

Doubly charged Higgs through photon-photon collisions in 3-3-1 models

J. E. Cieza Montalvo,¹ Nelson V. Cortez,² and M. D. Tonasse³

¹*Instituto de Física, Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, 20559-900 Rio de Janeiro, RJ, Brazil*

²*Justino Boschetti 40, 02205-050 São Paulo, SP, Brazil*

³*Unidade de Registro, Campus Experimental de Registro, Universidade Estadual Paulista, Rua Tamekishi Takano 5, 11900-000, Registro, SP, Brazil*

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We study the production and signatures of doubly charged Higgs bosons (DCHBs) in the process $\gamma\gamma \leftrightarrow H^{--}H^{++}$ at the e^-e^+ International Linear Collider and CERN Linear Collider, where the intermediate photons are given by the Weizsäcker-Williams and laser backscattering distributions.

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I. INTRODUCTION

The Higgs boson is the one missing piece, a critical ingredient to complete our understanding of the standard model (SM), the current theory of fundamental particles and how they interact. Different types of Higgs bosons, if they exist, may lead us into new realms of physics beyond the SM. Up to now, SM experiments have given no evidence for the presence of the SM Higgs boson. So, the observation of any kind of Higgs particle must be an important step forward in the understanding of physics in the electroweak sector or beyond the SM. In extended models doubly charged Higgs bosons (DCHBs) appear as being relatively light and typically as a component of triplet representations such as the left-right symmetric models [1]. They appear also as components of sextet ones such as $SU(3)_L \otimes U(1)_N$ (3-3-1) [2]. It was shown that the simplest 3-3-1 model, which is also the simplest chiral extension of the SM, is in triplet representation of $SU(3)$ of the matter field, so it incorporates the DCHBs naturally and the results in Refs. [3,4] indicate a sufficient number of events required for establishing the signal. Research has been done with DCHBs and a lower limit of 141 (114) GeV for their masses are obtained by Hera and Fermilab, respectively [5].

Independently of DCHBs, the 3-3-1 models have interesting features for the TeV scale energy. Let us briefly enumerate some notable points of them: (a) Because of cancellation of chiral anomalies, the number of generations N must be a multiple of 3, however, due to asymptotic freedom in QCD, which imposes that the number of generations must be $N \leq 5$, the unique allowed number of generations is $N = 3$; (b) Some of the 3-3-1 process accessible to the next generation of accelerators violates individual lepton numbers; (c) The quantization of electric charge is independent if neutrinos are Dirac or Majorana particles [6]; (d) The seesaw mechanism can be naturally incorporated in some version of the 3-3-1 models [7]; (e) The building of the supersymmetric version of a 3-3-1 gauge model [8]; (f) The models are nonperturbative at the TeV scale, as well as the supersymmetric version [9];

(g) We observe that the parameter $\sin^2\theta_W/(1 - 4\sin^2\theta_W)$ has a Landau pole, that is, $\sin^2\theta_W$ must be $< 1/4$, that is a good feature of the model, since when evolving the $\sin^2\theta_W$ to high values it is possible to find an upper bound to the masses of the new particles [9].

The simplest scalar sector of the 3-3-1 model has three Higgs triplets [10,11]. In this version the three scalar triplets are $\eta = (\eta^0 \ \eta_1^- \ \eta_2^+)^T$, $\rho = (\rho^+ \ \rho^0 \ \rho^{++})^T$, and $\chi = (\chi^- \ \chi^{--} \ \chi^0)^T$ transforming as $(\mathbf{3}, 0)$, $(\mathbf{3}, 1)$, and $(\mathbf{3}, -1)$, respectively. The neutral components of the scalars triplets η , ρ , and χ develop nonzero vacuum expectation values $\langle \eta^0 \rangle = v_\eta$, $\langle \rho^0 \rangle = v_\rho$, and $\langle \chi^0 \rangle = v_\chi$, with $v_\eta^2 + v_\rho^2 = v_W^2 = (246 \text{ GeV})^2$. The pattern of symmetry breaking is $SU(3)_L \otimes U(1)_N \xrightarrow{(\chi)} SU(2)_L \otimes U(1)_Y \xrightarrow{(\eta, \rho)} U(1)_{\text{em}}$. In the potential [12] f is a constant with dimension of mass and the λ_i ($i = 1, \dots, 9$) are adimensional constants, which represent the intensities of the quartic vertices of the Higgs, with $\lambda_3 < 0$ and $f < 0$ from the positivity of the scalar masses.

