

Comment on “General covariance, Lorentz covariance, the Lorentz force, and Maxwell equations,” by H. W. Crater [Am. J. Phys. 62 (10), 923–931 (1994)]

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where $k' = N' + 2l'$ (where N' is the dimension of the Coulomb space, and l' the corresponding orbital angular momentum quantum number).

Under the transformations: $s = \rho^2$, and $\psi = \rho^{1/2}\phi$, this yields

$$\left[-\frac{\hbar^2}{2m} \left(\frac{d^2}{d\rho^2} - \frac{(3/4) + (k'-1)(k'-3)}{\rho^2} \right) + 4B\rho^2 \right] \phi(\rho) = \frac{4e^2}{4\pi\epsilon_0} \phi(\rho). \quad (8)$$

One notes directly that if $k' = 1$ or 3, this reduces to Eq. (3). Comparison of Eqs. (1) and (8) shows there is a mapping when

$$k = 2k' - 2 \quad k' \geq 2,$$

or

$$k = 6 - 2k' \quad k' \leq 2. \quad (9)$$

The corresponding expression for the eigenfunctions is, as before Eq. (5), while for the binding energies

$$B = \frac{mc^2\alpha^2}{2(n+k/4)^2}, \quad n=0,1,2,\dots \quad (10)$$

Thus, if $k' = 1$, i.e., one has a 1-D system or if $k' = 3$, i.e., one has a 3-D system with $l' = 0$, then $k = 4$ (the result obtained above) and has identical energies for these two cases. If $k' = 2$, i.e., one has a 2-D system with $l' = 0$, then $k = 2$. If $k' = 4$, i.e., one has a 4-D system with $l' = 0$, then $k = 6$. If

$k' = 5$, i.e., one has a 5-D system with $l' = 0$, then $k = 8$, etc. If one has a 3-D system with $l' = 1$, then $k' = 5$, one has $k = 8$ and $n + k/4 = 2, 3, \dots$

This comment has pointed out two errors in the paper of Bateman *et al.* [namely, in their expressions for the analogs of Eqs. (1) and (4)], despite which, they end up with the right results, and two errors in the recent Pradhan note [namely, his deriving the wrong expression for k , and also his claiming that k must be an integer in order for one to be able to map Eq. (3) into Eq. (1)]. This comment has also detailed a very straightforward mapping procedure for a general N' -dimensional Coulomb potential into an oscillator, which subsumes as special cases the one-, two-, and three-dimensional Coulomb potentials.

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¹D. S. Bateman, C. Boyd, and B. Dutta-Roy, "The mapping of the Coulomb problem into the oscillator," *Am. J. Phys.* **60**, 833-836 (1992).

²References in Ref. 1.

³C. Müller, *Spherical Harmonics, Lecture Notes in Mathematics*, Vol. 17 (Springer-Verlag, Berlin, 1966).

⁴A. Chatterjee, "Large- N expansions in quantum mechanics," *Phys. Rep.* **186**, 249-370 (1990).

⁵H. A. Mavromatis, *Exercises in Quantum Mechanics* (Kluwer Academic, Dordrecht, The Netherlands, 1992), p. 120.

⁶P. Pradhan, "Comment on 'The mapping of the Coulomb problem into the oscillator,' by David S. Bateman, Clinton Boyd, and Binayak Dutta-Roy [Am. J. Phys. **60**(9), 833-836 (1992)]," *Am. J. Phys.* **63**, 664 (1995).

Comment on "General covariance, Lorentz covariance, the Lorentz force, and Maxwell equations," by H. W. Crater [Am. J. Phys. **62** (10), 923-931 (1994)]

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Recently Crater¹ has raised the potentially important point that there would exist an ambiguity in defining electric and magnetic three-vector fields in noninertial and curved spacetimes [see Eqs. (13a)–(13b) and (15a) and (15b) in Ref. 1]. Here, we would like to call attention to the fact that there is not any *real* ambiguity. Given the Faraday tensor field F^{ab} as a solution of the Maxwell equations in some curved spacetime, an arbitrary observer with four-velocity v^a can associate without any ambiguity the following electric E^a and magnetic B^a four-vector fields²

$$E^a = F^{ab}v_b, \quad (1)$$

and

$$B^a = -\frac{1}{2} \epsilon^{abcd} F_{bc}v_d, \quad (2)$$

where $v^av_a = -1$, and ϵ_{abcd} is the totally skew-symmetric Levi-Civita pseudotensor. Furthermore, since Eqs. (1) and (2) are covariant expressions, E^a and B^a do *not* depend on whether the Faraday tensor is given in its covariant, mixed or contravariant version. Although F^{ab} conveys all the information about the electromagnetic field, Eqs. (1) and (2) allow us to cast, for instance, the Lorentz force

$$K^a = qF^{ab}u_b \quad (3)$$

on a charge q with four-velocity u^a in an appealing form. As an example, let us consider in an *arbitrary* spacetime the Lorentz force acting on a charge as measured by a comoving observer ($v^a = u^a$). In this case, we have from Eqs. (1) and (3) that

