Microscopic approach for the *n-d* effective interaction

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(Received 20 July 1989)

A three-boson model is applied to the nucleon-deuteron (*n-d*) system to construct an effective energy-dependent two-body potential in configuration space. The three-nucleon observables at low energy are well reproduced with just one free parameter (related with the range of the nucleon-nucleon interaction). We show that the present results give support to a previous phenomenological *n-d* potential.

I. INTRODUCTION

The present status of theoretical study of three-nucleon systems requires one to solve the Faddeev equation with inclusion of three-nucleon forces added to a realistic two-nucleon interaction in order to reproduce the experimental three-nucleon observables. The extension of this procedure for more nucleons is a very difficult task because the number of degrees of freedom increases and the exact approach becomes more and more involved. For more complex systems, using a short-range nucleon-nucleon interaction, one can construct a microscopic nucleon-nucleus potential, which has a small diffusivity. A phenomenological nucleon-nucleus optical potential is usually constructed by fitting the parameters of a complex Woods-Saxon potential to the experimental elastic scattering data. But this simple method does not work for the spin doublet neutron-deuteron (*n-d*) system. In fact a short-ranged *n-d* optical potential fitted to reproduce correctly the triton binding energy (E_t) and the spin doublet scattering length (a_{nd}) fails dramatically on several accounts.[1] First, it fails to reproduce the low-energy scattering phase shifts and even produces an effective range of wrong sign. Second, it fails to produce the excited virtual state of triton. Finally, the pole in energy of the effective range function, which is expected to appear below the scattering threshold, appears in the wrong place. However, all these difficulties can be solved satisfactorily by a Faddeev calculation using three-body dynamics. The peculiar properties of the *n-d* system, such as, the behavior of the effective range function and the correlations among various low-energy observables led people to think that they were manifestations of the detailed three-body dynamics. Any contribution which shows that this long time belief may not be correct is important for the development of a microscopic *n-d* optical potential.

Recently, it has been shown[2] that the low-energy *n-d* system can be treated without using three-body dynamics. The important role of such a model was the introduction of a phenomenological *n-d* optical potential not limited to the usual class of potentials such as experimental, gaussian, Yukawa, and Woods'-Saxon types. The very small binding energy of deuteron on the nuclear scale and correspondingly its large size imposes that the *n-d* optical potential should not decay as rapidly as the usual potentials. Physically, the one-nucleon exchange part of the *n-d* optical potential should have a long-range tail: Two nucleons in the trinucleon system experience an effective interaction well outside the range of nucleon-nucleon interaction by exchanging the third nucleon.

It has been conjectured that in the two-body model of Ref. 2, the long-range part should have the form of $\exp(-\Lambda r)/r^2$, where Λ is given by the deuteron binding energy, as also suggested in Refs. 1 and 3. By "long range" we mean that the behavior of the interaction is exponentially much weaker than the normal nucleon-nucleon range of the interaction. The short-range part which depends on some average properties of the nucleon-nucleon interaction was chosen as constant. Holding the deuteron energy fixed and modifying these average properties correspond to changing the interior part of the neutron-deuteron optical potential. In calculations involving three-body dynamics these correspond to modifying the off-shell properties of the nucleon-nucleon interaction, introduction of a tensor force model, or a three-body force. The correlation between a_{nd} and E_t (Phillips' curve[4]) was very well reproduced by this model which has only two body dynamics.

After the use of the above model we posed the following question to ourselves: Would it be possible to derive from a three-body model (off-the-energy-shell effects included) any equivalent two-body effective interaction? To answer this question we followed a suggestive comment by Noyes,[5] and based on the zero range theory[6] we derived the tail of such an interaction,[7] with some approximations using as the effective n-d wave function the overlap of a deuteron-triton wave function. In the asymptotic limit this overlap function must be equal to the spectator function[8] which enters into the separable model potential equations for the triton. So, to get the effective long-range behavior of the n-d potential we conjectured that both approaches should be equivalent. In fact, the form of the n-d potentials presented in Refs. 2 and 7 are different but have the same qualitative features in their long-range behavior. But as the short range of the model of Ref. 7 is not yet parametrized with the essential information about the nucleon-nucleon force range it is not in a suitable form to produce the three-nucleon observables. Also the final drastic analytical approximation of the exchanged term for the case $E_d \neq 0$, in Ref. 7, does not seem to be enough to give us the correct effective radial dependence of the potential. These were the facts that led us to look for the present approach directly based on the spectator function.

The main purpose of this work is to give a clear justification of the phenomenological approach of Ref. 2, through the use of the spectator function[8] to obtain the effective two-body wave function. We modify the zero-range equations for a three-boson system introducing a cutoff[9] chosen as $(\beta^2+q^2)^{-1}$, where q is the n-d relative momentum. It is interesting to point out that the value of β that fit the three-nucleon observables is of the order of the nucleon-nucleon force range. In this aspect we interpret β as the short-range information that was missing in the zero-range model.

