Effective action for QED with fermion self-interaction in $D=2$ and $D=3$ dimensions

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In this work we discuss the effect of the quartic fermion self-interaction of Thirring type in QED in $D=2$ and $D=3$ dimensions. This is done through the computation of the effective action up to quadratic terms in the photon field. We analyze the corresponding nonlocal photon propagators nonperturbatively in k/m, where k is the photon momentum and m the fermion mass. The poles of the propagators were determined numerically by using the MATHEMATICA software. In $D=2$ there is always a massless pole whereas for strong enough Thirring coupling a massive pole may appear. For $D=3$ there are three regions in parameter space. We may have one or two massive poles or even no pole at all. The interquark static potential is computed analytically in $D=2$. We notice that the Thirring interaction contributes with a screening term to the confining linear potential of massive two-dimensional QED (QED$_2$). In $D=3$ the static potential must be calculated numerically. The screening nature of the massive QED$_3$ prevails at any distance, indicating that this is a universal feature of $D=3$ electromagnetic interaction. Our results become exact for an infinite number of fermion flavors.

I. INTRODUCTION

In two dimensions, bosonization is a powerful technique used in a variety of examples [1–3]. In the past years there have been many attempts to generalize those ideas to higher dimensions [4–12]. For instance, one can derive an effective action by integrating out the fermion degrees of freedom and studying the physical properties of the resulting bosonic effective theory. Such an approach has been used in [4,13–16] to show that the static potential in three-dimensional QED (QED$_3$) is of screening type. In [4] we have used the perturbative path integral bosonization in both $D=2$ and $D=3$ QED. It is remarkable that in QED$_2$ at the quadratic approximation in the gauge fields but without any expansion in k/m, there are only massless poles [4], which is in agreement with what has been observed in [17], but differs from the result obtained through perturbative (m/e) calculation of [18]. In three dimensions it was shown that there is a massive excitation which depends on the dimensionless parameter $16\pi m/e^2$ and a simple approximated expression for this function has been found [4]. This in fact generalizes the calculations of [5], which were obtained at leading order of the derivative expansion, and that of [19] carried out at a higher order in k/m, which in its turn is related to consistent higher derivative actions [20–24].

The aim of this work is to analyze the influence of adding a Thirring term to QED in the static potential as well as in the particle content of the theory. In particular, we conclude that such a term does not change the large distance physics.

We start by introducing the notation which will be used in both $D=2$ and $D=3$. The generating functional for QED with Thirring self-interaction is given by

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left\{ i \int d^D x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}^j \right.\right.$$
$$\left.\left. \times \left(i\partial\!\!\!/ - m - \frac{e}{\sqrt{N}}A\!\!\!/ \right) \psi^j - \frac{g^2}{2N} (\bar{\psi}^j \gamma^\mu \psi^j)^2 \right] \right\}, \quad (1)$$

where N is the number of fermion flavors. It is convenient to introduce an auxiliary vector field B_μ and work with the physically equivalent generating functional:

$$Z = \int \mathcal{D}A_\mu \mathcal{D}B_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left\{ i \int d^D x \left[-\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} B_\mu B^\mu \right.\right.$$
$$\left.\left. + \bar{\psi}^j \left(i\partial\!\!\!/ - m - \frac{e}{\sqrt{N}}A\!\!\!/ - \frac{g}{\sqrt{N}}B\!\!\!/ \right) \psi^j \right] \right\}. \quad (2)$$

After integration over the fermionic fields we obtain

$$Z = \int \mathcal{D}A_\mu \mathcal{D}B_\mu \exp\left\{ i \int d^n x \left[-\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} B_\mu B^\mu \right] \right\}$$
$$\times \det\left[i\partial\!\!\!/ - m - \frac{1}{\sqrt{N}}(e A\!\!\!/ + g B\!\!\!/) \right]^N. \quad (3)$$

The fermion determinant can be evaluated perturbatively in $1/N$ and Furry's theorem guarantees that only an even number of vertices contribute. Since each vertex is of order $1/\sqrt{N}$ the leading contribution with two vertices will be N-independent. The next to leading contribution with four vertices is of order $1/N$ and will be neglected henceforth [25]. Therefore, at leading order in $1/N$, we have the quadratic effective action:

