Dissipative three-wave structures in stimulated backscattering. I. A subluminous solitary attractor

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We present a solitary solution of the three-wave nonlinear partial differential equation (PDE) model—governing resonant space-time stimulated Brillouin or Raman backscattering—in the presence of a cw pump and dissipative material and Stokes waves. The study is motivated by pulse formation in optical fiber experiments. As a result of the instability any initial *bounded* Stokes signal is amplified and evolves to a subluminous backscattered Stokes pulse whose shape and velocity are uniquely determined by the damping coefficients and the cw-pump level. This asymptotically stable solitary three-wave structure is an attractor for any initial conditions in a compact support, in contrast to the known superluminous dissipative soliton solution which calls for an unbounded support. The linear asymptotic theory based on the Kolmogorov-Petrovskii-Piskunov assertion allows us to determine analytically the wave-front slope and the subluminous velocity, which are in remarkable agreement with the numerical computation of the nonlinear PDE model when the dynamics attains the asymptotic steady regime. [S1063-651X(97)13201-9]

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I. INTRODUCTION

Localized traveling backscattered Stokes pulses, well modeled by the three-wave nonlinear interaction between the pump wave E_p, the material wave E_a, and the backscattered Stokes wave E_S, have been obtained, in the ns range, in stimulated Brillouin backscattering (SBBS) experiments in liquids [1], gases [2], plasmas [3], and fiber-ring resonators [4–6]. Such spiked dynamics also appears in stimulated Raman backscattering (SRBS) experiments [7,8], in the ps range, due to the faster material response.

This resonant three-wave interaction problem is described by the nonlinear partial differential equations (PDE) model within the slowly varying envelope approximation; it has been the object of many theoretical studies and numerical simulations. For interacting waves whose profiles vanish at infinity (bounded envelopes for all three waves), the problem has been integrated by the inverse scattering transform (IST) in the nondissipative case [9–11], and solved numerically in the nonintegrable dissipative case [12], yielding no soliton solution for the backscattering problem: the collision of the pump wave with the material wave generates only a radiative backscattered wave.

However, we are concerned with nonconservative stimulated backscattering (SBBS or SRBS) in the presence of a continuous pump. This cw pump gives rise to two main nonlinear responses resulting from the instability. On the one hand, when it enters a semi-infinite material medium it generates a stationary distributed mirror where the monotonic backscattered amplification of the Stokes wave in the optical medium is saturated by the monotonic depletion of the forward-propagating cw pump [13]. On the other hand, in the unlimited three-wave interaction, the cw pump may generate a localized structure: Two kinds of initial conditions for the E_a or E_S signals will yield two classes of localized traveling structures which may be associated with the above-cited experiments.

(i) Initially *bounded* Stokes or material wave fluctuations yield, as we show in this paper, a *subluminous* backscattered three-wave solitary structure, whose constant velocity is uniquely determined by the damping coefficients and the cw-pump level, and which is an *attractor* for any initial condition in a compact support. This *Cauchy problem* of an initially bounded Stokes wave packet, propagating (in the absence of interaction) at the velocity of light, and colliding with a cw pump (Fig. 1), cannot yield a superluminous asymptotic traveling structure.

(ii) Initially *unbounded* Stokes conditions present well-known analytical *superluminous* three-wave soliton solutions [14–16], which are also available for dissipative E_a and E_S envelopes [5,17]. Superluminous motion of the three-wave localized structure does not contradict by any means the special theory of relativity. This motion can be viewed as the result of the convective amplification of the leading edges of the Stokes and material pulses, whereas their rears are at-

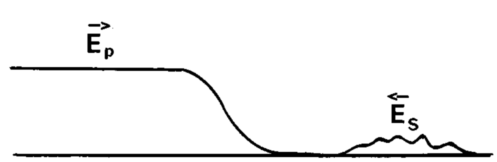

FIG. 1. Shape of the initially bounded envelope $E_S(x,0)$ counter propagating with respect to a cw pump $E_p(x,0)$.

