

Effects of dynamical masses of gluons and quarks on hadronic B decays

C. M. Zanetti* and A. A. Natale†

**Instituto de Física, Universidade de São Paulo, São Paulo, SP, Brazil*

†Instituto de Física Teórica, São Paulo State University, São Paulo, SP, Brazil

Abstract.

We study hadronic annihilation decays of B mesons within the perturbative QCD at collinear approximation. The regulation of endpoint divergences is performed with the help of an infrared finite gluon propagator characterized by a non-perturbative dynamical gluon mass. The divergences at twist-3 are regulated by a dynamical quark mass. Our results fit quite well the existent data of $B^0 \rightarrow D_s^- K^+$ and $B^0 \rightarrow \bar{D}_s^* K^+$ for the expected range of dynamical gluon masses. We also make predictions for the rare decays $B^0 \rightarrow K^- K^+$, $B_s^0 \rightarrow \pi^- \pi^+$, $\pi^0 \pi^0$, $B^+ \rightarrow D_s^{(*)+} \bar{K}^0$, $B^0 \rightarrow D_s^{\pm(*)} K^\mp$ and $B_s^0 \rightarrow D^{\pm(*)} \pi^\mp$, $D^0 \pi^0$.

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INTRODUCTION

Perturbative QCD has been largely applied in the study of B meson decays, specially in non-leptonic decays which requires the calculation of hadronic matrix elements. In such processes the long distance interactions between the initial and final state mesons impose a difficult obstacle in amplitudes calculations. In the pQCD approach the calculation of the matrix elements can be performed due to the large scale involved in the strong interactions ($\mathcal{O}(m_B)$). The perturbative dynamics can be factorized from the non-perturbative dynamics by applying the hard exclusive scattering Brodsky and Lepage's approach [1]. However, in the collinear approximation of pQCD, the amplitude calculation of non-factorizable diagrams (spectator interaction and annihilation) presents end-point divergences that can not be absorbed into the mesons wave functions to preserve the factorization as usual [2]. Although these contributions in general are power suppressed and are negligible in front of the factorizable contributions, it is important to have a reliable calculation of the annihilation diagrams due to the strong phases that are relevant in the determination of the CP violation parameters, and also in the evaluation of pure annihilation processes. These divergences can be treated by different approaches, for example [2, 3]. In this work, we present a study of the regulation of the divergences using the prescription of an IR finite gluon propagator, which appears as a non-perturbative solution of the gluonic Schwinger-Dyson equations (SDE) [4]. For consistency with the gluon dressing and in order to deal with the collinear divergences that appear at twist-3, we will also use the dynamical masses of the light quarks. We focus on pure annihilation decays, in which the dependence on the regulation will be more evident.

PERTURBATIVE QCD APPROACH

Weak decays of B mesons are described by an effective Hamiltonian at a renormalization scale $\mu \ll M_W$. This effective Hamiltonian is formally obtained using the Operator Product Expansion (OPE), describing effective interactions through four-quark operators [8]. The amplitude of the decay $B \rightarrow f$ is obtained from:

$$\mathcal{A}(B \rightarrow f) = \langle f | \mathcal{H}_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_i V_{\text{CKM}}^i C_i(\mu) \langle Q_i \rangle, \quad (1)$$

where G_F is the Fermi constant, V_{CKM} are Cabibbo-Kobayashi-Maskawa factors, Q_i are the four-quarks operators contributing to the decay and $C_i(\mu)$ are the Wilson coefficients; $\langle Q_i \rangle = \langle f | Q_i | B \rangle$ are the hadronic matrix elements between the initial and final states.

In the pQCD formalism the hadronic matrix elements are obtained through a convolution of the hard scattering kernel and the distribution amplitudes of the mesons involved in the process. In the case of a two-body non-leptonic decay $B \rightarrow M_1 M_2$ in the collinear approach, the matrix element of a given operator Q_i is [1]:

$$\langle Q_i \rangle = \int dx dy dz T_i(x, y, z) \Phi_{M_1}(x) \Phi_{M_2}(y) \Phi_B(z), \quad (2)$$

where x, y, z are the momentum fractions, $\Phi_M(x)$ are the light-cone distribution amplitudes for the quark-antiquark states of the mesons decomposed in terms of spin structure, which are non-perturbative functions of the momentum fraction carried by the partons; T_i is the hard scattering kernel that can be perturbatively computed as function of the light-cone momenta of collinear partons.