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$$S_{eff}^{(2)}[B,A] = \frac{1}{2} \int \frac{d^D k}{(2\pi)^D} \{ -\tilde{A}_\mu(-k)[g^{\mu\nu}k^2 - k^\mu k^\nu]\tilde{A}_\nu(k)$$

$$+ \tilde{B}^\mu(k)\tilde{B}_\mu(-k) + (e\,\tilde{A}_\mu + g\,\tilde{B}_\mu)(k)\Pi^{\mu\nu}(e\,\tilde{A}_\nu$$

$$+ g\,\tilde{B}_\nu)(-k) \}, \tag{4}$$

where $\tilde{A}_\mu(k)$ and $\tilde{B}_\mu(k)$ represent the Fourier transformations of $A_\mu(x)$ and $B_\mu(k)$, respectively, and $\Pi^{\mu\nu}$ is the polarization tensor:

$$\Pi^{\mu\nu}(k) = i \int \frac{d^D p}{(2\pi)^D} \text{tr} \left[\frac{1}{\not{p} - m + i\epsilon} \gamma^\mu \frac{1}{(\not{p}+\not{k}) - m + i\epsilon} \gamma^\nu \right]. \tag{5}$$

The action $S_{eff}^{(2)}[B,A]$ is exact in the $N \to \infty$ limit.

In order to proceed further we have to calculate $\Pi^{\mu\nu}$ which depends on the dimensionality of the space-time.

II. EFFECTIVE POTENTIAL IN $D=2$

Using dimensional regularization we have

$$\Pi_{D=2}^{\mu\nu}(k) = g^2 \tilde{\Pi}(k^2) \theta^{\mu\nu}, \tag{6}$$

where $\theta^{\mu\nu} = g^{\mu\nu} - k^\mu k^\nu/k^2$ and

$$\tilde{\Pi}(k^2) = \frac{1}{\pi} \left[1 + \frac{1}{2} \frac{4m^2/k^2}{(1-4m^2/k^2)^{1/2}} \ln \frac{(1-4m^2/k^2)^{1/2}+1}{(1-4m^2/k^2)^{1/2}-1} \right]. \tag{7}$$

Plugging back in $S_{eff}^{(2)}[B,A]$ and performing the Gaussian integral over B^μ we end up with the gauge invariant effective action for the gauge field:

$$S_{eff}^{(2)}[A] = \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \tilde{A}_\mu(-k) \left[-k^2 + e^2 \frac{\tilde{\Pi}}{1+g^2\tilde{\Pi}} \right]$$

$$\times \theta^{\mu\nu} \tilde{A}_\nu(k). \tag{8}$$

If $g \to 0$ we reproduce the QED_2 result of [4], and when $m \to 0$ we recover the result of the Schwinger-Thirring model [26]. Introducing a gauge fixing term we can obtain the photon propagator whose gauge invariant piece is given by

$$D_{\mu\nu}^\parallel(k) = \frac{(1+\tilde{\Pi}\,g^2)}{4\,m^2[g^2(z-a)\tilde{\Pi}+z]} g_{\mu\nu}, \tag{9}$$

where we define the dimensionless quantities which will be used also in $D=3$:

$$z = \frac{k^2}{4\,m^2}, \tag{10}$$

$$a = \frac{e^2}{4\,m^2 g^2}.$$

Notice that g is another dimensionless quantity in $D=2$.

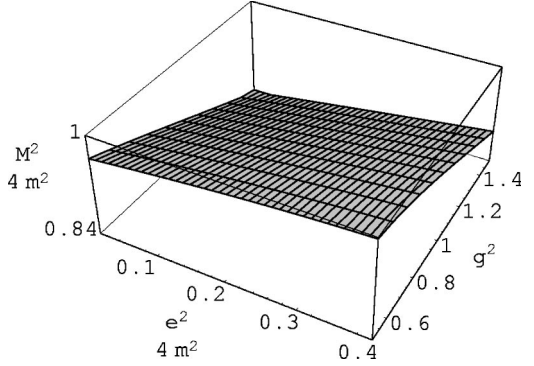FIG. 1. Photon effective mass in $D=2$ for $a<1$ and $0.25<g<1.5$.