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tenuated, since the pump wave is depleted during the interaction and totally or partially restored after that. No transportation of information can be obtained *via* this deformation process, which can only occur if a sufficiently extended background of Stokes light is available. This unbounded problem has often been considered in the literature, since it can be integrated by IST in the nondissipative case [18], and a perturbative theory has been developed for the weakly dissipative case [19]. To a certain extent, the infinite interaction problem can be associated with the periodical problem in a cavity, where, below a critical feedback, spontaneous structuration of superluminous solitary Stokes pulses [5] takes place from any initial conditions [6]. This case will be considered in paper II [20] in order to present the full family of nonintegrable superluminous or subluminous dissipative solitary attractors, dependent only on the wave-front slope of the backscattered Stokes wave.

In this paper we will consider only case (i): An initially bounded Stokes wave packet, associated with localized material fluctuations, grows backward at the expense of a maintained cw-pump input. This problem was already studied by including only the material wave damping [21,22], and could not lead to an asymptotic steady state. In the absence of any dissipation, or if only the material wave is assumed to be damped, the Stokes envelope exhibits unlimited amplification, compression, and frontal steepening, the maximum of the pulse shifting to its wave front with a decreasing velocity: no saturated steady behavior is obtained [23]. This nonstationary situation is unchanged even if the slowly varying envelope approximation for the material wave E_a is taken off [24].

However, it has been shown that, for the unbounded case (ii), steady traveling solitonlike solutions exist for the Stokes envelope when damping of both E_a and E_S are taken into account [5,17]. By analogy, the aim of the present paper is to revisit the initially bounded case (i), already studied in [22], but now taking into account both E_a and E_S dampings, in order to look for a steady traveling Stokes envelope solution as well. Indeed, we shall see in this case, by the numerical treatment of the nonlinear three-wave dynamics starting from any initial Stokes signal *bounded* in a compact support, that it evolves toward a backscattered pulse attractor with constant *subluminous* velocity, in contrast to the known *superluminous* dissipative soliton solution [5], which calls for an *unbounded* support. Our basic tool will be a powerful assertion of Kolmogorov, Petrovskii, and Piskunov (KPP) [25], which has been applied to several diffusion problems [26,27], but never, to our knowledge, in the context of nonlinear optics. They state that for nonlinear problems governed by the PDE and containing instability and dissipation, a transition from an initially bounded unstable solution to a stable steady-state solution continues a long time by means of stationary traveling waves, whose velocity is unambiguously determined by the *linear* behavior of the equations, while the shape of the front is determined by the solution of the corresponding ordinary differential equation (ODE). Our problem belongs to this class, being initially bounded, dissipative, and exponentially unstable in the initial parametric regime (undepleted pump approximation). Therefore the *linear* KPP asymptotic method allows us to obtain analytically the front slope and the velocity of the localized structure, which are uniquely determined by the damping coefficients and the cw-pump level. The strength of this result is that this velocity corresponds to the *nonlinear* asymptotic steady-state structure exhibiting pump depletion and resulting from a complicated transient evolution. The reason for this agreement lies in the fact that the front foot of the Stokes structure always remains in the linear parametric regime; the slope of this front edge asymptotically attains the slope of the whole nonlinear structure in the steady-state regime, and the velocity is determined by this slope.

II. NUMERICAL THREE-WAVE DYNAMICS

We deal with the (1D)space-(1D)time three-wave problem (1D is one dimensional), relevant, for instance, in single-mode optical fibers [22,4]. The nonlinear resonant SBBS (or SRBS) process couples through electrostriction (through optically induced polarizability variations) a pump wave $E_p(\omega_p, k_p)$ and a backscattered Stokes wave $E_S(\omega_S, k_S)$ of complex amplitudes to a material acoustic wave (or polarizability wave) $E_a(\omega_a = \omega_p - \omega_S, k_a = k_p + k_S)$. Neglecting the material wave propagation (since $c_a \ll c$ in SBBS), it yields, through the slowly varying envelope approximation, the three coupled equations in dimensionless units [4]

