UNIVERSIDADE ESTADUAL PAULISTA
“JÚLIO DE MESQUITA FILHO”
Campus de São José do Rio Preto

Gislaine Mara Melega

Problema Integrado de Dimensionamento de Lotes e Corte de Estoque: Modelagem Matemática e Métodos de Solução

Tese de Doutorado

São José do Rio Preto

2017

Gislaine Mara Melega

Problema Integrado de Dimensionamento de Lotes e Corte de Estoque: Modelagem Matemática e Métodos de Solução

Tese apresentada como parte dos requisitos para obtenção do título de Doutor em Matemática, junto ao Programa de Pós-Graduação em Matemática, do Instituto de Biociências, Letras e Ciências Exatas da Universidade Estadual Paulista "Júlio de Mesquita Filho", Campus de São José do Rio Preto.

Financiadora: FAPESP-Proc. 2012/20631-2

Orientador: Prof. Dr. Silvio Alexandre de Araujo

São José do Rio Preto

2017

Melega, Gislaine Mara.
Problema Integrado de Dimensionamento de Lotes e Corte de Estoque: Modelagem Matemática e Métodos de Solução / Gislaine Mara Melega. - São José do Rio Preto, 2017.
150 f. : il. ; 30 cm.

Orientador: Silvio Alexandre de Araujo.
Tese (doutorado) - Universidade Estadual Paulista "Júlio de Mesquita Filho", Instituto de Biociências, Letras e Ciências Exatas.

1. Matemática. 2. Pesquisa operacional. 3. Otimização Matemática. 4. Problema de dimensionamento de lotes. 5. Problema de corte de estoque. 6. Modelos matemáticos. 7. Programação heurística I. de Araujo, Silvio Alexandre II. Universidade Estadual Paulista "Júlio de Mesquita Filho", Instituto de Biociências, Letras e Ciências Exatas III. Título.

CDU - 518.73

Gislaine Mara Melega

Problema Integrado de Dimensionamento de Lotes e Corte de Estoque: Modelagem Matemática e Métodos de Solução

Tese apresentada como parte dos requisitos para obtenção do título de Doutor em Matemática, junto ao Programa de Pós-Graduação em Matemática, do Instituto de Biociências, Letras e Ciências Exatas da Universidade Estadual Paulista "Júlio de Mesquita Filho", Campus de São José do Rio Preto.

Financiadora: FAPESP-Proc. 2012/20631-2

Banca Examinadora

Prof. Dr. Silvio Alexandre de Araujo
UNESP - São José do Rio Preto
Orientador

Profa. Dra. Maria do Socorro Nogueira Rangel
UNESP - São José do Rio Preto

Profa. Dra. Kelly Cristina Poldi
UNICAMP - Campinas

Profa. Dra. Sonia Cristina Poltroniere Silva
UNESP - Bauru

Prof. Dra. Deisemara Ferreira
UFSCar - Sorocaba

São José do Rio Preto,
21 de Março de 2017.

Aos meus amados pais, Paulo e Elizabeth,
aos meus avós, Francisco e Maria,
ao meu grande parceiro Leandro e amigos,
em especial Seu Oto *(in memoriam)*
Dedico.

Agradecimentos

E assim se encerra mais uma etapa da minha vida e com ela ficam os ensinamentos e agradecimentos desta grande conquista. Alguém um dia me disse que "a gratidão é a lembrança do coração", faz sentido. Ao longo de nossas vidas, sempre aparecem pessoas, as quais eu intitulo "anjos da guarda", que nos ajudam e sem as quais nossos objetivos seriam muito difíceis, ou até impossíveis de serem alcançados. Por isso eu quero aqui expressar de coração, o meu profundo agradecimento a todos. Em especial agradeço:

A Deus, por todas as oportunidades oferecidas, pela sabedoria para superar os momentos difíceis, que não foram poucos, e pelas inúmeras felicidades que vivi durante a realização deste trabalho, nos quais tive o prazer de conhecer pessoas e lugares incríveis e que estarão em minha memória para sempre.

