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Pressure field in a tube with a general and arbitrary time- and position-dependent gas source

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In this article we present analytical and numerical results for a pressure profile along the axis of a tube with a general and arbitrary time- and position-dependent gas source. The model is able to determine the pressure values along the tube, once the pumping speed at each extremity and the gas sources are specified. The time evolution of the pressure along a tube is presented for situations commonly found in high-vacuum applications, such as particle accelerators, colliders, storage rings, and synchrotron light sources. © 2004 American Vacuum Society. [DOI: 10.1116/1.1778408]

I. INTRODUCTION

Several areas of applied physics deal with problems in high-vacuum technology that present tubular geometry, such as particle accelerators, colliders, and storage rings, and electron devices, such as klystrons and photomultipliers, as well as electron microscopes and mass spectrometers. In terms of vacuum modeling, one usually assumes a straight section with pumping at the extremes, and constant degassing rates. This kind of model is adequate for steady-state situations and where the materials used do not present very different types of general gas sources (e.g., constant degassing rates). Realistic situations in vacuum tubes require the use of parts with different degassing rates (e.g., insulating ceramics or bellows). The same phenomenon happens when beam particles or beam radiation strike the walls. In addition, the degassing rate is time dependent, and in this case, realistic modeling must take this into account.

In this article we present analytical results for a pressure profile in a tube with a general and arbitrary time- and position-dependent gas source. We also present results for two typical situations found in vacuum applications. The first situation deals with localized impulsive degassing, which may represent a situation where there is sudden intense radiation-induced degassing in a small area in the wall of the tube. The second situation deals with a tube with extensive axially dependent degassing, impulsive in time. We present results for the pressure profile and its time evolution for both cases.

The results are very powerful because they present means by which to deal analytically with generic degassing problems, for any extensive and time-dependent degassing rate can be represented by a linear combination of localized impulsive degassing rates.

II. ANALYTICAL SOLUTIONS

We treat the case of a tube with length L and diameter D with a constant degassing rate, plus an arbitrary gas source, which depends both on the time and position, as illustrated in Fig. 1.

The differential equation for the pressure in the molecular gas flow regime as a function of the position and time, $p(x,t)$, can be written as

$$c \frac{\partial^2 p(x,t)}{\partial x^2} = -q(x,t) + v \frac{\partial p(x,t)}{\partial t}, \quad (1)$$

where c is the specific conductance of the tube ($c=CL$, C is the total tube conductance); $q(x,t)$ includes both the steady-state degassing from the walls, q_s , and the time- and position-dependent source, $q_T(x,t)$; and v is the volume per unit length of the tube.¹ The gas source can be represented by the following function:

$$q(x,t) = q_s + q_T(x,t). \quad (2)$$

Since the differential equation is linear, the general solution will be the sum of the particular solutions for the impulsive sources and can be written as

$$p_G(x,t) = p_s(x) + p_T(x,t). \quad (3)$$

The steady-state part of the degassing will be responsible for the usual parabolic pressure profile, and the transient part can represent any gas source or process that can induce desorption from the walls. The following boundary conditions were assumed:²⁻⁴

- (1) For the steady-state solution, the pressure at the end of the tube, $x=L/2$, is the total throughput, $q_s L$, divided by the total pumping speed, $2S$.
- (2) All the gas reaching the pumps is pumped, both for the transient and the steady-state solutions, so that at $x=L/2$,

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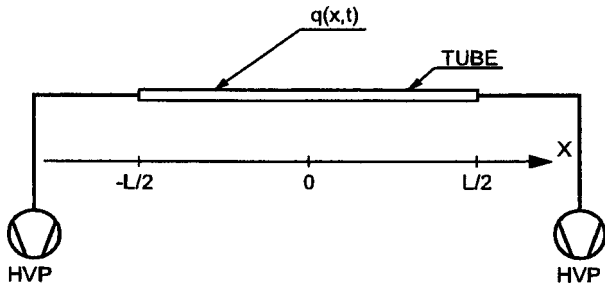


FIG. 1. Schematic drawing of the tubular geometry studied. HVP—A generic high-vacuum pump.

$$-c \left. \frac{\partial p_T(x,t)}{\partial x} \right|_{x=L/2} = S p_T(L/2, t), \quad \forall t \geq 0.$$

- (3) The geometry of the problem is symmetric, so we can treat it in the interval $0 \leq x \leq L/2$ and at $x=0$,

$$c \left. \frac{\partial p_T(x,t)}{\partial x} \right|_{x=0} = 0, \quad \forall t \geq 0.$$

- (4) The transient gas source occurs at $t=0$, so $p_T(x,0)=0$, $0 \leq x \leq L/2$.