In the case of elastic e^-e^+ scattering we will use the $\gamma\gamma$ differential luminosity which is given by

$$\left(\frac{dL^{e\ell}}{d\tau}\right)_{\gamma\gamma/\ell\ell} = \int_\tau^1 \frac{dx_1}{x_1} f_{\gamma/\ell}(x_1) f_{\gamma/\ell}(x_2 = \tau/x_1), \quad (1)$$

where $\tau = x_1x_2$ and $f_{\gamma/\ell}(x)$ are the distribution functions [13,14]. The presentation of this subject is given by [15].

Studies of DCHBs using specifically $\gamma\gamma$ collision were also done in Ref. [16].

In the present work we report about signals of this kind of bosons, which appear when we consider some particular values for the adimensional parameter λ_9 in the Higgs potential, the vacuum expectation value v_χ , and some masses of DCHBs.

II. CROSS SECTION PRODUCTION

We study the production of DCHBs in the process $e^-e^+ \rightarrow e^-e^+\gamma\gamma \rightarrow H^{\pm\pm}H^{\pm\pm}$, which occurs through the nonresonant contributions of the $H^{\pm\pm}$ and $U^{\pm\pm}$ in

the t and u channels and the quartic vertex $\gamma\gamma H^- H^{++}$. The intermediate photons are generated either by the Weizsäcker-Williams [13] or laser backscattering distribu-

tions [14]. The interaction Lagrangian is given in several papers (see, for example, Ref. [3]), then we evaluate the differential subprocess cross section for this reaction as

$$\begin{aligned} \frac{d\hat{\sigma}}{d\cos\theta} = & \frac{2\beta\alpha^4\pi^3(\Lambda_{U\gamma})^4}{\hat{s}} \left(\frac{\hat{t}^2}{m_{U^{\pm\pm}}^4} - \frac{2\hat{t}}{m_{U^{\pm\pm}}^2} + 4 \right) \left[\frac{1}{(\hat{t} - m_{U^{\pm\pm}}^2)^2} + \frac{1}{(\hat{u} - m_{U^{\pm\pm}}^2)^2} + \frac{4}{(\hat{t} - m_{U^{\pm\pm}}^2)(\hat{u} - m_{U^{\pm\pm}}^2)} \right] \\ & + \frac{\beta\alpha^2\pi(\Lambda_{U\gamma})^4}{8\hat{s}} \left[\frac{\hat{t}^2}{(\hat{t} - m_{H^{\pm\pm}}^2)^2} + \frac{\hat{u}^2}{(\hat{u} - m_{H^{\pm\pm}}^2)^2} + \frac{2m_{H^{\pm\pm}}^4}{(\hat{t} - m_{H^{\pm\pm}}^2)(\hat{u} - m_{H^{\pm\pm}}^2)} \right] \\ & + \frac{\beta\alpha^3\pi^3(\Lambda_{U\gamma})^2(\Lambda_{U\gamma})^2}{\hat{s}} \left[\left(\frac{\hat{t}^2}{m_{U^{\pm\pm}}^2} - \hat{t} \right) \left(\frac{1}{(\hat{t} - m_{U^{\pm\pm}}^2)(\hat{t} - m_{H^{\pm\pm}}^2)} \right) + \left(\frac{m_{H^{\pm\pm}}^4}{m_{U^{\pm\pm}}^2} - \hat{u} \right) \left(\frac{1}{(\hat{t} - m_{U^{\pm\pm}}^2)(\hat{u} - m_{H^{\pm\pm}}^2)} \right) \right. \\ & + \left. \left(\frac{\hat{t}^2}{m_{U^{\pm\pm}}^2} - \hat{t} \right) \left(\frac{1}{(\hat{u} - m_{U^{\pm\pm}}^2)(\hat{t} - m_{H^{\pm\pm}}^2)} \right) + \left(\frac{m_{H^{\pm\pm}}^4}{m_{U^{\pm\pm}}^2} - \hat{u} \right) \left(\frac{1}{(\hat{u} - m_{U^{\pm\pm}}^2)(\hat{u} - m_{H^{\pm\pm}}^2)} \right) \right] \\ & + \frac{4\pi\beta\alpha^2}{\hat{s}} \left[8 + 4\pi\alpha(\Lambda_{U\gamma})^2 \left(\frac{\hat{t}}{m_{U^{\pm\pm}}^2} - 4 \right) \left(\frac{1}{\hat{t} - m_{U^{\pm\pm}}^2} + \frac{1}{\hat{u} - m_{U^{\pm\pm}}^2} \right) - (\Lambda_{U\gamma})^2 \left(\frac{\hat{t}}{\hat{t} - m_{U^{\pm\pm}}^2} + \frac{\hat{u}}{\hat{u} - m_{U^{\pm\pm}}^2} \right) \right], \quad (2) \end{aligned}$$

