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MASSLESS PARTICLE CREATION IN A $F(R)$ ACCELERATING UNIVERSE

S. H. Pereira,[1] J. C. Z. Aguilar,[2] and E. C. Romão[3]

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RESUMEN

En este trabajo se presenta un mecanismo alternativo para generar partículas de materia oscura. Se discute el problema de la creación de partículas escalares de masa cero en los modelos de universo plano, homogéneo e isótropo, cuyo factor de escala sigue una teoría de tipo $f(R)$. Se encuentra que la principal contribución al número total de partículas y energía en general proviene de pequeños números de onda k, y partículas con valores grandes de k sólo se producen en el futuro. Si el mínimo k corresponde a la longitud de onda máxima dentro del horizonte de Hubble, se puede estimar la densidad actual de energía de las partículas sin masa como 10^{-120} veces la densidad actual crítica de energía. Estas partículas constituyen un campo similar a la radiación cósmica de fondo, lo que podría interpretarse como partículas de materia oscura. El estudio que aquí se presenta puede ser la base para investigaciones futuras relacionadas con la creación de la materia en el universo para varios modelos de evolución.

ABSTRACT

In this paper we present an alternative mechanism to generate dark matter particles. We discuss quantitatively the problem of massless particle creation in a flat, homogeneous and isotropic universe expanding with a scale factor which follows directly from a particular $f(R)$ theory of gravity. We find that the main contribution to the total number of particles and total energy comes from small wavenumbers k and particles with large values of k are produced only in the future. If we choose the minimal mode k as corresponding to the maximum wavelength inside the Hubble horizon, we can estimate the present energy density of such massless particles as 10^{-120} times the present critical energy density. Such particles form a background field similar to the cosmic microwave background radiation, which could be interpreted as dark matter particles. The study presented here might be the basis for future researches related to the creation of matter in the universe for various models of evolution.

Key Words: dark matter — early universe

1. INTRODUCTION

The phenomenon of particle creation in an expanding universe has been studied by several authors (Birrell & Davies 1982; Fulling 1989; Grib et al. 1994; Mukhanov & Winitzki 2007; Zeldovich 1970; Mamayev et al. 1976; Pavlov 2008; Grib & Pavlov 2005; Grib & Mamayev 1970; Grib et al. 1975; Barrow et al. 2008) after the pioneering work of Parker (Parker 1968, 1969, 1971, 1972, 1973). One of the most interesting results from Parker's work is that in a radiation dominated universe there is no creation of massless particles, either of zero or non-zero spin. In the description of particle production by the gravitational field, a widely used method is that of instantaneous Hamiltonian diagonalization (Pavlov 2008) suggested by Grib and Mamayev (Grib & Mamayev 1970; Grib et al. 1975). Of particular interest are the works relating the gravitational particle production at the end of the inflation as a possible mechanism to produce super-heavy particles of dark matter (Grib & Pavlov 2005; Chung et al. 2001).

All these searches occur in the framework of standard general relativity. Recently, non-standard gravity theories have been proposed as an alternative to explain the present accelerating stage of the universe only with cold dark matter, that is, with no appeal to the existence of dark energy. This is naturally obtained in $f(R)$ gravity theories

[1]Universidade Estadual Paulista Júlio de Mesquita Filho, Brazil.
[2]Federal University of São João del-Rei, Brazil.
[3]University of São Paulo, Brazil.

(for a review, see Sotiriou & Faraoni 2010). In such an approach, the curvature scalar R in the Einstein-Hilbert action is replaced by a general function $f(R)$, so that the Einstein field equation is recovered as a particular case (Vollick 2003; Amarzguioui et al. 2006; Allemandi et al. 2005, 2004; Sotiriou 2006; Meng & Wang 2004; Dolgov & Kawasaki 2003; Cembranos 2006; Gasperine & Veneziano 1992; Vilkovisky 1992). A review of various modified gravities considered as gravitational alternatives for dark energy, a unified description of the early-time inflation and the solutions of some related problems in $f(R)$ theories have been presented by Nojiri and Odintsov (2003, 2011). A recent work (Pereira et al. 2010) studied the quantum process of particle creation in a radiation dominated universe in the framework of a $f(R) = R + \beta R^n$ theory, which has the correct limit of the standard general relativity for $\beta = 0$. In that paper it was shown that both massive and massless scalar particles can be produced by purely expanding effects, contrary to Parker's results.