REGULATION OF DIVERGENCES

In this work we deal with the soft divergences due to the IR vanishing gluon using the non-perturbative propagator proposed by Cornwall [4]:

$$D_g(Q^2) = \frac{1}{Q^2 + M_g(Q^2)}, \quad M_g^2(Q^2) = m_g^2 \left[\ln \left(\frac{Q^2 + 4m_g^2}{\Lambda^2} \right) / \ln \left(\frac{4m_g^2}{\Lambda^2} \right) \right]^{-\frac{12}{11}}, \quad (3)$$

where $m_g = 500 \pm 200$ MeV is the dynamical gluon mass scale. A simple fit for the coupling constant that is factored out in this procedure is given by [4]

$$\bar{\alpha}_{sd}(q^2) = \frac{1}{4\pi b \ln[(4m_g^2 - q^2 - i\epsilon)/\Lambda^2]}, \quad (4)$$

where $b = (33 - 2n_f)/48\pi^2$. This prescription for the gluon propagator has already been successfully used in previously works (se e.g. [5], and references therein).

For the collinear divergences we use the dressed quark propagator that can be written as

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}, \quad (5)$$

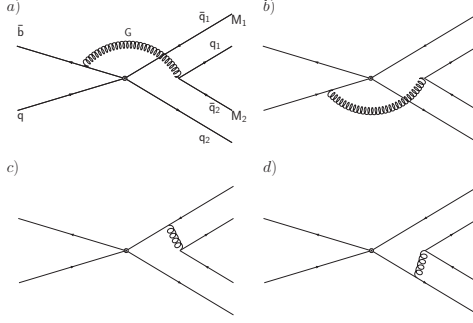


FIGURE 1. Feynman diagrams contributing to the amplitude of annihilation decays of B mesons.

where $Z(p^2)$ is the Lorentz scalar quark wave function renormalization and $M(p^2)$ is the quark dynamical mass. At large momenta, and in the chiral limit, it is known that the dynamical mass is related to the quark condensate ($\langle \bar{q}q \rangle$) as $M(p^2) \propto \langle \bar{q}q \rangle / p^2$ [6]. The many estimates of the dynamical quark mass give values between 250 MeV and 300 MeV [6, 7]. Here, since the mass function has a plateau at low momenta and decreases (in the chiral limit) very fast at high momenta [7], we can safely assume $Z(p^2) \approx 1$ and $S(p) = 1/(\not{p} + M(0))$.

ANNIHILATION DECAYS

We compute the amplitudes for pure annihilation decays in the collinear approximation of pQCD at twist-3 approximation. This type of process occurs at α_s order in perturbative QCD. A typical annihilation decay of a B meson is shown in Fig. 1, where the bottom quark decays through the W exchange or via a penguin process and a quark-antiquark pair is created in the final state through a gluon emission by any of the quarks involved in the process. The channels with these characteristics that we shall analyze are the following charmless channels: $B_s^0 \rightarrow \pi^+ \pi^-$, $\pi^0 \pi^0$, $B_d^0 \rightarrow K^+ K^-$; and charmed channels: $B^0 \rightarrow D_s^{\pm(*)} K^\mp$, $B^+ \rightarrow D_s^{+(*)} \bar{K}^0$, $B_s^0 \rightarrow D^{\pm(*)} \pi^\mp$, $D^0 \pi^0$, $\bar{D}^0 \pi^0$.

The branching ratios of each decay is calculated from

$$\mathcal{B}(B \rightarrow M_1 M_2) = \frac{\tau_B p_c}{8\pi m_B^2} |\mathcal{A}(B \rightarrow M_1 M_2)|^2, \quad (6)$$

where τ_B is the B meson lifetime, p_c is the momentum of the final state particles in the B meson rest frame.

We perform the numerical calculation considering the gluon mass scale with values $m_g = 400$ MeV, 500 MeV. The dynamical masses of the light quarks are $M(0) = m_u = m_d$, in the interval $250 \text{ MeV} < M(0) < 300 \text{ MeV}$, and the same value is assumed for the s quark. The Wilson's coefficients are computed using the equations given in the appendices of Ref. [9]. Other numerical parameters such as the masses, decay constants and life times of mesons, and the CKM parameters are taken from [10]. In Table 1 we show the results obtained and the experimental data available for each decay channel.

