

Soft superweak CP violation in a 3-3-1 model

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We show that it is possible to implement soft superweak CP violation in the context of a 3-3-1 model with only three triplets. All CP violation effects come from the exchange of singly and doubly charged scalars. We consider the implication of this mechanism in the quark and lepton sectors. In particular it is shown that, in this model, as in most of those which incorporate scalar mediated CP violation, it is possible to have large electric dipole moments for the muon and the tau lepton while keeping small those of the electron and neutron. The CKM mixing matrix is real at the tree level but gets a phase at the 1-up loop level. [S0556-2821(99)04117-X]

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I. INTRODUCTION

The origin and the smallness of CP violation are still open questions. In the context of the electroweak standard model [1] the only source of CP violation in the quark sector is the surviving phase in the charged current coupled to the vector boson W^\pm [2]. This is called explicit or hard CP violation. On the other hand there is no CP violation in the lepton sector at lower orders since neutrinos are massless.

Although this is an interesting feature of the model it leaves open the question of why CP is so feebly violated. It is well known that CP is softly broken if it occurs through a dimensional coupling in the bare Lagrangian and/or spontaneously. The possibility that CP nonconservation arises exclusively through Higgs boson exchange is a rather old one. It can be implemented in a spontaneous way as was proposed by Lee [3] or, explicitly in the parameters of the scalar potential, as proposed by Weinberg [4,5]. Of course, both possibilities can be mixed. Since the works of Lee and Weinberg it has been known that in renormalizable gauge theories the violation of CP has the right strength if it occurs through the exchange of a Higgs boson of mass M_H , i.e., it is proportional to $G_F m_f^2 / M_H^2$, where m_f is the fermion mass. Since then, there have been many realizations of that mechanism in extensions of the electroweak standard model. Even if we insist that the CP violation arises solely through the Higgs exchange we have several possibilities in the standard electroweak model with several doublets [6].

Some years ago it was proposed a model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ gauge symmetry with exotic charged leptons [7]. In this model since the scalar multiplets transform in a different way one from another the scalar potential is constrained in such a way that even with three triplets there is not spontaneous CP violations [8]. Here we will show that it is possible to have soft CP violation because the coupling constant of the trilinear term in the scalar potential is complex and the vacuum expectation value

(VEV) of the three triplets are also complex. Hence, CP is a symmetry in the full bare Lagrangian but in the trilinear term in the potential, in particular all Yukawa couplings are real at tree level. We will show that the model is a realization of a superweak CP violation [9] in the sense that all flavor changing phenomena other than CP violation are accurately described by the real Cabbibo-Kobayashi-Maskawa (CKM) matrix since CP violation is restricted to one operator of dimension three [10].

If the condition that except in the trilinear term CP is a symmetry of the Lagrangian is assumed, we have verified that it is possible to choose the physical phases in such a way that the only places of CP violation in the Lagrangian density are those related with the singly and doubly charged scalars which are present in the model.

This work is organized as follows. In Sec. II we review the Higgs sector of the model. We consider in particular the minimization of the scalar potential and we also give the definitions of the Goldstone and the physical scalar fields of the several charged sectors. In Sec. III we study the Yukawa interactions showing how the VEVs phases can be absorbed in the fermion fields in some places of the Lagrangian density and that they only survive in the sector involving both simply and doubly charged scalar fields. In Sec. IV we show that there is not CP violation in the vector boson-fermion interactions. The phenomenology of the model is considered in Sec. V while our conclusions are in the last section.

II. A MODEL WITH THREE SCALAR TRIPLETS

As we said before, here we will consider a model with $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ symmetry with both exotic quarks and charged leptons [7]. In this model in order to give mass to all fermions it is necessary to introduce three scalar triplets. They transform as

$$\begin{aligned} \chi &= \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (\mathbf{3}, -\mathbf{1}), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (\mathbf{3}, \mathbf{1}), \\ \eta &= \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (\mathbf{3}, \mathbf{0}). \end{aligned} \tag{1}$$

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The more general scalar potential invariant under the gauge symmetry is

$$\begin{aligned}
 V(\chi, \eta, \rho) = & \mu_1^2 \chi^\dagger \chi + \mu_2^2 \eta^\dagger \eta + \mu_3^2 \rho^\dagger \rho + (\alpha \epsilon_{ijk} \chi_i \rho_j \eta_k + \text{H.c.}) \\
 & + a_1 (\chi^\dagger \chi)^2 + a_2 (\eta^\dagger \eta)^2 + a_3 (\rho^\dagger \rho)^2 + a_4 (\chi^\dagger \chi) (\eta^\dagger \eta) + a_5 (\chi^\dagger \chi) (\rho^\dagger \rho) + a_6 (\rho^\dagger \rho) (\eta^\dagger \eta) + a_7 (\chi^\dagger \eta) (\eta^\dagger \chi) \\
 & + a_8 (\chi^\dagger \rho) (\rho^\dagger \chi) + a_9 (\rho^\dagger \eta) (\eta^\dagger \rho) + [a_{10} (\chi^\dagger \eta) (\rho^\dagger \eta) + \text{H.c.}].
 \end{aligned} \tag{2}$$

All terms of this potential but the a_{10} term conserve the total lepton number L (or $L+B$, where B is the baryonic number). The minimum of the potential must be studied after the shifting of the neutral components of the three scalar multiplets in Eq. (1). Hence, we redefine the neutral components as follows:

$$\begin{aligned}
 \eta^0 & \rightarrow \frac{v_\eta}{\sqrt{2}} \left(1 + \frac{X_\eta^0 + iI_\eta^0}{|v_\eta|} \right), & \rho^0 & \rightarrow \frac{v_\rho}{\sqrt{2}} \left(1 + \frac{X_\rho^0 + iI_\rho^0}{|v_\rho|} \right), \\
 \chi^0 & \rightarrow \frac{v_\chi}{\sqrt{2}} \left(1 + \frac{X_\chi^0 + iI_\chi^0}{|v_\chi|} \right),
 \end{aligned} \tag{3}$$

where $v_a = |v_a| e^{i\theta_a}$ with $a = \eta, \rho, \chi$.

The condition that the first derivative of the potential in Eq. (2) is zero (i.e., no linear terms in all neutral fields must survive) implies the following constraint equations:

$$\begin{aligned}
 \frac{a_4}{2} |v_\eta|^2 |v_\chi| + \frac{a_5}{2} |v_\rho|^2 |v_\chi| + \mu_1^2 |v_\chi| + a_1 |v_\chi|^3 \\
 + \frac{1}{\sqrt{2} |v_\chi|} \text{Re}(\alpha v_\chi v_\rho v_\eta) = 0,
 \end{aligned} \tag{4a}$$

$$\begin{aligned}
 a_2 |v_\eta|^3 + \frac{a_4}{2} |v_\chi|^2 |v_\eta| + \frac{a_6}{2} |v_\rho|^2 |v_\eta| + \mu_2^2 |v_\eta| \\
 + \frac{1}{\sqrt{2} |v_\eta|} \text{Re}(\alpha v_\chi v_\rho v_\eta) = 0,
 \end{aligned} \tag{4b}$$

$$\begin{aligned}
 \frac{a_6}{2} |v_\eta|^2 |v_\rho| + \frac{a_5}{2} |v_\rho| |v_\chi|^2 + \mu_3^2 |v_\rho| + a_3 |v_\rho|^3 \\
 + \frac{1}{\sqrt{2} |v_\rho|} \text{Re}(\alpha v_\chi v_\rho v_\eta) = 0,
 \end{aligned} \tag{4c}$$

and, finally

$$\text{Im}(\alpha v_\chi v_\rho v_\eta) = 0. \tag{4d}$$

Before considering how many physical phases will survive in the potential (this must be done by taking at the same time the Yukawa interactions) we will study all scalar mass eigenstates in the model. Recall that we have to verify where the VEV's phases appear in the several sectors of the Lagrangian that is, in vertices and in mixing matrices. Hence, we will firstly write down explicitly the mass and mixing matrices of each charged sector of the model. Two of the

phases in Eqs. (3) can be transformed away with a $SU(3)$ transformation; whenever we do that we will mention it explicitly and we will always choose $\theta_\eta = \theta_\rho = 0$.

A. Doubly charged scalars

In this sector we have the following mass matrix in the $(\rho^{++}, \chi^{++})^T$ basis

$$\begin{pmatrix} -\frac{a_8}{2} |v_\chi|^2 + \frac{A}{\sqrt{2} |v_\rho|^2} & \frac{\alpha_1^* v_\eta^*}{\sqrt{2}} - \frac{a_8}{2} v_\rho v_\chi \\ -\frac{a_8}{2} |v_\rho|^2 + \frac{A}{\sqrt{2} |v_\chi|^2} & \end{pmatrix} \tag{5}$$

where we have defined $A \equiv \text{Re}(\alpha v_\chi v_\eta v_\rho)$.