In the present work we investigate the particle production in a $f(R) = \beta R^n$ theory in a flat and matter dominated universe. We show that massless scalar particles can be produced as a possible alternative to create dark matter particles through the decay of these particles into particle-antiparticle pairs. Our quantitative analysis is done with the choice of $n = 4$, which leads to a scale factor of the type $t^{8/3}$. Such a scale factor leads to a deceleration parameter $q = -5/8 \simeq -0.62$, very closed to the best fit value for the ΛCDM model of cosmology, which provides $q_0 \simeq -0.58$ for a universe with 28% of dark matter and 72% of cosmological constant (Weinberg 2008). In this sense, our "toy model" can reproduce very well the current observations of an accelerated expansion of the universe without appealing to a dark energy component. Moreover it has the advantage of presenting an exact solution for the spectrum of created particles in terms of the real physical time t and with initial conditions at t_i. Thus, the treatment presented here can be the basis for future research related to the creation of matter in the universe for various models of evolution. It is important to note that most studies in the literature consider the creation process only in limiting cases, $t \to \infty$. Here we presents finite results concerning real and finite time t.

2. GENERAL THEORY OF SCALAR PARTICLE CREATION

The canonical quantisation of a real minimally coupled scalar field in curved backgrounds follows in straight analogy with the quantisation in a flat Minkowski background. The gravitational metric is treated as a classical external field which is generally non-homogeneous and non-stationary. The basic equation for the study of scalar particle creation in a spatially flat Friedmann-Robertson-Walker geometry is (Mukhanov & Winitzki 2007):

$$\chi_k''(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0\,, \tag{1}$$

with

$$\omega_k^2(\eta) \equiv k^2 + m_{\rm eff}^2 \qquad \text{and} \qquad m_{\rm eff}^2 \equiv m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)}\,, \tag{2}$$

where k is the Fourier mode or wavenumber of the particle, ω_k and $m_{\rm eff}$ represents the frequency and the effective mass of the particle, respectively. $a(\eta)$ is the cosmological scale factor in terms of the conformal time η and the prime denotes derivatives with respect to it. The conformal time η and the physical time t are related by

$$\eta \equiv \int \frac{dt}{a(t)}\,. \tag{3}$$

Following standard lines, the quantisation can be carried out by imposing equal-time commutation relations for the scalar field χ and its canonically conjugate momentum $\pi \equiv \chi'$, namely $[\chi(x,\eta)\,, \pi(y,\eta)] = i\delta(x-y)$, and by implementing secondary quantisation. After convenient Bogoliubov transformations, one obtains the transition amplitudes for the vacuum state and the associated spectrum of the produced particles in a non-stationary background (Grib et al. 1994; Mukhanov & Winitzki 2007).

Usually, the calculations of particle production deals with comparing the particle number at asymptotically early and late times, or with respect to the vacuum states defined in two different frames and does not involve any loop calculation. However, the main problem one encounters when treating quantisation in expanding backgrounds concerns the interpretation of the field theory in terms of particles. The absence of Poincaré group symmetry in curved space-time leads to the problem of the definition of particles and vacuum states. The problem may be solved by using the method of the diagonalization of the instantaneous Hamiltonian by a Bogoliubov transformation, which leads to finite results for the number of created particles.

