

Decay of protons and neutrons induced by acceleration

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(Received 27 October 1998; published 16 March 1999)

We investigate the decay of accelerated protons and neutrons. Calculations are carried out in the inertial and coaccelerated frames. Particle interpretation of these processes are quite different in each frame but the decay rates are verified to agree in both cases. For the sake of simplicity our calculations are performed in a two-dimensional spacetime since our conclusions are not conceptually affected by this.

[S0556-2821(99)03307-X]

PACS number(s): 13.30.-a, 04.62.+v, 12.15.Ji, 14.20.Dh

I. INTRODUCTION

It is well known that according to the standard model the mean proper lifetime of neutrons is about $\tau_n = 887$ s while protons are stable ($\tau_p > 1.6 \times 10^{25}$ yr) [1]. This is only true, however, for *inertial* nucleons. There are a number of high-energy phenomena where acceleration plays a crucial role (see Refs. [2] and [3,4] for comprehensive discussions on electron depolarization in storage rings and bremsstrahlung, respectively). The influence of acceleration in particle decay was only considered quite recently [5]. As was pointed out, acceleration effects are not expected to play a significant role in most particle decays observed in the laboratory. Notwithstanding, this might not be so under certain astrophysical and cosmological conditions. Muller has estimated [5] the time decay of accelerated μ^- , π^- , and p^+ via the following processes:

$$(i) \quad \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu,$$

$$(ii) \quad \pi^- \rightarrow \mu^- \bar{\nu}_\mu,$$

$$(iii) \quad p^+ \rightarrow n e^+ \nu_e,$$

as described in the laboratory frame. Here we analyze in more detail process (iii) and the related one

$$(iv) \quad n \rightarrow p^+ e^- \bar{\nu}_e.$$

Process (iii) is probably the most interesting one in the sense that the proton *must* be accelerated in order to give rise to a nonvanishing rate. In the remaining cases, nonvanishing rates are obtained even when the decaying particles (μ^- , π^- , n) are inertial. As a first approximation, Muller has considered that all particles involved are scalars. Here we shall treat e^- , ν_e , and the corresponding antiparticles as fermions while p^+ and n will be represented by a classical current. This is a suitable approximation as far as these nucleons are energetic enough to have a well defined trajectory. Moreover, we will analyze β and inverse β decays in the coaccelerated frame in addition to in the inertial frame. This is interesting because the particle content of these decays will be quite different in each one of these frames. This is a consequence of the fact that the Minkowski vacuum corresponds to a thermal state of Rindler particles [6,7]. We

have chosen to perform the calculations in a two-dimensional spacetime because there is no conceptual loss at all. A comprehensive (but restricted to the inertial frame) four-dimensional spectral analysis of the inverse β decay for accelerated protons and a discussion of its possible importance to cosmology and astrophysics will be presented elsewhere.

The paper is organized as follows. In Sec. II we introduce the classical current which suitably describes the decay of accelerated nucleons. Section III is devoted to calculate the β and inverse β decay rates in the inertial frame. In Sec. IV we review the quantization of the fermionic field in the coaccelerated frame. In Sec. V we compute the β and inverse β decay rates in the accelerated frame. For this purpose we must take into account the Fulling-Davies-Unruh thermal bath [6,7]. Finally, in Sec. VI we discuss our results and further perspectives. We will use natural units $k_B = c = \hbar = 1$ throughout this paper unless stated otherwise.

II. DECAYING-NUCLEON CURRENT

In order to describe the uniformly accelerated nucleon, it is convenient to introduce the Rindler wedge. The Rindler wedge is the portion of Minkowski spacetime defined by $z > |t|$ where (t, z) are the usual Minkowski coordinates. It is convenient to cover the Rindler wedge with Rindler coordinates (v, u) which are related with (t, z) by

$$t = u \sinh v, \quad z = u \cosh v, \quad (2.1)$$

where $0 < u < +\infty$ and $-\infty < v < +\infty$. As a result, the line element of the Rindler wedge is written as

$$ds^2 = u^2 dv^2 - du^2. \quad (2.2)$$

The world line of a uniformly accelerated particle with proper acceleration a is given in these coordinates by $u = a^{-1} = \text{const}$. Particles following this world line have proper time $\tau = v/a$. Thus let us describe a uniformly accelerated nucleon through the vector current

$$j^\mu = qu^\mu \delta(u - a^{-1}), \quad (2.3)$$

where q is a small coupling constant and u^μ is the nucleon's four-velocity: $u^\mu = (a, 0)$ and $u^\mu = (\sqrt{a^2 t^2 + 1}, at)$ in Rindler and Minkowski coordinates, respectively.

The current above is fine to describe stable accelerated nucleons but must be improved to allow nucleon-decay processes. For this purpose, let us consider the nucleon as a two-level system [7–9]. In this scenario, neutrons $|n\rangle$ and protons $|p\rangle$ are going to be seen as excited and unexcited states of the nucleon, respectively, and are assumed to be eigenstates of the nucleon Hamiltonian \hat{H} :

$$\hat{H}|n\rangle = m_n|n\rangle, \quad \hat{H}|p\rangle = m_p|p\rangle, \quad (2.4)$$

where m_n and m_p are the neutron and proton mass, respectively. Hence, in order to consider nucleon decay processes, we replace q in Eq. (2.3) by the Hermitian monopole

$$\hat{q}(\tau) \equiv e^{i\hat{H}\tau} \hat{q}_0 e^{-i\hat{H}\tau}. \quad (2.5)$$

Here $G_F \equiv |\langle m_p | \hat{q}_0 | m_n \rangle|$ will play the role of the Fermi constant in the two-dimensional Minkowski spacetime. As a result, current (2.3) will be replaced by

$$\hat{j}^\mu = \hat{q}(\tau) u^\mu \delta(u - a^{-1}). \quad (2.6)$$

III. INERTIAL FRAME CALCULATION OF THE β AND INVERSE β DECAY FOR ACCELERATED NUCLEONS

Let us firstly analyze the decay of uniformly accelerated protons and neutrons in the inertial frame [see processes (iii) and (iv) in Sec. I]. We shall describe electrons and neutrinos as fermionic fields:

$$\hat{\Psi}(t, z) = \sum_{\sigma=\pm} \int_{-\infty}^{+\infty} dk [\hat{b}_{k\sigma} \psi_{k\sigma}^{(+\omega)}(t, z) + \hat{d}_{k\sigma}^\dagger \psi_{-k-\sigma}^{(-\omega)}(t, z)], \quad (3.1)$$

where $\hat{b}_{k\sigma}$ and $\hat{d}_{k\sigma}^\dagger$ are annihilation and creation operators of fermions and antifermions, respectively, with momentum k and polarization σ . In the inertial frame, frequency, momentum and mass m are related as usual: $\omega = \sqrt{k^2 + m^2} > 0$. $\psi_{k\sigma}^{(+\omega)}$ and $\psi_{k\sigma}^{(-\omega)}$ are positive and negative frequency solutions of the Dirac equation $i\gamma^\mu \partial_\mu \psi_{k\sigma}^{(\pm\omega)} - m\psi_{k\sigma}^{(\pm\omega)} = 0$. By using the γ^μ matrices in the Dirac representation (see, e.g., Ref. [4]), we find

$$\psi_{k+}^{(\pm\omega)}(t, z) = \frac{e^{i(\mp\omega t + kz)}}{\sqrt{2\pi}} \begin{pmatrix} \pm \sqrt{(\omega \pm m)/2\omega} \\ 0 \\ k/\sqrt{2\omega(\omega \pm m)} \\ 0 \end{pmatrix} \quad (3.2)$$

and

$$\psi_{k-}^{(\pm\omega)}(t, z) = \frac{e^{i(\mp\omega t + kz)}}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \pm \sqrt{(\omega \pm m)/2\omega} \\ 0 \\ -k/\sqrt{2\omega(\omega \pm m)} \end{pmatrix}. \quad (3.3)$$