As expected, we have a doubly charged Goldstone boson G^{++} and a physical doubly charged scalar Y^{++}

$$\begin{pmatrix} \rho^{++} \\ \chi^{++} \end{pmatrix} = \frac{1}{(|v_\rho|^2 + |v_\chi|^2)^{1/2}} \begin{pmatrix} |v_\rho| & -|v_\chi| e^{-i\theta_\chi} \\ |v_\chi| e^{i\theta_\chi} & |v_\rho| \end{pmatrix} \times \begin{pmatrix} G^{++} \\ Y^{++} \end{pmatrix}, \tag{6}$$

with the mass square of the Y^{++} field given by

$$M_{Y^{++}}^2 = \frac{A}{\sqrt{2}} \left(\frac{1}{|v_\chi|^2} + \frac{1}{|v_\rho|^2} \right) - \frac{a_8}{2} (|v_\chi|^2 + |v_\rho|^2). \tag{7}$$

Notice that this mass is proportional to $|\alpha| v_\chi$ and $|v_\chi|^2$, here $|\alpha|$ is an arbitrary mass scale while $|v_\chi|$ is the VEV that is in control of the $SU(3)_L$ symmetry and so it is the largest mass scale of the model. So it means that the doubly charged scalar might be a heavy scalar.

B. Singly charged scalars

Next, for the simply charged scalar fields we have, in the $(\eta_1^+, \rho^+, \eta_2^+, \chi^+)^T$ basis,

$$\begin{pmatrix} -\frac{a_9}{2}|\mathbf{v}_\rho|^2 + \frac{A}{\sqrt{2}|\mathbf{v}_\eta|^2} & -\frac{a_9}{2}\mathbf{v}_\rho^* \mathbf{v}_\eta^* + \frac{\alpha \mathbf{v}_\chi}{\sqrt{2}} & -\frac{a_{10}}{2}\mathbf{v}_\rho^* \mathbf{v}_\chi^* & -\frac{a_{10}}{2}\mathbf{v}_\rho^* \mathbf{v}_\eta^* \\ & -\frac{a_9}{2}|\mathbf{v}_\eta|^2 + \frac{A}{\sqrt{2}|\mathbf{v}_\rho|^2} & -\frac{a_{10}}{2}\mathbf{v}_\chi^* \mathbf{v}_\eta & -\frac{a_{10}}{2}\mathbf{v}_\eta^2 \\ & & -\frac{a_7}{2}|\mathbf{v}_\chi|^2 + \frac{A}{\sqrt{2}|\mathbf{v}_\eta|^2} & -\frac{a_7}{2}\mathbf{v}_\chi \mathbf{v}_\eta + \frac{\alpha^* \mathbf{v}_\rho}{\sqrt{2}} \\ & & & -\frac{a_7}{2}|\mathbf{v}_\eta|^2 + \frac{A}{\sqrt{2}|\mathbf{v}_\chi|^2} \end{pmatrix}. \quad (8)$$

Notice that if $a_{10}=0$, η_1^+ and ρ^+ decouple from η_2^+ and χ^+ . Hence, the mass matrix in Eq. (8) is reduced to two 2×2 mass matrices and we have two Goldstone bosons G_1^+ and G_2^+ ; and two massive fields Y_1^+ and Y_2^+ ,

$$\begin{pmatrix} \eta_1^+ \\ \rho^+ \end{pmatrix} = \frac{1}{(|\mathbf{v}_\eta|^2 + |\mathbf{v}_\rho|^2)^{1/2}} \begin{pmatrix} -|\mathbf{v}_\eta| & |\mathbf{v}_\rho| \\ |\mathbf{v}_\rho| & |\mathbf{v}_\eta| \end{pmatrix} \begin{pmatrix} G_1^+ \\ Y_1^+ \end{pmatrix}, \quad (9)$$

with

$$m_{Y_1^+}^2 = \frac{A}{\sqrt{2}} \left(\frac{1}{|\mathbf{v}_\eta|^2} + \frac{1}{|\mathbf{v}_\rho|^2} \right) - \frac{a_9}{2} (|\mathbf{v}_\rho|^2 + |\mathbf{v}_\eta|^2), \quad (10)$$

and

$$\begin{pmatrix} \eta_2^+ \\ \chi^+ \end{pmatrix} = \frac{1}{(|\mathbf{v}_\eta|^2 + |\mathbf{v}_\chi|^2)^{1/2}} \begin{pmatrix} -|\mathbf{v}_\eta| & |\mathbf{v}_\chi| \\ -|\mathbf{v}_\chi| & |\mathbf{v}_\eta| \end{pmatrix} \begin{pmatrix} G_2^+ \\ Y_2^+ \end{pmatrix}, \quad (11)$$

with

$$m_{Y_2^+}^2 = \frac{A}{\sqrt{2}} \left(\frac{1}{|\mathbf{v}_\eta|^2} + \frac{1}{|\mathbf{v}_\chi|^2} \right) - \frac{a_7}{2} (|\mathbf{v}_\chi|^2 + |\mathbf{v}_\eta|^2). \quad (12)$$

There are 5 phases in the matrix in Eq. (8), two of them can be transformed away with a $SU(3)$ transformation (here when we do that we will choose $\theta_\eta = \theta_\rho = 0$). The other three phases can be absorbed by redefining three scalar fields. However these phases will appear in the Yukawa interactions or in the scalar potential in terms like $a_{10}\chi^+ \eta^0 \rho^- \eta^0$. Hence, there is CP violation in the propagators of the singly charged scalars if $a_{10} \neq 0$. Since we want CP softly broken we will consider here $a_{10}=0$ with the singly charged scalar given by the expressions above.

C. Neutral scalars

Finally, in the neutral sector we have CP even fields, denoted here by H_i^0 , and CP odd fields, $G_{1,2}^0, h^0$.

In the CP odd sector we have the following mass matrix:

$$\begin{pmatrix} \frac{A}{\sqrt{2}|\mathbf{v}_\eta|^2} & \frac{A}{\sqrt{2}|\mathbf{v}_\eta||\mathbf{v}_\rho|} & \frac{A}{\sqrt{2}|\mathbf{v}_\eta||\mathbf{v}_\chi|} \\ & \frac{A}{\sqrt{2}|\mathbf{v}_\rho|^2} & \frac{A}{\sqrt{2}|\mathbf{v}_\chi||\mathbf{v}_\rho|} \\ & & \frac{A}{\sqrt{2}|\mathbf{v}_\chi|^2} \end{pmatrix} \quad (13)$$

in the $(I_\eta, I_\rho, I_\chi)^T$ basis. There are in fact two neutral Goldstone bosons G_1^0 and G_2^0 (as it must be since we have two massive neutral vector bosons Z and Z') and a physical CP odd scalar field h^0 . Explicitly we have

$$\begin{pmatrix} I_\eta^0 \\ I_\rho^0 \\ I_\chi^0 \end{pmatrix} = \begin{pmatrix} -\frac{|\mathbf{v}_\eta|}{N_1} & -\frac{|\mathbf{v}_\rho|^2|\mathbf{v}_\eta|}{N_2} & \frac{|\mathbf{v}_\rho||\mathbf{v}_\chi|}{N_3} \\ -\frac{|\mathbf{v}_\rho|}{N_1} & -\frac{|\mathbf{v}_\eta|^2|\mathbf{v}_\rho|}{N_2} & \frac{|\mathbf{v}_\eta||\mathbf{v}_\chi|}{N_3} \\ 0 & \frac{|\mathbf{v}_\chi|N_1^2}{N_2} & \frac{|\mathbf{v}_\eta||\mathbf{v}_\rho|}{N_3} \end{pmatrix} \begin{pmatrix} G_1^0 \\ G_2^0 \\ h^0 \end{pmatrix}, \quad (14)$$

where

$$N_1 = (|\mathbf{v}_\eta|^2 + |\mathbf{v}_\rho|^2)^{1/2},$$

$$N_2 = (|\mathbf{v}_\eta|^2|\mathbf{v}_\rho|^4 + |\mathbf{v}_\eta|^4|\mathbf{v}_\rho|^2 + |\mathbf{v}_\chi|^2N_1^4)^{1/2}, \quad (15)$$

and

$$N_3 = (|\mathbf{v}_\chi|^2|\mathbf{v}_\rho|^2 + |\mathbf{v}_\chi|^2|\mathbf{v}_\eta|^2 + |\mathbf{v}_\rho|^2|\mathbf{v}_\eta|^2)^{1/2}, \quad (16)$$

and with the mass of the h^0 field given by

$$M_h^2 = \frac{A}{\sqrt{2}} \left(\frac{1}{|\mathbf{v}_\eta|^2} + \frac{1}{|\mathbf{v}_\chi|^2} + \frac{1}{|\mathbf{v}_\rho|^2} \right). \quad (17)$$

In the CP even sector we have the mass matrix

$$\begin{pmatrix} -2a_2|v_\eta|^2 + \frac{A}{\sqrt{2}|v_\eta|^2} & -a_6|v_\eta||v_\rho| - \frac{A}{\sqrt{2}|v_\eta||v_\rho|} & -a_4|v_\eta||v_\chi| - \frac{A}{\sqrt{2}|v_\eta||v_\chi|} \\ -2a_3|v_\rho|^2 + \frac{A}{\sqrt{2}|v_\rho|^2} & -a_5|v_\rho||v_\chi| - \frac{A}{\sqrt{2}|v_\rho||v_\chi|} & \\ -2a_1|v_\chi|^2 + \frac{A}{\sqrt{2}|v_\chi|^2} & & \end{pmatrix}, \quad (18)$$

in the $(X_\eta^0, X_\rho^0, X_\chi^0)^T$ basis. All those scalars are physical but we will not write the respective mass eigenstates [12]. It is enough to stress that $X_a^0 = O_{ai}^H H_i^0$ where $a = \eta, \rho, \chi$; $i = 1, 2, 3$ and O^H is an orthogonal 3×3 matrix.

