Spontaneous breaking of a global symmetry in a 3-3-1 model

J. C. Montero, C. A. de S. Pires, and V. Pleitez
Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145, 01405-900 São Paulo, S.P., Brazil
(Received 16 March 1999; published 29 October 1999)

In a 3-3-1 model in which the lepton masses arise from a scalar sextet it is possible to break spontaneously a global symmetry which implies in a pseudoscalar Majoron-like Goldstone boson. This Majoron does not mix with any other scalar fields and for this reason it does not couple, at the tree level, to either the charged leptons or to the quarks. Moreover, its interaction with neutrinos is diagonal. We also argue that there is a set of parameters in which the model can be consistent with the invisible Z^0 width and that heavy neutrinos can decay sufficiently rapid by Majoron emission, having a lifetime shorter than the age of the universe. [S0556-2821(99)02217-1]

PACS number(s): 14.80.Mz, 12.60.Fr

I. INTRODUCTION

In chiral electroweak model neutrinos can be massless at any order in perturbation theory if both conditions are supplied: no right-handed neutrinos are introduced and the total lepton number is conserved. If we do not assume lepton number conservation we have two possibilities: we break it by hand, i.e., explicit breaking or, we break it spontaneously. The later possibility implies the existence of a pseudoscalar Goldstone boson called a Majoron which was first suggested in Ref. [1] where a non-Hermitian scalar singlet (singlet Majoron model) was introduced, and in Ref. [2] where a non-Hermitian scalar triplet was introduced (triplet Majoron model). Since the data of the CERN e^+e^- collider LEP [3] the triplet majoron model was ruled out. The original model only considered one triplet and one doublet. In that model the majoron is a linear combination of doublet and triplet components but it is predominantly triplet. Hence, the lightest scalar (R^0) has a mass which is proportional to the vacuum expectation value (VEV) of the triplet and for this reason it is very small. Once we have the decay $Z^0 \to R^0 M^0$, where M^0 denotes the Majoron, and since the only decay of the light scalar is $R^0 \to M^0 M^0$, there is an extra contribution to the Z^0-invisible width. Its contribution is exactly twice the contribution of a simple neutrino. Since the Higgs scalars have only weak interactions they escape undetected. Hence, any experiment that counts the number of neutrino species by measuring the Z^0-invisible width automatically counts five neutrinos [4,5].

There are also possibilities involving only Higgs doublets and charged singlet scalars but they also need to include Dirac neutrino singlets. A minimal model of this sort was proposed in Ref. [6] where an extra doublet scalar-carrying lepton number was added (doublet Majoron model). The new doublet does not couple to leptons. The LEP data imply that at least a second doublet of the new type has to be introduced [7]. In this sort of model, since there are at least three doublets, the lightest scalar R^0 can be assumed naturally to be heavier than the Z^0 avoiding the decay $Z^0 \to R^0 M^0$. In this Majoron doublet model the lepton number violation takes place at the same scale of the electroweak symmetry breaking. It is also possible to consider a Majoron model with one complex singlet, a complex triplet, and the usual SU(2) doublet [8]. In this case the Majoron can evade the LEP data since it can be mainly a singlet.

II. THE MODEL

Here we will consider a model with $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ symmetry with both exotic quarks and only the known charged leptons [9]. In this model, in order to give mass to all fermions it is necessary to introduce three scalar triplets

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (\mathbf{3}, \mathbf{-1}), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (\mathbf{3}, \mathbf{1}),$$

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (\mathbf{3}, \mathbf{0}) \tag{1a}$$

and a sextet

$$S = \begin{pmatrix} \sigma_1^0 & \frac{h_2^-}{\sqrt{2}} & \frac{h_1^+}{\sqrt{2}} \\ \frac{h_2^-}{\sqrt{2}} & H_1^{--} & \frac{\sigma_2^0}{\sqrt{2}} \\ \frac{h_1^+}{\sqrt{2}} & \frac{\sigma_2^0}{\sqrt{2}} & H_2^{++} \end{pmatrix} \sim (\mathbf{6}, \mathbf{0}). \tag{1b}$$

Although we can assign a lepton number to several scalar fields we prefer to use the global quantum number $\mathcal{F} = L + B$ [10]. It is clear that the model needs only one global quantum number and not four as in the standard model, i.e., family lepton number $L_i, (i = e, \mu, \tau$ with $L = \Sigma_i L_i)$ and the baryonic number B. The quantum number $\mathcal{F}$ coincides with L and B for the known particles but implies the assignation of a single quantum number to the other particles.