In order to keep a unified procedure for inertial and accelerated frame calculations, we have orthonormalized modes (3.2), (3.3) according to the same inner product definition [9] that will be used in Sec. IV:

$$\begin{aligned} \langle \psi_{k\sigma}^{(\pm\omega)}, \psi_{k'\sigma'}^{(\pm\omega')} \rangle &\equiv \int_{\Sigma} d\Sigma_\mu \bar{\psi}_{k\sigma}^{(\pm\omega)} \gamma^\mu \psi_{k'\sigma'}^{(\pm\omega')} \\ &= \delta(k - k') \delta_{\sigma\sigma'} \delta_{\pm\omega \pm \omega'}, \end{aligned} \quad (3.4)$$

where $\bar{\psi} \equiv \psi^\dagger \gamma^0$, $d\Sigma_\mu \equiv n_\mu d\Sigma$ with n^μ being a unit vector orthogonal to Σ and pointing to the future, and Σ is an arbitrary spacelike hypersurface. (In this section, we have chosen $t = \text{const}$ for the hypersurface Σ .) As a consequence canonical anticommutation relations for fields and conjugate momenta lead to the following simple anticommutation relations for creation and annihilation operators

$$\{\hat{b}_{k\sigma}, \hat{b}_{k'\sigma'}^\dagger\} = \{\hat{d}_{k\sigma}, \hat{d}_{k'\sigma'}^\dagger\} = \delta(k - k') \delta_{\sigma\sigma'} \quad (3.5)$$

and

$$\{\hat{b}_{k\sigma}, \hat{b}_{k'\sigma'}\} = \{\hat{d}_{k\sigma}, \hat{d}_{k'\sigma'}\} = \{\hat{b}_{k\sigma}, \hat{d}_{k'\sigma'}^\dagger\} = \{\hat{b}_{k\sigma}, \hat{d}_{k'\sigma'}\} = 0. \quad (3.6)$$

Next we couple minimally electron $\hat{\Psi}_e$ and neutrino $\hat{\Psi}_\nu$ fields to the nucleon current (2.6) according to the Fermi action

$$\hat{S}_I = \int d^2x \sqrt{-g} j_\mu (\hat{\Psi}_\nu \gamma^\mu \hat{\Psi}_e + \hat{\Psi}_e \gamma^\mu \hat{\Psi}_\nu). \quad (3.7)$$

Note that the first and second terms inside the parenthesis at the right-hand side (RHS) of Eq. (3.7) vanish for the β decay [process (iv) in Sec. I] and inverse β decay [process (iii) in Sec. I], respectively.

Let us first consider the inverse β decay. The vacuum transition amplitude is given by

$$\mathcal{A}_{(iii)}^{p \rightarrow n} = \langle n | \otimes \langle e_{k_e \sigma_e}^+, \nu_{k_\nu \sigma_\nu} | \hat{S}_I | 0 \rangle \otimes | p \rangle. \quad (3.8)$$

By using current (2.6) in Eq. (3.7), and acting with \hat{S}_I on the nucleon states in Eq. (3.8), we obtain

$$\begin{aligned} \mathcal{A}_{(iii)}^{p \rightarrow n} &= G_F \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dz \frac{e^{i\Delta m \tau}}{\sqrt{a^2 t^2 + 1}} u_\mu \delta(z - \sqrt{t^2 + a^{-2}}) \\ &\quad \times \langle e_{k_e \sigma_e}^+, \nu_{k_\nu \sigma_\nu} | \hat{\Psi}_\nu \gamma^\mu \hat{\Psi}_e | 0 \rangle, \end{aligned} \quad (3.9)$$

where $\Delta m \equiv m_n - m_p$, $\tau = a^{-1} \sinh^{-1}(at)$ is the nucleon's proper time and we recall that in Minkowski coordinates the four-velocity is $u^\mu = (\sqrt{a^2 t^2 + 1}, at)$ [see Eq. (2.3) below]. The numerical value of the two-dimensional Fermi constant G_F will be fixed further. By using the fermionic field (3.1) in Eq. (3.9) and solving the integral in the z variable, we obtain

$$\mathcal{A}_{(iii)}^{p \rightarrow n} = \frac{-(G_F/4\pi) \delta_{\sigma_e, -\sigma_\nu}}{\sqrt{\omega_\nu \omega_e (\omega_\nu + m_\nu) (\omega_e - m_e)}} \int_{-\infty}^{+\infty} d\tau e^{i[\Delta m \tau + a^{-1}(\omega_e + \omega_\nu) \sinh a \tau - a^{-1}(k_e + k_\nu) \cosh a \tau]} \\ \times \{[(\omega_\nu + m_\nu)(\omega_e - m_e) + k_\nu k_e] \cosh a \tau - [(\omega_\nu + m_\nu)k_e + (\omega_e - m_e)k_\nu] \sinh a \tau\}.$$

The differential transition rate

$$\frac{d^2 \mathcal{P}_{in}^{p \rightarrow n}}{dk_e dk_\nu} = \sum_{\sigma_e = \pm} \sum_{\sigma_\nu = \pm} |\mathcal{A}_{(iii)}^{p \rightarrow n}|^2 \quad (3.10)$$

calculated in the inertial frame will thus be

$$\frac{d^2 \mathcal{P}_{in}^{p \rightarrow n}}{dk_e dk_\nu} = \frac{G_F^2}{8\pi^2} \int_{-\infty}^{+\infty} d\tau_1 \int_{-\infty}^{+\infty} d\tau_2 \exp[i\Delta m(\tau_1 - \tau_2)] \exp\{i(\omega_e + \omega_\nu)(\sinh a \tau_1 - \sinh a \tau_2)/a - i(k_e + k_\nu) \\ \times [(\cosh a \tau_1 - \cosh a \tau_2)/a]\} \left(\frac{(\omega_\nu + m_\nu)(\omega_e - m_e) + k_\nu k_e}{\sqrt{\omega_\nu \omega_e (\omega_\nu + m_\nu) (\omega_e - m_e)}} \cosh a \tau_1 - \frac{(\omega_\nu + m_\nu)k_e + (\omega_e - m_e)k_\nu}{\sqrt{\omega_\nu \omega_e (\omega_\nu + m_\nu) (\omega_e - m_e)}} \sinh a \tau_1 \right) \\ \times \left(\frac{(\omega_\nu + m_\nu)(\omega_e - m_e) + k_\nu k_e}{\sqrt{\omega_\nu \omega_e (\omega_\nu + m_\nu) (\omega_e - m_e)}} \cosh a \tau_2 - \frac{(\omega_\nu + m_\nu)k_e + (\omega_e - m_e)k_\nu}{\sqrt{\omega_\nu \omega_e (\omega_\nu + m_\nu) (\omega_e - m_e)}} \sinh a \tau_2 \right).$$

