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Oscillating holograms recorded in photorefractive crystals by a frequency detuned feedback loop

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We report an optoelectronic feedback loop suitable for generating noise-free interference patterns oscillating at arbitrary waveforms. The technique allows controlling the frequency detuning between the interfering beams through a phase modulator in a closed-loop interferometer. We use the dither signal method and propose a quasisynchronous demodulation scheme to create a phase modulated error signal for driving the loop. The dynamics of the interference fringes is easily controlled by a voltage waveform from a function generator, which is used in association with a time delay circuit for shifting the frequency of the reference signal used for lock-in demodulation. The technique is specially suited for applications involving low-frequency phase oscillations, such as those frequently encountered in the generation of space-charge waves in highly resistive photorefractive materials. The processing scheme allows real time monitoring of the hologram strength, and absolute values for the diffraction efficiency and the holographic phase shift can be obtained. Photorefractive wave oscillations ranging from approximately 100 mHz to 10 Hz were produced in a nominally undoped Bi₁₂TiO₂₀ sample. The technique can be readily applied to other fields of optical interferometry, such as for testing optical surfaces, optimizing adaptive holographic devices, measuring physical quantities, among other applications. © 2009 American Institute of Physics. [DOI: 10.1063/1.3072684]

I. INTRODUCTION

A volume hologram arises in a photorefractive crystal when it is illuminated by an interference pattern with proper light wavelength. The holographic recording evolves photo-excitation of free charge carriers (electrons and/or holes), which then move and are retrapped elsewhere. As a result, a spatially modulated electric field (space-charge field) appears and a refractive index grating (hologram) is established through the Pockels effect.^{1,2} Due to the dynamic nature of the photorefractive process, recording can take place both in the stationary regime (stationary interference pattern) or under nonstationary regime (moving interference pattern). In analyzing nonstationary holograms it is usual to use the concept of photorefractive waves (or space-charge waves). When the crystal is biased by an electric field, resonant excitation of photorefractive waves can be observed for a certain translation velocity of the interference pattern.^{3,4}

There have been quite detailed studies on the holographic recording when the writing interference pattern is moving at constant speed^{3–7} or when it is spatially oscillating.^{8–14} The usual procedure for generating dynamic interference patterns is to act on a phase-shifting device (placed in the path of one of the writing beams) using an open-loop configuration (no feedback). In this case, the phase modulation (or frequency detuning) produced in the

optical beam depends on the input signal (usually a voltage signal) and on the response of the phase modulator (half-wave voltage). The procedure is simple and easy to perform, but the final velocity of the light pattern is subjected to the phase noise acting in the optical paths of the interferometer. Phase noise can be especially deleterious to photorefractive waves due to the resonant nature of the grating strength. The effect of phase disturbances can be minimized by reducing the optical paths, controlling the laboratory environment, or by operating at high frequencies, among others. However, highly resistive photorefractive materials [such as Bi₁₂TiO₂₀ (BTO) and Bi₁₂SiO₂₀] can exhibit quite low eigenfrequency of the space-charge oscillations (typically in the range 1–300 Hz)^{3–15} depending on the experimental conditions, and are particularly affected by phase disturbances because environmental noise tends to be stronger at the low-frequency region of the noise spectrum.

Phase noise leads to annoying fluctuations in the position of the interference pattern and a number of stabilization methods were developed for compensating this unwanted effect.^{16–25} Stabilization of a holographic setup using a photorefractive crystal in a two-wave mixing experiment was first proposed by Kamshilin *et al.*¹⁶ The method is based on the active control of the phase shift φ between the transmitted and diffracted beams that propagate along one of the crystal output directions. Phase φ is linked to the holographic phase shift ϕ between the interference pattern and the refractive index modulation; for a pure index grating and in cases