As already stressed in [4] the expression (7) for $\tilde{\Pi}$ is correct for $k^2<0$ and we have used it to check that $D_{\mu\nu}$ is causal and no tachyonic poles appear. In analyzing the particle content of $S_{eff}^{(2)}[A]$ we restrict ourselves to the region $0 \le z < 1$, which is below the pair creation threshold ($z=1$). In that region $\tilde{\Pi}$ must be continued to

$$\tilde{\Pi}(z) = \frac{1}{\pi} \left[1 - \frac{1}{[z(1-z)]^{1/2}} \arctan \sqrt{\frac{z}{1-z}} \right],$$

$$0 \le z < 1. \tag{11}$$

For $z \to 0$, $\tilde{\Pi}$ becomes linear in z and, therefore, for arbitrary values of the dimensionless parameters a and g, we always have a massless simple pole $k^2=0$ as in pure QED_2 [4]. Since $\tilde{\Pi} \le 0$ for $0 \le z < 1$ it is clear that $z + g^2\tilde{\Pi}(z-a)$ will never vanish for $a \ge 1$ and we are left with only a massless pole. For $a<1$ there is always a massive pole at $k^2/2m^2 = M_{2D}(a,g)$, which was numerically evaluated and plotted in Fig. 1 in the region $0.1 \le a \le 0.93$ and $0.25 < g < 1.5$.

As we see in Fig. 1, when we decrease the Thirring coupling the mass of the pole tends to reach the pair creation threshold $z \to 1^-$ and becomes nonphysical.

At this point, we observe that the effect of the Thirring self-interaction in the pole structure of QED_2 is to introduce a massive pole if the Thirring coupling is strong enough, i.e., $mg^2/4\pi > e^2/16\pi m$. The massless pole of pure QED_2 with massive fermions remains untouched for any value of the coupling e and g, which is compatible with a recent study [27].

Now two comments are in order. First, though the Thirring interaction may introduce a mass in the photon propagator the gauge symmetry is not broken. As we see in Eq. (8) this is only possible because the action is nonlocal. If we try to make it local, for instance by making a derivative expansion for $m \to \infty$, we will miss the massive pole since the Thirring contribution will be neglected at first order in the derivative expansion. The massive pole will be only seen in the next to leading order in a higher derivative theory. Second, for any $g \ne 0$ it is always possible to find a value for k^2 such that $1 + \tilde{\Pi}\,g^2 = 0$ and therefore $D_{\mu\nu}^\parallel = 0$. That is, the

Thirring self-interaction originates a region in momentum space (hyperboloid) of forbidden momenta. We are not aware of similar observations in the literature and we do not have a deeper understanding of this fact.

Next we analyze the effect of Thirring interaction for the effective potential between two static charges. We assume that two charges Q and $-Q$ are located at $x=L/2$ and $x=-L/2$. From the equation of motion coming from $S_{eff}^{(2)}[A]$ we obtain the potential produced by the positive charge:

$$A_\mu(x)=\int \frac{d^2k}{(2\pi)^2}\int d^2x' D_{\mu\nu}^{\parallel}(k)e^{ik(x-x')}J^\nu(x'),\quad (12)$$

where

$$J^\nu(x')=Q\delta\left(x_1'-\frac{L}{2}\right)\delta^{\nu 0}\qquad (13)$$

and $D_{\mu\nu}^{\parallel}$ is given in Eq. (9). The only non-vanishing component of the potential is A_0, which can be obtained analytically through a contour integral:

$$V(L)=A_0(x=-L/2)$$

$$=Q\left[\frac{L}{2\left(1+\frac{2ag^2}{3\pi}\right)}+\frac{2a}{m\sqrt{u}}\frac{e^{-2m\sqrt{u}L}}{(u-a)^2\tilde{\Pi}(u)-a}\right]$$

$$(14)$$

where u is the non-vanishing solution of

$$g^2(z-a)\tilde{\Pi}(z)+z=0,\qquad (15)$$

with $\tilde{\Pi}$ given in Eq. (12). The *screening* effect, second term in $V(L)$, only exists for $a<1$ and is a pure consequence of the Thirring coupling. At large distances the *confining* nature massive QED$_2$ prevails and the influence of the Thirring term fades away.