In order to decouple the integrals above, it is convenient to introduce first new variables s and ξ such that $\tau_1 \equiv s + \xi/2$, $\tau_2 \equiv s - \xi/2$. After this, we write

$$\frac{d^2 \mathcal{P}_{in}^{p \rightarrow n}}{dk_e dk_\nu} = \frac{G_F^2}{4\pi^2 \omega_\nu \omega_e} \int_{-\infty}^{+\infty} ds \int_{-\infty}^{+\infty} d\xi e^{i\{\Delta m \xi + 2a^{-1} \sinh(a\xi/2)[(\omega_\nu + \omega_e) \cosh as - (k_\nu + k_e) \sinh as]\}} [(\omega_\nu \omega_e + k_\nu k_e) \cosh 2as \\ - (\omega_e k_\nu + \omega_\nu k_e) \sinh 2as - m_\nu m_e \cosh a\xi]. \quad (3.11)$$

Next, by defining a new change of variables,

$$k_{e(\nu)} \rightarrow k'_{e(\nu)} = -\omega_{e(\nu)} \sinh(as) + k_{e(\nu)} \cosh(as),$$

we are able to perform the integral in the s variable, and the differential transition rate (3.11) can be cast in the form

$$\frac{1}{T} \frac{d^2 \mathcal{P}_{in}^{p \rightarrow n}}{dk'_e dk'_\nu} = \frac{G_F^2}{4\pi^2 \omega'_e \omega'_\nu} \int_{-\infty}^{+\infty} d\xi \\ \times \exp[i\Delta m \xi + i2a^{-1}(\omega'_e + \omega'_\nu) \sinh(a\xi/2)] \\ \times (\omega'_\nu \omega'_e + k'_\nu k'_e - m_\nu m_e \cosh a\xi), \quad (3.12)$$

where $T \equiv \int_{-\infty}^{+\infty} ds$ is the total proper time and $\omega'_{e(\nu)} \equiv \sqrt{k'^2_{e(\nu)} + m^2_{e(\nu)}}$.

The total transition rate $\Gamma_{in}^{p \rightarrow n} = \mathcal{P}_{in}^{p \rightarrow n}/T$ is obtained after integrating Eq. (3.12) in both momentum variables. For this purpose it is useful to make the following change of variables:

$$k'_{e(\nu)} \rightarrow \tilde{k}_{e(\nu)} \equiv k'_{e(\nu)}/a, \quad \xi \rightarrow \lambda \equiv e^{a\xi/2}.$$

(Note that $\tilde{k}_{e(\nu)}$ is adimensional.) Hence we obtain

$$\Gamma_{in}^{p \rightarrow n} = \frac{G_F^2 a}{2\pi^2} \int_{-\infty}^{+\infty} \frac{d\tilde{k}_e}{\tilde{\omega}_e} \int_{-\infty}^{+\infty} \frac{d\tilde{k}_\nu}{\tilde{\omega}_\nu} \int_0^{+\infty} \frac{d\lambda}{\lambda^{1-i2\Delta m/a}} \\ \times \exp[i(\tilde{\omega}_e + \tilde{\omega}_\nu)(\lambda - \lambda^{-1})] \\ \times [\tilde{\omega}_\nu \tilde{\omega}_e + \tilde{k}_\nu \tilde{k}_e - m_\nu m_e (\lambda^2 + \lambda^{-2})/(2a^2)], \quad (3.13)$$

where $\tilde{\omega}_{e(\nu)} \equiv \sqrt{\tilde{k}_{e(\nu)}^2 + m^2_{e(\nu)}/a^2}$.

Let us assume at this point $m_\nu \rightarrow 0$. In this case, using Eqs. (3.871.3)–(3.871.4) of Ref. [10], we perform the integration in λ and obtain the following final expression for the proton decay rate:

$$\Gamma_{in}^{p \rightarrow n} = \frac{4G_F^2 a}{\pi^2 e^{\pi \Delta m/a}} \int_0^{+\infty} d\tilde{k}_e \int_0^{+\infty} d\tilde{k}_\nu \\ \times K_{i2\Delta m/a} [2(\sqrt{\tilde{k}_e^2 + m_e^2/a^2} + \tilde{k}_\nu)]. \quad (3.14)$$

Performing analogous calculation for the β decay, we obtain for the neutron differential and total decay rates the following expressions [see Eqs. (3.12) and (3.13)]:

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{\text{in}}^{n \rightarrow p}}{dk'_e dk'_\nu} &= \frac{G_F^2}{4\pi^2 \omega'_e \omega'_\nu} \int_{-\infty}^{+\infty} d\xi \exp[-i\Delta m \xi + i2a^{-1} \\ &\quad \times (\omega'_e + \omega'_\nu) \sinh(a\xi/2)] \\ &\quad \times (\omega'_\nu \omega'_e + k'_\nu k'_e - m_\nu m_e \cosh a\xi) \end{aligned} \quad (3.15)$$

and

$$\begin{aligned} \Gamma_{\text{in}}^{n \rightarrow p} &= \frac{G_F^2 a}{2\pi^2} \int_{-\infty}^{+\infty} \frac{d\tilde{k}_e}{\tilde{\omega}_e} \int_{-\infty}^{+\infty} \frac{d\tilde{k}_\nu}{\tilde{\omega}_\nu} \int_0^{+\infty} \frac{d\lambda}{\lambda^{1+i2\Delta m/a}} \\ &\quad \times \exp[i(\tilde{\omega}_e + \tilde{\omega}_\nu)(\lambda - \lambda^{-1})] \\ &\quad \times [\tilde{\omega}_\nu \tilde{\omega}_e + \tilde{k}_\nu \tilde{k}_e - m_\nu m_e (\lambda^2 + \lambda^{-2}) / (2a^2)]. \end{aligned} \quad (3.16)$$

By making $m_\nu \rightarrow 0$ in the expression above, we end up with

$$\begin{aligned} \Gamma_{\text{in}}^{n \rightarrow p} &= \frac{4G_F^2 a}{\pi^2 e^{-\pi\Delta m/a}} \int_0^{+\infty} d\tilde{k}_e \int_0^{+\infty} d\tilde{k}_\nu \\ &\quad \times K_{i2\Delta m/a} [2(\sqrt{\tilde{k}_e^2 + m_e^2/a^2} + \tilde{k}_\nu)]. \end{aligned} \quad (3.17)$$

In order to determine the value of our two-dimensional Fermi constant G_F we will impose the mean proper lifetime $\tau_n(a) = 1/\Gamma_{\text{in}}^{n \rightarrow p}$ of an inertial neutron to be 887 s [1]. By taking $a \rightarrow 0$ and integrating both sides of Eq. (3.15) with respect to the momentum variables, we obtain

$$\begin{aligned} \Gamma_{\text{in}}^{n \rightarrow p}|_{a \rightarrow 0} &= \frac{G_F^2}{4\pi^2} \int_{-\infty}^{+\infty} \frac{dk'_e}{\omega'_e} \int_{-\infty}^{+\infty} \frac{dk'_\nu}{\omega'_\nu} \int_{-\infty}^{+\infty} d\xi \\ &\quad \times \exp[i(\omega'_e + \omega'_\nu)\xi] \exp(-i\Delta m \xi) \\ &\quad \times (\omega'_\nu \omega'_e + k'_\nu k'_e - m_\nu m_e). \end{aligned} \quad (3.18)$$