Ao meu anjo da guarda por proteção e amparo.

Aos meus amados pais, Paulo e Elizabeth, meus avós Maria e Francisco, a minha Tia Lia e toda minha família pelo apoio, compreensão e carinho em todos os momentos difíceis e de alegrias que estiveram junto comigo. Obrigada por me incentivarem a chegar onde estou, grande parte disso devo a vocês, com vocês tudo é mais ameno e feliz. Mesmo as vezes sem saber ao certo o que estava acontecendo, vocês simplesmente me apoiaram e agradeço a Deus por ter vocês em minha vida.

Ao meu amado companheiro e amigo Leandro. Obrigada pela compreensão, pelo apoio, por me fazer acreditar que posso mais que imagino e por compreender todos os meus momentos difíceis. Com o seu companheirismo, amizade, paciência, compreensão, apoio, alegria e amor, este trabalho pode ser concretizado. Obrigada por ter feito do meu sonho o nosso sonho, te amo ainda mais.

Em especial ao meu orientador Silvio, por todos os ensinamentos, dedicação, amizade e, é claro, pela paciência durante todos estes anos que trabalhamos juntos, desde a Iniciação Científica. Muito obrigado por todas as oportunidades oferecidas, pelos puxões de orelha e pela confiança à mim depositada desde sempre.

Ao Prof. Raf Jans, pela valiosa contribuição para a realização deste trabalho. Agradeço pela sua receptividade, pela paciência e por todos os ensinamentos durante o período que estive em Montréal.

As minhas grandes amigas e irmãs Bruna, Jaquelini e Michelli que a vida me possibilitou conhecer, obrigada por me aguentarem com minhas dúvidas, frustrações e alegrias com vocês compartilhadas, amo vocês demais. Obrigada por me escutarem e me ajudarem mesmo estando longe, com nossas longas conversas, me dando apoio e incentivo sempre, obrigada.

Ao pessoal da salinha Willian (Paraguai), Eliel, Ismael (Pena), Moisés, Junior, Daniel, Cheienne, Fabiola, Gino, Jairo, Paola, Guto, Marcelo e em especial Michelli, Mari, Luana, Yen, Daniella e Diego, com quem passei grande parte de todos estes anos. Obrigada pelos bons momentos que passamos, pelas conversas, dando risada, tomando cafés e desabafando, foi muito bom conviver e conhecer vocês.

As minhas *roommates* de Montréal, Mrunalini e Teresita, obrigada por me aguentarem nos momentos de loucura, pelas conversas, risadas e jantas que tivemos juntas. A Lini em especial por ter sido minha família em Montrél desde o primeiro dia. Obrigada por ter me acalmado no dia que cheguei e ter me deixado fazer parte da sua vida quando ainda não conhecia ninguém. Obrigada pelas nossas conversas na cozinha e pelas noites assistindo séries, sem você Montréal não teria sido a mesma.

A todas as pessoas que conheci em Montréal, em especial, a Aurélien, Selene, Andrea, Vinínius, Serena, Gianluca, Giusy, Gia Como, Khalid, Ahmad, Luciano e Giulia e, todos os outros amigos que conheci no CIRRELT. Obrigada por terem sido minha família no Canadá e fazer com que esta experiência fosse uma das melhores da minha vida. Aos meus amigos dos cafés Serena, Gianluca, Giusy, Gia Como, Luciano e Giulia, obrigada pela companhia, conversas e risadas.

Aos professores de pós-graduação, que embora tenha passado por várias disciplinas duras, que muitas vezes tiraram meu sono, me trouxeram valiosos ensinamentos e me ajudaram no meu crescimento intelectual e profissional. Obrigada.

