Dynamical chiral-symmetry breaking and exotic quark representations

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Using the effective potential for composite operators to study the chiral-symmetry-breaking transition in QCD, we examine a conjecture made by Marciano concerning the mass scale at which such transition occurs for exotic quark representations.

Some years ago Marciano[1] speculated about the existence of exotic new species of quarks belonging to a higher representation of SU(3) in addition to the usual 3-plets that are needed to describe the hadronic sector. In this case no extrastrong interaction based on an unbroken gauge group SU(N) will be introduced as in the hypercolor scenario.[2,3]

The basis of Marciano's conjecture is that the quantity

$$C_2(R)\alpha_s(\mu) , \tag{1}$$

with $C_2(R)$ being the quadratic SU(3) Casimir invariant, is a measure of the effective color flux linking a quark-antiquark pair at mass scale μ. As the Casimir invariants of the R representation are bigger for the exotic representation than for the fundamental representation, they interact more strongly than ordinary quarks.

If μ_R and μ_3 are the mass scales of the R ($R\neq 3$) sector and 3-plet sector, respectively, at which the chiral-symmetry breaking occurs the question is as follows: can $\alpha_s(\mu_R)$ differ from $\alpha_s(\mu_3)$? In Ref. 1, two extreme possibilities were treated: (i) chiral symmetry is sector independent and (ii) chiral symmetry is sector dependent but such that

$$C_2(R)\alpha_s(\mu_R)=C_2(3)\alpha_s(\mu_3) . \tag{2}$$

The first possibility is not an interesting one, because it does not explain flavor-symmetry breaking in the Weinberg-Salam model. The second one, on the other hand, should allow a mass-scale hierarchy.

In this work we shall check if condition (2) is realized calculating $\alpha_s(\mu_R)$ and $\alpha_s(\mu_3)$ independently. For this we describe the chiral phase transformation using the effective potential for composite operators proposed some years ago for the authors of Ref. 4 and that have been used recently to study chiral-symmetry breaking in vectorlike theories.[5–9] There are some difficulties with this formalism.[6,10] However, we hope that the critical coupling constants are not affected by these problems.

The effective potential derived in Ref. 4 has the form

$$V(G)=-i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}(\ln S^{-1}G-S^{-1}G+1)+V_2(G) , \tag{3}$$

where G is the complete propagator of fermions and S the respective bare propagator. $V_2(G)$ is the sum of all two-particle-irreducible vacuum diagrams. We calculate $V(G)$ in the usual approximation:[5–9]

$$iG^{-1}=\not{p}-\Sigma(p) \tag{4}$$

with[5,9]

$$\Sigma(p)=\phi,\quad p\le p_c$$
$$=\phi\frac{p_c^2}{p^2}\left[\ln\frac{p_c}{\Lambda}\right]^{1-A/2}\left[\ln\frac{p}{\Lambda}\right]^{A/2-1},\quad p\ge p_c , \tag{5}$$

where

$$A=18C_2(R)/[11N-4n_fT(R)] ,$$

p_c is a momentum defining the infrared region and $\Lambda\approx 200$ MeV is the QCD scale. In (5), ϕ will be considered a variational parameter. We use, for the running coupling constant,[11]

$$\bar{g}^{\,2}(p,k)=\bar{g}^{\,2}(p),\quad p>k$$
$$=\bar{g}^{\,2}(k),\quad k>p \tag{6}$$

with

$$\bar{g}^{\,2}(p)=\frac{24\pi^2}{11N-4n_fT(R)}\times\begin{cases}\left[\ln\dfrac{p_c}{\Lambda}\right]^{-1}, & p_c\ge p ,\\ \left[\ln\dfrac{p}{\Lambda}\right]^{-1}, & p\ge p_c .\end{cases} \tag{7}$$