The most general gauge and $\mathcal{F}$-conserving scalar potential is

$$V=\mu_1^2\eta^\dagger\eta+\mu_2^2\rho^\dagger\rho+\mu_3^2\chi^\dagger\chi+\mu_4^2\mathrm{Tr}(S^\dagger S)+\lambda_1(\eta^\dagger\eta)^2+\lambda_2(\rho^\dagger\rho)^2+\lambda_3(\chi^\dagger\chi)^2+\eta^\dagger\eta[\lambda_4\rho^\dagger\rho+\lambda_5\chi^\dagger\chi]+\lambda_6(\rho^\dagger\rho)(\chi^\dagger\chi)$$
$$+\lambda_7(\eta^\dagger\rho)(\rho^\dagger\eta)+\lambda_8(\eta^\dagger\chi)(\chi^\dagger\eta)+\lambda_9(\rho^\dagger\chi)(\chi^\dagger\rho)+\lambda_{10}(\mathrm{Tr}S^\dagger S)^2+\lambda_{11}\mathrm{Tr}[(S^\dagger S)^2]+\mathrm{Tr}(S^\dagger S)[\lambda_{12}(\eta^\dagger\eta)+\lambda_{13}(\chi^\dagger\chi)$$
$$+\lambda_{14}(\rho^\dagger\rho)]+[\lambda_{15}\epsilon^{ijk}\chi_i(S\chi^\dagger)_j\eta_k+\lambda_{16}\epsilon^{ijk}\rho_i(S\rho^\dagger)_j\eta_k+\lambda_{17}\epsilon^{ijk}\epsilon^{lmn}S_{il}S_{jm}\eta_n\eta_k+\mathrm{H.c.}]+\lambda_{18}\chi^\dagger SS^\dagger\chi+\lambda_{19}\eta^\dagger SS^\dagger\eta$$
$$+\lambda_{20}\rho^\dagger SS^\dagger\rho+\left[\frac{f_1}{2}\epsilon^{ijk}\eta_i\rho_j\chi_k+\frac{f_2}{2}\chi^T S^\dagger\rho+\mathrm{H.c.}\right]. \quad (2)$$

Terms like the quartic $(\chi^\dagger\eta)(\rho^\dagger\eta)$, $\chi S\eta^\dagger\rho$, $\eta S\eta^\dagger\eta, \chi\rho SS$, and the trilinear $\eta S^\dagger\eta, SSS$ do not conserve the $\mathcal{F}$ quantum number (or the lepton L) and they will not be considered here.

The minimum of the potential must be studied after the shifting of the neutral components of the three scalar multiplets. Hence, we redefine the neutral components in Eqs. (1) as follows:

$$\eta^0\to\frac{1}{\sqrt{2}}(v_\eta+R_1^0+iI_1^0),\quad \rho^0\to\frac{1}{\sqrt{2}}(v_\rho+R_2^0+iI_2^0),$$

$$\chi^0\to\frac{1}{\sqrt{2}}(v_\chi+R_3^0+iI_3^0), \quad (3)$$

and

$$\sigma_1^0\to\frac{1}{\sqrt{2}}(v_{\sigma_1}+R_4^0+iI_4^0),\quad \sigma_2^0\to\frac{1}{\sqrt{2}}(v_{\sigma_2}+R_5^0+iI_5^0), \quad (4)$$

where v_a (with $a=\eta,\rho,\chi,\sigma_1,\sigma_2$) are considered real parameters for the sake of simplicity. The $\mathcal{F}$ number attribution is the following:

$$\mathcal{F}(U^{--})=\mathcal{F}(V^-)=-\mathcal{F}(J_1)=\mathcal{F}(J_{2,3})=\mathcal{F}(\rho^{--})=\mathcal{F}(\chi^{--})$$
$$=\mathcal{F}(\chi^{--})=\mathcal{F}(\sigma_1^0)=\mathcal{F}(h_2^-)=\mathcal{F}(H_1^{--})=\mathcal{F}(H_2^{--})$$
$$=2, \quad (5)$$

where J_1 ($J_{2,3}$) are exotic quarks of charge 5/3 (−4/3) present in the model and we have included them by completion. For leptons and the known quarks $\mathcal{F}$ coincides with the total lepton and baryon numbers, respectively. All the other fields have $\mathcal{F}=0$. As we said before, all terms in Eq. (2) conserve the $\mathcal{F}$ quantum number. However, if we assume that $\langle\sigma_1^0\rangle\neq 0$ we have the spontaneous breakdown of $\mathcal{F}$ and the corresponding pseudoscalar, the Majoron M^0, as we will show below [11].