III. YUKAWA INTERACTIONS

Here we will consider the most general Yukawa interaction with real coefficients which is invariant under the gauge symmetry. However, there are flavor changing neutral currents through the neutral scalars and since these fields get nonzero VEVs as in Eq. (3) it is not straightforward to see which phases can be absorbed by redefining the fermion fields or which of them, and where, survive in the Lagrangian density.

In this section we will show that it is possible to choose that all Yukawa interactions which have a counterpart in the standard model are CP conserving. Hence, all CP violation effects arise from the singly and/or doubly charged scalar-fermion interactions. The vector gauge interactions with the fermions are also CP conserving including the singly and doubly charged bileptons as we will show in Sec. IV.

A. Quark-scalar interactions

First, let us consider the quark-scalar interactions. The quark multiplets are the following:

$$Q'_{1L} = \begin{pmatrix} u'_1 \\ d'_1 \\ J'_1 \end{pmatrix}_L \sim (3, \frac{2}{3}), \quad Q'_{mL} = \begin{pmatrix} d'_m \\ u'_m \\ J'_m \end{pmatrix}_L \sim (3^*, -\frac{1}{3}), \quad (19a)$$

and the respective right-handed components in singlets

$$U'_{\alpha R} \sim (3, 1, 2/3), \quad D'_{\alpha R} \sim (3, 1, -1/3), \quad J'_{1R} \sim (3, 1, 5/3);$$

$$J'_{mR} \sim (3, 1, -4/3), \quad (19b)$$

where $m = 2, 3$ and $\alpha = 1, 2, 3$.

With the quark multiplets in Eqs. (19) and the scalar ones in Eq. (1) we have the Yukawa terms

$$\begin{aligned} -\mathcal{L}_Y = & \bar{Q}'_{1L} \sum_\alpha (G_{1\alpha} U'_{\alpha R} \eta + \tilde{G}_{1\alpha} D'_{\alpha R} \rho) \\ & + \sum_i \bar{Q}'_{iL} \sum_\alpha (F_{i\alpha} U'_{\alpha R} \rho^* + \tilde{F}_{i\alpha} D'_{\alpha R} \eta^*) \\ & + \lambda_1 \bar{Q}'_{1L} J'_{1R} \chi + \sum_{i,m} \lambda_{im} \bar{Q}'_{iL} J'_{mR} \chi^* + \text{H.c.}, \quad (20) \end{aligned}$$

here $i = 2, 3$ and we will assume that all coupling constants in Eq. (20) are real. In the following subsections we will analyze, case by case, all the charged sectors.

1. u -like sector—neutral scalar interactions

After the spontaneous symmetry breaking the VEVs of the neutral scalars are arbitrary complex numbers, as discussed in Sec. II. Using the shifted neutral fields, given in Eq. (3), from Eq. (20) we obtain the interactions of the quarks with the neutral scalars.

In the u -like sector we have the interactions

$$-\mathcal{L}_Y^u = \bar{u}'_{1L} \sum_\alpha G_{1\alpha} U'_{\alpha R} \eta^0 + \sum_m \bar{u}'_{mL} \sum_\alpha F_{i\alpha} U'_{\alpha R} \rho^{0*} + \text{H.c.} \quad (21)$$

In this and in the following sections we will not use the freedom of choosing two phases equal to zero because we want to see how many phases can be absorbed in the fermion fields. Redefining the phases of the following fields [11]:

$$U''_{\alpha R} \equiv e^{i\theta_\eta} U'_{\alpha R}, \quad u''_{mL} \equiv e^{-i(\theta_\eta + \theta_\rho)} u'_{mL}, \quad (22)$$

the mass matrix of the u -like sector becomes real and can be diagonalized by real orthogonal matrices $\mathcal{O}_{L,R}^u$ (recall that $\alpha = 1, 2, 3$ and $m = 2, 3$):

$$(\mathcal{O}_L^u)^T \Gamma^u \mathcal{O}_R^u = M^u = \text{diag}(m_u, m_c, m_t), \quad (23)$$

with

$$\Gamma^u = \frac{1}{\sqrt{2}} \begin{pmatrix} |v_\eta| G_{11} & |v_\eta| G_{12} & |v_\eta| G_{13} \\ |v_\rho| F_{21} & |v_\rho| F_{22} & |v_\rho| F_{23} \\ |v_\rho| F_{31} & |v_\rho| F_{32} & |v_\rho| F_{33} \end{pmatrix}. \quad (24)$$

Symmetry eigenstates (singly and doubly primed fields) are related to the mass eigenstates fields u, c, t as follows:

$$\begin{pmatrix} u'_{1L} \\ u'_{2L} \\ u'_{3L} \end{pmatrix} = \mathcal{O}_L^u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L, \quad \begin{pmatrix} U''_{1R} \\ U''_{2R} \\ U''_{3R} \end{pmatrix} = \mathcal{O}_R^u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R. \quad (25)$$

The Yukawa interaction in Eq. (21) can be written as

$$\begin{aligned} -\mathcal{L}_Y^u &= \bar{U}_L M^u U_R + \bar{U}_L M^u U_R \frac{X_\rho^0 - iI_\rho^0}{|v_\rho|} \\ &+ \bar{U}_L (\mathcal{O}_L^u)^T \Delta \mathcal{O}_L^u M^u U_R \left(\frac{X_\eta^0 + iI_\eta^0}{|v_\eta|} - \frac{X_\rho^0 - iI_\rho^0}{|v_\rho|} \right) + \text{H.c.}, \end{aligned} \quad (26)$$

with

$$\Delta \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (27)$$

Since there is no mixing among X 's and I 's we have no CP violation in this sector.

2. d -like sector—neutral scalar interactions

Similarly, in the d -like sector, the interaction with the neutral Higgs scalars are

$$-\mathcal{L}_Y^d = \bar{d}'_{1L} \sum_\alpha \tilde{G}_{1\alpha} D'_{\alpha R} \rho^0 + \sum_m \bar{d}'_{mL} \sum_\alpha \tilde{F}_{m\alpha} D'_{\alpha R} \eta^{0*} + \text{H.c.} \quad (28)$$

Making the following phase redefinition

$$D''_{\alpha R} \equiv e^{i\theta_\rho} D'_{\alpha R}, \quad d''_{mL} \equiv e^{-i(\theta_\rho + \theta_\eta)} d'_{mL}, \quad (29)$$

we obtain a real mass matrix which can be diagonalized with an orthogonal transformation

$$(\mathcal{O}_L^d)^T \Gamma^d \mathcal{O}_R^d = M^d = \text{diag}(m_d, m_s, m_b), \quad (30)$$

with

$$\Gamma^d \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} |v_\rho| \tilde{G}_{11} & |v_\rho| \tilde{G}_{12} & |v_\rho| \tilde{G}_{13} \\ |v_\eta| \tilde{F}_{21} & |v_\eta| \tilde{F}_{22} & |v_\eta| \tilde{F}_{23} \\ |v_\eta| \tilde{F}_{31} & |v_\eta| \tilde{F}_{32} & |v_\eta| \tilde{F}_{33} \end{pmatrix}. \quad (31)$$

The symmetry eigenstates (singly and doubly primed fields) are related to the mass eigenstates (unprimed fields) as follows:

$$\begin{pmatrix} d'_{1L} \\ d'_{2L} \\ d'_{3L} \end{pmatrix} = \mathcal{O}_L^d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \quad \begin{pmatrix} D''_{1R} \\ D''_{2R} \\ D''_{3R} \end{pmatrix} = \mathcal{O}_R^d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R, \quad (32)$$

and the interactions in Eq. (28) become

$$\begin{aligned} -\mathcal{L}_Y^d &= \bar{D}_L M^d D_R + \bar{D}_L M^d D_R \frac{X_\eta^0 - iI_\eta^0}{|v_\eta|} \\ &+ \bar{D}_L (\mathcal{O}_L^d)^T \Delta \mathcal{O}_L^d M^d D_R \left(\frac{X_\rho^0 + iI_\rho^0}{|v_\rho|} - \frac{X_\eta^0 - iI_\eta^0}{|v_\eta|} \right) + \text{H.c.} \end{aligned} \quad (33)$$

Again, we see that if X 's and I 's do not mix among them we have not CP violation through the exchange of neutral fields. This is the case of the model with only three triplets in Sec. II.