III. THE MASSIVE POLES IN $D=3$

Using a parity and gauge invariant regulator, we obtain, in this case,

$$\Pi^{\mu\nu}(k)=\Pi_1 i\,\epsilon^{\mu\nu\rho}k_\rho+\Pi_2 k^2\theta^{\mu\nu},$$

where, for $0\leq z<1$, we have the parametric functions:

$$8\pi\,\Pi_1=f_1(z)=-\frac{1}{z^{1/2}}\ln\left(\frac{1+z^{1/2}}{1-z^{1/2}}\right),$$

$$16\pi m\,\Pi_2=f_2(z)=\frac{1}{z}\left[1+\left(\frac{1+z}{2}\right)f_1\right].\qquad (16)$$

Again, plugging back in $S_{eff}^{(2)}[B,A]$ and integrating over B_μ we obtain

$$S_{eff}^{(2)}[A]=\int \frac{d^3k}{(2\pi)^3}\tilde{A}_\mu(-k)\Bigg\{\left[-k^2+\frac{e^2}{g^2}\right.$$

$$\left.\times\left(1-\frac{g^2k^2\Pi_2+1}{(g^2k^2\Pi_2+1)^2-g^4k^2\Pi_1^2}\right)\right]\theta^{\mu\nu}$$

$$+i\frac{e^2\Pi_1}{(g^2k^2\Pi_2+1)^2-g^4k^2\Pi_1^2}\epsilon^{\mu\nu\rho}k_\rho\Bigg\}\tilde{A}_\nu(k).$$

$$(17)$$

Once again we recover the pure QED$_3$ result for $g\to 0$. As in the $D=2$ case $S_{eff}^{(2)}[A]$ is gauge invariant and, after introducing a gauge fixing term, we can write the gauge invariant piece of the photon propagator as

$$D_{\mu\nu}=\frac{\mathcal{N}(a,b,z)}{4\,m^2 D_+D_-}g_{\mu\nu}-\frac{i\,e^2 f_1}{32\,\pi\,m^2 k^2 D_+D_-}\epsilon_{\mu\nu\rho}k^\rho,$$

$$(18)$$

where a is defined as in Eq. (10) and

$$b=\frac{m\,g^2}{4\pi},$$

$$\mathcal{N}=(1+b\,z\,f_2)[b(a-z)f_2-1]-b^2(a-z)f_1^2,\quad (19)$$

$$D_\pm=b(a-z)G_\pm-\sqrt{z},\qquad (20)$$

$$G_\pm=\frac{1}{\sqrt{z}}\left[1+\frac{(1\pm\sqrt{z})^2}{2}f_1\right].\qquad (21)$$

Both $G_\pm$ are monotonically decreasing functions which satisfy

$$1\leq G_-\leq 2\quad\text{and}\quad G_+\leq -2\qquad (22)$$

for $z\leq 1$. Therefore when looking for the poles of the propagator

$$D_+D_-=0,\qquad (23)$$

it is natural to split the analysis in two regions.

A. $a<1(e^2/16\pi m<m\,g^2/4\pi)$

In this case we can have $D_+=0$ for some $z>a$ and $D_-=0$ for some $z<a$. Indeed we have always been able to find massive poles in both regions ($z>a$ and $z<a$) for arbitrary values of b and $0<z<1$.

We might say that these poles have distinct origins. The first one is due to the fermion self-interaction, whereas the second one has its origin due to the dynamically generated Chern-Simons term. This can be seen if one works with the reducible representation for the gamma matrices (4×4). In this case the fermion mass term is invariant under parity transformation and, as a consequence, no parity-odd term is dynamically generated; then the gauge invariant piece of the propagator is given by

$$D_{\mu\nu} = \frac{(1 + b\, z\, f_2)}{4\, m^2 z [b(a-z)f_2 - 1]} g_{\mu\nu}$$

which is obtained by taking $\Pi_1 = 0$ in Eq. (17). Since $f_2 \leq 0$ for $0 \leq z < 1$ one can check that there is always a massless pole and another one at $z_o > a$ which satisfies

$$b(a - z_o)f_2(z_o) - 1 = 0.$$

In pure QED_3 with reducible representation no topological mass is dynamically generated for the gauge field, consequently the massive pole found from the equation above has its origin in the Thirring term.

