref. [4], we use for $F_5(p_1,p_2,q)$ and $F_V(p_1,p_2,q)$ the following expressions:

$$F_5(p_1,p_2,q) = F_V(p_1,p_2,q) = \frac{1}{1+|p_1^2/\Lambda^2|}\,\frac{1}{1+|q^2/\Lambda^2|}\,\frac{1}{1+|p_2^2/\Lambda^2|}\,, \tag{58}$$

where Λ is an ultraviolet cutoff.

The calculated spectral functions with use of the form factors are plotted in Figs. 5–7. We plotted A_R, $\rho_{\pi R}$, and $\rho_{\omega R}$ (again without the delta functions) for three representative values of cutoffs, $\Lambda = M$ (solid curves), $1.25M$ (long-dashed curves), and $\Lambda = \infty$ (short-dashed curves). The effect of the form factor is to increase $A_R(\kappa)$ for negative κ, a result already found in [4]. The effect on the meson spectral functions is to increase (decrease) $\rho_{R\pi}$ ($\rho_{R\omega}$).

In Ref. [4], it was found that for a $\Lambda < \Lambda_{\rm crit} \approx 1.75M$ the ghost poles in the nucleon propagator disappear. In the present case, we found that the self-consistency does not alter significantly this critical value; the ghost poles disappear in all propagators for a $\Lambda \lesssim 1.60M$.

We have also investigated the effect of the self-consistency on the ghost-free spectral functions; i.e., we compared the self-consistent and not self-consistent spectral functions for several values of Λ's smaller than $\Lambda_{\rm crit}$. We found the surprising result that the effect of the self-consistency is negligible in all spectral functions; the effect is almost invisible when one plots the spectral functions.

Although on physical grounds one expects that the cutoffs for the π and ω vertices have different values, we used the same value for both, since in this work we are mostly interested in the qualitative effects. The consequences of the modifications induced by the form factors on physical observables deserves a separate study. Work in this direction is in progress.

We conclude this section with the general remark that the self-consistency does not affect the spectral properties of the propagators.

V. CONCLUSIONS AND PERSPECTIVES

In this paper we have solved self-consistently the coupled set of Schwinger-Dyson equations for the nucleon and π and ω mesons in the vacuum. The set of equations was truncated by postulating a three-point meson-nucleon vertex function. The understanding of the vacuum properties of the nucleon and meson propagators is a

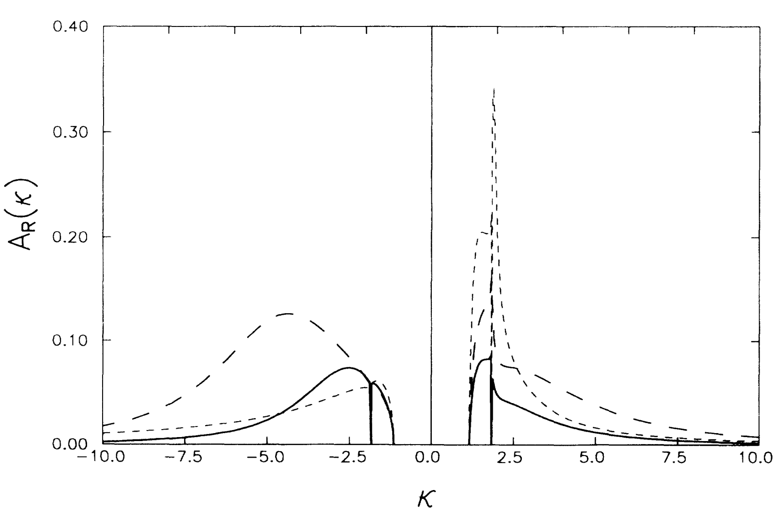

FIG. 5. $A_R(\kappa)$ for different values of the cutoff: $\Lambda = M$ (solid curve), $1.25M$ (long-dashed curve), and ∞ (short-dashed curve). Units are the same as in Fig. 2. The short-dashed curve is multiplied by 5 for negative κ.