3. Exotic quarks—neutral scalar interactions

In the sector involving exotic quarks we have

$$-\mathcal{L}_Y^J = \lambda_1 \bar{J}'_{1L} J'_{1R} \chi^0 + \sum_{i,m} \lambda_{im} \bar{J}'_{iL} J'_{mR} \chi^{0*} + \text{H.c.} \quad (34)$$

Making the redefinition of the right-handed components of the exotic quarks

$$J''_{1R} \equiv e^{i\theta_\chi} J'_{1R}, \quad J''_{mR} \equiv e^{-i\theta_\chi} J'_{mR}, \quad (35)$$

we have

$$\begin{aligned} -\mathcal{L}_Y^J &= \lambda_1 \bar{J}'_{1L} J''_{1R} \frac{|v_\chi|}{\sqrt{2}} \left(1 + \frac{X_\chi^0 + iI_\chi^0}{|v_\chi|} \right) \\ &+ \sum_{i,m} \lambda_{im} \bar{J}'_{iL} J''_{mR} \frac{|v_\chi|}{\sqrt{2}} \left(1 + \frac{X_\chi^0 - iI_\chi^0}{|v_\chi|} \right) + \text{H.c.} \end{aligned} \quad (36)$$

Notice that J'_1 (or J''_1) does not mix with any other quarks but $J'_{2,3}$ (or $J''_{2,3}$) mix between themselves since they have the same charge. Here we use, when necessary,

$$J \equiv J''_1 \quad \text{and} \quad (J''_{L,R})_m = (\mathcal{O}_{L,R}^J)_{mi} (J_{L,R})_i, \quad (37)$$

$\mathcal{O}_{L,R}^J$ are orthogonal 2×2 matrices; where J and j_i and now with $i=1,2$ denote mass eigenstates, i.e., the mass eigenstates in the exotic quark sector will be denoted when necessary J for the charge 5/3 quark and $j_{1,2}$ for the two charge $-4/3$ quarks. Hence Eq. (36) can be written in terms of mass eigenstates exotic fermions

$$\begin{aligned} -\mathcal{L}_Y^J &= m_J \bar{J}_L J_R \left(1 + \frac{X_\chi^0 + iI_\chi^0}{|v_\chi|} \right) + \bar{J}_L M^J j_R \left(1 + \frac{X_\chi^0 - iI_\chi^0}{|v_\chi|} \right) \\ &+ \text{H.c.}, \end{aligned} \quad (38)$$

where $M^J = \text{diag}(m_{j_1}, m_{j_2})$.

4. Singly charged scalar—quark interactions

The interaction Lagrangian is

$$\begin{aligned}
-\mathcal{L}_Y^{u-d} = & \sum_{\alpha} (\bar{d}'_{1L} G_{1\alpha} U'_{\alpha R} \eta_1^- + \bar{u}'_{1L} \tilde{G}_{1\alpha} D'_{\alpha R} \rho^+) \\
& + \sum_{m,\alpha} (\bar{d}'_{mL} F_{i\alpha} U'_{\alpha R} \rho^- + \bar{u}'_{mL} \tilde{F}_{i\alpha} D'_{\alpha R} \eta_1^+) + \text{H.c.}
\end{aligned} \quad (39)$$

Using the phase redefinition of Eqs. (22), (29), and (35) in Eq. (39) we have

$$\begin{aligned}
-\mathcal{L}_Y^{u-d} = & \frac{\sqrt{2}}{|\mathbf{v}_\rho|} \bar{D}_L (\mathcal{O}_L^d)^T K_1 \mathcal{O}_L^u M^u U_R \rho^- \\
& + \bar{U}_L (\mathcal{O}_L^u)^T K_2 \mathcal{O}_L^d M^d D_R \frac{\sqrt{2}}{|\mathbf{v}_\eta|} \eta_1^+ \\
& + \bar{D}_L (\mathcal{O}_L^d)^T K_1 \Delta \mathcal{O}_L^u M^u U_R \left[\frac{\sqrt{2}}{|\mathbf{v}_\eta|} \eta_1^- - \frac{\sqrt{2}}{|\mathbf{v}_\rho|} \rho^- \right] \\
& + \bar{U}_L (\mathcal{O}_L^u)^T K_2 \Delta \mathcal{O}_L^d M^d D_R \left[\frac{\sqrt{2}}{|\mathbf{v}_\rho|} \rho^+ - \frac{\sqrt{2}}{|\mathbf{v}_\eta|} \eta_1^+ \right] \\
& + \text{H.c.},
\end{aligned} \quad (40)$$

being

$$K_1 = \text{diag}(e^{-i\theta_\eta}, e^{i\theta_\rho}, e^{i\theta_\rho}) \quad \text{and} \quad K_2 = \text{diag}(e^{-i\theta_\rho}, e^{i\theta_\eta}, e^{i\theta_\eta}). \quad (41)$$

Notice that if we consider the more general potential, i.e., with the a_{10} term, we have, according to the Eq. (8) a general mixing among all singly charged scalars. There is CP violation through the exchange of a singly charged scalar in this case. However, when we consider the case when $a_{10}=0$ the mixing in that sector is given by Eqs. (9) and (11) and there is no CP violation in the interaction of Eq. (40) if we also chose that $\theta_\eta = \theta_\rho = 0$ by making a $SU(3)$ transformation.

Finally, we have the interaction involving the exotic quarks,

$$\begin{aligned}
-\mathcal{L}_Y^{J-\chi} = & \sum_{\alpha} \bar{J}_L G_{1\alpha} \mathcal{O}_{R\alpha\beta}^u U_{\beta R} \eta_2^+ e^{-i\theta_\eta} \\
& + \frac{m_J \sqrt{2}}{|\mathbf{v}_\chi|} \bar{U}_L (\mathcal{O}_L^u)^T \Delta J_R \chi^- e^{-i\theta_\chi} \\
& + \frac{\sqrt{2}}{|\mathbf{v}_\chi|} \bar{D}_L (\mathcal{O}_L^d)^T \bar{\Delta} (\mathcal{O}_R^J)^T \tilde{M}^J \tilde{J}_R \chi^+ e^{-i(\theta_\rho + \theta_\eta - \theta_\chi)} \\
& + \text{H.c.},
\end{aligned} \quad (42)$$

where we have already used Eqs. (22) and (29) and defined

$$\begin{aligned}
\bar{\Delta} & \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \tilde{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{j_1} & 0 \\ 0 & 0 & m_{j_2} \end{pmatrix}, \\
\tilde{J}_R & = \text{diag}(0, j_1, j_2).
\end{aligned} \quad (43)$$

The phases of the fields u'_{1L} and d'_{1L} (and those of the J'_{1L} and J'_{mL} too) are, in principle, still free. However, we can not change the phases of these fields unless we change the phases in the Eqs. (21) and (28) [or also in Eqs.(40)]. Notwithstanding, the phases in those equations have already being absorbed after the redefinition given in Eqs. (22) and (29), respectively. We can not absorb phases any more so we have CP violation through the exchange of the singly charged scalars which are coupled to the exotic and known quarks. Hence, we have shown that the only phase redefinition in the quark fields are those in Eqs. (22), (29), and (35). It means that the observable CP violating phase is $\text{Im} \mathbf{v}_\chi$ (or $-\text{Im} \alpha$). This will be also the case in the interactions with the doubly charged scalars.

5. Doubly charged scalar–quark interactions

The doubly charged Higgs scalars couple the known quarks with the exotic ones and by using Eqs. (22) and (29) we have

$$-\mathcal{L}_Y^{J-\rho} = \sum_{\alpha} \bar{J}_L \tilde{G}_{1\alpha} \mathcal{O}_{R\alpha\beta}^d D_{\beta R} \rho^{++} e^{-i\theta_\rho} + \sum_{i,l,\alpha,\beta} \bar{J}_L l (\mathcal{O}_L^J)^T_{li} F_{i\alpha} \mathcal{O}_{R\alpha\beta}^u U_{\beta R} \rho^{--} e^{-i\theta_\eta} + \text{H.c.}, \quad (44a)$$

$\beta = 1, 2, 3, l = 1, 2$, and

$$-\mathcal{L}_Y^{J-\chi} = \frac{m_J \sqrt{2}}{|\mathbf{v}_\chi|} \bar{D}_L (\mathcal{O}_L^d)^T \Delta J_R \chi^{--} e^{-i\theta_\chi} + \frac{\sqrt{2}}{|\mathbf{v}_\chi|} \bar{U}_L \bar{\Delta} (\mathcal{O}_L^u)^T \mathcal{O}_L^J \tilde{M}^J \tilde{J}_R \chi^{++} e^{i(\theta_\chi - \theta_\rho - \theta_\eta)} + \text{H.c.}, \quad (44b)$$

where we have omitted matrix indices in the last equation. From the same argument of the preceding subsections we can see that there is CP violation in the doubly charged Higgs exchange too. The fields ρ^{++} and χ^{++} are given in terms of the respective mass eigenstates in Eq. (6). Fermion fields are all the mass eigenstates.

B. Lepton-scalar interactions

Now, let us consider the leptonic sector. The leptons are assigned to the following representations:

$$\Psi_{aL} = \begin{pmatrix} \nu_{l_a} \\ l'^{-}_a \\ E'^{+}_a \end{pmatrix}_L \sim (\mathbf{3}, 0); \quad l'^{-}_{aR} \sim (\mathbf{1}, -1),$$

$$E'^{-}_{aR} \sim (\mathbf{1}, +1), \quad a = 1, 2, 3. \quad (45)$$

The Yukawa interactions in this sector are

$$-\mathcal{L}_Y^l = G_{ab}^e \bar{\psi}_{aL} l'^{-}_{bR} \rho + G_{ab}^\psi \bar{\psi}_{aL} E'^{+}_{bR} \chi + \text{H.c.} \quad (46)$$

Next, we will consider each type of interactions as we did in the quark sectors.