$$\begin{aligned}(\partial_t + \partial_x + \mu_p)E_p &= -E_S E_a ,\\ (\partial_t - \partial_x + \mu_S)E_S &= E_p E_a^* ,\\ (\partial_t + \mu_a)E_a &= E_p E_S^* ,\end{aligned} \tag{1}$$

where the envelope amplitudes E_i, the time t and space x variables, and the damping rates γ_i are normalized to the constant pump input E_{cw} and to the SBBS or SRBS coupling constant K [$E_i/E_{cw} \rightarrow E_i$, $tKE_{cw} \rightarrow t$, $xcKE_{cw}/n \rightarrow x$, and $\gamma_i(KE_{cw})^{-1} \rightarrow \mu_i$ $(i = p,S,a)$].

In order to look for steady localized traveling Stokes structures, we must assume a constant pump input compensating for Stokes and material losses, and therefore we neglect the damping of the pump wave ($\mu_p = 0$). This is locally legitimate if the structure's width is small compared to μ_p^{-1}. Therefore, considering that the threshold condition is satisfied

$$\mu_S \mu_a \equiv \frac{\gamma_S \gamma_a}{(KE_{\text{cw}})^2} < 1, \tag{2}$$

the E_S and E_a waves are unstable, exponentially growing in the linear parametric regime until a nonlinear stage is reached. The basic problem is to determine the nonlinear stage of this unstable process in the presence of dissipative E_S and E_a waves, when the depletion of the pump E_p saturates the instability. As we shall see, the nonlinear evolution of the initially bounded Stokes signal, exhibiting damping, yields a saturated localized three-wave structure in contrast to the unsaturated no-Stokes-damping case.

Let us consider the nonlinear space-time evolution of the three-wave system governed by Eqs. (1) for an initially bounded Stokes wave packet $E_S(x, t = 0)$ counterpropagating with respect to the pump $E_p(x, t = 0)$, as shown by Fig. 1. Since we are looking for solutions localized in the vicinity of the E_S characteristic, it is naturally to reduce the above

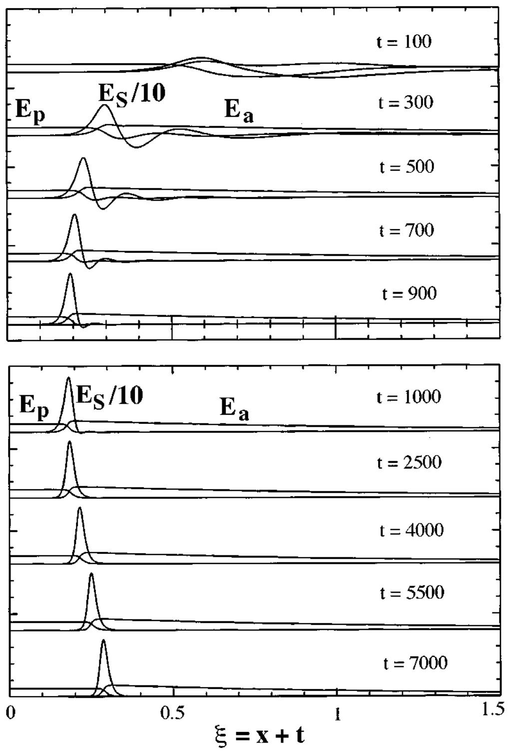

FIG. 2. Three-wave envelope evolution, obtained by numerical computation of Eqs. (3), in the comoving Stokes frame $\xi=x+t$ for $\mu_a=1$ and $\mu_S=0.01$, from time $t=100$ to time $t=7000$, where the Stokes peak velocity approaches the asymptotic subluminous velocity V predicted by the theory [and given by Eq. (12)], while its shape attains the steady state. (Characteristic time t for SBBS is 10–100 ns [6, 20]).