B. $a \geq 1 \, (e^2/16\pi m \geq m\, g^2/4\pi)$

In this QED_3 dominated region we can only have poles from $D_- = 0$ or equivalently:

$$b\, G_- = \frac{\sqrt{z}}{a - z}. \tag{24}$$

Since the right-hand side of Eq. (24) is a monotonically increasing function of z in the range $0 < z < 1$ and G_- is limited according to Eq. (22), it is clear that there are no solutions for $D_- = 0$ whenever $b \geq 1/(a-1)$; i.e., $a \geq 1 + 1/b$. Therefore in terms of QED_3 and Thirring couplings, if

$$\frac{e^2}{16\pi m} \geq \frac{m\, g^2}{4\pi} + 1, \tag{25}$$

the propagator (18) has no poles whatsoever; this may happen as an artifact of the approximation. It is possible that, as at this order no poles do appear, the usually negligible next perturbative contribution could introduce back the massive pole. We have found numerically that the above bound indeed exists. For any $b < 1/(a-1)$ we have always been able to find one massive pole at $D_- = 0$ for arbitrary values of the parameter a in the region $a > 1$.

Summarizing,

(i) If $e^2/16\pi m < m\, g^2/4\pi$, then two massive poles ($D_\pm = 0$) are present.

(ii) If $m\, g^2/4\pi \leq e^2/16\pi m < m\, g^2/4\pi + 1$, just one pole ($D_- = 0$) appears.

(iii) Finally, if $e^2/16\pi m \geq m\, g^2/4\pi + 1$, no poles appear at all.

Now two remarks follow. First, concerning the dependence on the QED_3 and Thirring couplings on the massive pole found from $D_- = 0$, for $b < 1/(a-1)$, we have found numerically and it is plotted in Fig. 2 that the mass increases along with the QED_3 coupling and decreases for growing Thirring coupling. If we take both small $m g^2$ and large $e^2/4m$ we tend to violate the condition $e^2/16\pi m < m\, g^2/4\pi + 1$ and the pole tends to go beyond the pair creation threshold ($z \geq 1$) as we see on the top of the hill in Fig. 2. The second comment regards the pure QED_3 limit ($g \rightarrow 0$) for which there is still a region without poles in the propagator, i.e., $e^2/16\pi m \geq 1$. This seems to have gone unnoticed in the literature [4,28,29]. Sometimes the quadratic approximation for $S_{eff}^{(2)}[A]$ is called a small coupling ap-
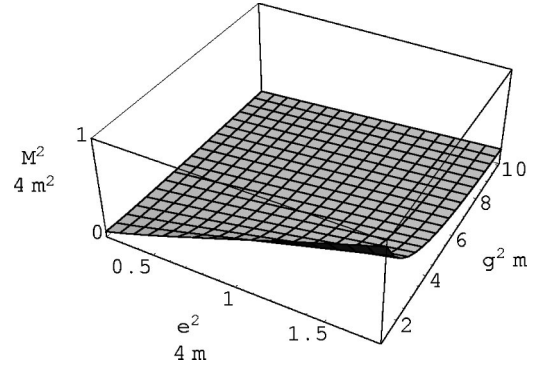

FIG. 2. Behavior of the photon effective mass in $D = 3$.

proximation in the literature, thus one might argue that our calculations only make sense for small coupling e, such that we are below the bound $e^2 = 16\pi m$. This is certainly sensible at the leading order in the derivative expansion, as in [13] since $m \rightarrow \infty$, but it is not true in general. In particular, for QED_3 with a large number of flavors, we have argued that the quadratic approximation for the effective action corresponds to the leading $1/N$ contribution and no restriction is required on e or m, therefore the problem persists.