TABLE 1. Branching ratios for B decays for the gluon mass scale $m_g = 400$ MeV and 500 MeV. The error corresponds to the variation of the dynamical quark mass, $250 \text{ MeV} < m_q < 300 \text{ MeV}$. The available experimental data are also shown.

Channel	$\mathcal{B}(m_g = 400 \text{ MeV})$	$\mathcal{B}(m_g = 500 \text{ MeV})$	Experimental data [10]
$B^0 \rightarrow K^+ K^-$	$(5.83 \pm 0.67) \times 10^{-8}$	$(4.34 \pm 0.01) \times 10^{-8}$	$< 4.1 \times 10^{-7}$
$B_s^0 \rightarrow \pi^+ \pi^-$	$(11.08 \pm 1.06) \times 10^{-7}$	$(5.65 \pm 0.27) \times 10^{-7}$	$< 1.7 \times 10^{-6}$
$B_s^0 \rightarrow \pi^0 \pi^0$	$(5.54 \pm 1.06) \times 10^{-7}$	$(2.83 \pm 0.27) \times 10^{-7}$	$< 2.1 \times 10^{-6}$
$B^0 \rightarrow D_s^- K^+$	$(5.67 \pm 0.73) \times 10^{-5}$	$(2.84 \pm 0.16) \times 10^{-5}$	$(2.9 \pm 0.5) \times 10^{-5}$
$B^0 \rightarrow D_s^+ K^-$	$(5.69 \pm 0.66) \times 10^{-8}$	$(2.71 \pm 0.22) \times 10^{-8}$	
$B^+ \rightarrow D_s^+ \bar{K}^0$	$(6.09 \pm 0.69) \times 10^{-8}$	$(1.34 \pm 0.22) \times 10^{-8}$	$< 9 \times 10^{-4}$
$B_s^0 \rightarrow D^- \pi^+$	$(2.41 \pm 0.05) \times 10^{-6}$	$(1.65 \pm 0.55) \times 10^{-6}$	
$B_s^0 \rightarrow \bar{D}^0 \pi^0$	$(1.21 \pm 0.03) \times 10^{-6}$	$(0.86 \pm 0.25) \times 10^{-6}$	
$B_s^0 \rightarrow D^+ \pi^-$	$(8.13 \pm 0.85) \times 10^{-7}$	$(3.52 \pm 1.35) \times 10^{-7}$	
$B_s^0 \rightarrow D^0 \pi^0$	$(4.04 \pm 0.44) \times 10^{-7}$	$(3.39 \pm 0.95) \times 10^{-7}$	
$B^0 \rightarrow D_s^{*-} K^+$	$(2.35 \pm 0.15) \times 10^{-5}$	$(1.56 \pm 0.05) \times 10^{-5}$	$(2.2 \pm 0.6) \times 10^{-5}$
$B^0 \rightarrow D_s^{*+} K^-$	$(5.55 \pm 0.41) \times 10^{-8}$	$(2.21 \pm 0.06) \times 10^{-8}$	
$B^+ \rightarrow D_s^{*+} \bar{K}^0$	$(5.51 \pm 0.43) \times 10^{-8}$	$(2.36 \pm 0.06) \times 10^{-8}$	$< 9 \times 10^{-4}$
$B_s^0 \rightarrow D^{*-} \pi^+$	$(3.36 \pm 0.06) \times 10^{-6}$	$(1.88 \pm 0.21) \times 10^{-6}$	
$B_s^0 \rightarrow D^{*+} \pi^-$	$(3.26 \pm 0.29) \times 10^{-8}$	$(3.91 \pm 1.31) \times 10^{-8}$	

In conclusion, we have studied some decay channels of B_s^0, B_d^0, B^+ mesons which occur through the annihilation diagrams. We have argued that infrared finite gluon propagators and running coupling constants obtained as solutions of the QCD Schwinger-Dyson equations, may serve as a natural cutoff for the end-point divergences that appear in the calculation of these decays. Also, we use the constituent quark mass for the light quarks in the propagators to be able to perform the twist-3 calculation of the amplitudes. Comparing to the data available, our calculations are very satisfactory, notably for the two channels with actual measurements $B^0 \rightarrow D_s^- K^+$ and $B^0 \rightarrow D_s^{*-} K^+$.

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