A todos os colegas, pessoas e funcionários do IBILCE que, direta ou indiretamente, contribuíram para a elaboração deste trabalho. Em especial, aos funcionários do DMAP e DCCE, Getúlio, Tiago pela ajuda e a Olga pelos almoços e cafés.

E por fim, agradeço à FAPESP pelo apoio financeiro (Processos $n^{\underline{o}}$ 2012/20631-2 e 2014/17273-2).

"Se fui capaz de ver mais longe,
é porque me apoiei em ombros de gigantes."
Isaac Newton

Resumo

Nesta tese, estamos interessados em tratar de maneira integrada dois conhecidos problemas da literatura. Esta integração é referida na literatura como problema integrado de dimensionamento de lotes e corte de estoque. A ideia consiste em considerar simultaneamente, as decisões relacionadas com ambos os problemas, de modo a capturar a interdependência entre estas decisões e, assim, obter uma melhor solução global. Propõe-se um modelo matemático geral para o problema integrado de dimensionamento de lotes e corte de estoque ($GILSCS$), que considera vários níveis de integração e nos permite classificar a literatura, em termos de modelos matemáticos, dos problemas integrados. A classificação é organizada a partir de dois principais aspectos de integração que são: a integração através dos períodos de tempo e a integração entre os níveis de produção . Em um horizonte de planejamento que considera vários períodos, o estoque fornece uma ligação entre os períodos. Esta integração, por períodos de tempo, constitui o primeiro tipo de integração. O problema geral também considera a produção em diferentes níveis: objetos são fabricados ou comprados e então são cortados para produzir peças menores e estas, por sua vez, constituem componentes para a produção dos produtos finais. A integração entre os diferentes níveis de produção consiste no segundo tipo de integração. A revisão da literatura também possibilita direcionar interessantes áreas para pesquisas futuras. O comportamento da solução para este tipo de problema, com três níveis e vários períodos, é estudado a partir do desenvolvimento de métodos de solução considerando abordagens que superam as dificuldades do problema, que consistem no alto número de padrões de corte, estruturas em vários níveis (multiestágios) e variáveis binárias de preparo. Os métodos de solução propostos para o problema $GILSCS$ são baseados em duas abordagens conhecidas da literatura, usadas com sucesso para resolver os problemas separadamente, que são o procedimento de geração de colunas e heurísticas de decomposição do tipo *relax-and-fix*. Estas estratégias e suas variações são combinadas à um pacote de otimização em um estudo computacional com dados gerados aleatoriamente. Uma revisão da literatura, em termos de métodos de solução, para o problema integrado também é apresentada. Outras contribuições desta tese consistem em propor diferentes modelos matemáticos para o problema integrado, combinando modelos alternativos para cada um dos problemas separadamente. Neste estudo, o objetivo é comparar e avaliar, com um extensivo estudo computacional, a qualidade e o impacto das diferentes formulações. O outro trabalho trata de uma aplicação do problema integrado em um indústria de móveis de pequeno porte, em que restrições específicas do ambiente industrial são abordadas, como estoque de segurança e ciclos da serra. A solução obtida pelo modelo proposto é comparada com uma simulação da prática da empresa.

Palavras Chave: Problema Integrado de Dimensionamento de Lotes e Corte de Estoque. Revisão e Classificação da Literatura. Geração de Colunas. *Relax-and-Fix*. Heurística Híbrida. Restrição de Ciclos da Serra. Indústria de Móveis.

