Remark on the Neves-Wotzasek constraints conversion method

Everton M. C. Abreu*
Departamento de Física e Química, Universidade Estadual Paulista, Av. Ariberto Pereira da Cunha 333, Guaratinguetá, 12500-000, São Paulo, SP, Brazil

Anderson Ilha†
Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21945-970, Rio de Janeiro, RJ, Brazil

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In this work we give an alternative way which generalizes the recently implemented Neves-Wotzasek method of conversion from second to first-class systems. We have proved that this generalization is correct reproducing the results given in the literature using the case of a sphere with an antisymmetric generator as an example.

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I. INTRODUCTION

Recently, Neves and Wotzasek (NW) [1] have developed a method to convert a second-class system into a first-class one. The NW formalism is mainly an extension of the iterative Wess-Zumino constraint conversion formalism [2] used previously to deal with anomalous gauge theories. The NW procedure brings also a two-way mapping which provides a relationship between the first and second-class theories. This mapping produces a noninvariant theory without performing the gauge fixing operation [3].

The NW method extends the Kovner-Rosenstein [4] technique, which discloses a hidden symmetry of the nonlinear sigma model, including WZ fields to convert the spherical constraints. Now, this extension introduces a new geometrical interpretation for the WZ gauge orbits. The geometrical features of the WZ gauge theory are disclosed by the set of linear-momentum first-class constraints whose gauge orbits are orthogonal to the original set of second-class surfaces. Such an identification could be made with the partial fixing of the WZ symmetries, leading to an identification of the first-class constraint models with the singular metric models.

In this work we want to comment on a shortcoming of the NW paper regarding the elimination of the WZ sector which is crucial for the geometrical interpretation proposed there. In this paper we propose an alternative method for such elimination which is quite unique instead of the case by case analysis of Ref. [1] and to do this we have organized the paper in the following sequence. In Sec. II we have introduced our generalization followed by an application accomplished in Sec. III; the conclusion is the last section.

II. THE GENERALIZATION OF THE NEVES-WOTZASEK METHOD

To this end we shall start with the theory with the gauge projector

$$L=\frac{1}{2}\dot{q}_k M_{km}\dot{q}_m, \tag{1}$$

where the gauge invariant projector

$$M_{km}=\delta_{km}-\frac{t_k t_m}{t^2};\quad t^2=t_n t_n. \tag{2}$$

The vector

$$t_k=\partial_k\Omega, \tag{3}$$

where $\partial\equiv\partial/\partial_k$, is normal to the constraint surface Ω and can be interpreted as a singular metric in Ref. [1]. It also provides the link between the invariant and noninvariant aspects of a theory. It appears as

$$t_k=f(q)T_{km}q_m, \tag{4}$$

where $f(q)$ is some function of the coordinates and the matrix T_{km} (symmetric or antisymmetric in the coordinate indices) is the surface generator element defining its properties. With these ingredients the computation of the sympletic matrix is an easy task [1]. For the case of spherical models with unitary radius, $\Omega=q^2-1$, we propose to deform the constraint in the following fashion:

$$q^2=f(\theta), \tag{5}$$

which is the generalized form of a spherical constraint. The $f(\theta)$ is still an arbitrarily function of a new variable θ that will play the role of WZ variable in the sequel. Differentiating this constraint we have that

$$q\dot{q}=\frac{1}{2}\frac{\partial f(\theta)}{\partial\theta}\dot{\theta}, \tag{6}$$

and with this equation we can write the action (1) as

$$L=\frac{1}{2}\dot{q}^2-\frac{1}{8}\frac{1}{f}\left(\frac{\partial f(\theta)}{\partial\theta}\right)^2\dot{\theta}^2+\lambda[q^2-f(\theta)], \tag{7}$$

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*Email address: everton@feg.unesp.br
†Email address: ais@if.ufrj.br

with λ imposing the ansatz (5). We can see that a proper choice of $f(\theta)$ affords a general study of various models. For example, if we choose $f(\theta)=e^{\theta}$, we can obtain easily the invariant theory proposed by Barcelos-Neto and Oliveira [5]. With $f=c-2\theta$ we can reproduce the results obtained for the Skyrme model [6].

Another interesting generalization is, to the case of different geometries of degree two, rewriting the constraint (5) as

$$q_k T_{km} q_m = f(\theta),$$

where T_{km}, in this case, is obviously a symmetrical generating matrix. More general cases, even that with antisymmetric generators, as shown in Eq. (4), can be obtained using that

$$\Omega(q) = f(\theta),$$

where $\Omega(q)$ is a function which details the kind of constraint. In its differential form we can write

$$f'(\theta)\dot{\theta} + \partial_k \Omega_k \dot{q}_k = 0, \tag{8}$$

which will be very useful as we will see in the next section.

III. AN EXAMPLE: THE ANTISYMMETRIC CASE

As an example of our generalization let us discuss the case of a sphere with an antisymmetric generator studied in Ref. [1]. Choosing

$$\theta + \Omega = 0, \tag{9}$$

with

$$\Omega = \arctan\left(\frac{q_2}{q_1}\right), \tag{10}$$

where the gradient vector of such surface is

$$t_k = \partial_k \Omega = -\frac{\epsilon_{km} q_m}{q^2}, \tag{11}$$

and using these last three equations in Eq. (8) we have that

$$\dot{\theta} = -\frac{\dot{q}\epsilon q}{q^2}. \tag{12}$$

Finally the Lagrangian is

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}\frac{(\dot{q}\epsilon q)^2}{q^2} = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2\dot{\theta}^2 - \lambda(\Omega + \theta), \tag{13}$$

which agrees with WZ model presented in Ref. [1] for the antisymmetric case.

IV. CONCLUSION

Concluding this Brief Report we can say that the NW method brings a relationship between gauge theories and nonlinear models of the second-class type that levels gauge and nongauge theories. The geometrical interpretation of the WZ orbits as trajectories crossing orthogonally the nonlinear surface promotes a new understanding of the gauge symmetry. Now we can see the gauge symmetry as a transformation that translates the physical space from one nonlinear surface to the other. The NW procedure ignores the idea of gauge fixing from the moment that it reduces the redundant degree of freedom without a specific choice of gauge. Furthermore, with this concept we can study the Gribov ambiguities. Without gauge fixing the Gribov copies do not come out. The solution of this problem can be hopefully obtained with NW conversion method because it is *a priori* a reduction without gauge fixing.

In this Brief Report we have made some observations regarding the transformation leading from the WZ theory to the singular metric model with a discussion at the Lagrangean level where the spherical coordinate plays the role of WZ variable. So, a general Lagrangian can be constructed having this arbitrary function as an input. This makes possible the application of the method to various cases and at the same time prove that the WZ theories are equivalent to theories with singular metric.

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