1. Lepton-neutral scalar interactions

In this sector the relation between the symmetry eigenstate fields (E'_a, l'_{aL}) and the mass eigenstate fields $(l_i = e, \mu, \tau; E_i = E_1, E_2, E_3)$ is obtained through orthogonal matrices (note that in the leptonic sector $i = 1, 2, 3$)

$$l'_{aL} = \mathcal{O}_{Lai}^e l_{iL}, \quad l'_{aR} = \mathcal{O}_{Rai}^e l_{iR},$$

$$E'_{aL} = \mathcal{O}_{Lai}^E E_{iL}, \quad E'_{aR} = \mathcal{O}_{Rai}^E E_{iR}. \quad (47)$$

The respective mass matrices are defined as follows:

$$M^e = \frac{|v_\rho|}{\sqrt{2}} (\mathcal{O}_L^e)^T G^e \mathcal{O}_R^e,$$

$$M^E = \frac{|v_\chi|}{\sqrt{2}} (\mathcal{O}_L^E)^T G^E \mathcal{O}_R^E, \quad (48)$$

with $M^e = \text{diag}(m_e, m_\mu, m_\tau)$ and $M^E = \text{diag}(m_{E_1}, m_{E_2}, m_{E_3})$. Notice that the exotic lepton masses are proportional to the larger VEV, v_χ . We see that there is no CP violation in this sector too.

In terms of the physical lepton fields we have

$$-\mathcal{L}_Y^l = \bar{l}_{eL} M^e l_{eR} + \bar{E}_L M^E E_R + \bar{l}_{eL} M^e l_{eR} \frac{X_\rho^0 + iI_\rho^0}{|v_\rho|}$$

$$+ \bar{E}_L M^E E_R \frac{X_\chi^0 + iI_\chi^0}{|v_\chi|} + \text{H.c.}, \quad (49)$$

and we have redefined the right-handed lepton components

$$l''_{iR} = e^{i\theta_\rho} l_{iR}, \quad E''_{iR} = e^{i\theta_\chi} E_{iR}, \quad (50)$$

but however we have omitted the double primed in Eq. (49).

2. Lepton-singly charged scalar interactions

The interactions involving singly charged scalars are

$$-\mathcal{L}_Y^{v-l,E} = \frac{\sqrt{2}}{|v_\rho|} \bar{\nu}_L^T M^e l_{iR} \rho^+ + \frac{\sqrt{2}}{|v_\chi|} \bar{\nu}_L^T \mathcal{K} M^E E_R \chi^- e^{-i(\theta_\chi - \theta_\rho)} + \text{H.c.}, \quad (51)$$

where we have defined

$$\bar{\nu}_L^T = \bar{\nu}_L \mathcal{O}_L^e e^{-i\theta_\rho}, \quad (52)$$

and $\mathcal{K} \equiv (\mathcal{O}_L^e)^T \mathcal{O}_L^E$. We see that even if we choose $\theta_\rho = 0$ the phase θ_χ survives and we have CP violation by the singly charged scalar exchanging, i.e., in the neutrino-exotic lepton vertex.

3. Lepton-doubly charged scalar interactions

The interactions with the doubly charged scalars in the lepton sector are given by

$$-\mathcal{L}_Y^{l-E} = \frac{\sqrt{2}}{|v_\rho|} \bar{E}_L \mathcal{K}^T M^e l_{iR} \rho^{++} e^{-i\theta_\rho} + \frac{\sqrt{2}}{|v_\chi|} \bar{E}_L \mathcal{K} M^E E_R \chi^{--} e^{-i\theta_\chi} + \text{H.c.} \quad (53)$$

As in the previous case we have CP violation in the doubly charged scalar sector even if we choose $\theta_\rho = 0$.

IV. GAUGE INTERACTIONS

Next, we will verify in what conditions all phase redefinitions that have been done in the previous section do not appear in the vector boson-fermion interactions.

A. Quark-vector boson interactions

With the redefinition of the phases in Eqs. (22) and (29) the mixing matrix in the charged currents coupled to the W^\pm is real

$$\mathcal{L}_Y^{W-q} = -\frac{g}{\sqrt{2}} \bar{U}_L \gamma^\mu V_{\text{CKM}} D_L W_\mu^+ + \text{H.c.}, \quad (54)$$

with the CKM matrix defined as $V_{\text{CKM}} = (\mathcal{O}_L^u)^T \mathcal{O}_L^d$. So we have no CP violations in this sector. Similarly, we have in the charged currents coupled to the vector bileptons V^+ and U^{--} ,

$$\mathcal{L}_Y^{V-q} = -\frac{g}{\sqrt{2}} \left(\bar{J}'_{1L} \gamma^\mu u'_{1L} - \sum_{i,m} \bar{d}''_{iL} \gamma^\mu J'_{mL} \right) V_\mu^+ e^{-i(\theta_\rho + \theta_\eta)} + \text{H.c.} \quad (55)$$

and

$$\mathcal{L}_Y^{U-q} = -\frac{g}{\sqrt{2}} \left(\bar{J}'_{1L} \gamma^\mu d'_{1L} - \sum_{i,m} \bar{u}''_{iL} \gamma^\mu J'_{mL} \right) U_\mu^{--} e^{-i(\theta_\rho + \theta_\eta)} + \text{H.c.} \quad (56)$$

However, we always can choose $v_\eta = v_\rho = 0$ by using a $SU(3)$ transformation. Hence, we will not have CP violation in the bilepton sector.

B. Lepton-vector boson interactions

The charged current interactions with the vector bosons in the leptonic sector are

$$\mathcal{L}_l^{CC} = -\frac{g}{\sqrt{2}} \sum_i (\bar{\nu}_{iL} \gamma^\mu l_{iL} W_\mu^+ + \bar{\psi}_{iL} \gamma^\mu \nu_{iL} V_\mu^+ + \bar{\psi}_{iL} \gamma^\mu l_{iL} U_\mu^{++} + \text{H.c.}), \quad (57)$$

where all fields are still symmetry eigenstates (but we have omitted the prime). Then, the charged current interactions in terms of the physical basis, using Eq. (47), is given by

$$\mathcal{L}_l^{CC} = -\frac{g}{\sqrt{2}} (\bar{\nu}_L \gamma^\mu l_L W_\mu^+ + \bar{\psi}_L \gamma^\mu \mathcal{K}^T \nu_L V_\mu^+ e^{i\theta_\rho} - \bar{\psi}_L \gamma^\mu \mathcal{K} l_L U_\mu^{++} + \text{H.c.}), \quad (58)$$

where $\mathcal{K} \equiv (\mathcal{O}_L^e)^T \mathcal{O}_L^\psi$ and we have used the redefinition of the neutrino fields in Eq. (52). Although a phase appears we can always choose it as being zero by an $SU(3)$ transformation.

We see from Eq. (58) that for massless neutrinos we have no mixing in the charged current coupled to W_μ^+ but we still have mixing in the charged currents coupled to V_μ^+ and U_μ^{++} . That is, if neutrinos are massless we can always choose $\bar{\nu}'_L \equiv \bar{\nu}_L \mathcal{O}_L^e$. However, the charged currents coupled to V_μ^+ and U_μ^{++} are not diagonal in flavor space since the mixing matrix \mathcal{K} survives. Thus, there is not CP violation in this sector if we use the freedom to choose $\theta_\rho = 0$.

The pure gauge boson interactions also conserve CP if the $SU(3)_L$ gauge and $U(1)_N$ vector bosons W^a with $a = 1, \dots, 8$, and B , respectively transform as

$$(W_\mu^1, W_\mu^2, W_\mu^3, W_\mu^4, W_\mu^5, W_\mu^6, W_\mu^7, W_\mu^8, B_\mu) \xrightarrow{CP} (W^{1\mu}, -W^{2\mu}, W^{3\mu}, -W^{4\mu}, W^{5\mu}, -W^{6\mu}, W^{7\mu}, W^{8\mu}, B^\mu). \quad (59)$$

It means that the physical fields transform as

$$(W_\mu^+, V_\mu^+, U_\mu^{++}, A_\mu, Z_\mu, Z'_\mu) \xrightarrow{CP} -(W^{\mu-}, -V^{\mu-}, -U^{\mu--}, A^\mu, Z^\mu, Z'^\mu). \quad (60)$$

We have shown that if all term, except the trilinear term in the scalar potential conserve CP this symmetry will be broken when the neutral scalars gain a complex VEV.

V. PHENOMENOLOGICAL CONSEQUENCES

As it is well known, the violation of the CP symmetry was discovered in 1964 in the $K^0 - \bar{K}^0$ system [13]. Up to now, it is only in this particular system in which CP violating effects has been seen. If the source of the CP violation is the weak interactions we expect also to see its effects in B decays. However, only a general discussion is presented here concerning the mesons case [14]. On the other hand a detailed study of the electric dipole moments of the neutron and charged leptons is shown.

A. Quark sector

1. CP violation in the neutral meson systems

In Fig. 1 we show the tree level contributions to the mass difference $\Delta M_K = 2\text{Re}M_{12}$ (where $M_{12} = \langle K^0 | \mathcal{H}_{\text{eff}} | \bar{K}^0 \rangle$). These diagrams exist because of the flavor changing neutral currents in Eq. (33) and in the couplings with the Z'^0 . The H^0 's contributions to ΔM_K have been considered in Ref.