Next, by performing the integral in ξ , we obtain

$$\begin{aligned} \Gamma_{\text{in}}^{n \rightarrow p}|_{a \rightarrow 0} &= \frac{2G_F^2}{\pi} \int_{m_e}^{+\infty} \frac{d\omega'_e}{\sqrt{\omega'^2_e - m_e^2}} \int_{m_\nu}^{+\infty} \frac{d\omega'_\nu}{\sqrt{\omega'^2_\nu - m_\nu^2}} \\ &\quad \times (\omega'_\nu \omega'_e - m_\nu m_e) \delta(\omega'_\nu + \omega'_e - \Delta m). \end{aligned} \quad (3.19)$$

Now it is easy to perform the integral in ω'_ν :

$$\begin{aligned} \Gamma_{\text{in}}^{n \rightarrow p}|_{a \rightarrow 0} &= \frac{2G_F^2}{\pi} \int_{m_e}^{\Delta m - m_\nu} \frac{d\omega'_e}{\sqrt{\omega'^2_e - m_e^2}} \\ &\quad \times \frac{\omega'_e(\Delta m - \omega'_e) - m_\nu m_e}{\sqrt{(\Delta m - \omega'_e)^2 - m_\nu^2}}. \end{aligned} \quad (3.20)$$

By integrating the right-hand side of Eq. (3.20) with $m_\nu \rightarrow 0$ and imposing $1/\Gamma_{\text{in}}^{n \rightarrow p}|_{a \rightarrow 0}$ to be 887 s, we obtain $G_F = 9.918 \times 10^{-13}$. Note that $G_F \ll 1$ which corroborates our perturbative approach. Now we are able to plot the neutron

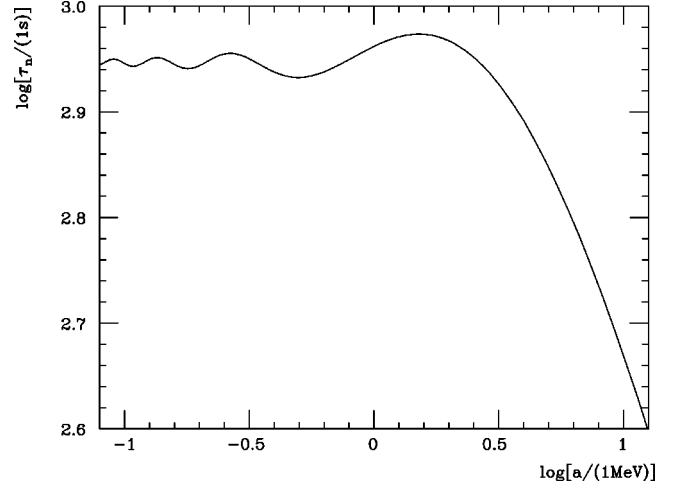


FIG. 1. The neutron mean proper lifetime is plotted as a function of the proper acceleration a . Note that $\tau_n \rightarrow 887$ s as the proper acceleration vanishes. After an oscillatory regime τ_n decreases steadily as the acceleration increases.

mean proper lifetime as a function of its proper acceleration a (see Fig. 1). Note that after an oscillatory regime it decays steadily. In Fig. 2 we plot the proton mean proper lifetime. The necessary energy to allow protons to decay is provided by the external accelerating agent. For accelerations such that the Fulling-Davies-Unruh (FDU) temperature (see discussion in Sec. V) is of order of $m_n + m_e - m_p$, i.e., $a/2\pi \approx 1.8$ MeV, we have that $\tau_p \approx \tau_n$ (see Figs. 1 and 2). Such accelerations are considerably high. Just for sake of comparison, protons at the CERN Large Hadron Collider (LHC) have a proper acceleration of about 10^{-8} MeV.

IV. FERMIONIC FIELD QUANTIZATION IN A TWO-DIMENSIONAL RINDLER WEDGE

We shall briefly review [11] the quantization of the fermionic field in the accelerated frame since this will be crucial

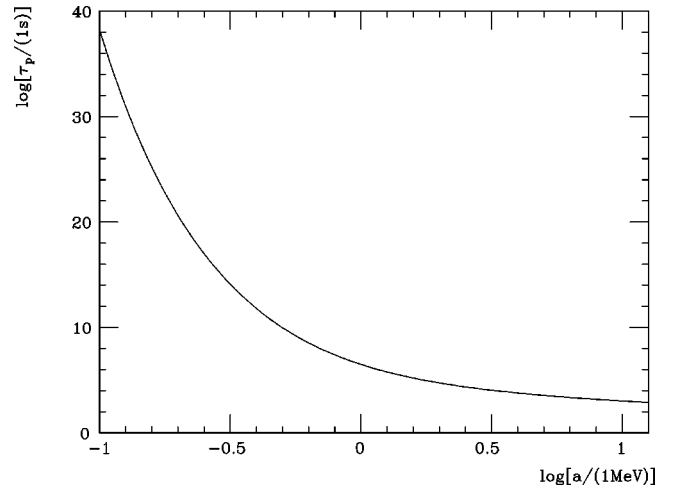


FIG. 2. The proton mean proper lifetime is plotted as a function of the proper acceleration a . $\tau_p \rightarrow +\infty$ for inertial protons ($a \rightarrow 0$). For accelerations $a \approx a_c \equiv 2\pi(m_n + m_e - m_p) \approx 11$ MeV we have that $\tau_p \approx \tau_n$.

for our further purposes. Let us consider the two-dimensional Rindler wedge described by the line element (2.2). The Dirac equation in curved spacetime is $(i\gamma_R^\mu \bar{\nabla}_\mu - m)\psi_{\omega\sigma} = 0$, where $\gamma_R^\mu \equiv (e_\alpha)^\mu \gamma^\alpha$ are the Dirac matrices in curved spacetime, $\bar{\nabla}_\mu \equiv \partial_\mu + \Gamma_\mu$ and $\Gamma_\mu = \frac{1}{8}[\gamma^\alpha, \gamma^\beta](e_\alpha)^\lambda \bar{\nabla}_\mu (e_\beta)_\lambda$ are the Fock-Kondratenko connections. (γ^μ are the usual flat-spacetime Dirac matrices.) In the Rindler wedge the relevant tetrads are $(e_0)^\mu = u^{-1} \delta_0^\mu$, $(e_i)^\mu = \delta_i^\mu$. As a consequence, the Dirac equation takes the form

$$i \frac{\partial \psi_{\omega\sigma}}{\partial v} = \left(\gamma^0 m u - \frac{i\alpha_3}{2} - i u \alpha_3 \frac{\partial}{\partial u} \right) \psi_{\omega\sigma}, \quad (4.1)$$

where $\alpha_i \equiv \gamma^0 \gamma^i$.