where

$$(\Lambda_{U\gamma}) = i \frac{v_\eta v_\chi}{\sin\theta_W} \sqrt{\frac{2}{v_\eta^2 + v_\chi^2}}, \quad (3)$$

$$(\Lambda_\gamma) = \frac{v_\chi^2 - v_\eta^2}{v_\chi^2 + v_\eta^2}. \quad (4)$$

α is the fine structure constant, which we take equal to $\alpha = 1/128$, $\sqrt{\hat{s}}$ is the center of mass (c.m.) energy of the $\gamma\gamma$ system, $\hat{t} = m_{H^{\pm\pm}}^2 - (1 - \beta \cos\theta)\hat{s}/2$ and $\hat{u} = m_{H^{\pm\pm}}^2 - (1 + \beta \cos\theta)\hat{s}/2$, with β being the velocity of the Higgs in the subprocess c.m. and θ its angle with respect to the incident γ in this frame, $\Lambda_{U\gamma}^{\mu\nu}$ is the vertex strength of bosons $U^{\pm\pm}$ to γ and $H^{\pm\pm}$ ($\Lambda_{U\gamma}^{\mu\nu} = \Lambda_{U\gamma} g^{\mu\nu}$), Λ_γ^μ is the vertex strength of the $H^{\pm\pm}$ to γ and $H^{\pm\pm}$ ($\Lambda_\gamma^\mu = \Lambda_\gamma(p_1 - q_1)^\mu$) with p_1 and q_1 being the momentum four-vectors of the γ , and $H^{\pm\pm}$ and the others couplings are given in [3,17] and the quartic vertex strength which is equal to $-4ie^2 g_{\mu\nu}$.

For the standard model parameters we assume PDG values, i.e., $\sin^2\theta_W = 0.2315$ [18]. We can obtain the total cross section for this process folding $\hat{\sigma}$ with the two photon luminosities

$$\begin{aligned} \sigma &= \int_{\tau_{\min}}^1 \frac{dL}{d\tau} d\tau \hat{\sigma}(\hat{s} = x_1 x_2 s) \\ &= \int_{\tau_{\min}}^1 \int_{\ln\sqrt{\tau}}^{-\ln\sqrt{\tau}} \frac{dx_1}{x_1} f_{\gamma/\ell}(x_1) f_{\gamma/\ell}(x_2) \int \frac{d\hat{\sigma}}{d\cos\theta} d\cos\theta. \quad (5) \end{aligned}$$