[15]. For $m_H \sim 150$ GeV the constraint coming from the experimental value of ΔM_K implies $(\mathcal{O}_L^d)_{11}(\mathcal{O}_L^d)_{12} \lesssim 0.01$. There are also tree level contributions to ΔM_K coming from the Z' exchange which were considered in Ref. [16]. Similarly for the mass difference of $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$, and $D^0 - \bar{D}^0$ systems. However, in the present model, the CP violating parameters like ε_K have only contributions coming from box diagrams involving two doubly charged scalars as can be seen from Fig. 2. The direct CP violation parameter ε'_K has contributions at the 1-loop level too, as is shown in Fig. 3. There are penguin contributions only at the 2-loop level as we will see later on.

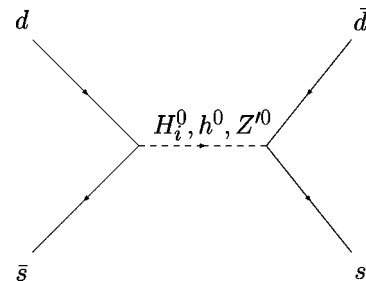
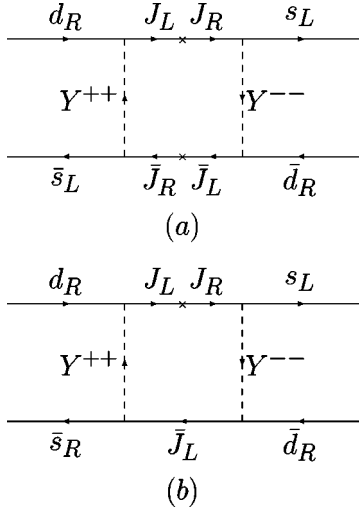


FIG. 1. Tree level contributions to ΔM_K .

FIG. 2. Some of the box diagram contributions to ϵ and M_{12} .

Although we will not make here a detailed calculation of ϵ_K and ϵ'_K and the respective parameters in the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ systems we can notice that in principle the model can give values for these CP violating parameters which are in accord with data since they depend on different mixing matrices $\mathcal{O}_{L,R}^{u,d}$. From Eqs. (44a) and (44b) we see that ϵ_K comes from diagrams like those shown in Figs. 2(a) and 2(b). (There are two other diagrams as the ones in Figs. 2 but with the lines of Y and J interchanged.) There are diagrams similar to those in Figs. 2 but with one of the scalar bileptons being changed by a vector bilepton and with no mass insertion. These contributions are less suppressed by the mixing angles than by the vector bilepton mass.

Similar diagrams do exist for the other B mesons. The coefficient of the amplitude produced by diagrams like that in Fig. 2 are proportional to the several mixing matrix elements and Yukawa couplings. For instance, for the neutral K system, from Fig. 2(a), up to a $\sin(4\theta_\alpha)$ factor the amplitude is proportional to

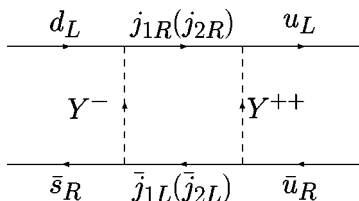
$$G_{1\alpha}(\mathcal{O}_R^d)_{\alpha 1}(\mathcal{O}_L^d)_{21}G_{1\beta}(\mathcal{O}_R^d)_{\beta 1}(\mathcal{O}_L^d)_{11}, \quad (61a)$$

while for B mesons, similar diagrams to that in Fig. 2(a) imply that the amplitudes are proportional to

$$G_{1\alpha}(\mathcal{O}_R^d)_{\alpha 1}(\mathcal{O}_L^d)_{31}G_{1\beta}(\mathcal{O}_R^d)_{\beta 1}(\mathcal{O}_L^d)_{11}, \quad (61b)$$

for the neutral B_d system, and

$$G_{1\alpha}(\mathcal{O}_R^d)_{\alpha 2}(\mathcal{O}_L^d)_{31}G_{1\beta}(\mathcal{O}_R^d)_{\beta 3}(\mathcal{O}_L^d)_{21}, \quad (61c)$$

FIG. 3. The box diagram contribution to ϵ' .

for the neutral B_s system. We see from Eqs. (61) that the orthogonality condition implies that if we have chosen two of the ϵ_{K,B_d,B_s} parameters the third one is fixed. A similar analysis follows from Fig. 2(b) and the equivalent diagrams for the B systems.

In the D mesons case the mixing matrices involved are different since in these models the left-handed mixing matrix \mathcal{O}_L^u survives in different places of the Lagrangian so it is not natural to set them equal to zero (see below). Hence, we see that all CP phenomenology in the meson systems can be accommodated, in principle, in the present model.

Concerning the direct CP violating parameters ϵ'_K its contribution for the neutral kaons comes from diagrams like the one in Fig. 3 (and the one with the line of Y s and $j_{1,2}$ interchanged). The vertices are given in Eqs. (42) and (44b) and we see that there is a GIM-like cancellation between the contributions of j_1 and j_2 . It means that the suppression of ϵ'_K does not give necessarily a strong constraint on the masses of $j_{1,2}$. There are similar diagrams with the Y^- substituted by a V^- vector bilepton and mass insertions in the exotic quarks lines. More details will be given elsewhere [14] however since the contributions given in Fig. 3 depend on the masses of the $j_{1,2}$ exotic quarks it seems no problem to produce the value

$$Re \left(\frac{\epsilon'_K}{\epsilon_K} \right) = 0.00200 \pm 0.00030 \text{ (stat)} \pm 0.00026 \text{ (syst)} \\ \pm 0.00019 \text{ (MC stat)} \quad (62)$$

obtained recently by the KTeV collaboration at Fermilab [17,18]. Hence in this model we have naturally that $0 \neq |\epsilon'| \ll |\epsilon|$.

As in the case of the ϵ parameters, once we have chosen the appropriate value of the mixing matrix elements (and Yukawa couplings) for explaining the observed value in the $K^0 - \bar{K}^0$ system, we can still choose the ϵ related to the B_d or to the B_s systems but not for both at the same time. The third one has to have a small value of ϵ' due to the orthogonality of the mixing matrix.

2. Electric dipole moment

It is well known that the discovery of a nonzero electric dipole moment (EDM) for the neutron (or another elementary nondegenerate system) would be a direct evidence for both CP and T violation. The current experimental upper bound is [19]

$$|d_n| < 6.3 \times 10^{-26} e \text{ cm}. \quad (63)$$

In the standard model the neutron EDM arises at the three-loop level and for this reason it is very small, $\sim 10^{-34} e \text{ cm}$ [20].

In the present model we can calculate the EDM of the d and u quarks at the one-loop level. In principle the contributions are those shown in Figs. 4 and 5. However, we can see from the interactions in Eqs. (40), (42), (44a), and (44b) with the phase convention $\theta_\eta = \theta_\rho = 0$ and the coefficient $a_{10} = 0$, that neither the diagram in Fig. 4(a) nor that in 4(b) contrib-

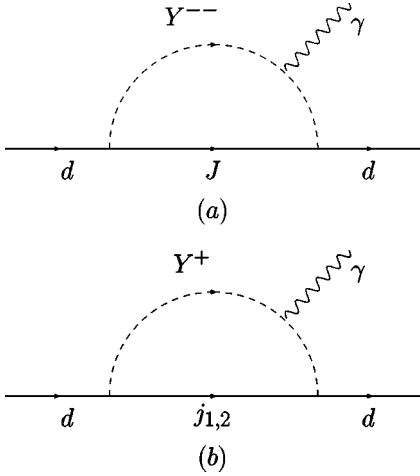


FIG. 4. Possible diagram contributing to the EDM of the quarks d .

ute to the EDM of the quark d , thus we have $d_d = 0$, at this level of approximation. For the u -like quarks we have not only the diagram in Fig. 5(a) but also those like the one in Fig. 5(b). Here we will show only the contributions from Fig. 5(a) and its related diagram with the photon attached to the internal fermion line (recall that $\theta_\chi = -\theta_\alpha$):

$$d_u = e \frac{\sqrt{2} G_{1\alpha} (\mathcal{O}_R^u)_{\alpha 1} (\mathcal{O}_L^u)_{11} m_J^2 |v_\eta|}{32\pi^2 (|v_\chi|^2 + |v_\eta|^2) m_{Y^+}^2} \times \left[F_+(m_u, m_J) + \frac{5}{3} F_-(m_u, m_J) \right] \sin\theta_\alpha, \quad (64)$$

where

$$F_\pm(m_u, m_J) = -\frac{m_{Y^+}^2}{2m_u^2} \ln \frac{m_{Y^+}^2}{m_J^2} + \frac{m_{Y^+}^2}{2m_u^2 \Delta_u} (m_{Y^+}^2 \pm m_u^2 - m_J^2) \times \ln \left[\frac{m_J^2 + m_{Y^+}^2 - m_u^2 + \Delta_u}{m_J^2 + m_{Y^+}^2 - m_u^2 - \Delta_u} \right], \quad (65)$$

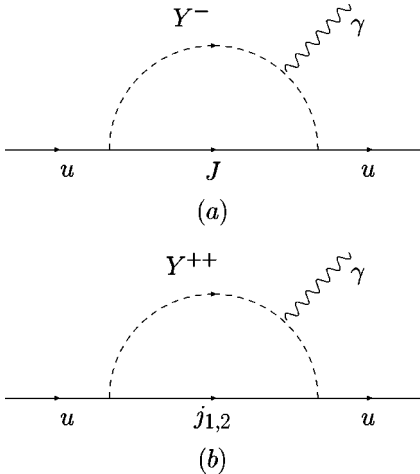


FIG. 5. Possible diagram contributing to the EDM of the quarks u .

and

$$\Delta_u^2 = [(m_{Y^+} + m_J)^2 - m_u^2][(m_{Y^+} - m_J)^2 - m_u^2], \quad (66)$$

the 5/3 factor is because of the charge of the quark- J .