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where self-diffraction effects can be neglected, $\varphi = \phi \pm \pi/2$.^{1,26} The holographic phase ϕ depends on the photorefractive charge transport mechanisms and on experimental conditions. For a stationary interference pattern, $\phi=0$ or π in the case of local photorefractive response (purely drift mechanism), whereas $\phi = \pm \pi/2$ for the nonlocal response (purely diffusive mechanism). It was found experimentally that stabilized (phase-locked) running holograms can be recorded in photorefractive materials if the phase φ (or ϕ) imposed through the feedback loop is different from the one corresponding to the stationary regime.²⁷ The method presented in Ref. 16 is suitable for operating the feedback loop with $\varphi=0$ (or π). Developments and/or improvements on different aspects of the technique extended its application to $\varphi = \pm \pi/2$,¹⁷ and later to any arbitrary φ value.¹⁸ By using these techniques, phase-locked photorefractive waves moving at constant velocities have been generated in different applications.^{28–30}

In a recent paper, Bryushinin and Sokolov³¹ presented an optoelectronic feedback loop suitable for generating photorefractive wave oscillations. The developed method is based on the effect of the non-steady-state photoelectromotive force (photo-emf) and positive feedback. The key parameter to be measured is the frequency of the oscillation, which is mainly defined by the nonlinearity of the photo-emf effect and not by the experimental apparatus (wideband amplifier). For example, the frequency of the oscillating photocurrent excited in the drift regime (for an appropriate phase shift of a wideband amplifier) equals the eigenfrequency of the space-charge oscillation. In this way, important photorefractive parameters of the material can be computed by using a simple frequency counter.

In this work, we describe a negative feedback loop suitable for generating stabilized holographic phase oscillations at arbitrary waveforms. The technique uses some ideas reported in Refs. 16–18 and introduces a quasisynchronous demodulation scheme to create a voltage-controlled phase term into the extracted error signal. This is done through a voltage waveform from a function generator and an electronic time delay circuit, which shift the frequency of the reference wave used for signal demodulation. When the loop is closed, the modulated phase term into the error signal is transferred to the phase shift φ between the interfering waves. The amplitude, frequency, and shape of the holographic phase oscillations are defined electronically by the voltage waveform, and not by the material properties. In addition, the technique allows real time monitoring of the grating diffraction efficiency during exposure. Typical space-charge oscillations produced in a BTO sample are shown.

II. OPTOELECTRONIC SCHEME

Figure 1 illustrates the interferometer and the signal processing block diagram. In the interferometer, R_0 and S_0 represent the complex amplitudes of the input optical waves, whereas R and S represent the complex amplitudes of the interfering (transmitted and diffracted) waves that are collected by a photodetector. Neglecting optical losses, the irradiances of the input and the output waves can be related

FIG. 1. Optoelectronic feedback loop. OSC- ω : sinusoidal oscillator operating at frequency ω , PMs: phase modulators, At: attenuator, $\delta \sin \omega t$: dither phase, ϕ_F : feedback phase, R_0 and S_0 : input optical waves, R and S : output optical waves, φ : phase shift between the R and S waves, D: photodetector, $\tilde{V}$: detector's output voltage, LP: low-pass filter, V : monitored voltage, BP: band-pass filter tuned at ω , A: amplifier, PS: phase-shifter, $\otimes$: multiplier, LA- 2ω : two-phase lock-in amplifier tuned at 2ω , V_X and V_Y : lock-in outputs, V_E : amplified V_X voltage used as an error signal to operate the feedback loop, and I: electronic integrator. The voltage-controlled time delay circuit employs voltage comparators (C1, C2, C3, and C4), ramp generators (RG1 and RG2), and a RS FF; the waveforms shown below the components represent the corresponding output signals.

through the expressions $I_R = I_R^0(1 - \eta)$ and $I_S = I_S^0\eta$, where $I_R = |R|^2$, $I_R^0 = |R_0|^2$, $I_S = |S|^2$, $I_S^0 = |S_0|^2$, and η represents the diffraction efficiency of the hologram. The phase shift between the R and S beams is represented by φ .

