Gerasimov-Drell-Hearn sum rule and the single-pion photoproduction multipole E_{0+} close to threshold

D. Drechsel and G. Krein*
Institut für Kernphysik, Universität Mainz, 55099 Mainz, Germany
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The long-standing discrepancy between the Gerasimov-Drell-Hearn sum rule and the analysis of pion photoproduction multipoles is greatly diminished by use of s-wave multipoles that are in accord with the predictions of chiral perturbation theory and describe the experimental data in the threshold region. The remaining difference may be due to contributions of channels with more pions and/or heavier mesons whose contributions to the sum rule remain to be investigated by a direct measurement of the photoabsorption cross sections. [S0556-2821(98)05721-X]

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A great deal of our knowledge about the nucleon's ground state and its excited states has been obtained through experiments with electromagnetic probes. The properties of the ground state can be related to photoabsorption cross sections through sum rules. The sum rule derived by Gerasimov, Drell and Hearn (GDH) [1] is one of the most important ones; it provides an astounding relationship between the anomalous magnetic moment κ of the nucleon and the photoabsorption cross sections for parallel and antiparallel alignments of the photon and photon helicities, $\sigma_{3/2}$ and $\sigma_{1/2}$, respectively. Specifically, the GDH sum rule is

$$-\frac{\kappa^2}{4}=\frac{M^2}{8\pi^2\alpha}\int_{\omega_{th}}^{\infty}\frac{\sigma_{1/2}(\omega)-\sigma_{3/2}(\omega)}{\omega}d\omega, \tag{1}$$

with ω_{th} the photoproduction threshold lab energy, $\alpha=e^2/4\pi=1/137$ the fine-structure constant, and M the nucleon mass. The importance of this sum rule is due to the fact that it is based on general principles of physics, such as Lorentz and gauge invariance, crossing symmetry, causality and unitarity. The sum rule has never been measured directly, but estimates for $\sigma_{1/2}$ and $\sigma_{3/2}$ have been made using pion photoproduction amplitudes [2]. The weighting factor $1/\omega$ in Eq. (1) indicates that the low-energy region is very important for the sum rule. It is therefore to be expected that a large fraction of the the sum rule is saturated by s-wave near-threshold pion photoproduction and by $\Delta(1232)$ resonance production.

There exists an extensive literature of studies carried out in this direction [2]. Karliner's work [3] is the first one to include an estimate of the two-pion contribution to the sum rule, and the most recent studies are from Workman and Arndt [4], Burkert and Z. Li [5], Sandorfi, Whisnant and Khandaker [6], and Arndt, Strakovsky and Workman [7]. These analyses are usually performed by an isospin decomposition of the photoproduction multipoles into isovector (VV), isoscalar (SS), and isovector-isoscalar (VS) components, so that the photoabsorption cross sections in Eq. (1) are given by $\sigma_{p,n}=\sigma^{VV}+\sigma^{SS}\pm\sigma^{VS}$ for protons and neutrons, respectively. Similarly we use, on the left hand side of Eq. (1), the relations $\kappa_{p,n}=(\kappa^S\pm\kappa^V)/2$. The general conclusions of these studies are that the SS component is very small and the VV component agrees reasonably well with the prediction of the sum rule, while there occurs an apparent discrepancy for the VS component, neither its magnitude nor its sign agree with the sum rule. The solution for this discrepancy has usually been looked for in phenomena occurring at the higher energies, e.g., in our poor knowledge of two-pion photoproduction or a possible failure of convergence of the GDH sum rule [3].

The purpose of the present communication is to draw attention to the somewhat unnoticed fact that the behavior of the E_{0+} photoproduction multipole in the low energy region, close to the single-pion production threshold is very important for this sum rule. In particular, we show that the use of an E_{0+} amplitude that is in accord with low-energy theorems and describes the experimental data diminishes considerably the discrepancies mentioned above.

The largest and most complete data base of photoproduction observables is provided by the VPI-SAID program [8]. Although there have been changes in the multipoles during the last years, mainly due to new experimental data and reexaminations of errors of older experiments, little has changed with respect to E_{0+}. Very recently, Hanstein, Drechsel and Tiator (HDT) [9–11] have analyzed pion photoproduction imposing constraints from fixed-t dispersion relations and unitarity. In the HDT approach, there are ten free parameters that are fitted to selected photoproduction data for photon energies in the range of 160–420 MeV. In particular, this data set contains the new data from MAMI for differential cross sections of π^0 photoproduction off the proton near threshold [12], and differential cross sections and beam asymmetries for π^+ and π^0 off the proton [13]. An interesting aspect of this approach is that the threshold region is not included in the data basis. Therefore, the threshold values obtained for the s-wave amplitudes are genuine predictions, in the sense that the cross sections above 160 MeV determine the threshold values by analytic continuation of the dispersion integrals. These predictions are in excellent

*Permanent address: Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona, 145 01405-900 São Paulo, SP-Brazil.

agreement with the results of chiral perturbation theory [14]. At threshold, the value of the amplitude $E_{0+}(n\pi^+)$ is 24.9 $\times 10^{-3}/m_{\pi^+}$ in the SAID analysis (version SP97K) and 28.4 $\times 10^{-3}/m_{\pi^+}$ for HDT, $28.4\times 10^{-3}/m_{\pi^+}$ predicted by ChPT [14] and $28.3\pm 0.2\times 10^{-3}/m_{\pi^+}$ according to the evaluation of an older experiment [15]. On the other hand, as has been clearly stated in Ref. [7], the analysis in the very-low energy region becomes very complicated because of the different thresholds for $\pi^0 p$ and $\pi^+ n$ production and therefore the SAID multipoles should not be used in the $\pi^+ n$ threshold region.