Similarly to the $D = 2$ case we now move to the calculation of the effect of the Thirring self-interaction on the potential between two static charges $+Q$ and $-Q$ located at $(x,y)_+ = (L/2, 0)$ and $(x,y)_- = (-L/2, 0)$. That is, the current of the positive charge is

$$J^\nu(x') = Q\, \delta\left(x' - \frac{L}{2}\right)\delta(y')\,\delta^{\nu 0}, \tag{26}$$

the potential produced by the above charge is

$$A_0(x) = \int d^3 x' \int \frac{d^3 k}{(2\pi)^3} e^{i\, k(x - x')} D_{0\alpha}(k) J^\alpha(x') \tag{27}$$

and

$$V(L) = A_0\left(x = -\frac{L}{2}, y = 0\right)$$

$$= \frac{Q}{4\, m^2}\int_0^\infty dk\, k\, \frac{J_0(k\, L)\, \mathcal{N}}{D_+ D_-}, \tag{28}$$

where $k = \sqrt{k_x^2 + k_y^2}$ and we have used $\epsilon_{0\alpha\gamma}\partial^\gamma J^\alpha = 0$. The fact that the current is static gives rise to a $\delta(k_0)$ upon integration over dx_0'. Since $k_0 = 0$ we have $k^2 = -(k_x^2 + k_y^2)$ and $\mathcal{N}/D_+ D_-$ in Eq. (28) are the continuations of the expressions (19) and (20) for the region $k^2 < 0$ according to the formula $\ln[(1 - \sqrt{z})/(1 + \sqrt{z})] = 2\, i \arctan(z)$. The Bessel function $J_0(k\, L)$ appears after integration over the angular variable. Different from $D = 2$ we are no longer able to calculate $V(L)$ analytically and we have to appeal to a numeric computation as in [30]. We have plotted the result in Fig. 3 for specific values of m, a and b. We have noticed that the screening form of the potential is insensitive to the parameters m, a and b, which is quite surprising in view of our
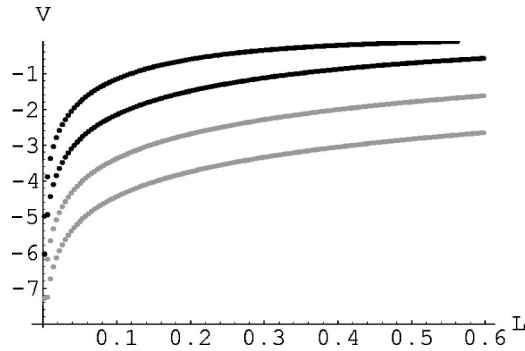
FIG. 3. Inter-fermion effective potential for $b=3$ and $m=1$ (black dotted line) and $m=0.1$ (gray dotted line). For each color, the upper curve corresponds to $a=4$, and the lower one to $a=1/2$.

FIG. 4. Logarithm of the absolute value of the inter-fermion effective potential for $b=3$, $m=1$ and $a=4$.

previous analysis of the pole content of the propagator. The presence of the Thirring self-interaction seems to be irrelevant for the static potential even at small distances. Our conclusion is in disagreement with [28] (see also [31]) who claims that because of the Thirring term, a repulsive barrier appears at low distances. The author of [28] makes use of the derivative expansion in the quadratic action $S_{eff}^{(2)}[A]$, which is presumably a good approximation for large fermion masses $k/2m \to 0$. We have also checked that $V(L)$ keeps its screening shape, even for large masses, for any distance L. Changing the values of the couplings g and e will not change the shape of $V(L)$ either (see Figs. 3 and 4 for the typical shape). The point 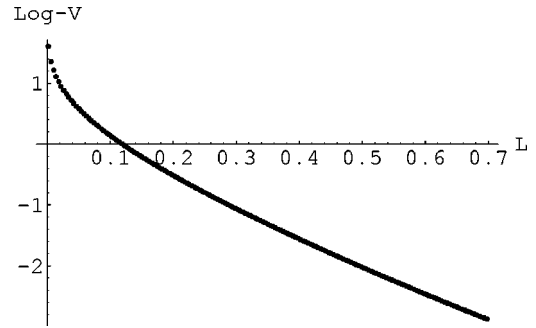is that the rapid oscillations of the Bessel function wash out any detail of the photon propagator leading always to a screening potential in $D=3$. Besides, it is worth mentioning that Fig. 4 indicates a nearly exponential asymptotic behavior for the screening potential. Finally, similar to $D=2$ it is always possible to find k^2 such that $\mathcal{N}(a,b,z)=0$ and the symmetric part of the photon propagator [see Eq. (18)] will vanish for those special values of momenta.

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