A phase modulator driven by an oscillator produces a dither signal $\delta \sin \omega t$ in one of the input beams, where $\delta \ll 1$ and $\omega \gg 1/\tau_{sc}$, with τ_{sc} the response time of the hologram. Thus, the output voltage of the photodetector can be expressed as

$$\tilde{V} = V_0[1 + m \cos(\tilde{\varphi})], \quad (1)$$

with

$$V_0 = \kappa(I_R + I_S), \quad m = 2(I_R I_S)^{1/2}/(I_R + I_S), \quad (2)$$

$$\tilde{\varphi} = \varphi + \delta \sin \omega t \quad (\delta \ll 1), \quad (3)$$

$$\varphi = \varphi(0) + \varphi_N + \varphi_F, \quad (4)$$

where m is the visibility of the collected pattern ($0 \leq m \leq 1$), κ is the irradiance-to-voltage conversion factor of the photodetector, $\varphi(0) = \varphi(t=0)$ is an initial phase, φ_N is a noise term introduced in the optical setup by environmental disturbances, and φ_F is a phase component added in one of the optical beams through the feedback loop; $\varphi_F=0$ in open-loop mode. Due to the dither signal, the Fourier spectrum of $\tilde{V}$

presents several harmonics of the fundamental frequency ω . Expanding Eq. (1) up to the second order we write

$$\tilde{V} = V + \tilde{V}_\omega + \tilde{V}_{2\omega}, \quad (5)$$

$$V = V_0[1 + mJ_0(\delta)\cos(\varphi)], \quad (6)$$

$$\tilde{V}_\omega = -2mV_0J_1(\delta)\sin(\varphi)\sin(\omega t), \quad (7)$$

$$\tilde{V}_{2\omega} = +2mV_0J_2(\delta)\cos(\varphi)\cos(2\omega t),$$

with V the output voltage of the photodetector (apart from fast components), $\tilde{V}_\omega$ the first harmonic component, $\tilde{V}_{2\omega}$ the second harmonic component, and $J_n(\delta)$ the first-kind Bessel function of order n with argument δ . The components of $\tilde{V}$ [Eq. (5)] are detected separately as shown in Fig. 1. Voltage V is readout using a low-pass filter. Voltage $\tilde{V}_{2\omega}$ is measured by a lock-in amplifier (A -input) with the (bandpass) input filter tuned at 2ω . Voltage $\tilde{V}_\omega$ is filtered out and multiplied with an amplified and phase shifted sinusoidal wave at ω from the local oscillator. It results in an alternate mixed signal, called $\omega 2$ -component, that can be written as

$$\tilde{V}_{\omega 2} = 2mV_0\alpha J_1(\delta)\sin(\varphi)\cos(2\omega t - \varepsilon), \quad (8)$$

where the amplification α and the phase shift ε can be adjusted electronically to $\alpha = J_2(\delta)/J_1(\delta)$, and $\varepsilon = \pi/2$. The $\omega 2$ -component feeds the B -input of the lock-in amplifier, which is operated in the differential ($A-B$) input mode. The input voltage at the lock-in amplifier contains the second harmonic term $\tilde{V}_2 = \tilde{V}_{2\omega} - \tilde{V}_{\omega 2}$,

$$\begin{aligned} \tilde{V}_2 &= 2mV_0J_2(\delta)\cos(2\omega t + \varphi) \quad \text{with} \quad mV_0 \\ &= 2\kappa(I_R^0 I_S^0)^{1/2}[\eta(1 - \eta)]^{1/2}. \end{aligned} \quad (9)$$

Let us consider a two-phase lock-in amplifier (containing two reference channels in phase quadrature), operated in the external $2F$ reference mode; in this mode, the frequency of the input reference signal is doubled. Assume that a voltage-controlled time delay circuit is employed as shown in Fig. 1. A time delay circuit can be built using comparators, ramp generators, and a reset-set flip-flop (RS-FF), as illustrated in the dashed box at the bottom of the figure.³² Comparators C1 and C2 produce two π -shifted ($T/2$ -delayed) square wave signals at the frequency ω , where the period $T = 2\pi/\omega$. RG1 and RG2 generate ramp waves at the trailing edges of the square waveforms. The $T/2$ -delayed ramp waves are compared to a reference voltage V_θ . The output from C3 is connected to the set input of the RS-FF and the output from C4 is connected to the reset input of the RS-FF. Comparator C3 (C4) produces a positive (negative) edge when V_θ is higher (lower) than the ramp voltage. These edges control the states of the FF outputs, resulting in delayed square wave signals. The time delay between the input signal (from C1 or C2) to the outputs (from FF) is proportional to V_θ (within the ramp limits) and the proportionality factor depends on the slope of the ramp waves, which can be adjusted electronically through a variable resistor. It follows that a time modulated signal $V_\theta = V_\theta(t)$ from a function generator produces a modu-