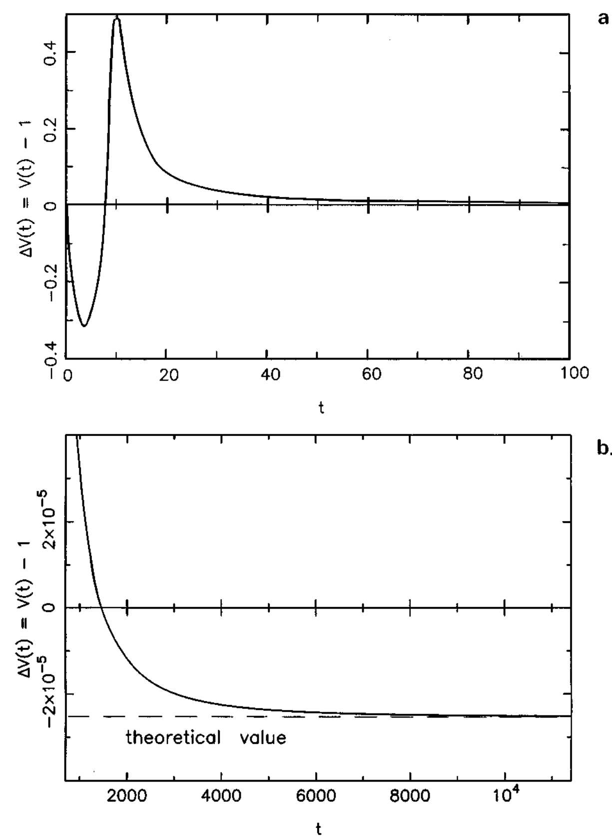

FIG. 3. Stokes peak velocity variation $\Delta V(t)=V(t)-1$ associated with the dynamical scenario of Fig. 2. (a) during the initial stage $(0<t<100)$, first showing transient subluminosity $(\Delta V(\mathrm{t})<0)$ and then superluminosity $(\Delta V(t)>0)$; and (b) in the asymptotic regime (different coordinate scales) for times $(1000<t<11\,200)$ where the velocity asymptotically approaches the subluminous theoretical value (12) of the attractor.

Cauchy problem to the initial-boundary value problem in the comoving E_S frame $(\xi=x+t,\tau=t)$, where Eqs. (1) read

$$(\partial_\tau+2\partial_\xi)E_p=-E_SE_a,$$

$$(\partial_\tau+\mu_S)E_S=E_pE_a^*, \tag{3}$$

$$(\partial_\tau+\partial_\xi+\mu_a)E_a=E_pE_S^*,$$

with the same initial data $E_S(\xi=x,\tau=0)\neq 0$ [or $E_a(\xi=x,\tau=0)\neq 0$] inside the compact support, and supplemented boundary data $E_p(\xi=0,\tau)=1$ and $E_S(\xi=0,\tau)=E_a(\xi=0,\tau)=0$. This initial-boundary-value problem well describes the evolution of the bounded backscattered envelope interacting with a cw-pump input, and is more suitable for long-time numerical integration.