To proceed, note that equation (1) is a second order differential equation with two independent solutions. Each solution χ_k must be normalised for all times according to

$$W_k(\eta) \equiv \chi_k(\eta){\chi^*}'_k(\eta) - \chi_k'(\eta)\chi_k^*(\eta) = -2i\,, \tag{4}$$

and they must also satisfy the initial conditions at the time η_i:

$$\chi_k(\eta_i) = 1/\sqrt{\omega_k(\eta_i)}\,, \qquad \chi'_k(\eta_i) = i\sqrt{\omega(\eta_i)}\,. \tag{5}$$

The Bogoliubov coefficients can be calculated and a straightforward calculation leads to the final expression for the total number of created particles and antiparticles in the k mode (Grib et al. 1994; Mukhanov & Winitzki 2007):

$$N_k(\eta) = \bar{N}_k(\eta) = \frac{1}{4\omega_k(\eta)}|\chi'_k(\eta)|^2 + \frac{\omega_k(\eta)}{4}|\chi_k(\eta)|^2 - \frac{1}{2}\,. \tag{6}$$

Another important quantity is the total energy per mode, $E_k(\eta)$. It is proportional to the product of the angular frequency ω_k and the total number of created particle in that mode,

$$E_k(\eta) = 2\omega_k(\eta)N_k(\eta), \tag{7}$$

where the factor 2 stands for the fact that the energies of particles and antiparticles are equal in this case.

Thus, the total number density of created particles n and the total energy density ε are readily obtained by integrating over all the modes (Grib et al. 1994; Mamayev et al. 1976)

$$n(\eta) = \frac{1}{2\pi^2 a(\eta)^3}\int_0^\infty k^2 N_k(\eta)dk\,, \tag{8}$$

$$\varepsilon(\eta) = \frac{1}{2\pi^2 a(\eta)^4}\int_0^\infty k^2 E_k(\eta)dk\,. \tag{9}$$

The expressions (6–9) can be rewritten in terms of the physical time t by using the relation (3).

3. PARTICLE CREATION IN A SIMPLE $F(R)$ THEORY

Now we will apply the above results to the study of massless scalar particle creation in a matter dominated and accelerating universe with a scale factor of the form $a(t) \propto t^{8/3}$, which follows readily from a simple $f(R)$ theory, discussed in detail by Allemandi et al. (2004), namely

$$f(R) = \beta R^n\,. \tag{10}$$

Taking into account the dominant energy condition, R^n must be positive definite for even integer values of n and β should be fixed as $\beta > 0$ for $n < 2$ and $\beta < 0$ for $n > 2$.

Now let us consider the case of a matter dominated universe ($\omega = 0$). Apart from integration constants, the modified Friedmann equations can be obtained as (Allemandi et al. 2004)

$$a(t) = \left(\frac{3\epsilon}{2n(3-n)}\right)^{n/3}\left[\frac{8\pi G\rho_0}{\beta(2-n)}\right]^{1/3}\left(\frac{t}{t_0}\right)^{2n/3}, \tag{11}$$

where $\epsilon = 1$ for odd values of n and $\epsilon \pm 1$ for even values. The dot denotes derivatives with respect to physical time t and ρ_0 is the present day value of the energy density. Note that the case $n = 3$ is singular. With this expression we can calculate the deceleration parameter,

$$q(t) \equiv -\left(\frac{\ddot{a}(t)a(t)}{\dot{a}(t)^2}\right) = \frac{3-2n}{2n}\,. \tag{12}$$

We see that accelerated solutions ($q < 0$) are obtained in the cases $n < 0$ or $n > 3/2$.

In order to illustrate the phenomenon of particle creation related to this theory we must to set particular values to the parameters n, β and ϵ. We choose $n = 4$, which corresponds to an accelerated $a(t)$ according to (12), $\beta < 0$ and $\epsilon = -1$. With such parameters, the scale factor is

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{8/3}, \qquad a_0 = \left(\frac{3}{8}\right)^{4/3}\left(\frac{4\pi G\rho_0}{-\beta}\right)^{1/3}, \tag{13}$$

and the deceleration parameter $q = -5/8 \simeq -0.62$. The Hubble parameter $H = \dot{a}/a = 8/3t$ determines the actual age of the universe as $t_0 = 8/3H_0 \simeq 18.3 \times 10^9$ years. It is interesting to note that it does not depends on the β parameter.