lated phase delay, which is equivalent to produce output signals that are shifted in frequency (detuned) with respect to the input at ω . As a result, it can be shown that the outputs from the lock-in demodulators, after filtering by low-pass filters, are slowly varying voltages that can be written as

$$V_X = V_2 \cos[\varphi - \theta(t) - \theta_0], \quad V_Y = V_2 \sin[\varphi - \theta(t) - \theta_0], \quad (10)$$

$$\theta(t) \equiv \rho H V_\theta(t), \quad \omega_D(t) = \dot{\theta}(t), \quad (11)$$

where $V_2 = \sqrt{2}mV_0J_2(\delta)$ is the root-mean-square value of $\tilde{V}_2$ [Eq. (9)], $\theta(t)$ is a time modulated phase term, θ_0 is a constant phase term selected in the internal reference channels of the lock-in amplifier, ρ is a constant conversion factor (in rad/V) characteristic of the time delay circuit, and H represents the (complex) response of the low-pass filters at the lock-in outputs. The angular frequency detuning between the input $\tilde{V}_2$ (at 2ω) and the (ω -detuned and frequency-doubled) reference signals of the lock-in amplifier is represented by ω_D in Eq. (11), with the upper dot meaning the time derivative.

In our circuit, factor ρ was adjusted electronically (through a variable resistor) to $\rho = 0.250 \pm 0.001$ rad/V. For the single delay line block (dashed box in Fig. 1) used in our experiments, the maximum phase excursion $\rho H V_\theta(t)$ is limited to 2π (from $-\pi$ to π), but larger amplitudes can be achieved by simply connecting several delay line blocks in cascade. In this case, supposing all blocks equally adjusted, factor ρ should be multiplied by the number of blocks.

The amplitude response of the low-pass filter is flat from dc to a point where it begins to roll off. The standard reference point is the cutoff frequency defined by $\Omega_C = 1/\tau$, where τ is the time constant of the low-pass filter. For a single pole filter, with 6 dB/octave rolloff, $H = 1/(1 + j\Omega\tau)$, with j the imaginary number and Ω the angular frequency of the oscillation. Therefore, a voltage waveform $V_\theta(t) = V_\Delta \sin \Omega t$ yields $\theta(t) = \Delta \sin(\Omega t - \xi)$, with $\Delta = \rho H_0 V_\Delta$, $H_0 = [1 + (\Omega\tau)^2]^{-1/2}$, and $\xi = \arctan(\Omega\tau)$. In practice, the filter response can be neglected ($H = 1$) whenever $\Omega < \Omega_C/10$; in this situation $H_0 \approx 1$ and $\xi \approx 0$. At the cutoff frequency $\Omega = \Omega_C$, the amplitude $H_0 = 0.707$ (-3 dB), and the phase delay $\xi = \pi/4$. Commercial lock-in amplifiers offer typically minimum $\tau = 1$ ms, whereas more expensive ones can reach $\tau = 1$ μ s. Taking the cutoff frequency as a reference, and for an ordinary lock-in at $\tau = 1$ ms, the maximum theoretical value for Ω is therefore limited to $\Omega_c/2\pi \approx 160$ Hz.

Suppose now that one of the lock-in outputs, say, V_X , is amplified and the corresponding analog voltage (V_E) is used as an error signal to operate the feedback loop. If the amplification (loop gain) is made large enough and the stability criteria are satisfied,³³ the system is forced to operate around the condition that makes $V_X = V_E = 0$, which means

$$\varphi(t) = \rho H V_\theta(t) + \theta_0 \pm \pi/2, \quad (12)$$

where one of the signs of $\pm \pi/2$ in Eq. (12) leads to the negative feedback condition. From Eqs. (4) and (12), the phase applied by the feedback loop $\varphi_F(t) = -\varphi(0) - \varphi_N(t) + \theta(t) + \theta_0 \pm \pi/2$. Thus, the initial component $\varphi(0)$ is can-