In a model with several complex scalar fields, as is the case of the 3-3-1 model [9], if the lepton number is spontaneously broken one of the neutral scalars is a Goldstone boson associated with the global symmetry breaking. With respect to the $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ gauge symmetry, this model has naturally three doublets $(\rho^+,\rho^0)^T,(\eta^0,\eta^-)^T$, and $(h^+,\sigma_2^0)^T$, one triplet

$$\begin{pmatrix}\sigma_1^0 & \frac{h_2^-}{\sqrt{2}}\\ \frac{h_2^-}{\sqrt{2}} & H_1^{--}\end{pmatrix}, \quad (6)$$

and two singlets χ_2^{--} and χ^0.

The mass matrix in the real sector in the basis $(R_1^0,R_2^0,R_3^0,R_4^0,R_5^0)^T$, is given by the symmetric matrix

$$m_{11}=\lambda_1 v_\eta^2-\frac{\lambda_{16}}{4\sqrt{2}}\frac{v_\rho^2 v_{\sigma_2}}{v_\eta}+\frac{1}{4\sqrt{2}v_\eta}(\lambda_{15}v_\chi v_{\sigma_2}-f_1 v_\rho)v_\chi+\frac{t_\eta}{2v_\eta},$$

$$m_{22}=\lambda_2 v_\rho^2-\frac{1}{8\sqrt{2}v_\rho}(2f_1 v_\eta+f_2 v_{\sigma_2})v_\chi+\frac{t_\rho}{2v_\rho},$$

$$m_{33}=\lambda_3 v_\chi^2-\frac{1}{8\sqrt{2}v_\chi}(2f_1 v_\eta+f_2 v_{\sigma_1})v_\rho+\frac{t_\chi}{2v_\chi},$$

$$m_{44}=(\lambda_{10}+\lambda_{11})v_{\sigma_1}^2+\frac{t_{\sigma_1}}{2v_{\sigma_1}},$$

$$m_{55}=\left(\lambda_{10}+\frac{\lambda_{11}}{2}\right)v_{\sigma_2}^2-\frac{1}{4\sqrt{2}v_{\sigma_2}}\lambda_{16}v_\rho^2 v_\eta-\frac{1}{8v_{\sigma_2}}(f_2 v_\rho+\sqrt{2}\lambda_{15}v_\eta)v_\chi+\frac{t_{\sigma_2}}{2v_{\sigma_2}},$$

$$m_{12}=\frac{1}{2}\left(\lambda_4 v_\eta+\frac{\lambda_{16}}{\sqrt{2}}v_{\sigma_2}\right)v_\rho+\frac{f_1}{4\sqrt{2}}v_\chi,$$

$$m_{13}=\frac{1}{2}\left(\lambda_5 v_\eta-\frac{\lambda_{15}}{\sqrt{2}}v_{\sigma_2}\right)v_\chi+\frac{f_1}{4\sqrt{2}}v_\rho,$$

$$m_{14}=(\lambda_{12}+\lambda_{19})\frac{v_\eta v_{\sigma_1}}{2},$$

$$m_{15}=\frac{1}{2}(\lambda_{12}-2\lambda_{17})v_\eta v_{\sigma_2}-\frac{\lambda_{15}}{4\sqrt{2}}v_\chi^2+\frac{\lambda_{16}}{4\sqrt{2}}v_\rho^2,$$

$$m_{23}=\frac{\lambda_6}{2}v_\rho v_\chi+\frac{f_1}{4\sqrt{2}}v_\eta+\frac{f_2}{8}v_{\sigma_2},$$

$$m_{24}=\frac{\lambda_{14}}{2}v_\rho v_{\sigma_1},$$

$$m_{25}=(2\lambda_{14}+\lambda_{20})\frac{v_\rho v_{\sigma_2}}{4}+\frac{\lambda_{16}}{2\sqrt{2}}v_\eta v_\rho+\frac{f_2}{8}v_\chi,$$

$$m_{34}=\frac{\lambda_{13}}{2}v_\chi v_{\sigma_1},$$

$$m_{35}=\frac{\lambda_{13}}{2}v_\rho v_{\sigma_2}-\frac{1}{4\sqrt{2}}(\lambda_{15}v_\eta-\lambda_{18}v_{\sigma_2})v_\chi+\frac{f_2}{8}v_\rho,$$