III. RESULTS AND CONCLUSIONS

In the following we present the cross section for the process $e^- e^+ \rightarrow e^- e^+ \gamma\gamma \rightarrow H^{\pm\pm} H^{\mp\mp}$ for the International Linear Collider (ILC) (1.5 TeV) and CERN Linear Collider (CLIC) (3 TeV), where we have chosen for the parameters, masses, and the vacuum expectation value (VEV), the following representative values: $\lambda_1 = -1.2$, $\lambda_2 = \lambda_3 = -\lambda_6 = \lambda_8 = -1$, $\lambda_4 = 2.98$, $\lambda_5 = -1.57$, $\lambda_7 = -2$, and $\lambda_9 = -1.2$; $v_\eta = 195$ GeV, $v_\chi = 1000$, and $v_\chi = 1500$ GeV and with other particles masses as given in Table I, it is noticeable that the value of λ_9 was chosen this way in order to guarantee the approximation $-f \simeq v_\chi$ [12], and that the masses of m_{h^0} , $m_{H_1^\pm}$, and $m_{H_2^\pm}$ depend on the parameter f and therefore they cannot be fixed by any value of v_χ , so when we have $M_{H^{\pm\pm}} = 500$ GeV, $v_\chi = 1000$ GeV, the masses of H_2^\pm and h are $m_{H_2^\pm}^\pm = 671.9$ GeV and $m_h = 1756.2$ GeV, and in the case of $v_\chi = 1500$ for $M_{H^{\pm\pm}} = 500$ GeV, the values

TABLE I. Values for v_χ , the masses of heavy leptons (E , M , and T), Higgs bosons (H_0^2 and H_0^3), gauge bosons (V , U , and Z'), and heavy quarks (J_1 , J_2 , and J_3) for $v_\eta = 195$ GeV. All the values in this table are given in GeV.

v_χ	m_E	m_M	m_T	$m_{H_0^2}$	$m_{H_0^3}$	m_V	m_U	$m_{Z'}$	m_{J_1}	m_{J_2}	m_{J_3}
1000	148.9	875	2000	1017.2	2000	467.5	464	1707.6	1000	1410	1410
1500	223.3	1312.5	3000	1525.8	3000	694.1	691.8	2561.3	1500	2115	2115

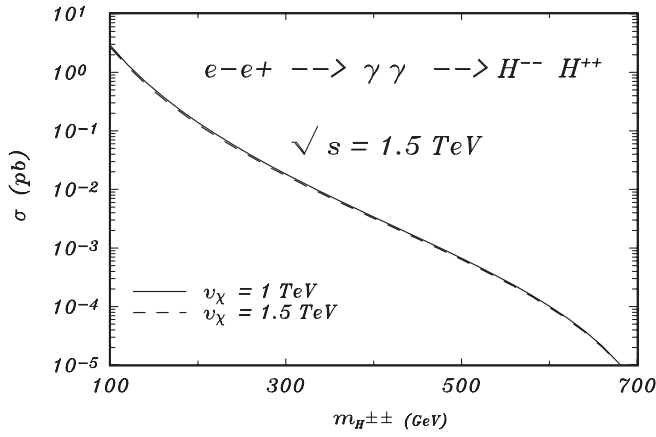


FIG. 1. Total cross section for the process $e^-e^+ \rightarrow e^-e^+\gamma\gamma \rightarrow H^{\pm\pm}H^{\mp\mp}$ as a function of $m_{H^{\pm\pm}}$ for bremsstrahlung at $\sqrt{s} = 1.5$ TeV (a) $v_\chi = 1$ TeV (solid line) and (b) $v_\chi = 1.5$ TeV (dashed line).

of the mass of H_2^\pm and h are $m_{H_2^\pm}^\pm = 901.6$ GeV and $m_h = 2802.7$ GeV, respectively.

The following figures will show the behavior of the cross section of the process $e^+e^- \rightarrow e^-e^+\gamma\gamma \rightarrow H^{\pm\pm}H^{\mp\mp}$ as a function of $m_{H^{\pm\pm}}$ for bremsstrahlung and laser backscattering photons, respectively. In that case, the cross section for the process initiated by backscattered photons is approximately one to two orders of magnitude larger than the one for bremsstrahlung photons due to the distribution of backscattered photons being harder than the one for bremsstrahlung. So in Fig. 1, we show the cross section for ILC for bremsstrahlung distribution. Considering that the expected integrated luminosity for the ILC collider will be of the order of 3.8×10^5 pb^{-1}/yr , then the statistics give a total of ≈ 250 events per year, if we take the mass of the boson $M_{H^{\pm\pm}} = 500$ GeV and $v_\chi = 1000$ GeV. Regarding the