We shall express the fermionic field as

$$\hat{\Psi}(v, u) = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega [\hat{b}_{\omega\sigma} \psi_{\omega\sigma}(v, u) + \hat{d}_{\omega\sigma}^\dagger \psi_{-\omega-\sigma}(v, u)], \quad (4.2)$$

where $\psi_{\omega\sigma} = f_{\omega\sigma}(u) e^{-i\omega v/a}$ are positive ($\omega > 0$) and negative ($\omega < 0$) energy solutions with respect to the boost Killing field $\partial/\partial v$ with polarization $\sigma = \pm$. From Eq. (4.1) we obtain

$$\hat{H}_u f_{\omega\sigma} = \omega f_{\omega\sigma}, \quad (4.3)$$

where

$$\hat{H}_u \equiv a \left[m u \gamma^0 - \frac{i\alpha_3}{2} - i u \alpha_3 \frac{\partial}{\partial u} \right]. \quad (4.4)$$

By ‘‘squaring’’ Eq. (4.3) and defining two-component spinors $\chi_j (j=1,2)$ through

$$f_{\omega\sigma}(u) \equiv \begin{pmatrix} \chi_1(u) \\ \chi_2(u) \end{pmatrix}, \quad (4.5)$$

we obtain

$$\left(u \frac{d}{du} u \frac{d}{du} \right) \chi_1 = \left[m^2 u^2 + \frac{1}{4} - \frac{\omega^2}{a^2} \right] \chi_1 - \frac{i\omega}{a} \sigma_3 \chi_2, \quad (4.6)$$

$$\left(u \frac{d}{du} u \frac{d}{du} \right) \chi_2 = \left[m^2 u^2 + \frac{1}{4} - \frac{\omega^2}{a^2} \right] \chi_2 - \frac{i\omega}{a} \sigma_3 \chi_1. \quad (4.7)$$

Next, by introducing $\phi^\pm \equiv \chi_1 \mp \chi_2$, we can define ξ^\pm and ζ^\pm through

$$\phi^\pm \equiv \begin{pmatrix} \xi^\pm(u) \\ \zeta^\pm(u) \end{pmatrix}. \quad (4.8)$$

In terms of these variables Eqs. (4.6),(4.7) become

$$\left(u \frac{d}{du} u \frac{d}{du} \right) \xi^\pm = [m^2 u^2 + (i\omega/a \pm 1/2)^2] \xi^\pm, \quad (4.9)$$

$$\left(u \frac{d}{du} u \frac{d}{du} \right) \zeta^\pm = [m^2 u^2 + (i\omega/a \mp 1/2)^2] \zeta^\pm. \quad (4.10)$$

The solutions of these differential equations can be written in terms of Hankel functions $H_{i\omega/a \pm 1/2}^{(j)}(imu)$, ($j=1,2$) [see Eq. (8.491.6) of Ref. [10]] or modified Bessel functions $K_{i\omega/a \pm 1/2}(mu)$, $I_{i\omega/a \pm 1/2}(mu)$ [see Eq. (8.494.1) of Ref. [10]]. Hence, by using Eqs. (4.5) and (4.8), and imposing that the solutions satisfy the first-order Eq. (4.3), we obtain

$$f_{\omega+}(u) = A_+ \begin{pmatrix} K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \\ -K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \end{pmatrix}, \quad (4.11)$$

$$f_{\omega-}(u) = A_- \begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \\ K_{i\omega/a+1/2}(mu) - iK_{i\omega/a-1/2}(mu) \end{pmatrix}. \quad (4.12)$$

Note that solutions involving $I_{i\omega/a \pm 1/2}$ turn out to be non-normalizable and thus must be neglected. In order to find the normalization constants

$$A_+ = A_- = \left[\frac{m \cosh(\pi\omega/a)}{2\pi^2 a} \right]^{1/2}, \quad (4.13)$$

we have used [9] [see also Eq. (3.4)]

$$\langle \psi_{\omega\sigma}, \psi_{\omega'\sigma'} \rangle \equiv \int_\Sigma d\Sigma_\mu \bar{\psi}_{\omega\sigma} \gamma_R^\mu \psi_{\omega'\sigma'} = \delta(\omega - \omega') \delta_{\sigma\sigma'}, \quad (4.14)$$

where $\bar{\psi} \equiv \psi^\dagger \gamma^0$ and Σ is set to be $v = \text{const}$. Thus the normal modes of the fermionic field (4.2) are

$$\psi_{\omega+} = \left[\frac{m \cosh(\pi\omega/a)}{2\pi^2 a} \right]^{1/2} \times \begin{pmatrix} K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \\ -K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \end{pmatrix} e^{-i\omega v/a}, \quad (4.15)$$

$$\psi_{\omega^-} = \left[\frac{m \cosh(\pi\omega/a)}{2\pi^2 a} \right]^{1/2} \times \begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(m\omega) + iK_{i\omega/a-1/2}(m\omega) \\ 0 \\ K_{i\omega/a+1/2}(m\omega) - iK_{i\omega/a-1/2}(m\omega) \end{pmatrix} e^{-i\omega v/a}. \quad (4.16)$$

As a consequence, canonical anticommutation relations for fields and conjugate momenta lead annihilation and creation operators to satisfy the following anticommutation relations:

$$\{\hat{b}_{\omega\sigma}, \hat{b}_{\omega'\sigma'}^\dagger\} = \{\hat{d}_{\omega\sigma}, \hat{d}_{\omega'\sigma'}^\dagger\} = \delta(\omega - \omega') \delta_{\sigma\sigma'} \quad (4.17)$$

and

$$\begin{aligned} \{\hat{b}_{\omega\sigma}, \hat{b}_{\omega'\sigma'}\} &= \{\hat{d}_{\omega\sigma}, \hat{d}_{\omega'\sigma'}\} = \{\hat{b}_{\omega\sigma}, \hat{d}_{\omega'\sigma'}\} \\ &= \{\hat{b}_{\omega\sigma}, \hat{d}_{\omega'\sigma'}^\dagger\} = 0. \end{aligned} \quad (4.18)$$

V. RINDLER FRAME CALCULATION OF THE β AND INVERSE β DECAY FOR ACCELERATED NUCLEONS

Now we analyze the β and inverse β decay of accelerated nucleons from the point of view of the uniformly accelerated frame. Mean proper lifetimes must be the same of the ones obtained in Sec. III but particle interpretation changes significantly. This is so because uniformly accelerated particles in the Minkowski vacuum are immersed in the FDU thermal bath characterized by a temperature $T = a/2\pi$ [6,7]. As it will be shown, the proton decay which is represented in the inertial frame, in terms of Minkowski particles, by process (iii) will be represented in the uniformly accelerated frame, in terms of Rindler particles, as the combination of the following processes:

$$(v) \ p^+ e^- \rightarrow n \nu, \quad (vi) \ p^+ \bar{\nu} \rightarrow n e^+, \quad (vii) \ p^+ e^- \bar{\nu} \rightarrow n.$$

Processes (v)–(vii) are characterized by the conversion of protons in neutrons due to the absorption of e^- and $\bar{\nu}$, and emission of e^+ and ν from and to the FDU thermal bath. Note that process (iii) is forbidden in terms of Rindler particles because the proton is static in the Rindler frame.

Let us calculate firstly the transition amplitude for process (v):

$$\mathcal{A}_{(v)}^{p \rightarrow n} = \langle n | \otimes \langle \nu_{\omega\nu\sigma\nu} | \hat{S}_I | e_{\omega_e^- \sigma_e^-}^- \rangle \otimes | p \rangle, \quad (5.1)$$

where \hat{S}_I is given by Eq. (3.7) with γ^μ replaced by γ_R^μ and our current is given by Eq. (2.6). Thus, we obtain [we recall that in Rindler coordinates $u^\mu = (a, 0)$]

$$\begin{aligned} \mathcal{A}_{(v)}^{p \rightarrow n} &= \frac{G_F}{a} \int_{-\infty}^{+\infty} dv e^{i\Delta m v/a} \langle \nu_{\omega\nu\sigma\nu} | \hat{\Psi}_\nu^\dagger(v, a^{-1}) \\ &\quad \times \hat{\Psi}_e(v, a^{-1}) | e_{\omega_e^- \sigma_e^-}^- \rangle, \end{aligned} \quad (5.2)$$

where we note that the second term in the parenthesis of Eq. (3.7) does not contribute. Next, by using Eq. (4.2), we obtain

$$\begin{aligned} \mathcal{A}_{(v)}^{p \rightarrow n} &= \frac{G_F}{a} \delta_{\sigma_e^-, \sigma_\nu} \int_{-\infty}^{+\infty} dv e^{i\Delta m v/a} \\ &\quad \times \psi_{\omega_\nu \sigma_\nu}^\dagger(v, a^{-1}) \psi_{\omega_e^- \sigma_e^-}(v, a^{-1}). \end{aligned} \quad (5.3)$$