The measured parameter is the EDM of the neutron which in the quark model is given in terms of the constituent quarks's EDM:

$$d_n = \frac{4}{3} d_d - \frac{1}{3} d_u = -\frac{1}{3} d_u \simeq -0.486 \times 10^{-21} G_{\alpha 1} (\mathcal{O}_R^u)_{1\alpha} (\mathcal{O}_L^u)_{11} \sin\theta_\alpha \text{ cm}, \quad (67)$$

where we have made the approximation $|v_\eta| \approx |v_\rho|$ in order to use the doubly charged vector bilepton mass M_U , given by $M_U^2 = (g^2/4)(|v_\chi|^2 + |v_\rho|^2)$, instead of $|v_\chi|^2 + |v_\eta|^2$ in Eq. (64); and $G_F/\sqrt{2} = g^2/8M_W^2$. We have used $M_U = 300$ GeV, $M_{Y^+} = 100$ GeV; $m_J = 50$ GeV and $|v_\eta| = 100$ GeV. However, the value of d_u (or d_n) is not sensible to the masses of the exotic particles m_{Y^+} , M_U and m_J , at least with M_U lesser than 10 TeV. It means that the factor $G_{1\alpha} (\mathcal{O}_R^u)_{\alpha 1} (\mathcal{O}_L^u)_{11}$, for any value of $\sin\theta_\alpha$, have to be invoked in order to obtain an EDM of the neutron compatible with the data in Eq. (63). This is not in conflict with the CKM mixing matrix in the charged current coupled to the vector boson W^\pm since the later is defined as $V_{\text{CKM}} = (\mathcal{O}_L^u)^T \mathcal{O}_L^d$. Since in Eq. (67) only the matrix element $(\mathcal{O}_{R,L}^u)_{11}$ related to the u -like quarks appears and, with the phase convention used here, the mixing matrices related to the d -like quarks do not enter at all in this sort of models and we cannot use $V_{\text{CKM}} = \mathcal{O}_L^d$ as is usually done in the standard electroweak model. Notice that the mass of the quark u does not enter in the EDM. This is because the interaction in Eq. (42) are not proportional to the mass of the u -like quarks.

In fact, the matrices $\mathcal{O}_L^{u,d}$ will appear in the neutral currents coupled to the extra neutral vector boson Z'^0 . We have

$$\begin{aligned} \mathcal{L}_{Z'} = & -\frac{g}{2c_W} [\bar{U}_L \gamma^\mu \mathcal{O}_L^u Y_L^U(Z') \mathcal{O}_L^u U_L \\ & + \bar{U}_R \gamma^\mu \mathcal{O}_R^u Y_R^U(Z') \mathcal{O}_R^u U_R + \bar{D}_L \gamma^\mu \mathcal{O}_L^d Y_L^D(Z') \mathcal{O}_L^d D_L \\ & + \bar{D}_R \gamma^\mu \mathcal{O}_R^d Y_R^D(Z') \mathcal{O}_R^d D_R] Z'_\mu, \end{aligned} \quad (68)$$

with the matrices

$$Y_L^U(Z') = Y_L^D(Z') = -\frac{1}{\sqrt{3}h(s_W)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2s_W - 1 & 0 \\ 0 & 0 & 2s_W - 1 \end{pmatrix} \quad (69)$$

and

$$Y_R^U(Z') = -\frac{4s_W}{\sqrt{3}h(s_W)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (70)$$

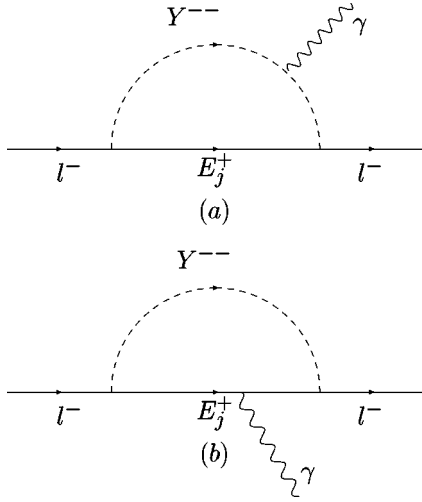


FIG. 6. Diagrams contributing to the EDM of the charged leptons.

$$Y_R^D(Z') = \frac{2s_W}{\sqrt{3}h(s_W)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $h(s_W) = (1 - 4s_W^2)^{1/2}$. Notice that the right-handed neutral currents remain diagonal, but it is not so for the left-handed ones. Here the matrices \mathcal{O}_L^u and \mathcal{O}_L^d survive in a different form than they appear in the V_{CKM} matrix. These matrices \mathcal{O}_L^u , \mathcal{O}_L^d , and V_{CKM} have to be only determined from experiment since we cannot set none of them to be diagonal in a natural way.

B. Lepton sector

Beside the neutron EDM, other ones which are experimentally well studied (alas not definitively yet) is the EDM of the electron and the muon. It is interesting that the EDM of the electron (d_e) in the standard model is rather small, of the order of magnitude of $2 \times 10^{-38} e$ cm [21]. However, the value of d_e could be even smaller, of the order of $10^{-41} e$ cm if there is a cancellation of the three-loop diagrams [22] even if QCD corrections are included [23]. On the other hand, the experimental upper limit is $\leq 4 \times 10^{-27} e$ cm [24].

Hence, a measurement of a nonzero large electron EDM (and other elementary particles like muon, tau, and neutron, see below) would indicate new physics beyond the standard model.

Although we could have CP violation in the singly charged scalar sector (if $a_{10} \neq 0$), there would be no contribution to the EDM of the leptons at the one-loop level since neutrinos are massless. If neutrinos remain massless the contribution to the electric dipole moment will arise at the three-loop level (this also happens in multi-Higgs extensions of the standard model [25]). This is not the case for diagrams involving doubly charged scalars and the known leptons and the exotic ones (see Fig. 6). It is always possible to choose the doubly charged scalars those fields which carry this phase, i.e., the CP violation in the leptonic sector occurs

only through the exchange of both exotic leptons and of doubly charged scalar fields.

The Yukawa couplings of leptons with the doubly charged scalars are given in Eq. (53) where the scalar fields are still symmetry eigenstates. In fact, there is one Goldstone boson G^{++} and a physical one Y^{++} ; we denote its mass by $m_{Y^{++}}$. (In this model there is not lepton number violation in the interactions with the neutral scalars.) As we have shown in Secs. III B and IV B it is not possible to absorb all phases in the complete lepton Lagrangian density. Since neutrinos are considered massless here, the only contributions to the leptonic EDM arise from the doubly charged scalars as shown in Fig. 6. From this we obtain

$$d_l = - \frac{em_l}{32\pi^2 m_{Y^{++}}^2} \sqrt{2} M_W^2 G_F \mathbf{O}_l \sin(2\theta_\alpha), \quad l = e, \mu, \tau; \quad (71)$$

where we have defined

$$\mathbf{O}_l = \sum_j \mathcal{K}_{lj}^2 \frac{4m_{E_j}^2}{M_U^2} [F_+(m_l, m_{E_j}) + F_-(m_l, m_{E_j})], \quad (72)$$

the matrix \mathcal{K} has been introduced in Eqs. (51) and (53);

$$F_\pm(m_l, m_{E_j}) = - \frac{m_{Y^{++}}^2}{2m_l^2} \ln \frac{m_{Y^{++}}^2}{m_{E_j}^2} + \frac{m_{Y^{++}}^2}{2m_l^2 \Delta_l} (m_{Y^{++}}^2 \pm m_l^2 - m_{E_j}^2) \times \ln \left[\frac{m_{E_j}^2 + m_{Y^{++}}^2 - m_l^2 + \Delta_l}{m_{E_j}^2 + m_{Y^{++}}^2 - m_l^2 - \Delta_l} \right], \quad (73)$$

and

$$\Delta_l^2 = [(m_{Y^{++}} + m_{E_j})^2 - m_l^2][(m_{Y^{++}} - m_{E_j})^2 - m_l^2]. \quad (74)$$

For nondegenerate heavy leptons the mixing angles remain in Eq. (72). In the following, for the sake of simplicity, we will assume that $m_{E_1} < m_{E_2} < m_{E_3}$ and these heavy leptons couple mainly with electron, muon, and tau respectively. For instance, the contribution of E_1 to the electron EDM, using $m_{E_1} = 50$ GeV and $m_{Y^{++}} = 100$ GeV [26] is given by

$$d_e \approx 10^{-25} \left(\frac{M_W^2}{M_U^2} \right) \mathcal{K}_{e1}^2 \sin(2\theta_\alpha) e \text{ cm}. \quad (75)$$

Assuming $M_U = 300$ GeV and the factor with the mixing angles $\mathcal{K}_{e1}^2 < 10^{-1}$ we obtain $d_e \approx 10^{-27} e$ cm for any value of θ_α , which is compatible with the experimental upper limit

of $10^{-27}e\text{ cm}$ [24]. However, if we have used $M_U=3000$ GeV we obtain a d_e compatible with the experimental data for $\mathcal{K}_{e1}^2 \approx 1$.