The explicit form of the general solution is shown below:

$$p_G(x,t) = -\frac{q_S}{2c}x^2 + \frac{q_S L}{2} \left(\frac{1}{S} + \frac{L}{4c} \right) + \sum_{m=1}^{\infty} \exp(-\alpha \beta_m^2 t) K(\beta_m, x) \times \int_0^t \exp(-\alpha \beta_m^2 t') \frac{\alpha}{c} \bar{q}(\beta_m, t') dt'. \quad (4)$$

The first two terms correspond to solution of the steady-state case; the last term corresponds to solution of the transient case, where

$$K(\beta_m, x) = \sqrt{2} \left[\frac{\beta_m^2 + H_2^2}{L/2(\beta_m^2 + H_2^2) + H_2} \right]^{1/2} \cos(\beta_m x)$$

and

$$\bar{q}(\beta_m, t') = \int_0^{L/2} K(\beta_m, x') q(x', t') dx',$$

where β_m are solutions of the transcendental equation,

$$\beta_m \tan\left(\beta_m \frac{L}{2}\right) = \frac{S}{c} = H_2,$$

and $q(x,t)$ is the function that represents the transient gas source for $0 \leq x \leq L/2$ and $t \geq 0$.

The throughput at each point of the tube is given by the following expression:

$$Q(x,t) = -c \frac{\partial p(x,t)}{\partial x}. \quad (5)$$

We assume that any amount of gas that reaches a vacuum pump is pumped, so that the integral of the throughput in a given time interval is given by

$$Q_T(L/2, t) = \int_0^t -c \left. \frac{\partial p(x, t')}{\partial x} \right|_{x=L/2} dt', \quad (6)$$

where Q_T is given in mbar l. This expression can be used to evaluate the lifetime of ionic, nonevaporable getter (NEG), or cryogenic pumps commonly used in accelerators.

A. Impulsive gas source, localized in position and time

We consider a straight tube section with an axially dependent degassing rate, impulsive in time, superimposed on the uniform degassing background of the tube walls. In this case, we can represent the total degassing rate by the following function:

$$q(x,t) = q_S + q' \delta(x-x') \delta(t-t'), \quad (7)$$

where q' represents the amount of gas liberated at $x=x'$ in $t=t'$. Considering this degassing rate as occurring at $x'=0$ and $t'=0$, Eq. (4) yields the following solution:

$$p_G(x,t) = -\frac{q_S}{2c}x^2 + \frac{q_S L}{2} \left(\frac{1}{S} + \frac{L}{4c} \right) + \frac{q'}{2v} \sum_{m=1}^{\infty} B_m^2 \cos(\beta_m x) \exp(-\alpha \beta_m^2 t), \quad (8)$$

with $\alpha = c/v$ and

$$B_m^2 = 2 \frac{\beta_m^2 + H_2^2}{[L/2(\beta_m^2 + H_2^2) + H_2]}.$$

For the case described by Eq. (8), the total amount of gas reaching the pump is given by the solution of Eq. (6):

$$Q_T\left(\frac{L}{2}, t\right) = q_S \frac{L}{2} t + \frac{q'}{2} \sum_{m=1}^{\infty} \frac{B_m^2}{\beta_m} \sin\left(\beta_m \frac{L}{2}\right) \times [1 - \exp(-\alpha \beta_m^2 t)]. \quad (9)$$

The part of Eq. (9) that represents the amount of gas coming from the transient source is

$$\lim_{t \rightarrow \infty} \left\{ \frac{q'}{2} \sum_{m=1}^{\infty} \frac{B_m^2}{\beta_m} \sin\left(\beta_m \frac{L}{2}\right) [1 - \exp(-\alpha \beta_m^2 t)] \right\} = \frac{q'}{2},$$

showing that one half of the amount of gas produced by the source reaches the pump at $x=L/2$.