Using now Eqs. (4.15) and (4.16) and performing the integral, we obtain

$$\begin{aligned} \mathcal{A}_{(v)}^{p \rightarrow n} &= \frac{4G_F}{\pi a} \sqrt{m_e m_\nu \cosh(\pi\omega_e^-/a) \cosh(\pi\omega_\nu/a)} \\ &\quad \times \text{Re}[K_{i\omega_\nu/a-1/2}(m_\nu/a) K_{i\omega_e^-/a+1/2}(m_e/a)] \\ &\quad \times \delta_{\sigma_e^-, \sigma_\nu} \delta(\omega_e^- - \omega_\nu - \Delta m). \end{aligned} \quad (5.4)$$

Analogous calculations lead to the following amplitudes for processes (vi) and (vii):

$$\begin{aligned} \mathcal{A}_{(vi)}^{p \rightarrow n} &= \frac{4G_F}{\pi a} \sqrt{m_e m_\nu \cosh(\pi\omega_e^+/a) \cosh(\pi\omega_\nu^-/a)} \\ &\quad \times \text{Re}[K_{i\omega_e^+/a-1/2}(m_e/a) K_{i\omega_\nu^-/a+1/2}(m_\nu/a)] \\ &\quad \times \delta_{\sigma_e^+, \sigma_\nu^-} \delta(\omega_\nu^- - \omega_e^+ - \Delta m), \end{aligned} \quad (5.5)$$

$$\begin{aligned} \mathcal{A}_{(vii)}^{p \rightarrow n} &= \frac{4G_F}{\pi a} \sqrt{m_e m_\nu \cosh(\pi\omega_e^-/a) \cosh(\pi\omega_\nu^-/a)} \\ &\quad \times \text{Re}[K_{i\omega_e^-/a+1/2}(m_e/a) K_{i\omega_\nu^-/a+1/2}(m_\nu/a)] \\ &\quad \times \delta_{\sigma_e^-, \sigma_\nu^-} \delta(\omega_\nu^- + \omega_e^- - \Delta m). \end{aligned} \quad (5.6)$$

The differential transition rates per absorbed and emitted particle energies associated with processes (v)–(vii) are given by

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{(v)}^{p \rightarrow n}}{d\omega_e^- d\omega_\nu} &= \frac{1}{T} \sum_{\sigma_e^- = \pm} \sum_{\sigma_\nu = \pm} |\mathcal{A}_{(v)}^{p \rightarrow n}|^2 n_F(\omega_e^-) [1 \\ &\quad - n_F(\omega_\nu)], \end{aligned} \quad (5.7)$$

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{(vi)}^{p \rightarrow n}}{d\omega_e^+ d\omega_\nu^-} &= \frac{1}{T} \sum_{\sigma_e^+ = \pm} \sum_{\sigma_\nu^- = \pm} |\mathcal{A}_{(vi)}^{p \rightarrow n}|^2 n_F(\omega_\nu^-) [1 \\ &\quad - n_F(\omega_e^+)], \end{aligned} \quad (5.8)$$

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{(vii)}^{p \rightarrow n}}{d\omega_e^- d\omega_\nu^-} &= \frac{1}{T} \sum_{\sigma_e^- = \pm} \sum_{\sigma_\nu^- = \pm} |\mathcal{A}_{(vii)}^{p \rightarrow n}|^2 n_F(\omega_e^-) n_F(\omega_\nu^-), \end{aligned} \quad (5.9)$$

where

$$n_F(\omega) \equiv \frac{1}{1 + e^{2\pi\omega/a}} \quad (5.10)$$

is the fermionic thermal factor associated with the FDU thermal bath and $T = 2\pi\delta(0)$ is the nucleon proper time. By using Eqs. (5.4)–(5.6) in Eqs. (5.7)–(5.9) we obtain

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{(v)}^{p \rightarrow n}}{d\omega_e - d\omega_\nu} &= \frac{4G_F^2 m_e m_\nu}{\pi^3} \left(\frac{m_e m_\nu}{a^2} \right) e^{-\pi\Delta m/a} \delta(\omega_e - \omega_\nu - \Delta m) \\ &\times \{ \text{Re}[K_{i\omega_\nu/a - 1/2}(m_\nu/a)] \\ &\times K_{i\omega_e/a + 1/2}(m_e/a) \}^2, \end{aligned} \quad (5.11)$$

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{(vi)}^{p \rightarrow n}}{d\omega_e + d\omega_{\bar{\nu}}} &= \frac{4G_F^2 m_e m_\nu}{\pi^3} \left(\frac{m_e m_\nu}{a^2} \right) e^{-\pi\Delta m/a} \delta(\omega_{\bar{\nu}} - \omega_e + \Delta m) \\ &\times \{ \text{Re}[K_{i\omega_{\bar{\nu}}/a + 1/2}(m_\nu/a)] \\ &\times K_{i\omega_e/a - 1/2}(m_e/a) \}^2, \end{aligned} \quad (5.12)$$

$$\begin{aligned} \frac{1}{T} \frac{d^2 \mathcal{P}_{(vii)}^{p \rightarrow n}}{d\omega_e - d\omega_{\bar{\nu}}} &= \frac{4G_F^2 m_e m_\nu}{\pi^3} \left(\frac{m_e m_\nu}{a^2} \right) e^{-\pi\Delta m/a} \delta(\omega_e + \omega_{\bar{\nu}} - \Delta m) \\ &\times \{ \text{Re}[K_{i\omega_{\bar{\nu}}/a + 1/2}(m_\nu/a)] \\ &\times K_{i\omega_e/a + 1/2}(m_e/a) \}^2. \end{aligned} \quad (5.13)$$

By integrating Eqs. (5.11)–(5.13) in frequencies ω_ν and $\omega_{\bar{\nu}}$ where it is appropriate, we obtain the following transition rates associated with each process:

$$\begin{aligned} \Gamma_{(v)}^{p \rightarrow n} &= \frac{4G_F^2 m_e m_\nu}{\pi^3 a^2 e^{\pi\Delta m/a}} \int_{\Delta m}^{+\infty} d\omega_e - \\ &\times \{ \text{Re}[K_{i(\omega_e - \Delta m)/a - 1/2}(m_\nu/a)] K_{i\omega_e/a + 1/2}(m_e/a) \}^2, \end{aligned}$$

$$\begin{aligned} \Gamma_{(vi)}^{p \rightarrow n} &= \frac{4G_F^2 m_e m_\nu}{\pi^3 a^2 e^{\pi\Delta m/a}} \int_0^{+\infty} d\omega_e + \\ &\times \{ \text{Re}[K_{i(\omega_e + \Delta m)/a + 1/2}(m_\nu/a)] K_{i\omega_e/a - 1/2}(m_e/a) \}^2, \end{aligned}$$

$$\begin{aligned} \Gamma_{(vii)}^{p \rightarrow n} &= \frac{4G_F^2 m_e m_\nu}{\pi^3 a^2 e^{\pi\Delta m/a}} \int_0^{\Delta m} d\omega_e - \\ &\times \{ \text{Re}[K_{i(\omega_e - \Delta m)/a - 1/2}(m_\nu/a)] K_{i\omega_e/a + 1/2}(m_e/a) \}^2. \end{aligned}$$

We recall that Rindler frequencies may assume arbitrary positive real values. (In particular there are massive Rindler particles with zero frequency. See Ref. [12] for a discussion on zero-frequency Rindler particles with finite transverse and angular momentum.)