$$m_{45}=\lambda_{10}v_{\sigma_1}v_{\sigma_2}. \tag{7}$$

The tadpole equations t_a where $a=\eta,\rho,\chi,\sigma_1,\sigma_2$ are given in the Appendix. The conditions for an extreme of the potential are $t_a=0$. Assuming that the matrix m_{ij} above is diagonalized by an orthogonal matrix $\mathcal{O}$, the relation among symmetry (R^0_i) and mass (H^0_j) eigenstates is $R^0_i=\mathcal{O}_{ij}H^0_j$; $i,j=1,2,3,4,5$. The masses m_{H_j} can vary, depending on a fine tuning of the parameters, from a few GeV's up to 2 or 3 TeV [typical values of the energy scale at which the break down of the $SU(3)_L$ symmetry does occur]. Also the arbitrary orthogonal matrix $\mathcal{O}$ is not necessarily almost diagonal. Denoting the lightest Higgs boson as H^0_1 two extreme possibilities are compatible with the LEP data: $R_4=(\mathcal{O}^{-1})_{41}H^0_1+\cdots$, with $(\mathcal{O}^{-1})_{41}\ll 1$, if $m_{H_1}<M_Z$; or $R_4\approx H^0_1+\cdots$, that is $(\mathcal{O}^{-1})_{41}\approx 1$, if $M_{H_1}>M_Z$. Intermediate values for the mass m_{H_1} and the mixing angles have been ruled out by the LEP data (see below).

The symmetric mass matrix of the imaginary part in the basis $(I^0_1,I^0_2,I^0_3,I^0_4,I^0_5)^T$ reads

$$M_{11}=-\frac{\lambda_{16}}{4\sqrt{2}}\frac{v^2_\rho v_{\sigma_2}}{v_\eta}+\lambda_{17}v^2_{\sigma_2}+\frac{1}{4\sqrt{2}}(\lambda_{15}v_\chi v_\eta-f_1v_\rho)\frac{v_\chi}{v_\eta}+\frac{t_\eta}{2v_\eta},$$

$$M_{22}=-\frac{1}{8}\left(\sqrt{2}f_1v_\eta+f_2v_{\sigma_2}\frac{v_\chi}{v_\rho}\right)+\frac{t_\rho}{2v_\rho},$$

$$M_{33}=-\frac{1}{8}(\sqrt{2}f_1v_\eta+f_2v_{\sigma_2})\frac{v_\rho}{v_\chi}+\frac{t_\chi}{2v_\chi},$$

$$M_{44}=\frac{t_{\sigma_1}}{2v_{\sigma_1}},$$

$$M_{55}=\frac{1}{4\sqrt{2}}(\lambda_{15}v^2_\chi-\lambda_{16}v^2_\rho)\frac{v_\eta}{v_{\sigma_2}}+\lambda_{17}v^2_\eta-\frac{f_2}{8}\frac{v_\rho v_\chi}{v_{\sigma_2}}+\frac{t_{\sigma_2}}{2v_{\sigma_2}},$$

$$M_{12}=-\frac{f_1}{4\sqrt{2}}v_\chi,\quad M_{13}=-\frac{f_1}{4\sqrt{2}}v_\rho,\quad M_{14}=0,$$

$$M_{15}=\frac{\lambda_{15}}{4\sqrt{2}}v_\eta-\frac{f_2}{8}v_{\sigma_2},$$

$$M_{23}=-\frac{1}{8}(\sqrt{2}f_1v_\eta+f_2v_{\sigma_2}),\quad M_{24}=0,$$

$$M_{25}=\frac{f_2}{8}v_\chi,\quad M_{34}=0,\quad M_{35}=\frac{f_2}{8}v_\rho,$$

$$M_{45}=0. \tag{8}$$

In Eqs. (7) and (8) when all $v_a\neq 0,a=\eta,\rho,\chi,\sigma_1,\sigma_2$ then we can use $t_a=0$. In Eqs. (8) there are three Goldstone boson. Notice, however, that since $M_{4i}=0$, the component I^0_4 has a zero mass, i.e., it is an extra Goldstone boson which decouples in the sense that it does not mix with the other CP-odd scalars. Hence, I^0_4 is the Majoron field. Hereafter, it will be denoted M^0. The submatrix 4×4 has still two other Goldstone bosons which are related to the masses of Z^0 and Z'^0. Hence, although the Majoron in the present model is a triplet under the subgroup SU(2), it does not mix with the other imaginary fields.