Abstract

In this thesis, the subject of interest is in treating, in an integrated way, two well-known problems in the literature. This integration is referred in the literature as the integrated lot-sizing and cutting stock problem. The basic idea is to consider, simultaneously, the decisions related to both problems so as to capture the interdependency between these decisions in order to obtain a better global solution. We propose a mathematical model for a general integrated lot-sizing and cutting stock (*GILSCS*) problem. This model considers multiple dimensions of integration and enables us to classify the current literature, in terms of mathematical models, in this field. The main classification of the literature is organized around two types of integration. In a planning horizon which consists of multiple periods, the inventory provides a link between the periods. This integration across time periods constitutes the first type of integration. The general problem also considers the production in different levels: objects are fabricated or purchased and then, they are cut to produce the pieces which are then assembled as components in the production of final products. The integration between these production levels constitutes the second type of integration. The literature review also enables us to point out interesting areas for future research. The behavior of a solution to this type of problem, with three levels of production and several time periods, is studied considering the development of solution approaches that overcome the difficulties of the problem, which are the high number of cutting patterns, multi-level structures and the binary values of the setup variables. The solution methods proposed to the *GILSCS* problem are based on two known strategies from the literature which are used successfully to solve the problems separately, which are the column generation procedure and decomposition heuristics based on relax-and-fix procedure. These strategies and their variations are combined into an optimization package in a computational study with randomly generated data. A literature review, in terms of solution methods, to the integrated problem, is also presented. Other contributions of this thesis consist of proposing different mathematical models for the integrated problem combining alternative models for each one of the problems separately. In this study, the aim is to compare and evaluate, with an extensive computational study, the quality and the impact of these different formulations. Another study is an application of the integrated problem in a small furniture factory, in which specific constraints related to the industrial environment are addressed, such as, safety stock level constraints and saw cycles constraints. The solution obtained from the proposed model is compared to a simulation of the common practice in the company.

Keywords: Integrated Lot-sizing and Cutting Stock Problems. Review and Classification of the Literature. Column Generation. Relax-and-Fix. Hybrid Heuristic. Saw Cycles Constraints. Furniture Factory.

UNIVERSIDADE ESTADUAL PAULISTA
"JÚLIO DE MESQUITA FILHO"
Campus de São José do Rio Preto

Gislaine Mara Melega

A General Integrated Lot-Sizing and Cutting Stock Problem: Mathematical Modelling and Solution Methods

PhD Thesis

São José do Rio Preto
2017

Contents

List of Tables

List of Figures

List of Symbols

Sets:

$T = \{1, \ldots, s\}$:	set of time periods (index t);
$I = \{1, \ldots, q\}$:	set of items (index i);
$O = \{1, \ldots, l\}$:	set of different types of objects (index o);
$P = \{l+1, \ldots, l+m\}$:	set of pieces (index p);
$F = \{l+m+1, \ldots, l+m+n\}$:	set of final products (index f);
$J = \{1, \ldots, N\}$:	set of all possible cutting patterns (index j);
$J_o = \{1, \ldots, N_o\}$:	set of cutting patterns for object type o (index j);

Parameters:

sc_t^i:	setup cost of item i in period t;
vc_t^i:	unit production cost of item i in period t;
hc_t^i:	unit holding cost of item i in period t;
st_t^i:	setup time of item i in period t;
vt_t^i:	unit production time of item i in period t;
d_t^i:	demand of item i in period t;
$sd_{t\tau}^i$:	sum of demands of item i from period t to period τ;
Cap_t:	capacity (time limit) available to produce the items in period t;
$S(i)$:	set of direct successors of item i (index l);
r_l^i:	number of items i used in one unit of item l;
$cv_{it\tau}$:	total production and inventory holding cost for producing item i in period t at a quantity that meets the demands for the periods from t until τ;
UB^o:	upper bound on the number of objects needed;
L:	object length;
W:	object width;
l_p:	length of piece p;
w_p:	width of piece p;
d^p:	demand of piece p;