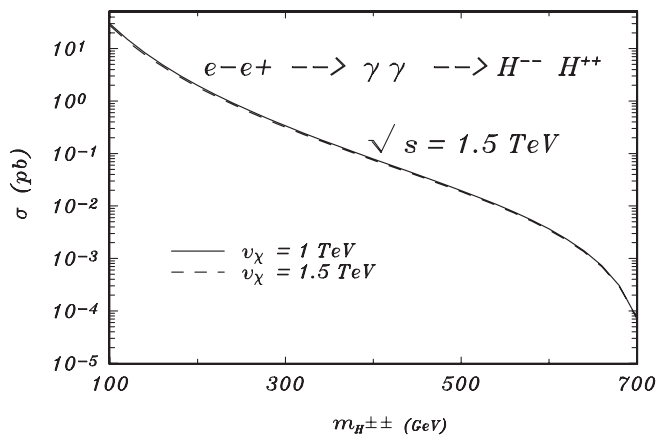


FIG. 2. Total cross section for the process $e^-e^+ \rightarrow e^-e^+\gamma\gamma \rightarrow H^{\pm\pm}H^{\mp\mp}$ as a function of $m_{H^{\pm\pm}}$ for backscattered photons at $\sqrt{s} = 1.5$ TeV (a) $v_\chi = 1$ TeV (solid line) and (b) $v_\chi = 1.5$ TeV (dashed line).

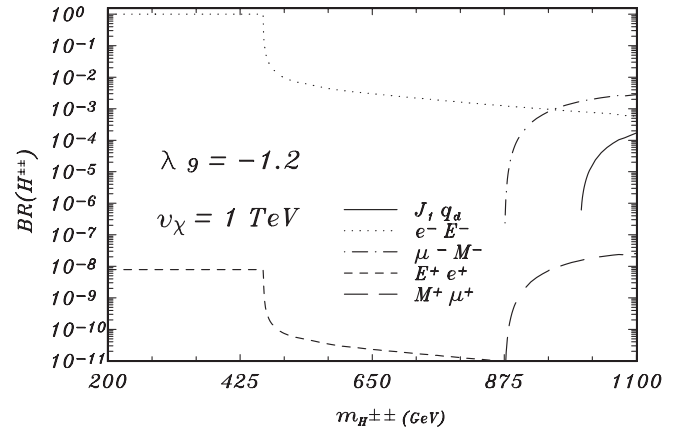


FIG. 3. Branching ratios for the doubly charged Higgs decays as functions of $m_{H^{\pm\pm}}$ for $\lambda_9 = -1.2$, $v_\chi = 1$ TeV for the quarks ($J_1 q_d$) and leptons (e^-E^- , μ^-M^- , E^+e^+ , and $M^+\mu^+$).

vacuum expectation value $v_\chi = 1500$ GeV for the same mass it will give a total of ≈ 240 events per year, in respect to the backscattered photons (see Fig. 2); the statistics are the following. Taking the mass of the boson $M_{H^{\pm\pm}} = 500$ GeV and $v_\chi = 1000$ GeV, we will have a total of $\approx 7.6 \times 10^3$ doubly charged Higgs produced per year. Regarding the vacuum expectation value $v_\chi = 1500$ GeV and for the same mass of the boson $H^{\pm\pm}$, it will give a total of $\approx 7.2 \times 10^3$ events per year, so we have that for the ILC (for bremsstrahlung and laser backscattering photons) the number of events is sufficiently appreciable, which we are going to analyze now to detect the signal.

Considering that the signal for $H^{\mp\mp}$ are $U^{\mp\mp}\gamma$ and $U^{\pm\pm}\gamma$ [17] and taking into account that the branching ratios for these particles would be $\text{BR}(H^{\mp\mp} \rightarrow U^{\mp\mp}\gamma) = 49.5\%$ and $\text{BR}(H^{\pm\pm} \rightarrow U^{\pm\pm}\gamma) = 49.5\%$, see Figs. 3–5, for the mass of the Higgs boson $m_{H^{\pm\pm}} = 500$ GeV,