The proton decay rate is given by adding up all contributions: $\Gamma_{\text{acc}}^{p \rightarrow n} = \Gamma_{(v)}^{p \rightarrow n} + \Gamma_{(vi)}^{p \rightarrow n} + \Gamma_{(vii)}^{p \rightarrow n}$. This can be written in a compact form as

$$\begin{aligned} \Gamma_{\text{acc}}^{p \rightarrow n} &= \frac{4G_F^2 m_e m_\nu}{\pi^3 a^2 e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega \\ &\times \{ \text{Re}[K_{i(\omega - \Delta m)/a - 1/2}(m_\nu/a)] K_{i\omega/a + 1/2}(m_e/a) \}^2. \end{aligned} \quad (5.14)$$

At this point we take the limit $m_\nu \rightarrow 0$. For this purpose, it is useful to note that [see Eqs. (8.407.1), (8.405.1), and (8.403.1) in Ref. [10]]

$$K_\nu(z) = \frac{\pi}{2 \sin \nu\pi} (J_{-\nu}(iz) e^{i\nu\pi/2} - J_\nu(iz) e^{-i\nu\pi/2}), \quad (5.15)$$

where ν is noninteger, and $|\arg iz| < \pi$. Using this expression in conjunction with Eq. (8.402) of Ref. [10], we have for small $|z|$ that

$$\begin{aligned} K_\nu(z) &\approx \frac{\pi}{2 \sin \nu\pi} [(iz/2)^{-\nu} \Gamma^{-1}(-\nu+1) e^{i\nu\pi/2} \\ &- (iz/2)^\nu \Gamma^{-1}(\nu+1) e^{-i\nu\pi/2}]. \end{aligned} \quad (5.16)$$

By using Eq. (5.16), we can show that

$$\begin{aligned} \frac{m_\nu}{a} K_{i(\omega - \Delta m)/a + 1/2}(m_\nu/a) K_{i(\omega - \Delta m)/a - 1/2}(m_\nu/a) \\ \xrightarrow{m_\nu \rightarrow 0} \frac{\pi}{2 \cosh[\pi(\omega - \Delta m)/a]}. \end{aligned} \quad (5.17)$$

It is now possible to obtain the following partial and total transition rates:

$$\begin{aligned} \Gamma_{(v)}^{p \rightarrow n} &= \frac{G_F^2 m_e}{\pi^2 a e^{\pi\Delta m/a}} \int_{\Delta m}^{+\infty} d\omega_e - \\ &\times \frac{K_{i\omega_e/a + 1/2}(m_e/a) K_{i\omega_e/a - 1/2}(m_e/a)}{\cosh[\pi(\omega_e - \Delta m)/a]}, \end{aligned} \quad (5.18)$$

$$\begin{aligned} \Gamma_{(vi)}^{p \rightarrow n} &= \frac{G_F^2 m_e}{\pi^2 a e^{\pi\Delta m/a}} \int_0^{+\infty} d\omega_e + \\ &\times \frac{K_{i\omega_e/a + 1/2}(m_e/a) K_{i\omega_e/a - 1/2}(m_e/a)}{\cosh[\pi(\omega_e + \Delta m)/a]}, \end{aligned} \quad (5.19)$$

$$\begin{aligned} \Gamma_{(vii)}^{p \rightarrow n} &= \frac{G_F^2 m_e}{\pi^2 a e^{\pi\Delta m/a}} \int_0^{\Delta m} d\omega_e - \\ &\times \frac{K_{i\omega_e/a + 1/2}(m_e/a) K_{i\omega_e/a - 1/2}(m_e/a)}{\cosh[\pi(\omega_e - \Delta m)/a]}, \end{aligned} \quad (5.20)$$

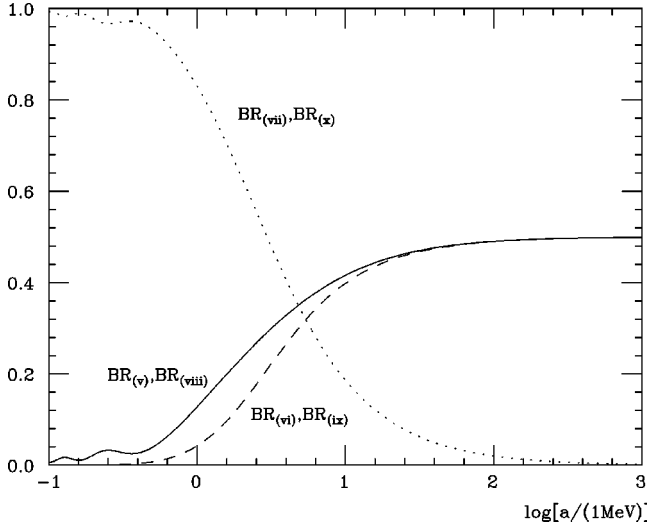


FIG. 3. Branching ratios $BR_{(v)}, BR_{(vi)}, BR_{(vii)}, BR_{(viii)}, BR_{(ix)}, BR_{(x)}$ are plotted. For protons, process (vii) dominates over processes (v) and (vi) for small accelerations, while processes (v) and (vi) dominate over process (vii) for high accelerations. For neutrons, process (x) dominates over processes (viii) and (ix) for small accelerations, while processes (viii) and (ix) dominate over process (x) for high accelerations.

and

$$\Gamma_{\text{acc}}^{p \rightarrow n} = \frac{G_F^2 m_e}{\pi^2 a e^{-\pi \Delta m/a}} \int_{-\infty}^{+\infty} d\omega \times \frac{K_{i\omega/a+1/2}(m_e/a) K_{i\omega/a-1/2}(m_e/a)}{\cosh[\pi(\omega - \Delta m)/a]}, \quad (5.21)$$

respectively. It is interesting to note that although transition rates have fairly distinct interpretations in the inertial and accelerated frames, mean proper lifetimes are scalars and must be the same in both frames. Indeed, by plotting $\tau_p(a) = 1/\Gamma_{\text{acc}}^{p \rightarrow n}$ as a function of acceleration, we do reproduce Fig. 2. In Fig. 3 we plot the branching ratios

$$BR_{(v)} \equiv \Gamma_{(v)}^{p \rightarrow n} / \Gamma_{\text{acc}}^{p \rightarrow n}, \quad BR_{(vi)} \equiv \Gamma_{(vi)}^{p \rightarrow n} / \Gamma_{\text{acc}}^{p \rightarrow n},$$

$$BR_{(vii)} \equiv \Gamma_{(vii)}^{p \rightarrow n} / \Gamma_{\text{acc}}^{p \rightarrow n}.$$

We note that for small accelerations, where ‘‘few’’ high-energy particles are available in the FDU thermal bath, process (vii) dominates over processes (v) and (vi), while for high accelerations, processes (v) and (vi) dominate over process (vii).