We integrate Eqs. (3) by following the characteristics [11,22] and using a standard four-step Runge-Kutta algorithm, which proved to be remarkably stable for very long interaction times. The numerical results show that any initial Stokes signal *bounded* in a compact support evolves to a backscattered pulse attractor of constant *subluminous* velocity. The whole pulse only reaches this velocity in the full nonlinear asymptotic regime after a complicated transient. The velocity of the nonlinear structure in the numerical dynamics is evaluated from the displacement of the Stokes peak maximum, in the comoving backscattered wave frame where we represent the dynamics. In Fig. 2 we plot the three-wave dynamical evolution, and in Figs. 3(a) and 3(b) the time velocity deviation $\Delta V(t)=V(t)-1$ of the Stokes peak (with respect to the luminous velocity $V=1$) for the case $(\mu_S=10^{-2},\ \mu_a=1)$. During the initial stage following the collision of the pump with the Stokes signal (Fig. 1), stimulation of the resonant E_a wave spreads the E_S envelope by slowing its peak. The result is a transient subluminous motion during the initial stage until time $t=8$, as shown in Fig. 3(a). Then the depletion of the pump causes the Stokes front to be more amplified than the rear part, and the Stokes peak to drift forward leading to a superluminous motion [Fig. 2, upper half; Fig. 3(a)]. Note that these large variations of the velocity are associated with the transient deformation of the pulse, the velocity corresponding only to the Stokes peak motion. In the nonlinear stage, the Stokes peak continues to have a transient superluminous velocity $(V>1)$ at long times $(t\simeq 1300)$, finally decreasing again (Fig. 2, lower half), and asymptotically reaching the subluminous steady state. This complex transient in the nonlinear stage presents some similarities with the previously studied dynamics in the absence of Stokes damping [22], but now, at some later time, the presence of the Stokes damping, even extremely small, is responsible for a completely different asymptotic behavior. Indeed, the velocity again becomes subluminous $(V<1)$ until reaching a stable localized steady state at constant velocity [Fig. 3(b)].

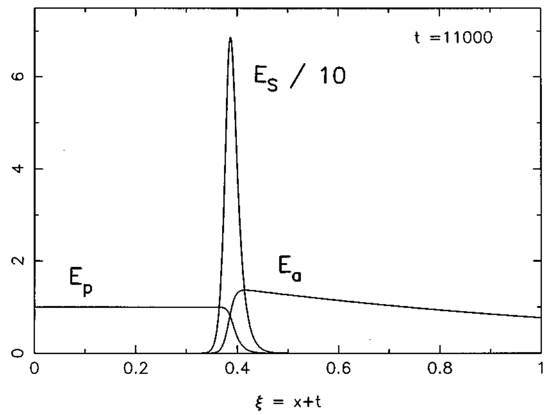

FIG. 4. Steady asymptotic three-wave profiles moving at the velocity V predicted by the theory.

Asymptotics

Thus our conjecture is verified: The damping of the Stokes wave saturates its amplitude, yielding a steady localized structure in contrast to the unlimited amplification and compression of the pulse in the no-Stokes-damping case [22]. In Fig. 4 we plot this dissipative three-wave steady solution presenting asymmetry and moving at constant velocity $V=1+\Delta V$ in the far asymptotic stage. The numerical value of the velocity deviation computed in this regime (interaction time $t=11\,000$) is $\Delta V_{\rm num} = -2.510\times10^{-5}$.

Therefore, this nonlinear (nonintegrable) problem, governed by the PDE, presenting instability and dissipation, and leading to a steady asymptotic behavior, contains all the ingredients required by the KPP procedure in order to obtain the expression for the velocity analytically.

III. ASYMPTOTIC THEORY: DETERMINATION OF THE PULSE'S VELOCITY AND SLOPE

Starting from localized initial Stokes- or material-wave fluctuations, which are unstable and exponentially growing at the expense of a cw-pump E_p, counterpropagating with respect to the Stokes component E_S, the KPP procedure allows us to look for localized traveling wave solutions. This asymptotic procedure, based on the steepest descent method, analytically determines the velocity and the slope of the wave front in the linear regime of undepleted pump, *which turns to be the value of the velocity of the nonlinear steady state* of strong pump depletion, since the front edge of the whole three-wave steady structure remains in the linear undepleted pump regime every time.

Let us stress this key point. We have seen that the *nonlinear* process saturates the instability, and leads to a nonlinear localized steady structure. Because the slope of its front, determining the velocity of the whole steady state, asymptotically rejoins the slope of its front foot, which always remains in the linear undepleted pump regime, the *linear* asymptotic behavior may allow us to determine the steady velocity through the KPP procedure.