Hence, as in the singlet Majoron model ours has no couplings with fermions (charged leptons and quarks). Moreover, as we said before, the real component can be heavier than the Z^0. It is easy to understand this. If $v_{\sigma^0_1}=0$, the tadpole equation in Eq. (A4) must be replaced in the mass matrices in Eqs. (7) and (8). In this case the σ^0_1 fields consists of two mass-degenerate fields R^0_4 and I^0_4 with mass

$$m^2_{R_4}=m^2_{I_4}=\lambda_{10}v^2_{\sigma_2}+\frac{1}{2}(\lambda_{12}+\lambda_{19})v^2_\eta+\frac{\lambda_{13}}{4}v^2_\chi+\frac{\lambda_{18}}{2}v^2_\rho. \tag{9}$$

The mass in Eq. (9) can be large because it depends on v^2_χ. When $v_{\sigma_1}\neq 0$ is used, the degeneration in mass of R^0_4 and I^0_4 is broken, the imaginary part becomes the Majoron and the real part has a mixing with the other real neutral components, which include several fields transforming under $SU(2)_L\otimes U(1)_Y$ as doublets $(\eta^0,\rho^0,\sigma^0_2)$, and one singlet (χ^0). This also happens in the one-singlet–one-doublet–one-triplet model when the triplet does not gain a VEV [12]. Notice that unlike v_{σ_1}, if $v_{\sigma_2}=0$ the condition in Eq. (A5) forces $f_2=0$. All the other VEV's have to be nonzero in order to have a consistent breaking of the SU(3) symmetry.

III. PHENOMENOLOGICAL CONSEQUENCES AND CONCLUSIONS

In the present model the interaction of the Majoron with the Z^0 (which is of the form $Z^0M^0H_j^0$), is given by

$$\mathcal{O}_{4j}\frac{(\sqrt{2}G_F)^{1/2}}{c_W}M_W(p_{M^0}-p_j)^\mu, \tag{10}$$

where p_{M^0} and p_j are the momenta of the Majoron and the physical real scalars H_j^0, respectively. We see that if it is allowed, the contribution of the decay mode $Z^0\to H_1^0M^0$, where H_1^0 is the lightest Higgs scalar, is $2|\mathcal{O}_{41}|^2$ times that of the neutrino antineutrino. Hence, as we said before, the value of the mixing matrix element $\mathcal{O}_{4j}$ is constrained appropriately: $2\mathcal{O}_{4j}^2\sim 10^{-4}$, to make the model consistent with the LEP data, i.e., now $\Gamma_Z\to H_1^0M^0$ (where H_1^0 is assumed the lightest scalar) could be reduced to an acceptable level. More interesting, however, is the fact that in this model the $Z^0\to H_1^0M^0$ might be kinematically forbidden since H_1 can be heavier than the Z^0 as was discussed above.

It is well known that if neutrinos are massive particles the thermal history of the universe strongly constrains the mass of the stable neutrinos, i.e., $m_\nu<100$ eV for light neutrinos or a few GeV for heavy ones [13]. One of the ways in which the cosmological constraints on neutrino masses can be altered is when the lepton number is broken globally, giving rise to the Majoron field: heavy neutrinos can decay rapidly by Majoron emission, thereby giving negligible contributions to the mass density of the universe [14]. Let us denote ν_h (ν_l) heavy (light) neutrinos and look for $\nu\to\nu'+M^0$ decays in the present model. Those decays, as in the triplet Majoron model, are completely forbidden at the tree level too (there is neither $\nu\to\nu'+\gamma$ nor $\nu\to 3\nu'$ decays at the lowest order).

Here we will denote, as usual, W^+ the vector boson which coincides with the respective boson of the standard model, i.e., it couples to the usual charged current in the lepton and the quark sectors and also satisfies $M_W^2/M_Z^2=c_W^2$; and V^+ denotes the vector boson which couples charged leptons with antineutrinos or the known quarks with the exotic ones. If the lepton number is not spontaneously broken W^+ and V^+ do not couple to one another. However, a mixing between both W^+ and V^+ arises when the lepton number is spontaneously broken. Let us consider this more in detail. In the basis $(W^+\ \ V^+)^T$ the mass square matrix is given by

$$\frac{g^2}{4}\begin{pmatrix} A+v_\rho^2/2 & \sqrt{2}v_{\sigma_1}v_{\sigma_2}\\ \sqrt{2}v_{\sigma_1}v_{\sigma_2} & A+v_\chi^2/2\end{pmatrix}, \tag{11}$$

where $A=(v_\eta^2+v_{\sigma_2}^2+2v_{\sigma_1}^2)/2$. We see from Eq. (11) that if $v_{\sigma_1}=0$ there is no mixing between W^+ and V^+. The mass eigenstates are given by $W_i^+=\mathcal{U}_{ij}B_j^+$, where $i,j=1,2$ and $B_1^+=W^+$, $B_2^+=V^+$, and the orthogonal matrix is given by (N is a normalization factor)