To follow up we need the scale factor $a(t)$ in terms of the conformal time η. This can be obtained by integrating equation (3) and rewriting $a(t)$ in terms of η,

$$a(\eta) = a_0 \left(\frac{3t_0}{5a_0}\right)^{8/5} \frac{1}{\eta^{8/5}} , \qquad 0 < \eta < \infty . \tag{14}$$

In terms of the conformal time, the early universe (the past) corresponds to $\eta \to \infty$, and the late universe (the future) corresponds to $\eta \to 0$.

Before we proceed with the calculation of the spectrum of non-massive particles created during the evolution of the universe, we must choose a value for the β parameter. We will choose β such that $a(t_0) \equiv a_0 = 1$, for $t_0 = 8/3H_0$, i.e., the scale factor is normalised to unity for the present time. To satisfy this condition we must have

$$\beta = -4\pi G \left(\frac{3}{8}\right)^{12} \rho_0 . \tag{15}$$

This gives $\beta \approx -6.6 \times 10^{-86}\text{GeV}^2$ if we set ρ_0 of the order of the critical density. Thus the scale factor is

$$a(\eta) = \left(\frac{8}{5H_0}\right)^{8/5} \frac{1}{\eta^{8/5}} . \tag{16}$$

It is important to note that we can recover all the expressions in terms of the physical time t by the substitution

$$\eta = \frac{3t_0}{5a_0} \left(\frac{t}{t_0}\right)^{-5/3} . \tag{17}$$

The equation (2) for the mode function in the case of a massless particle ($m = 0$) is

$$\chi_k''(\eta) + \left[k^2 - \frac{104}{25\eta^2}\right] \chi_k(\eta) = 0 . \tag{18}$$

In order to satisfy the initial conditions (5), the solution of this equation is given by

$$\chi_k(\eta) = A(k, \eta_i)\sqrt{\eta}\, Y_{\frac{21}{10}}(k\eta) - B(k, \eta_i)\sqrt{\eta}\, J_{\frac{21}{10}}(k\eta) , \tag{19}$$

where J_ν and Y_ν are the Bessel functions of the first and second kind, respectively, ν is its order, and A and B are complex constants depending on k and on the initial time η_i,

$$A(k, \eta_i) = \frac{8J_{\frac{31}{10}}(k\eta_i) - 13J_{\frac{11}{10}}(k\eta_i) + I\, 21J_{\frac{21}{10}}(k\eta_i)}{21\sqrt{k\eta_i}[J_{\frac{21}{10}}(k\eta_i)Y_{\frac{11}{10}}(k\eta_i) - Y_{\frac{21}{10}}(k\eta_i)J_{\frac{11}{10}}(k\eta_i)]} ,$$

$$B(k, \eta_i) = \frac{8Y_{\frac{31}{10}}(k\eta_i) - 13Y_{\frac{11}{10}}(k\eta_i) + I\, 21Y_{\frac{21}{10}}(k\eta_i)}{21\sqrt{k\eta_i}[J_{\frac{21}{10}}(k\eta_i)Y_{\frac{11}{10}}(k\eta_i) - Y_{\frac{21}{10}}(k\eta_i)J_{\frac{11}{10}}(k\eta_i)]} , \tag{20}$$

where $I \equiv \sqrt{-1}$.

By fixing η_i and taking some particular values of k and η it is easy to check that this solution correctly satisfies the normalisation condition (4).

The solution presented in equation (19) is exact and, at first, it may be used to calculate the total number density of particles equation (8) and the total energy density (equation 9) for any instant of time η after the initial time η_i. To recover the expressions as a function of physical time t we use equation (3); all quantities can be described in terms of t. In fact, the final analytical expression for N_k is quite complicated, but can be obtained in a relatively simple manner with the help of an algebraic manipulation software. Thus, the exact behavior and several interesting features of the functions (6–9) can be obtained by graphical analysis of the spectrum of created particles. This is what we will do in the next section.