celed out, the noise term $\varphi_N(t)$ is compensated in real time, and φ oscillates according to $\theta(t)$ around the position determined by $\theta_0 \pm \pi/2$. Also, the use of a two-phase lock-in amplifier allows real time monitoring of the diffraction efficiency through $V_Y = V_2 \propto [\eta(1 - \eta)]^{1/2}$. From Eqs. (9)–(12), it follows

$$\left(\frac{\eta}{1 - \eta}\right)^{1/2} = \beta \frac{V_Y}{V_R} \quad \text{with} \quad \beta = \frac{(I_R^0/I_S^0)^{1/2}}{2\sqrt{2J_2(\delta)}} \quad \text{and} \quad V_R = \kappa I_R^0(1 - \eta). \quad (13)$$

Note that absolute values for η can be calculated if one knows the (adimensional) parameter β and measures voltages V_Y and V_R . Parameter β is determined by the ratio between the input irradiances (I_R^0/I_S^0), and by the amplitude δ of the dither signal, whereas voltage V_R corresponds to the detector's output V when the S beam is cut off ($I_S^0 = I_S = 0$). Note also that in experiments where the diffracted beam is much weaker than the transmitted one ($I_S \ll I_R$), as usually occurs when $\eta \ll 1$, the detector's output $V \approx V_0 \approx V_R$, so that there is no need to cut off beam S to measure V_R . The diffraction efficiency computed from Eq. (13) neglects optical losses, and it is equivalent to the ratio between the diffracted and the sum of the diffracted and transmitted light irradiances generated by a single beam (R or S) entering the crystal in the Bragg angle.

III. EXPERIMENTAL RESULTS

The light source is an argon-ion laser operating at $\lambda = 514.5$ nm. Two piezoelectrically supported mirrors (PZT's) were employed as phase modulators, with the dither and the feedback signals applied independently. In our setup, the dither frequency ω is limited by the PZT-mirror: At frequencies above ≈ 2 kHz it begins to exhibit nonlinear response, thus introducing distortions in the Fourier components of the detected signal. So, in all experiments reported here, $\omega/2\pi = 1.7$ kHz, with the dither amplitude $\delta = 0.25$ rad. The time constant $\tau = 10$ ms, with 6 dB/octave rolloff, and therefore the cutoff frequency $\Omega_C/2\pi \approx 16$ Hz.

First we carried out experiments using a partially reflecting interface (semitransparent mirror) adjusted to produce the interference between the reflected wave (S -beam) with the transmitted one (R -beam). These experiments helped us to make the electronic calibrations for α and ε [Eq. (8)] and to analyze the operation of the system. The spatial period of the light pattern projected on the photodetector is much larger than the dimensions of the detector's active area, so phase φ is well defined. In this case, η is constant and it represents the reflectance of the interface. The collected irradiance is monitored by voltage V , acquired in 100 Hz bandwidth (see Fig. 1). Figure 2 shows the operation of the interferometer in open-loop mode (first half of the figure) and closed-loop mode (second half). In this experiment $V_\theta(t) = V_\Delta \sin(\Omega t)$, with $V_\Delta = 4.00 \pm 0.04$ V and $\Omega/2\pi = 1$ Hz [Fig. 2(a)]. Phase $\theta(t) = \Delta \sin(\Omega t - \xi)$, with $\Delta = 1.00 \pm 0.02$ rad and $\xi = 0.0627$ rad ($\approx 3.6^\circ$). In open-loop mode $\varphi_F = 0$, and the phase shift between the optical waves varies due to phase noise: $\varphi = \varphi(0) + \varphi_N$. Voltage V [Fig. 2(b)] varies ran-

FIG. 2. Open-loop and closed-loop modes. Voltage $V_\theta(t) = V_\Delta \sin(\Omega t)$ V, with $\Omega/2\pi = 1$ Hz and $V_\Delta = 4.0$ V (a), so that $\Delta = 1.0$ rad. In the first half of the figure, phase φ is subject to phase noise: $\varphi = \varphi(0) + \varphi_N$ (open loop). Output V (b) varies randomly between 1.29 (bright fringe) and 0.19 V (dark fringe), whereas V_X and V_Y (c) are modulated through $V_\theta(t)$. When the loop is closed, $V_X = 0$, V_Y is maximum, φ_N is compensated in real time, and the modulation is transferred to φ .

