Statistical quark models for the nucleon structure functions

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Abstract. We consider some existing relativistic models for the nucleon structure functions, relying on statistical approaches instead of perturbative ones. These models are based on the Fermi-Dirac distribution for the confined quarks, where a density of energy levels is obtained from an effective confining potential. In this context, it is presented some results obtained with a recent statistical quark model for the sea-quark asymmetry in the nucleon. It is shown, within this model, that experimental available observables, such as the ratio and difference between proton and neutron structure functions, are quite well reproduced with just three parameters: two chemical potentials used to reproduce the valence up and down quark numbers in the nucleon, and a temperature that is being used to reproduce the Gottfried sum rule violation.

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INTRODUCTION

The first model considering confinement in the calculation of structure functions, proposed in the deep inelastic scattering, is reported in [1]. In such work, it was not considered the existence of the sea-quark of the nucleon. After that, statistical (thermal) models were proposed by Cleymans and Thews [2] and by Mac and Ugaz [3], by considering a thermodynamical treatment for the partons confined in the hadron. Such models were also inspired in the Feynman's [4] idea to explain the observed nucleon sea asymmetry based on the Pauli principle: The proton has two valence quarks up (u) and one valence quark down (d). Then, by considering the Pauli principle, the creation of quark-antiquark pairs by gluons should be enhanced in case of $d\bar{d}$ pairs, in comparison with $u\bar{(u)}$ pairs.

In statistical thermal models, the hadron is made by a gas of partons confined in a finite volume (bag). In such models we have a relativistic gas of quarks, antiquarks and gluons, at a certain finite temperature, statistically distributed according to the Bose-Einstein (in case bosons) or Fermi-Dirac (in case of fermions) distribution laws.

The models of [2] and [3], together with a Feynman's suggestion, have been considered in a further development of a statistical quark model for the nucleon, as given in [5, 6, 7], where the aim was to describe the nucleon structure function and the sea-quark asymmetry, by considering a density of quark levels generated by a central confining interaction.

In the model of [5, 6, 7], to generate the quark energy levels and the corresponding wave functions, it was considered the relativistic linear confining potential proposed by

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Ferreira et al. [8], where the wave-function are given in coordinate space. Next, it was performed a Fourier transformation of the functions to the momentum space, in order to derive the observables for the quarks inside the nucleon, which are given in terms of the momentum fraction x. In this approach, once the energy levels are given by a relativistic quark potential model, a temperature parameter T is adjusted to obtain the violation of the Gottfried sum rule [9]. Two other parameters, the chemical potentials, are considered to obtain the normalization of the up and down valence quarks in the nucleon (proton and neutron). Among the nucleon observables, obtained within the model of [6], we report here recent improvements for the ratios and differences between the structure functions of the proton and the neutron, given by F_2^n/F_2^p and $F_2^p - F_2^n$, respectively. Such model results, as it will be shown, are in good agreement with the available experimental data.

The sea-quark distribution in the nucleon, the strange quark content, as well as sea-quark asymetries in the baryons, have also being considered in the chiral constituent quark model shown in Refs. [10].

For the sea-quark asymmetry of the nucleon, it was also derived within the statistical model of [6] the anti-down and anti-up quark functions, $\bar{d} - \bar{u}$ and $\bar{d}/\bar{u}$, which are shown as functions of the momentum fraction x.

Next, we outline the models of Cleymans-Thews and Mac-Ugaz, within a discussion on the approach developed in [5, 6], followed by recent model results on the neutron and proton structure functions, given by F_2^n/F_2^p and $F_2^p - F_2^n$, which are being compared with available experimental results.

THE CLEYMANS-THEWS AND MAC-UGAZ MODELS

In the Cleymans-Thews model [2], the hadron is defined by a gas of confined quarks, with the corresponding levels given by a continuum spectrum.

The inelastic structure function of the nucleon is calculated by performing a thermal average (sum averaging over flavors, color and spin of the quarks and antiquarks) on the hadronic tensor expressed in terms of the invariant structure functions, considering a process of quark-lepton interaction applied to the Born term of the electron-quark scattering.

The above resulting averaged function is multiplied by the thermal Fermi-Dirac distribution in case of fermions (the model does not take into account explicit gluon contributions), with the energy density characterized by a temperature T and a chemical potential μ.

The model describes the nucleon as composed basically by quarks and antiquarks, at a temperature $T \sim 40 - 50 MeV$, with a chemical potential $\mu = 200 MeV$. These parameters are determined by the rate of exponential decay of the inelastic structure function for the proton, F_2^p, at high values of the momentum fraction x. We should clarify that, the valued of the chemical potential μ is obtained by the rate of exponential decay ot the structure function, together with the normalization of quarks and antiquarks in the proton. In the region of low values of x ($x < 0.4$), the proton structure function is not well described, because $F_2^p \to 0$, when $x \to 0$. This thermal model was followed by the Mac and Ugaz model [3], that have considered additional effects as explained in the following.