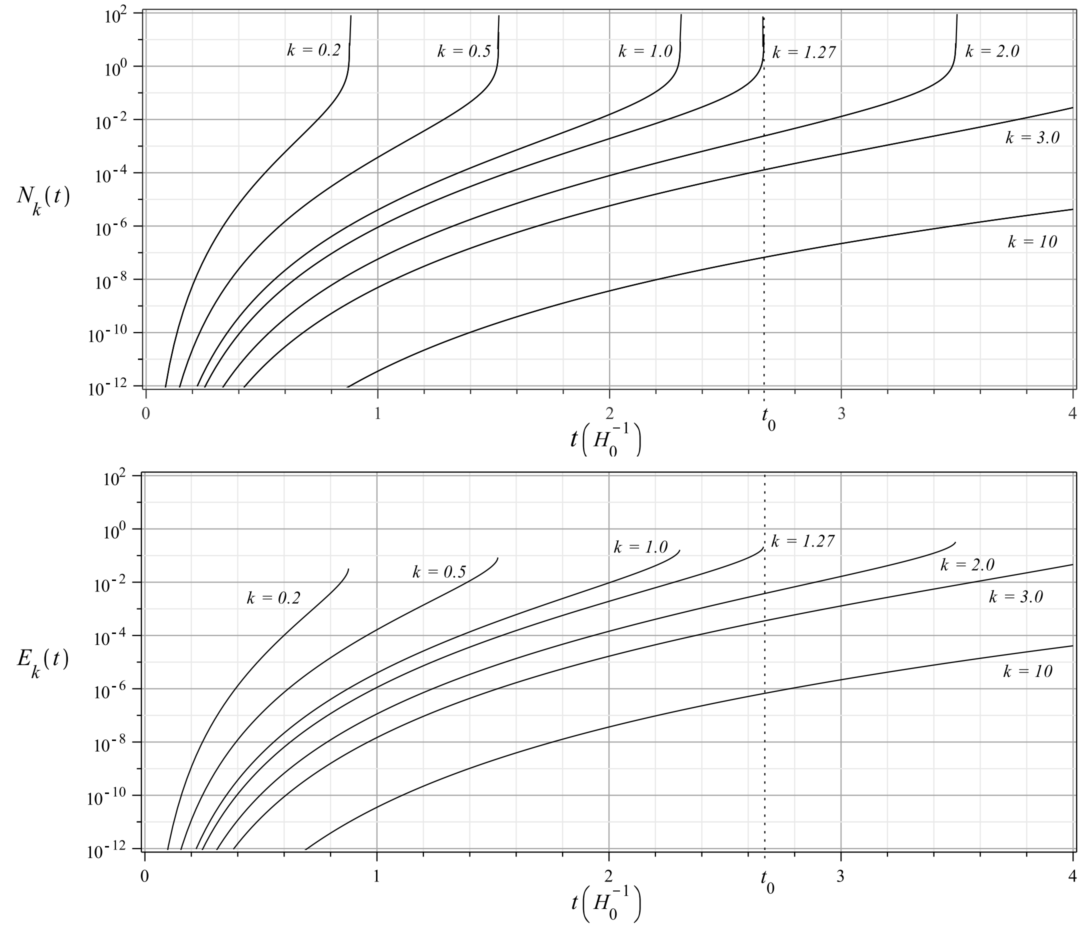

Fig. 1. Spectrum of the total number (top) and total energy (bottom) of created particles as function of the physical time t in units of H_0^{-1} for different values of the mode k. The present time is indicated by the vertical dashed line at t_0.

4. SPECTRUM OF CREATED PARTICLES

Now let us study the evolution in time of the total particle number N_k and the total energy per mode E_k, and try to estimate n and ε for such particles.

First we need to establish an appropriate time scale in order to perform the analytical study. As we have seen, the present time corresponds to $t_0 = 8/3H_0 = 5.8 \times 10^{17}$s, so that an appropriate time scale is in terms of H_0^{-1}. Using equation (17) we can estimate this time as $\eta_0 = 1.6$. Now we will establish the value of the initial time η_i. As we are dealing with a universe dominated by matter and we know (Kolb & Turner 1990) that the matter and radiation decouple at about $t_i = 10^{13}$ s $\approx 4.5 \times 10^{-5} H_0^{-1}$, by using equation (17) we can estimate $\eta_i = 1.4 \times 10^8$ (remember that, in the conformal time, the absolute past corresponds to $\eta \to \infty$). Thus we are interested in the interval $1.6 < \eta < 1.4 \times 10^8$, which corresponds to $4.5 \times 10^{-5} H_0^{-1} < t < (8/3) H_0^{-1}$.