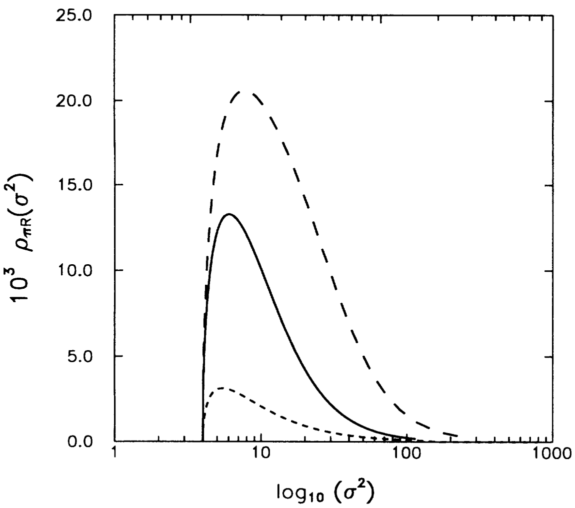

FIG. 6. $\rho_{\pi R}(\sigma^2)$ for same Λ's as in Fig. 5. Units are the same as in Fig. 3.

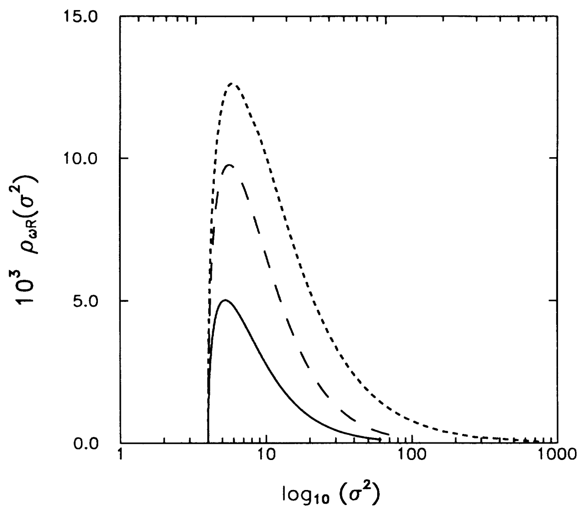

FIG. 7. Same as in Fig. 6 for $\rho_{\omega R}(\sigma^2)$.

necessary first step towards the study of the properties of nucleon and meson in nuclear matter, as well as those of nuclear matter and finite nuclei. Although many of such properties have been studied using relativistic quantum field models, the vacuum polarization effects in medium have invariably been neglected.

The main conclusion of our investigation is the surprising result that the self-consistency does not modify significantly the spectral properties of the propagators. The appearance or disappearance of the ghost poles in the propagators is not affected by the self-consistency.

One important aspect regarding the vacuum of meson-nucleon effective theories that was not yet satisfactorily investigated is the role of the three-point meson-nucleon vertex functions. In particular, the interplay of the infrared and ultraviolet sectors of the ω-nucleon three-point vertex is extremely important to the problem of ghosts poles, as shown in the recent studies of Refs. [8,4]. The constraint of current conservation on the three-point function is certainly an important aspect of the problem which also deserves more study. In this respect, an interesting possibility to implement the Ward-Takahashi identity is the use of the so called gauge technique, invented by Salam a long time ago [20]. This technique is particularly well suited for our formulation of the problem since it postulates a spectral function for the vertex which contains the spectral function of the fermion propagator. In this formulation, the Ward-Takahashi identity is automatically satisfied. In the past, this technique has been employed to investigate the problem of ghosts in the electron propagator [21]. Work in this direction is in progress.

The effects of the self-consistency on the nucleon and meson propagators in nuclear matter, in connection to the problem of ghost poles, remains an open problem, although work in this direction has recently been communicated [22].

Another important aspect is the role of chiral symmetry in hadronic models. This is a separate subject, with its own problems. Much remains to be done in this respect, both in vacuum and in nuclear matter.

ACKNOWLEDGMENTS

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