Thus, assuming an undepleted pump wave ($E_p=1$) throughout the whole linear interaction range, the linearized Eqs. (1) yield

$$(\partial_t-\partial_x+\mu_S)(\partial_t+\mu_a)E_{S,a}=E_{S,a}, \tag{4}$$

whose complex characteristic equation for an exponential dependence $[E_{S,a}\propto \exp(\gamma t+px)]$ reads

$$(\gamma-p+\mu_S)(\gamma+\mu_a)=1, \tag{5}$$

in which we keep only the unstable root (Re$\gamma>0$)

$$\gamma=\frac{p-\mu_a-\mu_S}{2}+\frac{\sqrt{(p+\mu_a-\mu_S)^2+4}}{2}. \tag{6}$$

For a large class of spatially bounded conditions, we may give the linear solution of Eqs. (1) by means of the Laplace transform

$$E_{S,a}(x,t)=\frac{1}{2i\pi}\int_{-i\infty+\sigma}^{i\infty+\sigma}\tilde{E}_{S,a}(p,t)\exp(px)dp;$$

$\tilde{E}_{S,a}(p,t)=\tilde{E}_{S,a}(p)\exp[\gamma(p)t]$, where $\gamma(p)$ is given by Eq. (6); and

$$\tilde{E}_{S,a}(p)=\int_0^\infty E_{S,a}(x,0)\exp(-px)dx.$$

Let us look for traveling waves with velocity V

$$z=x+Vt, \tag{7}$$

yielding

$$E_{S,a}(z,t)=\frac{1}{2i\pi}\int_{-i\infty+\sigma}^{+i\infty+\sigma}\tilde{E}_{S,a}(p)\exp[\gamma(p)-pV]t \times\exp(pz)dp. \tag{8}$$

This linear solution holds for long times t, allowing us to obtain the *asymptotic behavior* of the far pulse wave front where *the linear undepleted pump approximation always remains valid*, while the interaction is already strong enough so that, in our problem, the exponent function $f(p)=\gamma(p)-pV$ has a saddle point $p=p_\sigma$. Therefore, we are able to apply the steepest descent method to compute the asymptotic behavior [28], σ being chosen so that the integral path contains the saddle point

$$[\partial\gamma(p)/\partial p]_{p=p_\sigma}=V. \tag{9}$$

Expanding $f(p)$ around $p=p_\sigma$, and integrating, we obtain

$$E_{S,a}(z,t)\propto(tf''_{p=p_\sigma})^{-1/2}\tilde{E}_{S,a}(p_\sigma)\exp[\gamma(p_\sigma)-p_\sigma V]t \times\exp(p_\sigma z). \tag{10}$$

For times long enough to justify the asymptotic treatment, the stationarity of Eq. (10) implies a velocity of the pulse given by

$$\gamma(p_\sigma)=p_\sigma V \tag{11}$$

at the order of $(\ln t)/t$. For any other velocity the pulse is not steady. From Eq. (10), a velocity lower than that given by Eq. (11) yields an unstable pulse exponentially growing until its slope reaches the critical value p_σ, and therefore the critical velocity V. In the opposite case a greater velocity drives a damping. This steady state becomes the *attractor* solution of the whole nonlinear three-wave PDE dynamics proven by the numerical computation. Equations (6), (9), and (11) de-

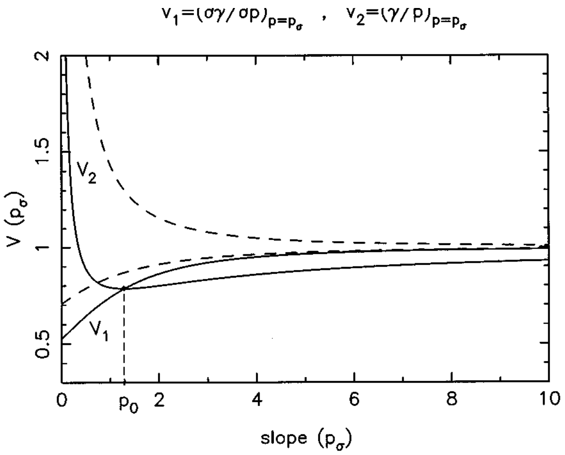

FIG. 5. Curves $V_1(p_\sigma)$ and $V_2(p_\sigma)$, respectively, given by expressions (15) and (16). The intersection point determines the steady subluminous velocity V, given by Eq. (12), of the solitary three-wave attractor. In the absence of Stokes damping ($\mu_S=0$), there is no intersection point (dashed curves), and thus no steady traveling structure [22].