domly from the maximum value $V_M = V_0(1 + m) = 1.29$ V (bright fringe) to the minimum $V_m = V_0(1 - m) = 0.19$ V (dark fringe). From these data we calculate $V_0 = (V_M + V_m)/2 = 0.74$ V, and the visibility $m = (V_M - V_m)/(V_M + V_m) = 0.74$. Voltages V_X and V_Y [Fig. 2(c)] are modulated through $V_\theta(t)$ and also by the noise term φ_N that exists in φ . When the loop is closed, $V_X = 0$ and $V_Y = V_2$, as shown in the second half of Fig. 2. In this case, the internal reference phase of the lock-in amplifier is adjusted ($\theta_0 = 0$) to obtain the light pattern oscillating around $\varphi = \pi/2$ (position of maximum sensitivity of the interferometer). Amplitude Δ was checked directly from V by using Eq. (6), with the parameters $V_0 = 0.74$ V and $m = 0.74$, which leads to 1 rad ($\pm 5\%$), in accordance with the preset value.

Stabilized phase oscillations at different waveforms (sinusoidal, triangular, and square) are illustrated in Fig. 3. Voltage outputs V , V_X , and V_Y are shown. The modulation amplitude is 1 rad ($V_\Delta = 4$ V) and the frequency $\Omega/2\pi = 1$ Hz for all waveforms. In the first half of the figure the sinusoidal oscillation is illustrated in three different cases (from $t = 0$ s to $t = 8$ s), where phase θ_0 is varied in steps of $\pi/2$: In sequence, $\theta_0 = \pi/2$ (pattern oscillating around a dark fringe), $\theta_0 = 0$ (maximum sensitivity and linear response), and $\theta_0 = -\pi/2$ (bright fringe). At $t = 8$ s the interferometer is set to maximum sensitivity again ($\theta_0 = 0$) and the waveform generated by $V_\theta(t)$ is switched from sinusoidal to triangular. The last part of Fig. 3 illustrates the outputs for a square waveform modulation.

Further, oscillating photorefractive waves were investigated using a nominally undoped BTO sample (BTO-8). The coherent and collimated input beams have irradiances $I_R^0 = 1$ mW/cm² and $I_S^0 = 24$ mW/cm². The beams intersect symmetrically in the crystal to record a grating with spatial frequency $K/2\pi = 160$ mm⁻¹ and the velocity of the interfer-

FIG. 3. Stabilized fringe oscillations at different waveforms. The frequency is 1 Hz and the amplitude is 1 rad for all waveforms. A sinusoidal oscillation is illustrated from $t=0$ s to $t=8$ s, with the internal reference phase of the lock-in amplifier varied in steps of $\pi/2$; starting with $\theta_0=\pi/2$ (oscillation centered on a dark fringe), $\theta_0=0$ (position of maximum phase sensitivity), and $\theta_0=-\pi/2$ (centered on a bright fringe). At $t=8$ s the waveform generated by $V_\theta(t)$ is shifted from sinusoidal to triangular and after that to square waveform, with $\theta_0=0$. In all cases, $V_X=0$ and V_Y is maximum (neglecting the transient terms).