We intend to compare the present model with that employed in Ref. 2, but this approach gives an energy-dependent potential.[5,7] So we have to clarify how a Phillips' curve is generated in this case. For a given binding energy of the triton, we calculate numerically the corresponding n-d potential by choosing the β parameter in order to fit the experimental value of a_{nd}. This process is further applied for distinct values of the triton binding energy keeping the β value fixed and obtaining distinct values of a_{nd}.

Our results show a good agreement with the Phillips' curve calculated previously by other models, enforcing the argument that the correlation between E_t and a_{nd} is not dependent on three-body dynamics. Another important aspect of such effective n-d interaction is its application for the construction of microscopic deuteron-nucleus optical potentials as already discussed in Ref. 2.

We present the model in Sec. II; the numerical results in Sec. III and our conclusions in Sec. IV.

II. THE MICROSCOPIC MODEL FOR THE n-d POTENTIAL

As in a previous work[7] our derivation starts from the zero-range model (ZRM). Here we briefly describe the ZRM which exploits the limiting case of zero-range forces between the nucleons. In this model the nucleons have free propagation except when they overlap. As a consequence of this hypothesis, if two nucleons interact, the third nucleon (spectator) receives from the remaining pair of nucleons only the asymptotic information containing in the on-the-energy shell two-nucleon scattering amplitude. The ZRM is the simplest approach for handling the three-nucleon system involving its dynamics. In terms of the three-nucleon Faddeev equations, the ZRM means replacing the complete off-the-energy shell two-nucleon t matrix by the on-the-energy-shell t matrix.

In the ZRM the equation for the spectator function for a three-boson system in the bound state reads[6]

$$\chi(\mathbf{q})=\frac{1}{2\pi^2}\frac{(E_d)^{1/2}+(E_t+\frac{3}{4}q^2)^{1/2}}{\mu^2+q^2} \times \int \frac{d^3p}{E_t+q^2+p^2+\mathbf{q}\cdot\mathbf{p}}\chi(\mathbf{p}) , \tag{1}$$

where E_d is the deuteron binding energy, $\mu^2=\frac{4}{3}(E_t-E_d)$. Our units are such that $\hbar$ and the nucleon mass are equal one. The three-boson system in the ZRM has a bound state collapse (Thomas Effect)[10] unless a cutoff in the momentum integration is introduced.[9] As the third particle approaches the two-particle subsystem there is gradually an increase in the interaction with each particle individually, and in Eq. (1) the free propagation of the system must be modified for typical distances less than the two-particle interaction range. The cutoff has this meaning. Also it plays the same role as the parameter that has dimension of inverse range in the case of the simple Yamaguchi form factor, in the three-body system. This means that if the range goes to zero, the three-nucleon system collapses.[10] In dealing with such an equation we choose a Yamaguchi form factor to take care of the short-range region in the three-nucleon coordinate space. In our model we simulate the triton as a three-boson system to simplify the calculation but retain its essential physics. The minimal physical ingredient of $\chi(\mathbf{q})$ is a simple pole at $q=i\mu$. The integration in Eq. (1) is performed by replacing $\chi(\mathbf{p})$ by $(\mu^2+p^2)^{-1}$ on the right-hand side, and inserting the cut as $(\beta^2+p^2)^{-1}$:

$$\chi(\mathbf{q})=\frac{(E_d)^{1/2}+(E_t+\frac{3}{4}q^2)^{1/2}}{q(\mu^2+q^2)(\beta^2+q^2)}\arctan\left\{\frac{2q(\beta-\mu)}{q^2+4[\beta+(E_t+\frac{3}{4}q^2)^{1/2}][\mu+(E_t+\frac{3}{4}q^2)^{1/2}]}\right\} . \tag{2}$$

The above approximation for the spectator function incorporates two important features of the three-boson system, namely the Efimov and Thomas effects[10]. The first one is connected with the change of the asymptotic form of the wave function which (for finite-range potential) is given by the pole at $q=i\mu$. One can see the Efimov limit as the limit when E_d goes to zero. In this case, the Eq. (2) no longer has the simple pole related to the energy characteristic of the short range of the interaction, and a longer range starts to appear which can easily be seen by taking its Fourier transform. In the Efimov limit, we have just a square-root cut starting at $q=i\mu$ modifying the asymptotic form of the wave function. This gives rise to a long-range potential. The Thomas effect is related to the limit of β going to infinity. In this case, for q going to infinity, the spectator function exhibits q^{-2} behavior, characteristic of a Dirac delta potential type, which for three dimensions causes the binding to collapse.