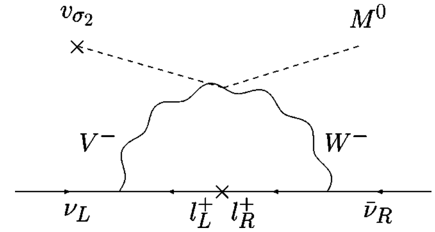

FIG. 1. One-loop contribution to the process $\nu_L\to\overline{\nu_R}+M^0$.

$$\mathcal{U}=\frac{1}{N}\begin{pmatrix}\dfrac{r-s-\sqrt{4b^2+(r-s)^2}}{2b} & 1\\ \dfrac{r-s+\sqrt{4b^2+(r-s)^2}}{2b} & 1\end{pmatrix}\approx\frac{1}{\sqrt{s^2+r^2}}\begin{pmatrix}-s & b\\ b & s\end{pmatrix}, \tag{12}$$

where $r=v_\rho^2/2$, $s=v_\chi^2/2$ and $b=\sqrt{2}v_{\sigma_1}v_{\sigma_2}$.

This mixing may have interesting cosmological consequences since there are interactions like $\kappa(g^2/\sqrt{2})\overline{l_L}\gamma^\mu\nu^cW_\mu^-$. Notice that its strength depends on the small parameter $\kappa\propto v_{\sigma_1}v_{\sigma_2}/v_\chi^2$ and it can be neglected in the usual processes. In fact, the model has three different mass scales since $v_{\sigma_1}\ll v_i\ll v_\chi$ with $v_i=v_\eta,v_\rho,v_{\sigma_2}$ [15]. It means that $W_1^+\approx W^+$, $W_2^+\approx V^+$ with $M_W^2/M_Z^2c_W$ compatible with the experimental data if $v_{\sigma_1}\leqslant 3.89$ GeV [15]. However, the mixing between W^+ and V^+ is interesting once there are new contributions to the Majoron emission. In fact, because of the $W^+W^-M^0$ vertex we have the neutrino transitions $(\nu_h)_L\to(\nu_l)_LM^0$; because of the vertex $V^+V^-M^0$ we have antineutrino transitions $\overline{(\nu_h)_R}\to\overline{(\nu_l)_R}$. Both contributions could be negligible since they are proportional to v_{σ_1}. More interesting is that possible to have neutrino–anti-neutrino transitions like the decay $(\nu_h)_L\to\overline{(\nu_l)_R}M^0$ mediated by the mixing between W^+ and V^+ as shown in Fig. 1. This diagram is ultraviolet finite in the Feynman gauge and depends quadratically on a low-energy scale that we have chosen, conservatively, as being the m_τ mass. The latter process implies a neutrino width which is, in a suitable approximation, dominated by the τ lepton contribution and is given by

$$\Gamma=\frac{1}{8\pi^5}G_F^4|\mathcal{K}_{\tau 3}|^2|\mathcal{K}_{\tau 1}|^2m_{\nu_h}m_\tau^6v_{\sigma_2}^2\left(\frac{M_W}{M_V}\right)^4, \tag{13}$$

where we have neglected a logarithmic dependence on m_{ν_h} (ν_h can be ν_τ or ν_μ with $\nu_l=\nu_\mu,\nu_e$ in the first case and $\nu_l=\nu_e$ in the second one). With reasonable values for the masses in Eq. (13), that is $m_{\nu_h}\approx 1$ MeV for the case of the τ neutrino, $v_{\sigma_2}\approx 100$ GeV, and $M_V\approx 400$ GeV, we can get a width of the order of 10^{-21} MeV (up to the suppression of the mixing matrix $\mathcal{K}$). The age of the universe has a correspondent width of 10^{-39} MeV, thus it means that the decay can have a lifetime less than the age of the universe and could be of cosmological interest. From the cosmological point of view there are also the processes $\nu_h+\nu_h\to M^0\to\nu_l+\nu_l$ and $\nu_l+\nu_l\to M^0+M^0$, which occur at the tree level approximation. The cosmological effect of these processes are the same as in Ref. [4].

If the parameters in this model are such that the Majoron is irrelevant from the cosmological point of view, there is still the possibility that the Majoron may be detected by its influence in neutrinoless double β decay with Majoron emission $nn\to e^-e^-M^0$ [denoted by $(\beta\beta)_{0\nu M}$]. However, it needs the Majoron-neutrino couplings in the range $m_\nu/v_{\sigma_1}\sim10^{-5}-10^{-3}$ in order to have the Majoron emission experimentally observable [16]. Notice that in the present model the accompanying 0^+ scalar, which is by definition the lightest scalar H_1^0, may not be emitted in $(\beta\beta)_{0\nu M}$ if it is a heavy scalar or it is very suppressed by the mixing factor.