ence pattern with respect to the grating is given by $u(t) = \omega_D(t)/K$ [Eq. (11)]. The plane containing the grating vector and [001]-axis corresponds to the (110) crystal face. The grating vector lies in a direction perpendicular to the crystallographic [001]-axis. An external dc voltage of 4.5 kV is applied using silver painted electrodes on the lateral surfaces of the crystal, resulting nominally in an electric field of 9.5 kV/cm parallel to the grating vector. The crystal is 4.75 mm wide (distance between the electrodes) and it is 0.95 mm thick (along the beam direction). The polarization of the light input beams is linear and oriented to be at 45° to the [001]-axis in the middle slice of the sample; this orientation prevents practically any change in the linear state of polarization due to the induced birefringence and results in parallel-polarized transmitted and diffracted beams at the crystal outputs.³⁴ In the example illustrated in Fig. 4, $V_\theta(t) = V_\Delta \sin(\Omega t)$ V, with $\Omega/2\pi=1$ Hz and $V_\Delta=2.0$ V [solid curve in Fig. 4(a)]. Phase $\theta_0=0$ and $\theta(t)=\Delta \sin(\Omega t - \xi)$, with $\Delta \approx 29^\circ$ and $\xi \approx 3.6^\circ$. The secondary vertical axis in Fig. 4(a) is used to plot phase φ (dashed curve). Figure 4(b) shows the output V_Y (solid curve), illustrating the typical oscillatory behavior of the hologram strength. The secondary vertical axis in Fig. 4(b) plots $\eta^{1/2}$ calculated through Eq. (13). The dashed line in this figure is a sinusoidal curve fitting (shifted up for clarity) of the experimental data. Statistically averaged values for the minimum (η_m) and maximum (η_M) diffraction efficiencies can be obtained in this way; in the example $\eta_M^{1/2} = 6.4 \times 10^{-2}$ ($\eta_M=0.41\%$) and $\eta_m^{1/2} = 4.1 \times 10^{-2}$ ($\eta_m=0.17\%$). The phase delay between the φ -modulation [Fig. 4(a)] and the η -modulation [Fig. 4(b)] can also be measured; it is 144° in this case (represented by the separation between the vertical lines). In our setup, the dominant noise sources are air currents and temperature gradients in the optical paths (≈ 1 m long), which are acting typically in the hertz and millihertz range. Figure 4(c) shows the feedback voltage applied directly to the phase modulator ($V_{PZT} \propto \varphi_F$); the impor-

FIG. 4. Typical photorefractive response for a modulation of the type $V_\theta(t) = V_\Delta \sin(\Omega t)$ V, with $\Omega/2\pi=1$ Hz and $V_\Delta=2.0$ V [solid curve in (a)]. The secondary vertical axis in (a) plots $\varphi(t)$ calculated according to Eq. (12) (dashed curve). The amplitude of the phase oscillation is $\Delta=0.5$ rad ($\approx 29^\circ$). The solid curve in (b) shows the typical V_Y output, whereas the dashed curve is a sinusoidal fitting (shifted up for clarity) of the experimental data. The secondary vertical axis in (b) plots $\eta^{1/2}$ calculated through Eq. (13). (c) shows the feedback voltage applied directly to the phase modulator (V_{PZT}), illustrating noise-compensation.

tance of noise compensation for generating stable phase oscillations at low frequencies is clear in this example.

The response of our BTO sample was analyzed by scanning the frequency Ω . Note that in self-stabilized regime the feedback loop uses a dynamic hologram as a reference for positioning the interference pattern. It means that for low Ω 's, the hologram moves nearly together with the light pattern and there is a maximum Δ that can be accomplished by the system. If the value selected for Δ is larger than this limit, the pattern and the hologram are forced to run indefinitely in one direction, limited by the dynamic range of the phase modulator. In order to measure the movement of the interference pattern with respect to a stationary (spatially fixed) coordinate system, we placed a small piece of glassplate by the side of the crystal. The glassplate was adjusted to interfere the reflected beam with the transmitted one (as previously described) and the optically mixed signal was used just to monitor the spatial position of the interference pattern. Figure 5 shows the experimental results (dots), with $\Omega/2\pi$ ranging from ≈ 100 mHz to ≈ 10 Hz. The interference pattern oscillates at Ω with phase amplitude Δ with respect to the hologram and phase $\theta_0=0$. In these experiments, Δ was adjusted (through V_Δ) so that the amplitude of the phase oscillation relative to the glassplate was 0.5 rad (kept constant for all Ω 's). The modulation of the hologram strength is measured by the difference between the maximum and minimum η -values. Results show the peak of the modulation located at $\Omega/2\pi \approx 2.5$ Hz. The low frequency of resonance observed for this crystal is attributed mainly to the short drift length of the photoelectrons (the mobility-lifetime product for the sample is estimated to be $\approx 0.6 \times 10^{-12}$ m²/V).¹⁵ Besides, the crystal is 0.95 mm thick and the bulk intensity absorption coefficient at this wavelength is high (≈ 1.7 mm⁻¹), so that the irradiance at the output face is only 20% of the intensity at the input face of the