$G = (V, A)$:	directed acyclic graph;
$V =$	$\{0, 1, \ldots, L\}$;
$A =$	$\{(g, h); 0 \leq g < h < L$ and $h - g = l_p$ for all $p \in P\}$
a_j^p:	number of pieces p cut in the cutting pattern j;
M:	large number;
L^o:	object length of type o;
sc_t^o:	setup cost/fixed ordered cost for object type o in period t;
vc_t^o:	unit production cost/purchase cost of object type o in period t;
hc_t^o:	unit holding cost of object type o in period t;
d_t^o:	independent demand of object type o in period t;
st_t^o:	setup time for object type o in period t;
vt_t^o:	unit production time of object type o in period t;
$CapO_t$:	production capacity (in time units) available to produce the objects in period t;
u_j^o:	setup cost for cutting pattern j for object type o;
c_j^o:	cost of cutting an object type o according to cutting pattern j;
hc_t^p:	unit holding cost of piece p in period t;
d_t^p:	independent demand of piece p in period t;
r_f^p:	number of pieces of type p required in the final product f;
a_{oj}^p:	number of pieces p cut from object type o using the cutting pattern j;
st_{jt}^o:	setup time of the object type o cut according to cutting pattern j in period t;
vt_{jt}^o:	production time to cut object type o according to cutting pattern j in period t;
$CapP_t$:	cutting capacity (in time units) available in period t;
sc_t^f:	setup cost of final product f in period t;
vc_t^f:	unit production cost of final product f in period t;
hc_t^f:	unit holding cost of final product f in period t;
st_t^f:	setup time of final product f in period t;
vt_t^f:	unit production time of final product f in period t;
d_t^f:	demand of final product f in period t;
$CapF_t$:	production capacity (in time units) available to produce the final products in period t;

Decision Variables:

X_t^i:	production quantity (in units) of item i in period t;
S_t^i:	inventory (in units) for item i at the end of period t;
Y_t^i:	binary variable indicating the production or not of item i in period t;
$zv_{it\tau}$:	fraction of the production plan for item i to meet demand from period t to period τ;
y_o:	binary variable that indicates whether object o is used or not;
h_o^p:	number of units of piece p cut from object o;
f:	flow through the network;
z_{gh}:	number of cutting patterns which have a piece of size $(h-g)$ allocated at a distance g from the beginning of the object;
Z_j:	number of objects cut according to cutting pattern j;
W_j:	binary variable indicating if cutting pattern j is used or not;
X_t^o:	production/purchase quantity (in units) of object o in period t;
S_t^o:	inventory (in units) of object o at the end of period t;
Y_t^o:	binary variable indicating the production/purchase or not of object o in period t;
X_t^p:	production quantity (in units) of piece p in period t;
S_t^p:	inventory (in units) of piece p at the end of period t;
Z_{jt}^o:	number of objects of type o cut according to cutting pattern j in period t;
W_{jt}^o:	binary variable indicating the setup or not of cutting pattern j for object type o in period t;
X_t^f:	production quantity (in units) of final product f in period t;
S_t^f:	inventory (in units) of final product f at the end of period t;
Y_t^f:	binary variable indicating the setup or not of final product f in period t.

Chapter 1

Introduction

Continuously, the representation of reality is a necessity of modern society, either because of the inability to deal directly with this, or by economic aspects, or by its complexity. Therefore, the industries face the challenge of improving their competitiveness searching for the representation of reality through models which are well structured and as representative as possible, in order to produce more, with better quality and lower costs. This competitiveness increasingly faster has expanded demand for new tools to support decision-making, consequently induces academic research of optimization models related to planning and controlling of production systems. Among the various decision processes, this research is part of the tactical/operational planning of production related to two problems known as lot-sizing problem and cutting stock problem.

The lot-sizing problem and the cutting stock problem have been the object of extensive research for more than 50 years. Much progress has been made with respect to formulations and solution methods for these two problems. Most of the research has been focused on solving these problems separately, since each problem is itself difficult to solve. However, with the fast progress in optimization theory, software, hardware and, a better understanding of the individual problems, as well as the dependencies among decisions observed in practical cases, more attention has been paid to the integration of these two problems in recent years. This integration is the subject of interest in this thesis and it is referred to in the literature as the integrated/combined lot-sizing and cutting stock problem. In this thesis, the problem is referred to as integrated problem.