The wave function in configuration space is given by the Fourier transform of Eq. (2)

$$\Psi(\mathbf{r})=\int \exp(i\mathbf{q}\cdot\mathbf{r})\chi(\mathbf{q})d^3q \ , \tag{3}$$

where $r=|\mathbf{r}|$ is the n-d relative distance. The effective n-d interaction is therefore obtained by inverting a two-body Schrödinger equation for Ψ,

$$V(r)=\frac{1}{\Phi(r)}\frac{d^2\Phi(r)}{dr^2}-\mu^2 \ , \tag{4}$$

where $\Phi(r)=r\Psi(\mathbf{r})$.

The present model has only one free parameter β, that can be associated with the range of the two nucleon interaction, as we observe in the next section.

III. NUMERICAL RESULTS

The value of the only parameter (β) we have in the model is fixed by the experimental value of the nucleon-deuteron scattering length ($a_{nd}=0.65$ fm), which is numerically calculated using the potential, Eq. (4), with the experimental values of E_d(2.226 MeV) and E_t(8.48 MeV). The fitted value of β is 1.19 fm^{-1}, and the corresponding value of β^{-1} is consistent with the range of the nucleon-nucleon interaction. We have also calculated the asymptotic normalization parameter for the triton, and we obtain 1.9, in agreement with the accepted value[11] of 1.82. The numerical integration of Eq. (3) needs some care for r equal to zero. In this limit we introduce in Eq. (3) a high-momentum cutoff of Gaussian type $\exp(-\alpha q^2)$, with $\alpha=0.001\ \text{fm}^2$. This was necessary due to our numerical integration method, but we have checked that the results we get for Eq. (3) are cutoff independently by performing the same calculation using $\alpha=0.01\ \text{fm}^2$ with the same results.

The numerical n-d potential we have obtained for $E_t=8.48$ MeV is shown in Fig. 1(a) in comparison with that proposed in Ref. 2. The curves display a different behavior up to $r=2$ fm. As we pointed out before, the region ($r<2$ fm) where the information about two nucleons is present is not important for the three-nucleon observables once the triton binding is given. We have also displayed in Fig. 1(a) our early analytical results obtained

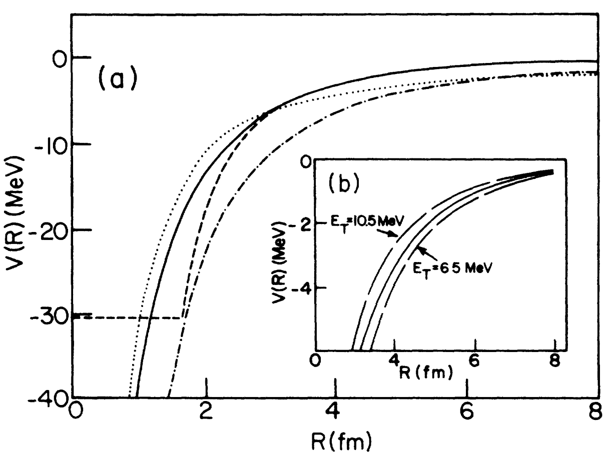

FIG. 1. The nucleon-deuteron potential. (a) Solid line—the present result for $E_t=8.48$ MeV; dashed line—the phenomenological potential of Ref. 2. Also, the dotted line reflects the potential of Ref. 7 for $E_d=0$, $V(r)=-(3\mu)/(4r)-3/(8r^2)$. The dashed-dotted line is the direct term of the potential given by Eq. (7) of Ref. 7 [in this question a multiplicative factor equal to $(\frac{3}{4})^{1.75}$ is missing]. (b) Medium-range behavior of the present result for $E_t=10.5$, 8.5, and 6.5 MeV as indicated.

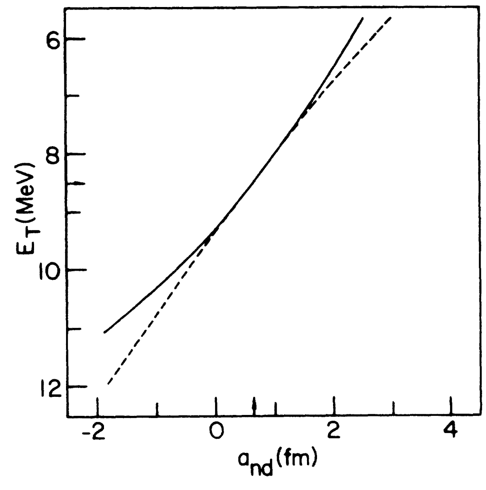

FIG. 2. The Phillips' plot for E_t against a_{nd}: In the present result (solid line) and in the calculation of Ref. 2 (dashed line).