FIG. 5. Frequency response of the photorefractive waves. The dots are the experimental data and the dashed lines are guides for the eyes. The modulation of the hologram strength is measured by the difference between the maximum (η_M) and minimum (η_m) values of the diffraction efficiency. The interference pattern oscillates at the angular frequency Ω and phase amplitude Δ with respect to the (dynamic) hologram. The amplitude of the pattern oscillation with respect to a stationary reference (glassplate) was kept constant and equal to 0.5 rad at all frequencies.

crystal. This causes a broadening of the resonance peak and an asymmetric shape of the response, as observed in Fig. 5.³⁵

It is worth to mention that a slow drift of the interference pattern can be detected when the oscillating photorefractive waves are observed for several minutes, which suggests that the pattern (and the phase-locked grating) are not only oscillating, but also moving with small (not null) average velocity. We observed experimentally that a stationary hologram (in average) is produced when $\theta_0 = (2.5 \pm 1.0)^\circ$. This occurs probably because phase $\theta_0 = 0$ leads to a center value $\varphi = \pi/2$, which produces a stationary hologram only if the diffusion transport mechanism can be neglected. In the experiments, the diffusion field $E_D = Kk_B T/e \approx 255$ V/cm, whereas the externally applied field was nominally $E_E = 9.5$ kV/cm. So, in theory, $\theta_0 \approx E_D/E_E \approx 0.027 (\approx 1.5^\circ)$ should produce a stationary hologram, which is in agreement with the experimental value considering that electrical charges associated with the photorefractive centers may accumulate near the electrodes and thereby act to screen the externally applied field E_E , increasing the theoretical θ_0 value.³⁶

IV. DISCUSSION

Through the proposed technique, the dynamics of the interference fringes is controlled by a voltage signal from a function generator and the voltage-to-phase conversion factor is determined by the time delay circuit. So, the amplitude, frequency, and shape (waveform) of the phase modulation can be chosen with good accuracy and independently of the response (half-wave voltage) of the phase modulator; note that the feedback system automatically imposes to the modulator the signal required for keeping the loop stable. The performance of the system depends essentially on the signal-to-noise voltage ratio (SNR) of the lock-in outputs, and the two most important instrumental parameters determining the SNR are δ and τ . For weak dither amplitudes ($\delta \ll 1$), the minimum detectable signal is limited essentially by the second harmonic component ($\tilde{V}_{2\omega} \propto J_2(\delta) \approx \delta^2/8$), whereas the background noise increases with the square root of the frequency bandwidth ($\Omega_C^{1/2} = \tau^{-1/2}$). Therefore, exceedingly small values for δ and τ may lead to problems due to poor

SNR $\propto \delta^2 \tau^{1/2}$. Fortunately, in most applications, it is not too much a problem to increase δ because the main effect of the dither signal (besides generating the Fourier components) is a small reduction in the visibility of the interference pattern, proportional to $J_0(\delta)$. So, for instance, if δ is doubled from 0.25 to 0.50 rad [$J_0(0.25) \approx 0.98$ and $J_0(0.5) \approx 0.94$], and τ is reduced from 10 to 1 ms, the SNR remains approximately unaltered. Note that τ cannot be arbitrarily reduced in our setup because high-frequency components (multiples of ω) begin to appear in the lock-in output signals, degrading the performance of the system. For operation with $\tau = 1$ ms, dither frequencies in the range of some tenths of kilohertz should be used, which can be easily achieved by optoelectronic phase modulators.

In conclusion, the goal of this paper was to describe a feedback loop suitable for stabilizing photorefractive wave oscillations at low frequencies. The technique can also be applied to other fields of optical interferometry, such as for testing optical surfaces, optimizing adaptive holographic devices, among others. We use the dither signal method and develop a quasisynchronous demodulation scheme to create a modulated phase term into the extracted error signal. This was done by slightly changing the frequency of an external wave used to feed the reference channels of a two-phase lock-in amplifier. Through the presented electronic processing scheme, the characteristics of the phase oscillation can be precisely known and varied over wide limits. A simple procedure for measuring the diffraction efficiency of the hologram was also described.

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