A similar analysis can be performed for uniformly accelerated neutrons. According to coaccelerated observers, the neutron decay will be described by a combination of the following processes:

$$(viii) \quad n\nu \rightarrow p^+ e^-, \quad (ix) \quad ne^+ \rightarrow p^+ \bar{\nu}, \quad (x) \quad n \rightarrow p^+ e^- \bar{\nu}.$$

The corresponding partial and total transition rates are

$$\Gamma_{(viii)}^{n \rightarrow p} = \frac{G_F^2 m_e}{\pi^2 a e^{-\pi \Delta m/a}} \int_{\Delta m}^{+\infty} d\omega_{e^-} \times \frac{K_{i\omega_{e^-}/a+1/2}(m_e/a) K_{i\omega_{e^-}/a-1/2}(m_e/a)}{\cosh[\pi(\omega_{e^-} - \Delta m)/a]}, \quad (5.22)$$

$$\Gamma_{(ix)}^{n \rightarrow p} = \frac{G_F^2 m_e}{\pi^2 a e^{-\pi \Delta m/a}} \int_0^{+\infty} d\omega_{e^+} \times \frac{K_{i\omega_{e^+}/a+1/2}(m_e/a) K_{i\omega_{e^+}/a-1/2}(m_e/a)}{\cosh[\pi(\omega_{e^+} + \Delta m)/a]}, \quad (5.23)$$

$$\Gamma_{(x)}^{n \rightarrow p} = \frac{G_F^2 m_e}{\pi^2 a e^{-\pi \Delta m/a}} \int_0^{\Delta m} d\omega_{e^-} \times \frac{K_{i\omega_{e^-}/a+1/2}(m_e/a) K_{i\omega_{e^-}/a-1/2}(m_e/a)}{\cosh[\pi(\omega_{e^-} - \Delta m)/a]}, \quad (5.24)$$

and

$$\Gamma_{\text{acc}}^{n \rightarrow p} = \frac{G_F^2 m_e}{\pi^2 a e^{-\pi \Delta m/a}} \int_{-\infty}^{+\infty} d\omega \times \frac{K_{i\omega/a+1/2}(m_e/a) K_{i\omega/a-1/2}(m_e/a)}{\cosh[\pi(\omega - \Delta m)/a]}, \quad (5.25)$$

respectively. Figure 1 gives the neutron mean proper lifetime $\tau_n(a) = 1/\Gamma_{\text{acc}}^{n \rightarrow p}$ which coincides with the one calculated in the inertial frame. In Fig. 3 we plot the branching ratios

$$BR_{(viii)} \equiv \Gamma_{(viii)}^{n \rightarrow p} / \Gamma_{\text{acc}}^{n \rightarrow p}, \quad BR_{(ix)} \equiv \Gamma_{(ix)}^{n \rightarrow p} / \Gamma_{\text{acc}}^{n \rightarrow p},$$

$$BR_{(x)} \equiv \Gamma_{(x)}^{n \rightarrow p} / \Gamma_{\text{acc}}^{n \rightarrow p}.$$

It is easy to see that $BR_{(viii)} = BR_{(v)}, BR_{(ix)} = BR_{(vi)}$, and $BR_{(x)} = BR_{(vii)}$.

VI. DISCUSSIONS

We have analyzed the decay of accelerated protons and neutrons. We have compared the particle interpretation of these decays in the inertial [see processes (iii) and (iv)] and accelerated [see processes (v)–(vii) and (viii)–(x)] frames. They were shown to be quite distinct. Branching ratios were also evaluated. For protons with small accelerations, process (vii) dominates over processes (v) and (vi), while for protons with high accelerations processes (v) and (vi) dominate over process (vii). For neutrons with small accelerations, process (x) dominates over processes (viii) and (ix), while for neutrons with high accelerations, processes (viii) and (ix) dominate over process (x). Mean proper lifetimes of the nucleons as a function of their proper acceleration were plotted in

Figs. 1 and 2. For accelerations such that $a \approx a_c \equiv 2\pi(m_n + m_e - m_p)$ we have that $\tau_p \approx \tau_n$. Although such accelerations are quite beyond present technology, decay of accelerated nucleons might be of some importance in astrophysics and cosmology. We have performed our calculations using Fermi theory in a two-dimensional spacetime. Although this is suitable to provide us with a qualitative understanding of many conceptual aspects underlying β and inverse β decay induced by acceleration, precise physical values will only be obtained after a more realistic analysis is performed. A four-dimensional calculation (but restricted to the inertial frame)

and its application to astrophysics and cosmology are presently under consideration and will be presented somewhere else.

ACKNOWLEDGMENTS

The authors are thankful to Dr. A. Higuchi for discussions in early stages of this work. G.M. was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico while D.V. was fully supported by Fundação de Amparo à Pesquisa do Estado de São Paulo.

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- [1] Particle Data Group, C. Caso *et al.*, *Eur. Phys. J. C* **3**, 1 (1998).
- [2] V.N. Baier, *Usp. Fiz. Nauk.* **105**, 441 (1971) [*Sov. Phys. Usp.* **14**, 695 (1972)]; D. Potaux, *Proceedings of the 8th International Conference on High Energy Accelerators*, edited by M.H. Blewett (CERN, Geneva, 1971); J.G. Learned, L.K. Resvanis, and C.M. Spencer, *Phys. Rev. Lett.* **35**, 1688 (1975); J.R. Johnson *et al.*, *Nucl. Instrum. Methods* **204**, 261 (1983); L. Knudsen *et al.*, *Phys. Lett. B* **270**, 97 (1991); R. Assmann *et al.*, *High Energy Spin Physics: Proceedings*, edited by K.J. Heller and S.L. Smith (AIP, New York, 1995); D.P. Barber *et al.*, *Phys. Lett. B* **343**, 436 (1995); A.A. Sokolov and I.M. Ternov, *Dokl. Akad. Nauk. SSSR* **153**, 1052 (1963) [*Sov. Phys. Dokl.* **8**, 1203 (1964)]; Ya.S. Derbenev and A.M. Kondratenko, *Zh. Eksp. Teor. Fiz.* **64**, 1918 (1973) [*Sov. Phys. JETP* **37**, 968 (1973)]; J.D. Jackson, *Rev. Mod. Phys.* **48**, 417 (1976).
- [3] J.D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).
- [4] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
- [5] R. Muller, *Phys. Rev. D* **56**, 953 (1997).
- [6] S.A. Fulling, *Phys. Rev. D* **7**, 2850 (1973); P.C.W. Davies, *J. Phys. A* **8**, 609 (1975).
- [7] W.G. Unruh, *Phys. Rev. D* **14**, 870 (1976).
- [8] B.S. DeWitt, *General Relativity*, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979); W.G. Unruh and R.M. Wald, *Phys. Rev. D* **29**, 1047 (1984).
- [9] N.D. Birrell and P.C.W. Davies, *Quantum Field Theory in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [10] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1980).
- [11] P. Candelas and D. Deutsch, *Proc. R. Soc. London* **A362**, 251 (1978); M. Soffel, B. Muller, and W. Greiner, *Phys. Rev. D* **22**, 1935 (1980); R. Járegui, M. Torres, and S. Hacyan, *ibid.* **43**, 3979 (1991); E. Bautista, *ibid.* **48**, 783 (1993).
- [12] A. Higuchi, G.E.A. Matsas, and D. Sudarsky, *Phys. Rev. D* **45**, R3308 (1992); *ibid.* **46**, 3450 (1992); *ibid.* **56**, R6071 (1997); L.C.B. Crispino, A. Higuchi, and G.E.A. Matsas, *ibid.* **58**, 084027 (1998).