Now we can insert equation (19) into equation (6) to obtain the spectrum of the total number of particles created for each mode k. With this we use equation (17) to write the result as a function of physical time t. The spectrum of the total number of created particles N_k for some values of the k mode is represented in Figure 1. By analysing the figure, we can see that for every mode k there is a critical value of the time t for which the number N_k of created particles grows abruptly and diverges. After that, the creation process stops for the corresponding mode. Mathematically this occurs because in the expression (6) the frequency ω_k appears in the denominator. Let us express ω_k in terms of the physical time:

$$\omega_k(t) = \sqrt{k^2 - 3^{1/3} \frac{351}{8192} t^{10/3}}\,. \tag{21}$$

We can see that when $k^2 = 3^{1/3}(351/8192)t^{10/3}$ we have $\omega_k = 0$ and the first term of equation (6) diverges. Physically, the significance of this divergence is that, for a given mode k, the values of $t > (8192/351)^{3/10} 3^{-1/10} k^{3/5}$

imply a negative frequency squared, $\omega_k^2 < 0$, and consequently the state of minimum energy and the quantum vacuum are not well-defined and the creation process ceases for these modes. Considering the present time in units of H_0^{-1}, the limit of k is $k_0 \simeq 1.2747549$. This indicates that up to the present time particles with modes $k < k_0$ do not exist. We will see later that such modes correspond to a wavelength greater than the Hubble horizon. The modes with $k > k_0$ started to be created in the past and the divergence will occur only in the future.

Note that an infinite number of particles per mode can be created until the process stops. In Figure 1, for instance, this is represented by the modes $k = 0.5$ and $k = 1.0$, where the divergence occurs in the past. For the mode $k = 1.27$ the divergence is occurring today. For the modes $k = 2.0$, $k = 3.0$ and $k = 10$ a significant amount of particles will be created only in the future. We see that for large values of k, the divergence will occur in the future, meaning that particles of higher energies can be created only in the future.

Is the total number of created particles per mode really divergent? Mathematically the expression (6) tells us that this happens due to the factor ω_k in the denominator. But perhaps there is some mechanism capable of interrupting the process of creation before it goes to infinity, so that the total number of created particles per mode is finite. We believe that such a hypothesis is true based on the spectrum analysis of the total energy of created particles per mode. The expression (7) contains the factor ω_k multiplying the total number of particles N_k, just cancelling the factor ω_k in the denominator of the first term of N_k. So we can conclude that the total energy of created particles per mode k must be finite. In fact, the graphical analysis shows that. In Figure 1 we also present the energy spectrum of created particles per mode as a function of the physical time t. We clearly see that the energy grows up to a maximum value for each mode k but it does not diverges. For larger values of the time, the frequency becomes negative and the creation process stops, as discussed earlier.

The divergence of the total number of created particle and a possible mechanism to stop such a process can also be analysed from another perspective. The mathematical reason for the divergence is the vanishing of the ω_k for a given mode k. From the definition of ω_k, equation (2), we see that, for a massless particle, ω_k vanishes due to the term $k^2 - a''/a$. Thus we see that if we have a non-vanishing mass this problem does not occur, and than we can interpret this divergence as a consequence of the zero mass limit. But from a quantum mechanical point of view, due to quantum fluctuations on small scales, we can think that before massless particles were created (which correspond just to relativistic particles with momentum k) the quantum process created a very light massive particle at rest (stopping the creation process for that mode before the divergence occurs) which then decayed into massless particles conserving the energy. As we have seen, the total energy per mode is finite. Such a mechanism could be an analog to a cut-off mechanism. We can also propose a second mechanism in order to explain this result. If we suppose that N^* light massive particles with mass m^* are created at rest due to the quantum fluctuations in a very short time, the total energy of the system is $E = N^* m^* c^2$. When the particles decay to massless particles, the limit $m^* \to 0$ must be accompanied by the limit $N^* \to \infty$ in order to keep the total energy finite, and that is what we see from our analysis.