In our model both the usual neutrinoless $(\beta\beta)_{0\nu}$ decay and also the decay $(\beta\beta)_{0\nu M}$ have new contributions. If $\mathcal{F}$ is conserved in the scalar potential or $v_{\sigma_1}=0$ the mixing among singly charged scalars occurs with η_1^- and ρ^- and between η_2^- and χ^-. However, if we allow $\mathcal{F}$-breaking terms in the scalar potential or $v_{\sigma_1}\neq0$ there is a general mixing among the scalar fields of the same charge. For instance, the trilinear term $f_2\chi^TS^\dagger\rho$ in the potential in Eq. (2) implies the trilinear interaction $f_2h_2^-h_2^-\chi^{++}$, and since there is a general mixing among all scalars of the same charge it means that there are processes where the vector bosons are substituted by scalars since the vertex $h_2^+e^-\nu$ does exist and h_2^+ mixes with all the other singly charged scalars. There are also trilinear contributions that arise because of the vertices $W^-V^-H_1^{++}$ and $h_2^-h_2^-H_1^{++}$ as in Refs. [17,18]. There is also the vertex $h_2^-h_2^+M^0$ contributing to the $(\beta\beta)_{0\nu M}$. It seems that the analysis of both $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{0\nu M}$ decays is more complicated than those considered in Refs. [15,19].

There are also phenomenological constraints on Majoron models coming from a search in the laboratory of flavor changing currents like $\mu\to e+M^0$ [20] or in astrophysics through processes like $\gamma+e\to e+M^0$ which contributes to the energy-loss mechanism of stars [4]. However, in the present model the Majoron couples only to neutrinos, for quarks and electrons the couplings arise only at the one-loop level. Hence, all these processes do not constrain the Majoron couplings at all (at the lowest order). The interaction of the Majoron with neutrinos is diagonal in flavor. The coupling between the Majoron and the real scalar field H_j^0, of the form $M^0M^0H_j^0$, is

$$i\mathcal{O}_{4j}(\lambda_{10}+\lambda_{11})\frac{v_{\sigma_1}}{2}, \tag{14}$$

which is a small coupling. Note that since the Majoron decouples from the other imaginary parts of the neutral scalars there are no trilinear couplings like $M^0A^0H_j^0$, where A^0 denotes a massive pseudoscalar, hence the model does not have the phenomenological consequences in accelerator physics as the seesaw Majoron model does [12].

Finally, we remark that here we have assumed that there is no spontaneous CP violation. Hence, all vacuum expectation values are real. If we allow complex VEV it has been shown that CP is violated spontaneously [21]. If this is the case, we have a mixing among all the scalars fields and also the majoron mixes with all the other CP-even and CP-odd scalars.

ACKNOWLEDGMENTS

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Conselho Nacional de Ciência e Tecnologia (CNPq) and by Programa de Apoio a Núcleos de Excelência (PRONEX). C.P. would like to thank Coordenadoria de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) for financial support.

APPENDIX: CONSTRAINT EQUATIONS

$$t_\eta=\mu_1^2v_\eta+\lambda_1v_\eta^3+\frac{\lambda_4}{2}v_\rho^2v_\eta+\frac{\lambda_5}{2}v_\chi^2v_\eta+\frac{\lambda_{12}}{2}(v_{\sigma_1}^2+v_{\sigma_2}^2)v_\eta-\lambda_{17}v_{\sigma_2}^2v_\eta+\frac{\lambda_{19}}{2}v_{\sigma_1}^2v_\eta-\frac{\lambda_{15}}{2\sqrt{2}}v_\chi^2v_{\sigma_2}+\frac{\lambda_{16}}{2\sqrt{2}}v_\rho^2v_{\sigma_2}+\frac{f_1}{2\sqrt{2}}v_\rho v_\chi, \tag{A1}$$

$$t_\rho=\mu_2^2v_\rho+\lambda_2v_\rho^3+\frac{\lambda_4}{2}v_\eta^2v_\rho+\frac{\lambda_6}{2}v_\chi^2v_\rho+\frac{\lambda_{14}}{2}(v_{\sigma_1}^2+v_{\sigma_2}^2)v_\rho+\frac{\lambda_{16}}{\sqrt{2}}v_{\sigma_2}v_\eta v_\rho+\frac{\lambda_{20}}{4}v_{\sigma_2}^2v_\rho+\frac{f_1}{2\sqrt{2}}v_\eta v_\chi+\frac{f_2}{4}v_{\sigma_2}v_\chi, \tag{A2}$$