For the muon we have an experimental EDM's upper limits of $d_\mu < 10^{-19}e\text{ cm}$ [27]. With $m_{E_2}=500$ GeV we obtain

$$d_\mu \approx 10^{-23} \mathcal{K}_{\mu 2}^2 \sin(2\theta_\alpha) e\text{ cm}. \quad (76)$$

It means a constraint in $\mathcal{K}_{\mu 2}^2 \approx 1$ and any θ_α we obtain a value smaller than the experimental upper limit.

For the tau lepton a limit of $10^{-17}e\text{ cm}$ is derived from $\Gamma(Z \rightarrow \tau^+ \tau^-)$ [28,29]. On the other hand, analyzing the process $e^+ e^- \rightarrow \tau \tau \gamma$, the L3 Collaboration has obtained the value $d_\tau = (0.0 \pm 1.5 \pm 1.3) \times 10^{-16}e\text{ cm}$ [30]. In the present model assuming $m_{E_3}=3000$ GeV we obtain

$$d_\tau \approx 10^{-21} \mathcal{K}_{\tau 1}^2 \sin(2\theta_\alpha) e\text{ cm}. \quad (77)$$

It means that even if $\mathcal{K}_{\tau 1} \approx 1$ for any value of θ_α give a value smaller than the experimental limit. It means that, the tau lepton has not a EDM of the order of the present experimental upper limit, however it still may induce anomalous couplings of the Z boson to fermions which could be seen in tau-charm factories [31].

In this model there is no rare decays such as $\mu \rightarrow 3e$ at tree level. However, the same loop diagrams that contribute for the EDM of the leptons imply transition magnetic and electric moments, like $\mu \rightarrow e \gamma$. However, it will constrain only the matrix elements $[(\mathcal{O}_L^e)^T \mathcal{O}_L^E]_{\mu 1}^2$.

VI. CONCLUSIONS

The model of CP violation of this work seems like an admixture of both spontaneous breaking as in Lee's two doublets model [3] and the charged-Higgs-boson exchange of Weinberg's three doublets one [4]. However, some differences are important to be pointed out. In a pure charged-scalar-exchange where the phases are in the coupling constants of the scalar potential it was shown that there is also CP violation through the exchange of neutral Higgs bosons [32]. This is correct in models with $SU(2)_L \otimes U(1)_Y$ symmetry with several scalar doublets, that is, with all of them having the same quantum number. For this reason a more general mixing among the scalar fields of the same charge is possible. In the present model all triplets have different $U(1)_N$ charges and this constrains their interaction terms. Hence, there is not mixing among the real and imaginary part of the neutral scalar fields even with three scalar triplets as can be seen from Eqs. (13) and (18). If we had considered $a_{10} \neq 0$ in the scalar potential in Eq. (2) we would have CP violation in the propagator of single charged scalars like in the Weinberg model; however even in this case since the a_{10} term in the scalar potential does not contribute to the mass matrix of the neutral Higgs there is not mixing among CP -even and CP -odd neutral scalars. We see that in this model the CP violation comes only from charged scalars exchange.

There are other interesting features of this model: (a) Notice that since we have no CP violation in the neutral scalar

sector the Weinberg's three-gluon operators involving neutral Higgs boson exchanges do not contribute to the EDM of the neutron at all [33]. There is also no contribution to the EDM of the electron and neutron coming from the diagrams of Ref. [34] which involve CP violation in the propagator of the neutral Higgs bosons and it does not depend on the phase convention since in Eqs. (26) and (33) phases do not appear at all; (b) it was suggested in Ref. [34] that similar contributions to the ones given in Refs. [33,34] but mediated by charged scalar could exist. In the present model two loop contribution to the EDMs as those in Ref. [35] do exist but with top quark substituted by an exotic quark or lepton. However, when they involve a single charged scalars the W^- is substituted by V^- and when they involve a doubly charged scalars W^- is substituted by an U^{--} ; hence they are suppressed with respect to the one-loop level contributions by a factor $(M_W/M_V)^2$ in the first case or by $(M_W/M_U)^2$ in the second one for both the quark u and charged leptons; however, these are the main contributions to the EDM of the d -quark since for this quark the one-loop contribution vanish at one-level as discussed above; (c) the same diagrams (but without the photon) that induce a EDM for fermions will induce phases in the mixing matrices. For instance, radiative corrections at one-loop level will induce phases in the mixing matrix in the charge $2/3$ sector. However, for the charge $-1/3$ sector the phases will be induced only at two-loop level. The usual Cabibbo-Kobayashi-Maskawa matrix is defined as $V_{CKM} = (\mathcal{O}_L^u)^T \mathcal{O}_L^d$ then we can see that phases in this matrix are induced at 1-loop level. Moreover, since this type of diagrams are the same which would contribute to the penguin diagram, say of the ϵ'_K parameter, hence there is also penguin contributions to the CP violating parameters however these are 2-loop contributions. Hence, it means that although $\arg \det M$ vanishes at tree level, it has contributions for the charge $2/3$ sector at the one-loop order, and for the charge $-1/3$ sector at the two-loop level; (iv) if we assume that CP is also conserved at the tree level in the pure QCD part then $\theta_{QCD}=0$ at the tree level and $\bar{\theta} = \theta_{QCD} + \arg \det M$ will be finite and calculable. Of course, this is not such a natural solution to the θ -vacua problem but at least it is in the same foot than the assumption that all Yukawa couplings are real at the tree level. In the 3-3-1 model with a scalar sextet which is needed in order to give to the charged lepton a mass, it was shown by Pal [36] that a Peccei-Quinn symmetry arise naturally if the trilinear $\eta \rho \chi$ is drops out in the scalar potential. However in the present model no sextet has been added, so there is no such a global symmetry and the referred trilinear is necessary in order to have the correct VEVs pattern in Eqs. (3).

Notice that in the u -quark EDM diagrams in Fig. 5 there is a vertex that does not depend on any of the fermion mass because in the model there is flavor violating neutral currents. For this reason the EDM is proportional to the VEV v_η . This feature is model dependent, for instance, in the model of Ref. [37] the electron's EDM is proportional to the tau lepton mass because none of the Yukawa interaction is proportional to the electron mass. In our case in the lepton sector the EDM's are inversely proportional to the lepton

masses, as can be seen from Eqs. (71), (72), and (73). This happens because in our model: (i) only one of the vertex in Figs. 6 depend on the known lepton masses; (ii) there is a mass insertion of the exotic lepton; and (iii) the other vertex which is proportional to the exotic lepton mass. Hence we have a factor $m_l m_{E_j}^2$. Notice that if the contributions in Fig. 6(a) would involve a neutral scalar the lepton in the internal line could be a known lepton, say, the same that in the external line, and in the case in the numerator of the expression for d_l it will appear $m_l m_{E_j}^2 \rightarrow m_l^3$ as is usually the case in models where CP violation arises in the neutral scalar sector as in the first paper in Ref. [3]. On the other hand, the same mass dependence appear in the model of the second reference of Ref. [3]. All these possibilities occur in scalar mediated CP violation since in this case an explicit Higgs-fermion coupling flip the chirality. However, it does not mean that we have a EDM singularity for massless fermions since a careful study of the EDM expressions shows that we still have a zero EDM when the fermion masses go to zero. For instance, in the case $m_Y \gg m_E, m_l$ the $F_+ + F_-$ function defined in Eq. (73) behaves as $\ln(m_Y^2/m_E^2) - 2$; on the other hand for $m_E \gg m_Y, m_l$ we have $F_+ + F_- \propto 2m_Y^2/m_E^2 + (m_Y^2/m_E^2)\ln(m_Y^2/m_E^2)$; hence in both cases $d_l \propto m_l$.

This model of CP violation seems also like a particular realization of the soft superweak model proposed recently by some authors [35,38]. In that model the violation of CP is due to the couplings of the usual left-handed doublets to a heavy sector. In our model, the heavy sector corresponds to

the exotic quarks J, j_i , the exotic leptons E_j and the scalars Y^+ and Y^{++} ; the only complex numbers are the phase θ_α appearing in the trilinear term in the scalar potential and the complex VEVs, eventually we have only one phase, say θ_χ , after using the SU(3) freedom to eliminate two phases and the constrain equation in Eq. (4d). As it was emphasize in Ref. [35] the calculation of the electron EDM due to *solely* to CP violation in the charged Higgs sector has not been study in literature (up to their work). Our model it is perhaps, the first renormalizable electroweak model in which the CP violation arise, at the tree level, *only* through that sector (the models in Refs. [35,38] are effective ones).

With three triplets and one sextet which are needed in the model of Ref. [39] it is possible to have truly spontaneous violation of the CP symmetry [8]. In this case, the minimization condition of the scalar potential implies more complicated constraint equations on the imaginary part of the neutral scalars so that two of the phases of the VEV survive in the Lagrangian density. The phenomenology of this model has been studied in Ref. [15].

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