Another interesting analysis based on the analytical results is the study of the behaviour of N_k and E_k as a function of k for some fixed time. This is shown in Figure 2 for three different instants of time, namely $t = t_0/2$ (past), $t = t_0$ (present) and $t = 2t_0$ (future). For N_k, as expected, the total number of particles per mode appears to diverge for a given value of the mode k, but decreases with increasing k. The same goes for E_k, although the total energy per mode is always finite, as expected. The decrease of these values with the increase of the mode k is an interesting feature, indicating that higher modes contribute much less to the total number of particles than to the total energy.

We have seen that the total number of particles is infinite, and the same will happen with the total number density of particles (equation 8), unless some mechanism stops the creation process before the divergence occurs. We believe that this hypothesis is true, since the total energy per mode is finite. Let us estimate the total energy density (equation 9) for the present time t_0. Since we do not have a mass scale, we need an appropriate energy scale for the modes in order to do the integration.

From the above analysis we conclude that for the present time the value of the lowest mode is $k_0 \simeq 1.2747549$. For lower values of k the frequency becomes negative and there is no creation of these modes. We have also seen that the contribution to the total energy decreases with increasing k, so the integration in equation (9) can be truncated at some value of k in the case of a numerical integration. First remember that the mode k is related to physical wavenumber $k_{\rm ph}$ by $k = a(t)k_{\rm ph}$. The physical mode $k_{\rm ph}$ is related to the physical wavelength $\lambda_{\rm ph}$ by $k_{\rm ph} = 2\pi/\lambda_{\rm ph}$. Therefore a minimum value of k corresponds to a maximum value of the physical wavelength $\lambda_{\rm ph}$,

$$k^{\rm min} = a(t)\frac{2\pi}{\lambda_{\rm ph}^{\rm max}} \,. \tag{22}$$

We choose the maximum value of λ as the size of the Hubble horizon today, $\lambda_{\rm ph}^{\rm max} = ct_0 = c8/3H_0 \simeq 3.52 \times 10^{28}$cm.

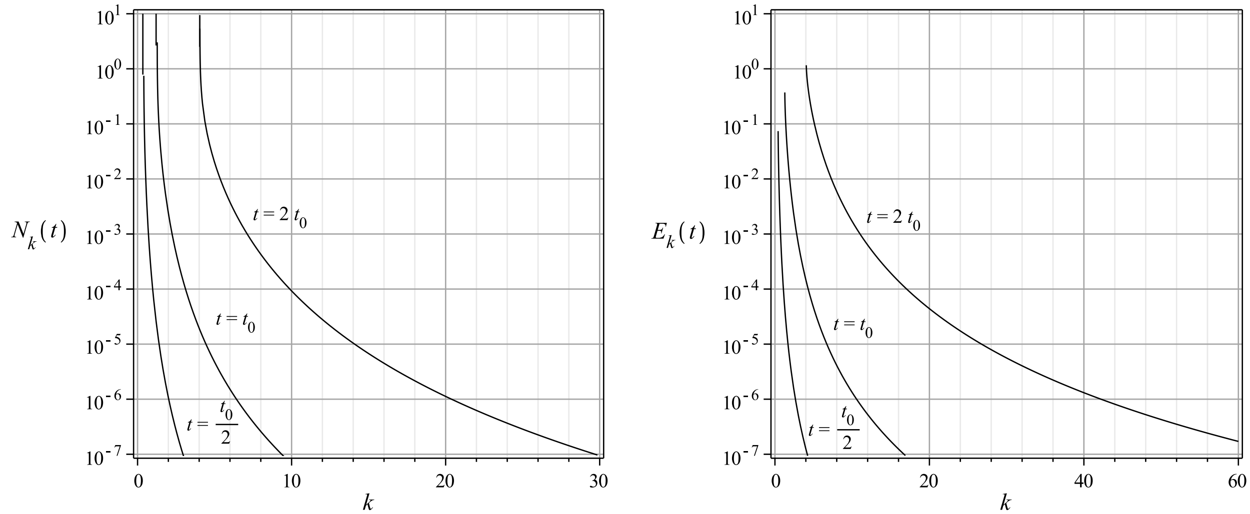

Fig. 2. Spectrum of the total number (left) and the total energy (right) of created particles as function of the mode k for different values of the time (t_0 represents the present time).