The multipole decomposition of the numerator of the integrand of the GDH sum rule is [2]

$$\begin{aligned}\Delta\sigma &\equiv \sigma_{1/2}-\sigma_{3/2}\\ &= 8\pi\frac{q}{k}\sum_{l\geqslant 0}\frac{l+1}{2}[(l+2)(|E_{l+}|^2+|M_{(l+1)-}|^2)\\ &\quad -l(|M_{l+}|^2+|E_{(l+1)-}|^2)\\ &\quad +2l(l+2)(E^*_{l+}M_{l+}-E^*_{(l+1)-}M_{(l+1)-})]\\ &= 8\pi\frac{q}{k}(|E_{0+}|^2+3|E_{1+}|^2+6E^*_{1+}M_{1+}-|M_{1+}|^2\\ &\quad +|M_{1-}|^2+\cdots),\end{aligned} \tag{2}$$

where q and k are the c.m. momenta of the pion and the photon, respectively. Note that $\Delta\sigma$ corresponds to $-2\sigma_{TT'}$ of Ref. [2].

The HDT analysis is limited to photon energies up to 500 MeV, s, p, and d waves for isospin 1/2, and s and p waves for isospin 3/2. For energies above 400 MeV, the differences between the SAID and HDT multipoles are very small. However, large differences occur for the E_{0+} multipole for π^+ production close to threshold, together with some minor differences for M_{1+} below 300 MeV. In Fig. 1 we present the comparison of both analyses for the integrand of Eq. (1) up to 500 MeV for the proton. More specifically, in Fig. 1(a) we plot the contribution of E_{0+}, given by $8\pi(q/\omega k)|E_{0+}|^2$, and in Fig. 1(b) we plot the contribution of M_{1+}, $-8\pi(q/\omega k)|M_{1+}|^2$. In Fig. 1(c) we plot the sum of all multipoles to the integrand. As may be seen from Fig. 1(a), the HDT value for the E_{0+} contribution is substantially larger than in the case of SAID, in accordance with the threshold behavior of this amplitude as discussed above. Together with a much smaller (but opposite) effect for the M_{1+} multipole [see Fig. 1(b)], this clearly leads to a larger integrand in the case of HDT as shown in Fig. 1(c).

As a result the observed difference in these two multipoles, the value of the integral of Eq. (1) for the proton,

$$I_p=\int_{\omega_{th}}^{\infty}\frac{\sigma^p_{1/2}(\omega)-\sigma^p_{3/2}(\omega)}{\omega}d\omega, \tag{3}$$

is changed by 20 μb. Using the estimate of Karliner [3] for the two-pion contribution, $I_p(2\pi)=-65\ \mu b$ and the SAID multipoles (SP97K solution) for the single-pion production, which gives $I_p(1\pi)=-216\ \mu b$, one obtains $I_p=-281\ \mu b$ (Sandorfi *et al.* [6] obtain $I_p=-289\ \mu b$ using another solution of the SAID multipoles). This has to be compared to the GDH value, $I_p=-281\ \mu b$. Correcting then the one-pion contribution for the proper low energy behavior, we predict $I_p(1\pi)=-196\ \mu b$ and $I_p=-261\ \mu b$, if we include the two-pion contribution as estimated by Karliner. Expressed in different words, the discrepancy reduces from 38% to 28% by use of s-wave multipoles that are in accord with the low energy theorems and describe the experimental