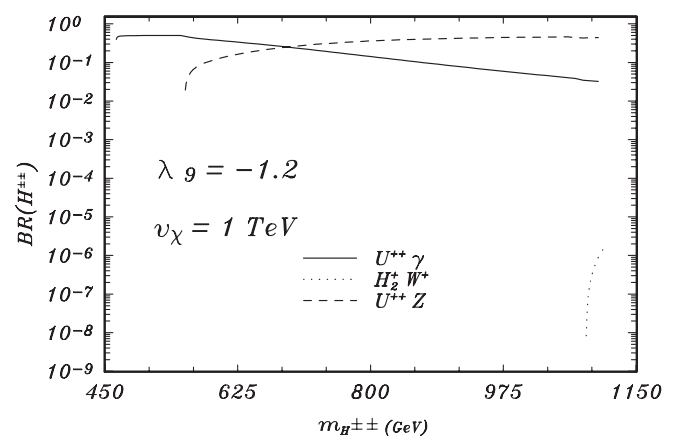


FIG. 4. Branching ratios for the doubly charged Higgs decays as functions of $m_{H^{\pm\pm}}$ for $\lambda_9 = -1.2$, $v_\chi = 1$ TeV for the new gauge bosons and photon, Z ($U^{\pm\pm}\gamma$ and Z), for the Higgs boson and gauge boson ($H_2^+W^+$).

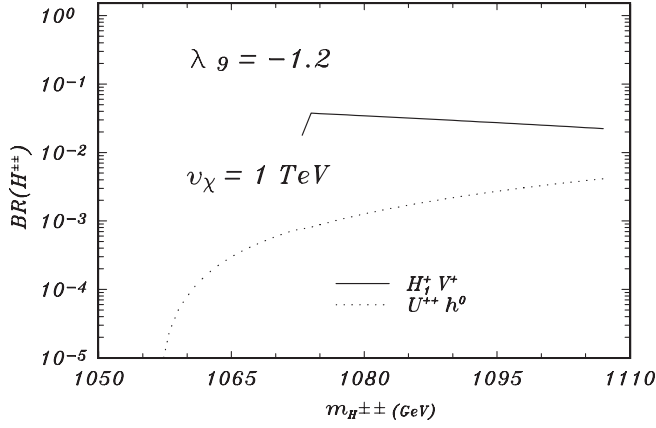


FIG. 5. Branching ratios for the doubly charged Higgs decays as functions of $m_{H^{\pm\pm}}$ for $\lambda_g = -1.2$, $v_\chi = 1$ TeV for $H_1^{\pm} V^{\pm}$.

$v_\chi = 1000$ GeV, and that the particles $U^{\mp\mp}$ decay into $e^- P^-$ and $e^+ P^+$, whose branching ratios for these particles would be $\text{BR}(U^{--} \rightarrow e^- P^-) = 50\%$ and $\text{BR}(U^{++} \rightarrow e^+ P^+) = 50\%$, see Figs. 6 and 7, then we would have approximately 15 events per year for the ILC for bremsstrahlung photons. Regarding the vacuum expectation value $v_\chi = 1500$ GeV it will not give any event because it is restricted by the values of $m_{U^{\pm\pm}}$ which in this case give $m_{U^{\pm\pm}} = 691.8$ GeV (see Table I). Considering the backscattered photons, we will have for this signal a total of $\approx 4.6 \times 10^2$ events per year for the mass of the Higgs boson $m_{H^{\pm\pm}} = 500$ GeV and $v_\chi = 1000$ GeV. Regarding the vacuum expectation value $v_\chi = 1500$ GeV, it will not give any event for the same reasons given above.

The cross section for the CLIC, which integrated luminosity will be of the order of 3×10^6 pb $^{-1}$ /yr, is restricted by the mass of the doubly charged Higgs, because for $v_\chi = 1000(1500)$ and $\lambda_g = -1.2$ the acceptable masses are up