$$K^a = qF^{ab}v_b = qE^a. \quad (4)$$

Hence, according to this covariant approach, the Lorentz force ascribed by an observer comoving with a charge is *purely electric* and has the form (4) rather than (24a) or (24b) of Ref. 1. In general, where charge and observer have distinct worldlines the Lorentz force can be written simply as

$$K^a = K_{\perp}^a + K_{\parallel}^a \quad (5)$$

The v^a -orthogonal component is given by

$$K_{\perp}^a \equiv q[(-v^b u_b)E^a + \tilde{\epsilon}_{bc}^a u^b B^c] \quad (6)$$

where $\tilde{\epsilon}_{abc} \equiv \epsilon_{dabc}v^d$ is the totally skew-symmetric Levi-Civita pseudotensor induced on the hypersurface orthogonal to v^a , while the v^a -parallel component is given by³

$$K_{\parallel}^a \equiv q(E^b u_b)v^a. \quad (7)$$

One can also calculate immediately from Eqs. (5)–(7) the proper acceleration of the charge $a = \sqrt{K^a K_a}/m$, where m is its rest mass.

The ambiguity previously pointed out¹ is a consequence of trying to ascribe a direct physical meaning to the components of the Faraday tensor in an arbitrary reference frame *without specifying the observer who is going to perform the measurement* [see (13a), (13b), (15a), and (15b) of Ref. 1 which are reproduced below]:

$$E_1 = F^{01}, \quad E_2 = F^{02}, \quad E_3 = F^{03}, \quad (8)$$

$$B_1 = F^{23}, \quad B_2 = F^{31}, \quad B_3 = F^{12}, \quad (9)$$

$$E_1 = -F_{01}, \quad E_2 = -F_{02}, \quad E_3 = -F_{03}, \quad (10)$$

$$B_1 = F_{23}, \quad B_2 = F_{31}, \quad B_3 = F_{12}. \quad (11)$$

According to the covariant procedure presented above, Eqs. (8) and (9), and (10) and (11) are not a matter of definition, but should be tested either true or false for each family of observers through Eqs. (1) and (2). In the Minkowski space,

for example, we tacitly assume that it is the inertial observers which are the natural ones. In some Cartesian coordinate system, where the metric is $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$, the four-velocity of one such a family of observers is $v^a = (1, 0, 0, 0)$. By using it in Eqs. (1) and (2), we obtain exactly (8) and (9), and (10) and (11). However, it is not so for a general observer in an arbitrary spacetime.

Summarizing, in order to generalize the electromagnetism in curved spaces one must use a *covariant* procedure, otherwise one may be driven into ambiguities as shown in Ref. 1. The Faraday tensor F^{ab} conveys all the information about the electromagnetic field. Finally, an arbitrary observer with four-velocity v^a can ascribe without any ambiguity, through F^{ab} , electric and magnetic fields as given by Eqs. (1) and (2). Notwithstanding, it is worthwhile to comment that there is a still remaining question left to be decided by experiment that concerns the determination of the exact form of the Maxwell equations in curved spacetimes. On theoretical grounds we must require, for instance, that any generalization of the Maxwell equations must lead to the usual electromagnetic field equations in the flat spacetime limit. The most popular prescription to obtain such a generalization is the so-called *minimal coupling procedure* ($\eta_{ab} \rightarrow g_{ab}$, and $\partial_a \rightarrow \nabla_a$). However, this is far from being unique.⁴ *After all, the laws of nature are more than pure math.*

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¹H. W. Crater, Am. J. Phys. **62**, 923 (1994).

²R. M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984), p. 64.

³Frame-oriented people would interpret in a Local Lorentz Frame K_{\perp}^a as being the usual three-dimensional Lorentz force lying on the three-dimensional hypersurface *orthogonal* to v^a , and K_{\parallel}^a as being related with the work done by the field on the charge.

⁴R. M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984), p. 71.

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During the following weeks Dirac tried to connect Heisenberg's quantum-theoretical re-interpretation of kinematical quantities with the action-angle variables of the Hamilton-Jacobi theory. 'I worked on it intensively from September 1925', Dirac said. 'During a long walk on a Sunday it occurred to me that the commutator might be the analogue of the Poisson bracket, but I did not know very well then what a Poisson bracket was. I had just read a bit about it, and forgotten most of what I had read. I wanted to check up on this idea, but I could not do so because I did not have any book at home which gave Poisson brackets, and all the libraries were closed. So I had to wait impatiently until Monday morning when the libraries were open and check on what Poisson bracket really was. Then I found that they would fit, but I had one impatient night of waiting.'

Jagdish Mehra, "Dirac's Contribution to the Early Development of Quantum Mechanics," in *Tributes to Paul Dirac*, edited by J. G. Taylor (Adam Hilger, Bristol, 1987), p. 65.