termine the velocity V, the slope of its wave front p_0 and the growth rate $\gamma(p_0)$ of the localized traveling solution $E_{S,a}(x+Vt)$, as functions of the damping parameters μ_S and μ_a:

$$V=\frac{2+\mu_a^2-\mu_S\mu_a+2\sqrt{1-\mu_S\mu_a}}{4+(\mu_a-\mu_S)^2}, \tag{12}$$

$$p_0=\frac{1}{\mu_S}+\mu_S-\frac{1}{\mu_a}-\mu_a+\frac{\sqrt{1-\mu_S\mu_a}}{\mu_a}+\frac{\sqrt{1-\mu_S\mu_a}}{\mu_S}, \tag{13}$$

$$\gamma(p_0)=\frac{1-\mu_S\mu_a+\sqrt{1-\mu_S\mu_a}}{\mu_S}. \tag{14}$$

Let us plot in Fig. 5 the curves $V_1(p_\sigma)$ and $V_2(p_\sigma)$, whose intersection at $p_\sigma=p_0$ determines the steady velocity V. Curve $V_1(p_\sigma)$ is obtained from the saddle point condition (9), with $\gamma(p)$ given by solution (6),

$$V_1=\frac{1}{2}+\frac{p_\sigma+\mu_a-\mu_S}{2\sqrt{(\mu_a-\mu_S+p_\sigma)^2+4}} \tag{15}$$

and $V_2(p_\sigma)$ is obtained from the stationarity condition (11) with $\gamma(p)$ still given by Eq. (6),

$$V_2=\frac{1}{2}-\frac{\mu_S+\mu_a}{2p_\sigma}+\frac{\sqrt{(\mu_a-\mu_S+p_\sigma)^2+4}}{2p_\sigma}. \tag{16}$$

The minimum of the curve $V_2(p_\sigma)$ just corresponds to the steady growth rate $p_\sigma=p_0$. This attractor point is removed to $+\infty$ when $\mu_S\to 0$, the limit point where the dashed lines of Fig. 5 intersect. Therefore no steady solution is attainable in the absence of Stokes damping, in accordance with the result of [22].

We have tested the accuracy of Eq. (12) for several pairs (μ_S,μ_a), and we always find an excellent agreement; e.g., for the case described in Sec. II ($\mu_S=10^{-2};\mu_a=1$), the numerical value for the velocity computed in the far asymptotic stage was $\Delta V_{\text{num}} = -2.510\times 10^{-5}$, while the theoretical value reads $\Delta V_{\text{theor}} = -2.512\times 10^{-5}$, given by Eq. (12) and plotted in Fig. 3(b).

IV. CONCLUSION

We have found a three-wave dissipative solitary structure for an initially bounded Stokes wave packet. The unstable stimulated backscattering process in the presence of a cw pump and material-wave damping, exhibiting a long and complicated space-time nonlinear transient, is finally saturated by even an extremely small Stokes damping, leading to a steady attractor. Let us point out the remarkable agreement of the pulse's characteristic parameters (velocity, slope) obtained from the nonlinear PDE dynamics, in the asymptotic steady state, with the values given by the linear asymptotic KPP procedure. The reason for this agreement lies in the fact that the front slope of the asymptotic steady Stokes structure, which determines the subluminous velocity, coincides with that of its front foot, which always remains in the linear parametric regime. Thus we obtain a steady subluminous three-wave structure, uniquely determined by the material- and Stokes-wave dampings and the cw-pump level, in opposition to the unlimited pulse amplification and compression in the absence of Stokes damping [22], and to the superluminous solitonlike solution [5] which calls for an unbounded Stokes support, and which stability will be analyzed in paper II [20].

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