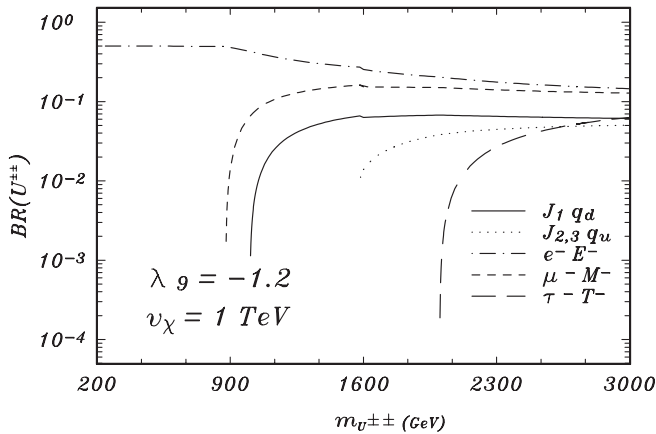


FIG. 6. Branching ratios for the doubly charged gauge bosons decays as functions of $m_{H^{\pm\pm}}$ for $\lambda_g = -1.2$, $v_\chi = 1$ TeV for the quarks ($J_1 q_d$, $J_{2,3} q_u$) and leptons ($e^- E^-$, $\mu^- M^-$, and $\tau^- T^-$).

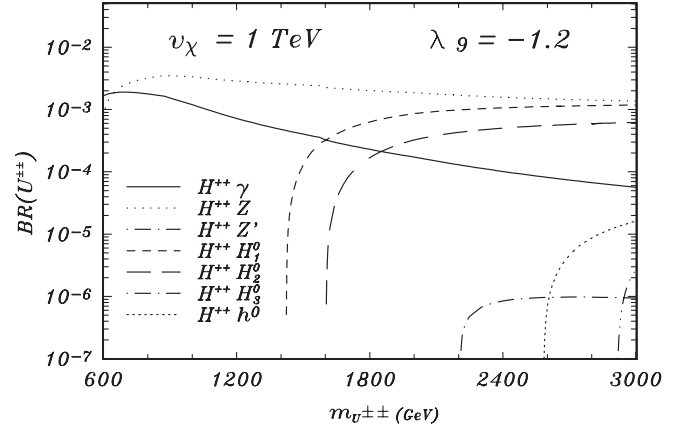


FIG. 7. Branching ratios for the doubly charged gauge bosons decays as functions of $m_{H^{\pm\pm}}$ for $\lambda_g = -1.2$, $v_\chi = 1$ TeV for the doubly charged Higgs and γ , Z , Z' , H_1^0 , H_2^0 , H_3^0 , h^0 .

to $m_{H^{\pm\pm}} \approx 1107(1651)$ GeV, see Ref. [4]. Taking this into account we obtain for bremsstrahlung distribution, see Fig. 8, a total of $\approx 3.9 \times 10^4$ of DCHBs produced per year, if we take the mass of the boson $m_{H^{\pm\pm}} = 500$ GeV and $v_\chi = 1000$ GeV. Taking the same signal as above, that is, $\text{BR}(H^{--} \rightarrow U^{--} \gamma) = 49.5\%$ and $\text{BR}(H^{++} \rightarrow U^{++} \gamma) = 49.5\%$, see Figs. 3 and 4, and considering that the particles $U^{\mp\pm}$ decay into $e^- P^-$ and $e^+ P^+$, whose branching ratios for these particles would be $\text{BR}(U^{--} \rightarrow e^- P^-) = 50\%$ and $\text{BR}(U^{++} \rightarrow e^+ P^+) = 50\%$, see Figs. 6 and 7, then we would have approximately $\approx 2.3 \times 10^3$ events per year for the CLIC for bremsstrahlung photons. Regarding the backscattered photons, see Fig. 9, we would have approximately 7.2×10^5 pairs of doubly charged Higgs produced per year for the same mass of the Higgs boson $m_{H^{\pm\pm}} = 500$ GeV and $v_\chi = 1000$ GeV, taking now the same signal as above we will have a total of approximately 4.4×10^4 events per year for the CLIC