$$t_\chi=\mu_3^2v_\chi+\lambda_3v_\chi^2+\frac{\lambda_5}{2}v_\eta^2v_\chi+\frac{\lambda_6}{2}v_\rho^2v_\chi+\frac{\lambda_{13}}{4}(v_{\sigma_1}^2+v_{\sigma_2}^2)v_\chi-\frac{\lambda_{15}}{\sqrt{2}}v_{\sigma_2}v_\eta v_\chi+\frac{\lambda_{18}}{4}v_{\sigma_2}^2v_\chi+\frac{f_1}{2\sqrt{2}}v_\eta v_\rho+\frac{f_2}{4}v_{\sigma_2}v_\rho, \tag{A3}$$

$$t_{\sigma_1}=\mu_4^2v_{\sigma_1}+\lambda_{10}(v_{\sigma_1}^2+v_{\sigma_2}^2)v_{\sigma_1}+\lambda_{11}v_{\sigma_1}^3+\frac{\lambda_{12}}{2}v_\eta^2v_{\sigma_1}+\frac{\lambda_{13}}{4}v_\chi^2v_{\sigma_1}+\frac{\lambda_{14}}{2}v_\rho v_{\sigma_1}+\frac{\lambda_{19}}{2}v_\eta^2v_{\sigma_1}, \tag{A4}$$

$$t_{\sigma_2}=\mu_4^2v_{\sigma_2}+\lambda_{10}(v_{\sigma_2}^2+v_{\sigma_1}^2)v_{\sigma_2}+\frac{\lambda_{11}}{2}v_{\sigma_2}^3+\frac{\lambda_{12}}{2}v_\eta^2v_{\sigma_2}+\frac{\lambda_{13}}{2}v_\chi^2v_{\sigma_2}+\frac{\lambda_{14}}{2}v_\rho^2v_{\sigma_2}-\lambda_{17}v_\eta^2v_{\sigma_2}\frac{\lambda_{18}}{4}v_\chi^2v_{\sigma_2}+\frac{\lambda_{20}}{4}v_\rho^2v_{\sigma_2}-\frac{\lambda_{15}}{2\sqrt{2}}v_\chi^2v_\eta+\frac{\lambda_{16}}{2\sqrt{2}}v_\rho^2v_\eta+\frac{f_2}{4}v_\rho v_\chi. \tag{A5}$$

[1] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. **98B**, 265 (1981).
[2] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).
[3] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[4] H. Georgi, S. L. Glashow, and S. Nussinov, Nucl. Phys. **B193**, 297 (1981).
[5] M. C. Gonzalez-Garcia and Y. Nir, Phys. Lett. B **232**, 383 (1989).
[6] S. Bertolini and A. Santamaria, Nucl. Phys. **B310**, 714 (1988).
[7] H. Kikuchi and E. Ma, Phys. Lett. B **335**, 444 (1994).
[8] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 774 (1982).
[9] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); R. Foot, O. F. Hernández, F. Pisano, and V. Pleitez, *ibid.* **47**, 4158 (1993); see also, P. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).
[10] V. Pleitez and M. D. Tonasse, Phys. Rev. D **48**, 5274 (1993).
[11] This case was considered briefly by M. B. Tully and G. C. Joshi, hep-ph/9810282, but no detail of the Majoron was shown.
[12] M. A. Diaz, M. A. Garcia-Jareno, D. A. Restrepo, and J. F. W. Valle, Nucl. Phys. **B527**, 44 (1998).
[13] See, E. Kolb and M. Turner, *The Early Universe* (Addison-Wesley, New York, 1990) and references therein.
[14] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Rev. Lett. **45**, 1926 (1981).
[15] J. C. Montero, C. A. de S. Pires, and V. Pleitez, Phys. Rev. D **60**, 098701 (1999).
[16] Z. G. Berezhiani, A. Yu. Smirnov, and J. W. F. Valle, Phys. Lett. B **291**, 99 (1992).
[17] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 2951 (1982).
[18] C. O. Escobar and V. Pleitez, Phys. Rev. D **28**, 1166 (1983).
[19] F. Pisano and S. Shelly Sharma, Phys. Rev. D **57**, 5670 (1998).
[20] D. A. Bryman and E. T. H. Clifford, Phys. Rev. Lett. **57**, 2787 (1986).
[21] L. Epele, H. Fanchiotti, C. García Canal, and D. Gómez Dumm, Phys. Lett. B **343**, 291 (1995).