B. Extensive gas source, position dependent and impulsive in time

In this case, we consider a straight tube section with an extensive gas source in space and impulsive in time super-

imposed on the uniform background degassing of the tube walls. The throughput per unit length is given by

$$q(x, t) = q_S + q'(x) \delta(t - t'). \quad (10)$$

In some situations in high-vacuum applications, for instance, in particle accelerators or storage rings, beam particles or beam radiation may hit the walls and produce a burst of gas. We can represent this situation with a transient gas source, extensive in space and impulsive in time, as shown below:

$$q'(x) = \begin{cases} 0, & \text{for } -L/2 \leq x < -a, \\ q', & \text{for } -a \leq x \leq a, \\ 0, & \text{for } a < x \leq +L/2. \end{cases} \quad (11)$$

The general solution can be written as

$$p_G(x, t) = -\frac{q_S}{2c} x^2 + \frac{q_S L}{2} \left(\frac{1}{S} + \frac{L}{4c} \right) + q'/v \sum_{m=1}^{\infty} \frac{B_m^2}{\beta_m} \sin(\beta_m a) \cos(\beta_m x) \exp(-\alpha \beta_m^2 t). \quad (12)$$

Like in the previous case, the total amount of gas reaching the pump at $x=L/2$ can be calculated as

$$Q_T(L/2, t) = q_S L/2t + q' \sum_{m=1}^{\infty} \frac{B_m^2}{\beta_m^2} \sin(\beta_m a) \sin(\beta_m L/2) \times [1 - \exp(-\alpha \beta_m^2 t)]. \quad (13)$$

The part of Eq. (13) that represents the amount of gas coming from the transient source is

$$\lim_{t \rightarrow \infty} \left\{ q' \sum_{m=1}^{\infty} \frac{B_m^2}{\beta_m^2} \sin\left(\beta_m \frac{L}{2}\right) \sin(\beta_m a) [1 - \exp(-\alpha \beta_m^2 t)] \right\} = q' a,$$

showing that one half of the amount of gas produced by the source reaches the pump at $x=L/2$.

III. RESULTS AND DISCUSSION

We present two case studies, using realistic parameters,^{5,6} to exemplify the results obtained in Sec. II. These examples illustrate situations commonly found in high-vacuum technology, mainly in electron tube and particle accelerator devices. Other possible applications in high-vacuum equipment may be treated as well.

The reader should be aware that the model assumes that all the gas arriving at the extremities is pumped, which is not a realistic hypothesis and depends on the amount of gas $q(x, t)$ and the specific conductance of the tube. The first parameter is important because it will define what happens with the pressure at the entrance of the pump. If the throughput is larger than the pressure times the pumping speed, the pressure will rise. The dynamics of this process will be defined by how the pumping speed depends on the pressure,

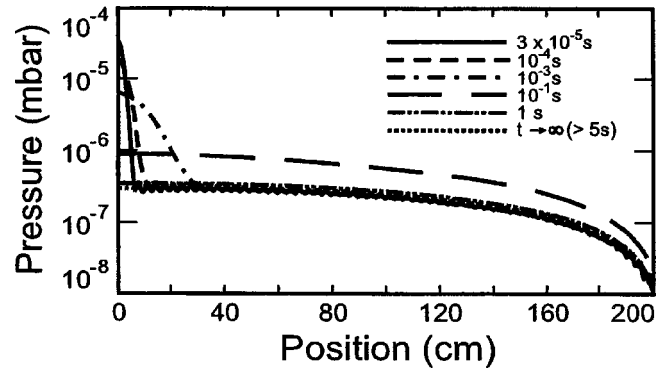


FIG. 2. Pressure field along the tube for $x \geq 0$ at different times.

which is a characteristic of the pump. So, each case should be treated according to specific conditions of the pumping system.

A. Impulsive gas source, localized in position and time

In this case, we deal with a tube that has a constant degassing rate along the whole length, plus localized impulsive degassing occurring at $x=0$ cm and $t=0$ s. The parameters are the following: $L=400$ cm; $D=3$ cm; $q_S=4.7 \times 10^{-9}$ mbar $\text{l s}^{-1} \text{ cm}^{-1}$; $q'=1.0 \times 10^{-6}$ mbar l ; $c=324$ $\text{l s}^{-1} \text{ cm}$; $v=7.1 \times 10^{-3}$ l cm^{-1} ; $q_S L=1.9 \times 10^{-5}$ mbar l s^{-1} ; $\alpha=4.56 \times 10^4$ $\text{cm}^2 \text{ s}^{-1}$; and $S=100$ l s^{-1} .