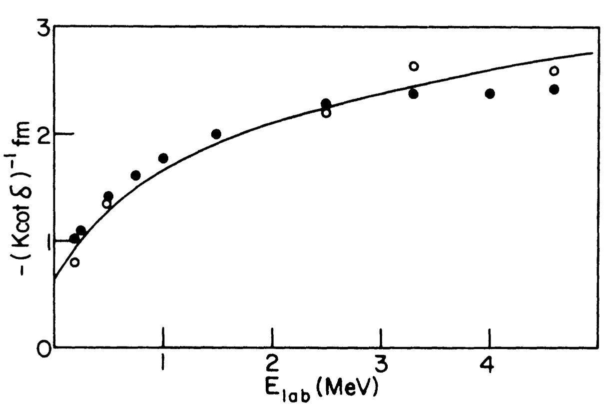

FIG. 3. $(k\cot\delta)^{-1}$ as a function of the neutron lab energy; in the present result for $E_t=8.48$ MeV and $a_{nd}=0.65$ fm (solid line), Ref. 12 (solid circles), and the experimental results from Ref. 14 (empty circles).

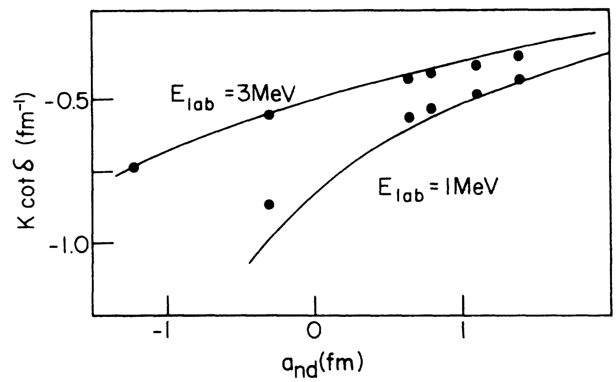

FIG. 4. The correlation between ($k\cot\delta$) and the *n-d* scattering length: Our results (solid line) and in the calculation of Ref. 12 for two energies as shown in the figure (solid circles).

in Ref. 7. As can be seen in this figure, just the direct term obtained in Ref. 7 yields good qualitative behavior for the potential. In the limit $E_d \rightarrow 0$ we have deduced only the direct part of the contribution for the effective potential. The simple analytical expression for $E_d \neq 0$ in Ref. 7 gives us the longer-range contribution in terms of the nucleon-deuteron distance. But, as such a term is strongly energy dependent it may not be enough to give us the correct effective *r* dependence. (In this respect we observe also that for $E_d = 0$ the energy dependent term of the approximation obtained in Ref. 7 alone does not yield the correct behavior of the effective potential.) Other terms may be also important. Thus, if we include the exchanged term of Ref. 7 for $E_d = 2.225$ MeV the result becomes poor. This fact, however, does not appear in the present context, using the spectator function. The present potential for medium range ($2\ \text{fm} < r < 6\ \text{fm}$) is very sensitive to the values of the triton binding, as is shown in Fig. 1(b) for $E_t = 10.5$, 8.48, and 6.5 MeV.

In Fig. 2, we show the Phillips plot for our potential. The small deviation from a straight line is due to the variation of the form of the potential in the medium-range radius with the triton energy. The calculation of Ref. 2 changes the form of the potential at short range. The results for $k\cot\delta$ for the doublet state are shown in Fig. 3 and are again in a good agreement with a separable model calculation[12] which yields $a_{nd} = 0.65$ fm. We notice a deviation near the deuteron breakup threshold because at this point an irregularity appears[1] and our model does not take account of the breakup channel. The $k\cot\delta$ dependence of the scattering length for laboratory energies of 1 and 3 MeV, presented in Fig. 4, reproduces previous three-nucleon separable model calculation.[12,13] We can also observe the consistency with the experimental data[14] below the threshold.

IV. CONCLUSIONS

In this work we have used for the three-boson system a model based on the zero-range theory[6] as a starting point to get an effective *n-d* potential. In this framework we need to introduce one free parameter, related to the range of the nucleon-nucleon force, which is obtained by the known values of the triton and deuteron bindings energies and doublet *n-d* scattering length.

Our numerical calculation gives a justification of an early phenomenological potential[2] used for the low-energy three-nucleon system. We note a difference coming from the dependence of the potential on the triton binding in the region of 2–6 fm. We have obtained a good description of the low-energy correlations between various observables. This emphasizes the origin of these correlations as coming from the existence of a universal long-range interaction for the tri-nucleon system. Some improvements are still needed in order to go further in the energy necessary to complete this description above the breakup, where, for example, a curious irregularity appears.[1] Work is in progress along these lines.

ACKNOWLEDGMENTS

Our thanks to Dr. Jishnu Dey for reading this manuscript and suggestions. This work was supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico of Brasil.

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