Introducing

$$t=\frac{\Lambda^2}{p^2},\quad z=\frac{p_c}{\Lambda},\quad v=\frac{\phi}{\Lambda} \tag{8}$$

and

$$\tilde{g}_c^{\,2}=(\ln z)^{-1},\quad \tilde{g}^{\,2}(t)=2\left[\ln\frac{1}{t}\right]^{-1} \tag{9}$$

we can use (5)–(9) in (3) in order to obtain V as a function of $v=\phi/\Lambda$. In this work we are only interested in determining the critical value for z (z_c). A necessary condition to have a transition is that there is a change of sign in the effective potential as we change the parameters of the theories. This fact can be seen already in the limit $v \ll 1$. Making a tedious but straightforward algebra[6-8] we obtain

$$16\pi^2\Lambda^{-4}V(v \ll 1)=v^2 n_f d(R)F(z,n_f,C_2(R)) \quad (10)$$

with

$$F(z,n_f,C_2(R))=2z^2+4I_1 - \frac{72C_2(R)}{4(11N-2n_f)}[(\tilde{g}_c^2+2I_2)z^2+4I_3] \quad (11)$$

and with

$$I_1=\frac{z^4}{2}(2\ln z)^{2-A}\Gamma(A-1,2\ln z)\,, \quad (12a)$$

$$I_2=z^2(2\ln z)^{1-A/2}\Gamma\left[\frac{A}{2}-1,2\ln z\right]\,, \quad (12b)$$

$$I_3=\frac{-z^4}{A}(2\ln z)^{2-A}\Gamma(A-1,2\ln z)+\frac{2z^2\ln z}{A}I_2 \quad (12c)$$

with $\Gamma(\alpha,x)$ an incomplete gamma function. The critical value for z (z_c) is such that $F(z_c,n_f,C_2(R))=0$. The expression (11) can be calculated numerically. We have done it using the different representation-dependent parameters. For example, for $n_f=6$, QCD can accommodate just two 6-plets and still retain asymptotic freedom.[1] For the last representation we have $C_2(6)=\frac{10}{3}$, $d(R)=6$, $T(R)=\frac{5}{2}$.

The critical value z_c for the 3-plets representation is $z_c=1.88$ and for the 6-plets is 21.51. In Table I we show the values of $C_2(6)\alpha_s(\mu_6)$ for those representations as well as the relation between the two mass scales at which chiral-symmetry breaking occurs for the $R=3$ and 6 given by the respective values of z_c. From the values shown in Table I we see that relation (2) is satisfied within 17% of error but contrary to what is expected by Marciano the strong-coupling regime for the fundamental representation implies the weak-coupling regime for exotic quarks. Note also that $\mu_6/\mu_3 \approx 10$; that means that the chiral-symmetry-breaking scale for the 6-plets is of the order of 1 GeV and that there is no hierarchy between these mass scales. We recall that the analysis made in Ref. 1 was based on the hypothesis that $\alpha_c(\mu_3)$ is small enough to start trusting perturbation theory. The effective potential for composite operators we have used with the ansatz (5) may be sensitive, in a nontrivial way, to nonperturbative effects.

TABLE I. The values of $\alpha_c C_2$ for $R=3$, with $n_f(3)=6$, and $R=6$, with $n_f(6)=2$, and the corresponding ratio μ_6/μ_3.

R	$C_2(R)$	α_c	$\alpha_c C_2(R)$	μ_6/μ_3
3	4/3	1.42	1.89	11.44
6	10/3	0.47	1.58	11.44

The values shown in Table I have only a qualitative status with respect to what is really realized. For example, with only a 6-plet we obtain $z_c \simeq 4.89$ [or $\alpha_c(6)\simeq 0.52$] and $C_2(6)\alpha_s(6)\simeq 1.72$; that is, relation (2) is satisfied within 9% of error.

We have also tried higher representations and found their critical couplings. Obviously to take them seriously we must look for a larger group in which to embed a QCD method that can accommodate these representations. For example, with one 8-plet we obtain $z_c=4.7434$ ($\alpha_c=0.58$) and relation (2) is satisfied within 7% of error.

Only for the case of three 6-plets or one 10-plet does a large mass hierarchy $\mu_{6,10}/\mu_3 \gg 1$ arise.

In this case we must also look for a group larger than SU(3). This could be the case in grand unified theories as was also suggested by Marciano.[1]

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