Taking those values into Eq. (8), we obtain the results shown in Fig. 2 which shows the pressure only in one half of the tube, because the geometry is symmetrical.

One can see from Fig. 2 that for times just above $t=0$ s, the pressure profile is represented by the steady-state parabola plus the rise in pressure due to localized burst of gas (solid line, $t=3 \times 10^{-5}$ s). A very short time later, one can see the rise in pressure becoming smoother, because the gas is already diffusing along the tube (dashed line, $t=10^{-4}$ s). After this, we can see the pressure decreasing around $x=0$ cm, because the gas is moving toward the pump (dot-dashed line, $t=10^{-3}$ s). At later times, we can see the gas reaching the pump, with a subsequent rise in pressure (long dashed line, $t=10^{-1}$ s), and the gas being pumped, with the pressure going down toward the parabolic background profile (dot-dot dashed line, $t=1$ s), then reaching the background profile after $t=3$ s. The oscillations observed on the $t=3 \times 10^{-5}$ s curve (solid line) are due to summation of Eq. (8), which was limited to 120 terms. To represent the steep peak in pressure from the burst of gas at times close to $t=0$ s, it would be necessary to use a larger number of terms. One should notice that for $t=10^{-4}$ s (and the same 120 terms), this artifact does not occur.

Figure 3 shows the time evolution of the pressure at five different positions along the tube ($x=0, 50, 100, 150$, and 200 cm). The plot shows the pressure only in one half of the tube, since the geometry is symmetrical.

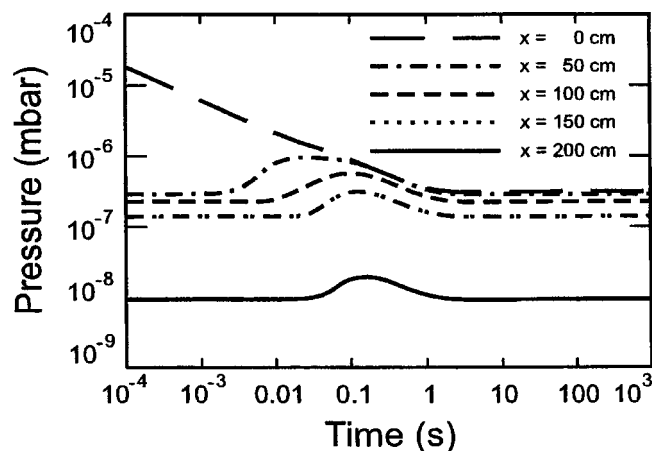


FIG. 3. Time evolution of the pressure in the tube observed at different positions.

We can notice two different kinds of behavior in the time evolution of the pressure along the tube. For the center ($x = 0$ cm), the pressure rises suddenly and decays to the background value in about 1 s. In the other parts of the tube, the pressure presents more general behavior: it stays at the background value while the gas diffuses to that position, rises due to the arrival of the gas, and decays back to the steady-state value. After about $t = 1$ s, the pressure tends toward the values observed before the impulsive degassing at $t = 0$ s, and presents a parabolic profile along the tube. We have not considered absorption of the gas by the tube walls, but this effect can be included in the model by changing the source term in Eq. (2). A negative term in the gas source equation can be physically interpreted as adsorption of gases and vapors by the walls (pumping effect of the walls).

B. Extensive gas source, position dependent and impulsive in time

Results for a tube with extensive axially dependent degassing, impulsive in time, are presented below. The parameters, typical of particle accelerators, are the following:⁵ $L = 400$ cm; $D = 3$ cm; $a = 30$ cm; $q_s = 4.7 \times 10^{-9}$ mbar l s⁻¹ cm⁻¹; $q' = 1.0 \times 10^{-6}$ mbar l cm; $v = 7.1 \times 10^{-3}$ l cm⁻¹; $c = 324$ l s⁻¹ cm; $q_s L = 1.9 \times 10^{-5}$ mbar l s⁻¹; $\alpha = 4.56 \times 10^4$ cm² s⁻¹; and $S = 100$ l s⁻¹.

Substituting these values in Eq. (12), we obtain the pressure field along the tube, shown in Fig. 4. The plot shows only the pressure in one half of the tube, since the geometry is symmetrical.