With this we are requiring that only the modes with wavelengths shorter than the actual size of the universe contribute to the total energy, which seems plausible, since we are restricting ourselves to the modes inside the Hubble horizon. Modes with longer wavelengths have $k < k_0$ and are not allowed. Thus we have an appropriate scale of energy to estimate the total energy density of the particles being created at present. The minimum value of the physical wavenumber is $k_{\rm ph}^{\rm min} = 2\pi/ct_0 \simeq 3.5 \times 10^{-42}$GeV. With this, we make a change of variables in (9) such that k_0 corresponds to $k^{\rm min}$ and the energy density for the present time can be calculated numerically as

$$\varepsilon_0 = \frac{1}{2\pi^2 a_0^4}\int_{k^{\rm min}}^{\infty} k^2 E_k(t_0)dk \;\approx\; 10^{-168}\ \mathrm{GeV}^4\,. \tag{23}$$

This result is very small compared to the value of the critical energy density, $\rho_0 = 4.0 \times 10^{-47}$GeV^4.

To finish, let us apply the method of a space-time correlation function suggested in Mamayev & Trunov (1983) in order to distinguish if such non-massive created particles are real or virtual ones. This method was generalised by Pavlov (2008) for a scalar field with non-conformal coupling. It is easy to check that for the present time the metric (14) is changing adiabatically for modes $k \gg k_0$,

$$\left|\frac{\omega_k^{'}}{\omega_k^2}\right| \ll 1\,, \tag{24}$$

and furthermore we have $a'(\eta_i) \to 0$ for $\eta_i \to \infty$, so that the analysis of Pavlov (2008) is valid here, and we conclude that such pairs of created particles should be interpreted as virtual particles. As they are virtual particles, their total energy does not contribute to the total energy of the universe.

5. CONCLUDING REMARKS

We have addressed the problem of minimally coupled massless particle creation in a $f(R) = \beta R^4$ theory for a matter dominated universe. We have verified that this particular type of $f(R)$ correctly reproduces a universe that evolves in accelerated manner ($q \simeq -0.62$). In addition we have shown that an infinite number of non-massive virtual particles with mode $k < k_0$ has been created in this particular model, although we believe that some mechanism stops the creation process before the number of total particles goes to infinity, as discussed in §3. Such an idea is based on the fact that the total energy per mode is finite, so we think that the total number of particles is also finite. Making an appropriate choice of the energy scale for the minimum mode k as that corresponding to the possible wavelengths within the Hubble horizon, the estimate for the total energy density of the virtual particles pairs being created today shows that the value is about 10^{-120} times lower than the critical energy density. As they are non-massive virtual particles, their total energy does not contribute to the total energy of the universe, but in the framework of the quantum mechanics, these particles could decay into massive real particle-antiparticle pairs, as occurs in pair production of electron-positron in the decay of an energetic photon. This opens the possibility that these virtual particles being created today could give rise to the dark matter in the universe. In order to be a source of dark matter particles, the mechanism of decay of these particles into massive particle-antiparticle pairs needs to be studied more carefully.

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J. C. Z. Aguilar, Department of Mathematics and Statistics, Federal University of São João del-Rei 36307-352, São João del-Rei, MG, Brazil.
S. H. Pereira, Faculdade de Engenharia de Guaratinguetá, UNESP, Univ. Estadual Paulista Júlio de Mesquita Filho, Departamento de Física e Química, Av. Dr. Ariberto Pereira da Cunha 333, Pedregulho 12516-410, Guaratinguetá, SP, Brazil (shpereira@gmail.com).
E. C. Romão, Department of Basic and Environmental Sciences, Lorena School of Engineering, University of São Paulo 12602-810, Lorena, SP, Brazil.