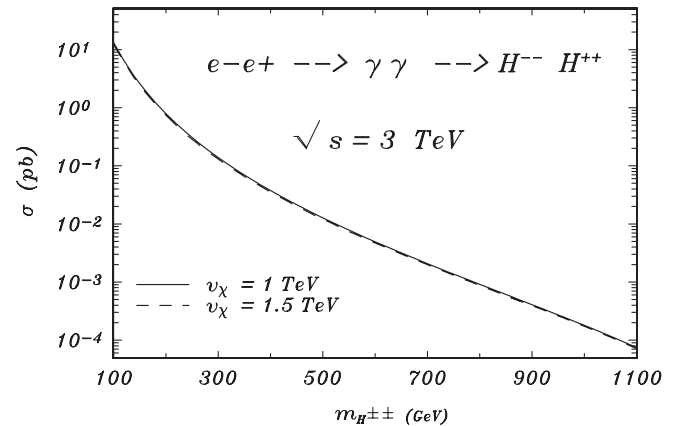


FIG. 8. Total cross section for the process $e^- e^+ \rightarrow e^- e^+ \gamma \gamma \rightarrow H^{\pm\pm} H^{\mp\pm}$ as a function of $m_{H^{\pm\pm}}$ for bremsstrahlung at $\sqrt{s} = 3$ TeV (a) $v_\chi = 1$ TeV (solid line) and (b) $v_\chi = 1.5$ TeV (dashed line).

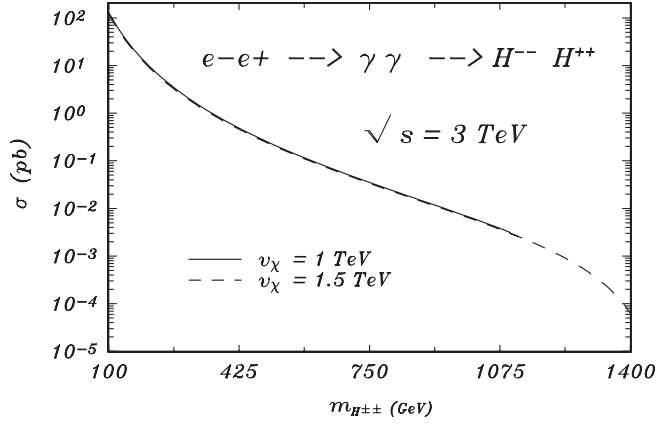


FIG. 9. Total cross section for the process $e^-e^+ \rightarrow e^-e^+\gamma\gamma \rightarrow H^{\pm\pm}H^{\mp\mp}$ as a function of $m_{H^{\pm\pm}}$ for backscattered photons at $\sqrt{s} = 3$ TeV (a) $v_\chi = 1$ TeV (solid line) and (b) $v_\chi = 1.5$ TeV (dashed line).

collider. We need to mention here that P_a^+ can decay also as $P_a^+ \rightarrow \ell^- \ell^+ P_b^+$, where the index a denotes the flavor. Therefore we will have eight charged leptons, see Ref. [19].

We still mention here that there exists an asymmetry between H^{--} and H^{++} in respect to decay rates we have not considered, such as $H^{\pm\pm} \rightarrow e^\pm P^\pm$, where the rate of $H^{--} \rightarrow e^- P^-$ is larger than the rate of $H^{++} \rightarrow e^+ P^+$

(Fig. 3), which indicates that in the 3-3-1 model the H^{--} generates matter in a large rate compared to the rate of the H^{++} , which generates antimatter. In this way symmetry does not exist between the two DCHBs in respect to decay rates. We can do an indirect measurement of this fact measuring the other rates such as $H^{\pm\pm} \rightarrow U^{\pm\pm} \gamma \rightarrow e^\pm P^\pm$, which was calculated here.

So the $\gamma\gamma$ collisions can be also a plentiful source of DCHBs [16,20]. In relation to the signal, $H^{\pm\pm} \rightarrow U^{\pm\pm} \gamma$ and $U^{\pm\pm} \rightarrow e^\pm P^\pm$, we conclude that it is a very striking and important signal; the doubly charged Higgs boson will deposit 6 times the ionization energy than the characteristic single-charged particle, that is, if we see this signal we will not only be seeing the doubly charged Higgs but also the doubly charged gauge bosons and heavy leptons.

It is noticeable here that there are no backgrounds to this signal. The discovery of a pair of DCHBs will be without any doubt of great importance for the physics beyond the SM because of the confirmation of the Higgs triplet representation and indirect verification that there is asymmetry in decay rates between matter and antimatter.

In summary, through this work we have shown that in the context of the 3-3-1 model the signatures for DCHBs can be significant for $\gamma\gamma$ collisions obtained by bremsstrahlung and backward Compton scattering of laser. Our study indicates the possibility of obtaining a clear signal of these new particles with a satisfactory number of events.

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