One can see from Fig. 4 that, for times just above $t = 0$ s, the pressure profile is represented by the steady-state parabola plus the rise in pressure due to the extensive burst of gas (solid line, $t = 3 \times 10^{-5}$ s). A short time later, one can see the rise in pressure becoming smoother, because the gas is diffusing along the tube (long dashed line, $t = 10^{-4}$ s and dotted line, $t = 10^{-3}$ s). After this, we can see the pressure decreasing around $x = 0$ cm, because the gas is moving toward the pump, and subsequent rise in pressure at $x = 200$ cm (dot-dot-dashed line, $t = 10^{-1}$ s). At later times, we can see the gas

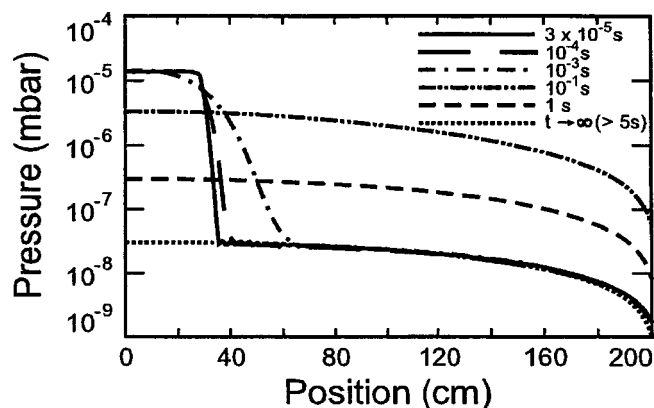


FIG. 4. Pressure field along the tube for $x \geq 0$ at different times.

being pumped, with the pressure going down toward the parabolic background profile (dashed line, $t = 1$ s and dotted line $t = 5$ s). The same oscillatory behavior noticed in Fig. 2 appears here, for the same reasons. Nevertheless, one should notice that the artifact is sensitive to both the maximum pressure and the extent of the burst. So, for a given amount of gas liberated by the transient gas source, if it is extensive, the number of terms needed to smooth the curve is larger.

Figure 5 shows the time evolution of the pressure at five different positions along the tube ($x = 0, 50, 100, 150$, and 200 cm).

The overall behavior in this situation is analogous to the localized (in space) burst that was shown in Fig. 3, but is more intense due to the larger amount of gas liberated. There is also a small numerical artifact that appears as higher background pressure at times less than 0.1 s for positions close to the extreme of the tube ($x > 180$ cm). This can also be seen in Fig. 4, where the background parabolas for times less than 0.1 s show a slight rise compared to the background pressure profile ($t \rightarrow \infty$) for positions above $x = 180$ cm.

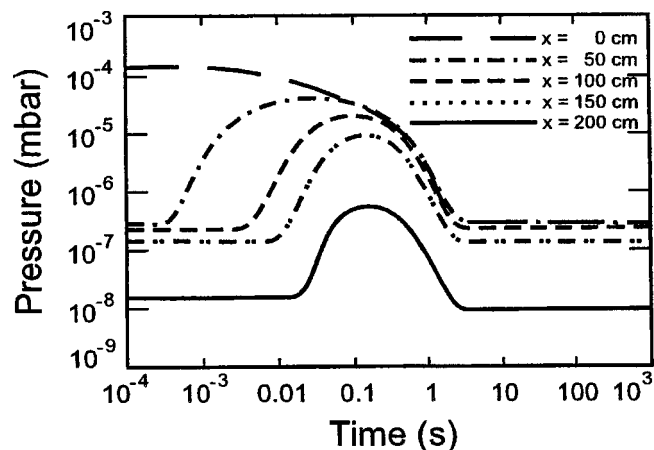


FIG. 5. Time evolution of the pressure in the tube observed at different positions.

IV. CONCLUSIONS

This model can be used to represent transient-state situations where a localized impulsive burst of gas occurs within the system, as in the case, for instance, of a particle beam or radiation beam striking the wall of the tube.³ It should be noted that this model presents a wide range of applications, because it can be generalized to conform to any degassing profile the burst may present. Any given time or spatial pressure profile of a burst can be represented by a general function like Eq. (4). Depending on details for the model, one might have to use numerical methods to solve the differential equation of Eq. (4).

To improve the model used to calculate the pressure profile along the tube, we can also consider the effect of natural adsorption of gas, important in the case of superconducting devices, in the wall of the tube. In this case, the degassed part of the wall serves as a vacuum pump, and the background pressure with a parabolic profile is reached later

when the process of slow uniform desorption of gas trapped in the walls is established (steady state).

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