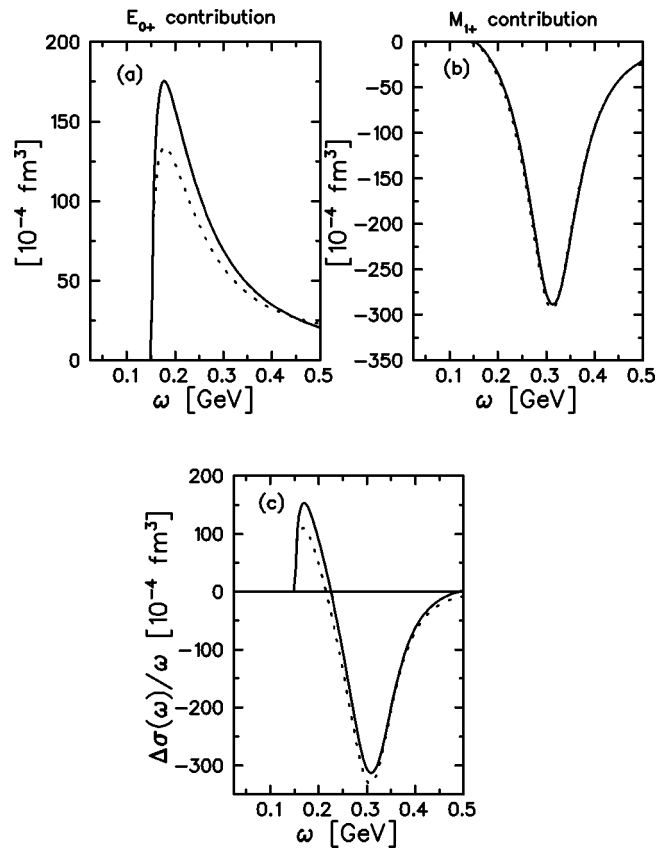

FIG. 1. (a) Contribution of the multipole E_{0+} to the integrand of Eq. (3), $8\pi(q/\omega k)|E_{0+}|^2$, (b) the corresponding contribution of M_{1+}, $-8\pi(q/\omega k)|M_{1+}|^2$, and (c) the integrand for the complete calculation including all partial waves. The solid lines correspond to the HDT multipoles and the dotted to the SAID multipoles.

TABLE I. Predictions from various models and data analyses for the GDH integral for proton (I_p), neutron (I_n), and the difference I_p-I_n in units of μb. The results, with exception of the ones of Ref. [5], include the two-pion background as estimated by Karliner [3].

	I_p	I_n	I_p-I_n
GDH integral	−204.5	−232.8	28.3
Karliner [3]	−261	−183	−78
Workman and Arndt [4]	−260	−157	−103
Burkert and Li [5]	−203	–	–
Sandorfi *et al.* [6]	−289	−160	−129
This work	−261	−180	−81

data in the threshold region. Concerning the remaining discrepancy it has to be said the estimate of the two-pion contribution of Ref. [3] relies heavily on the assumption that the two-pion contribution is generated by the resonances, and that its helicity structure follows the known behavior of the one-pion contribution as given by Eq. (2). It is not obvious, however, that the two-pion background has to be resonance dominated, and it will be most interesting to see the outcome of the GDH experiment scheduled at MAMI and ELSA [16]. For the neutron, the difference in the multipoles leads to a change by 17 μb. This is a substantial improvement, but still not enough to reverse the sign of I^{VS}.

In Table I we present the results of different studies of the sum rule. With the exception of the values given by Burkert and Li [5], all results in Table I include the estimate of Karliner [3] for the two-pion background. The results of the most recent analysis of Arndt, Strakovsky and Workman [7] are not presented in Table I because the authors do not quote their numbers, but do mention that their results are not very different from the ones of Ref. [6]. It is interesting to notice that if one uses the estimate of Burkert and Li for the contributions beyond one-pion production, $-32\ \mu b$, together with the one-pion results of the HDT multipoles, the results are still closer to the prediction of the sum rule. The discrepancy in this case would fall to 12%.

In conclusion, we would like to draw attention to the somewhat unnoticed fact that a precise threshold of the E_{0+} single-pion photoproduction multipole is quite essential for the GDH sum rule. The remaining discrepancies might be due to the non-resonant backgrounds.

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[1] S. B. Gerasimov, Sov. J. Nucl. Phys. **2**, 430 (1966), S. D. Drell and A. C. Hearn, Phys. Rev. Lett. **16**, 908 (1966).
[2] For a recent review see, D. Drechsel, Prog. Part. Nucl. Phys. **34**, 181 (1995).
[3] I. Karliner, Phys. Rev. D **7**, 2717 (1973).
[4] R. L. Workman and R. A. Arndt, Phys. Rev. D **45**, 1789 (1992).
[5] V. Burkert and Z. Li, Phys. Rev. D **47**, 46 (1993).
[6] A. M. Sandorfi, C. S. Whisnant, and M. Khandaker, Phys. Rev. D **50**, R6681 (1994).
[7] R. A. Arndt, I. I. Strakovsky, and R. Workman, Phys. Rev. C **53**, 430 (1996).
[8] VPI-SAID program: The Scattering Analysis Interactive Dial-in program, data available via telnet to VTINTE.PHYS.VT.EDU
[9] O. Hanstein, D. Drechsel, and L. Tiator, Phys. Lett. B **385**, 45 (1996).
[10] O. Hanstein, D. Drechsel, and L. Tiator, Phys. Lett. B **399**, 13 (1996).
[11] O. Hanstein, D. Drechsel, and L. Tiator, Nucl. Phys. **A632**, 561 (1998).
[12] M. Fuchs *et al.*, Phys. Lett. B **368**, 20 (1996).
[13] R. Beck *et al.*, Phys. Rev. Lett. **78**, 606 (1997); F. Härter, Ph.D. thesis, Mainz, 1996.
[14] V. Bernard, N. Kaiser, and U.-G. Meissner, Nucl. Phys. **B383**, 442 (1992).
[15] M. I. Adamovich, Proc. P. N. Lebedev Phys. Inst. **71**, 119 (1976).
[16] GDH Collaboration, J. Ahrends *et al.*, ELSA and MAMI proposal (A2/2-95).