In the model of Mac-Ugaz [3], the hadron is defined by means of a relativistic gas, confined in a volume V, in which the energy spectrum of quarks and gluons follow the MIT's bag model [11]. The volume V associated to the gas is given by the corresponding hadron size. By considering an effective temperature T, the statistics of Fermi-Dirac is applied to the confined quarks, with the Bose-Einstein statistic applied to the gluons.

So, in addition to the Cleymans-Thews model, in their statistical quark model distribution, Mac and Ugaz have also considered gluon distribution in the nucleon, confinement, and first order QCD perturbative corrections (where, from gluonic effects it is generated quark-antiquark pairs). No corrections exist to higher order perturbative QCD in the model. Finally, a simple relation between temperature T, chemical potential μ and volume V is obtained, given by $[V/(3\pi^2)]\left(\mu^3+\pi\mu T^2\right)$, where the chemical potential and temperature are parameters adjusted in their model. The model gives a quite high value for the nucleon radius, of about 2.5 fm, which is due to the continuus energy level distribution that was used.

THE NUCLEON STRUCTURE FUNCTION WITHIN THE STATISTICAL MODEL

Next, we show recent improved results of a statistical quark model given in refs. [6], which is being derived as extension of the above models. The model considered in [6] was previously outlined in [5], where the strangeness content of the nucleon was derived. In the phenomenological statistical model presented in [5] it was considered linear confined quarks to obtain the flavor asymmetry in the sea of the nucleon. Two different chemical potentials are required to describe the nucleon, which fixes the valence quark normalizations in the proton and neutron. Instead of assuming continuum levels for the quark energies, the levels are given by a linear Dirac confining potential. The up and down quark masses are assumed to be zero, with the corresponding strengths of the potential assumed to be the same. The main motivation in the model considered in [5, 6] is to deal directly with fundamental degrees of freedom of QCD.

As shown in [6], the statistical model can be improved by considering gluonic splitting processes and instanton induced processes. Once included such effects, the model presents quite good results for the sea-quark asymmetry in the nucleon, as compared with available experimental data. The model also presents good fits for the neutron to proton structure ratios F_2^n/F_2^p. These results have been extended to the corresponding neutron-proton difference $F_2^n-F_2^p$ of the structure functions. Recently improved results are shown in Fig. 1, compared with actual experimental data.

CONCLUSION

In the present report, we briefly review some statistical quark models, along with an idea of Feynman, which have inspired a new statistical model for the nucleon structure functions, where discrete quark levels are obtained by confining relativistic potentials. The nucleon structure functions are obtained by adjusting the model parameters, given by two chemical potentials and a temperature. The chemical potentials are adjusted by the valence up and down quark numbers in the proton and neutron, with the temperature

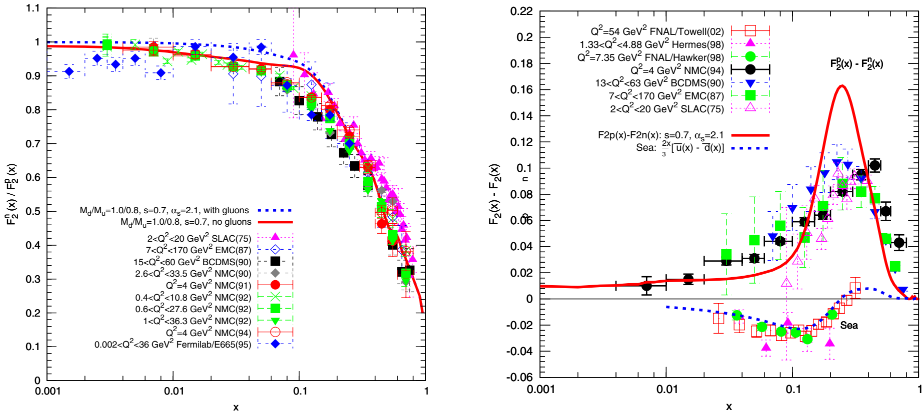

FIGURE 1. Ratio (left) and difference (right) of the structure functions of proton and neutron, given by F_2^n/F_2^p and $F_2^p - F_2^n$, respectively. For the references, given inside this figure, see [6, 7] and references therein

being adjusted by the Gottfried sum rule violation. Finally, by including gluonic effects, as well as instanton induced effects, the model allows one to describe quite well some experimental results for the nucleon-sea asymmetry, as shown in the results we present for F_2^n/F_2^p and $F_2^n - F_2^p$.

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