The interest in these problems often originates from direct practical applications of the integrated environments in various industries. For example, in the paper industry, large coils are manufactured to then be cut into smaller coils (reel) that correspond to customer's requests. In the furniture industry, wooden plates are cut into several wooden parts to be assembled into final products. In the fiber glass industry, fiber glass plates are cut to manufacture printed circuit boards and in the aluminum industry, aluminum profiles are cut to make several window types.

The Lot-Sizing Problem (*LSP*) considers the tradeoff between the setup and inventory holding costs to determine the minimal cost of a production plan for one (or several) machine(s) in order to meet the demand for each item. The *LSP* can be classified according to several characteristics, such as, the number of levels in the production structure (single level or multi-level), demand (constant or dynamic), time horizon (finite or infinite) and the consideration of capacity constraints and setup times. In the literature, many papers address the lot-zing problem in different environments, among which we highlight the review papers of Karimi et al. (2003), Brahimi et al. (2006), Buschkühl et al. (2008), Jans and Degraeve (2008) and Robinson et al. (2009). There is a EURO Working Group on Lot-Sizing (*EWG LOT*) (https://www.euro-online.org/web/ewg/37/ewg-on-lot-sizing-ewg-lot). A thorough discussion of lot-sizing problems is provided in the book by Pochet and Wolsey (2006) and a review of solution approaches can be found in Jans and Degraeve (2007).

The Cutting Stock Problem (*CSP*) involves the cutting of large objects available in stock into smaller pieces, in order to meet the demand of the pieces and optimize an objective function, such as the minimization of the total waste, the minimization of the costs of the used objects, or the maximization of profit. The economic and operational importance of the cutting stock problem and the difficulties in solving it, have motivated the academic community in this area to develop efficient solution methods, as can be seen in the review papers and special editions of Hinxman (1980), Dyckhoff et al. (1985), Dyckhoff et al. (1997), Arenales et al. (1999), Hifi (2002), Wang and Wäscher (2002), Oliveira and Wäscher (2007) and Morabito et al. (2009). There is also a EURO Special Interest Group on Cutting and Packing *ESICUP* (http://paginas.fe.up.pt/esicup/). In order to synthesize and classify the literature, Dyckhoff (1990) introduced a typology for the cutting stock problem. The typology is based on four characteristics of the problem, which are dimensionality (i.e. the number of relevant dimensions in the cutting process), type of assignment (i.e. the selection of objects and pieces), assortment of large objects and small pieces. Subsequently Wäscher et al. (2007) presented changes in Dyckhoff's typology, refining aspects of the problems analyzed and considered other problems.

The literature mostly deals with the lot-sizing problem and cutting stock problem separately through models that capture just the main trade-off in each problem. However, some reviews papers (Thomas and Griffin, 1996; Drexl and Kimms, 1997; Jans and Degraeve, 2008) have pointed out a tendency that dealing with various problems in an integrated way is an important aspect for future research. Over the last years, this tendency was observed for the lot-sizing and cutting stock problems, and the analysis of practical cases focusing more on the incorporation of relevant industrial concerns provided a further impetus. Therefore, this thesis aligns with this trend of evolution in dealing with the problems in an integrated way. The basic idea of the integrated lot-sizing and cutting stock problem is to consider, simultaneously, the decisions related to both problems so as to capture the interdependency between these decisions in order to obtain a better global solution.

Chapters of this thesis are organized as follows. In Chapter 2, we propose a general formulation for the integrated lot-sizing and cutting stock problem that considers two main aspects of integration, which are the integration across time periods and the integration between production levels. This model provides an instrument that allows us to classify the various models proposed in the literature. The concern in this thesis in classifying the integrated problems from the literature is due to the fact that when the authors refer to an integrated problem, they often consider different assumptions with respect to the level of integration considered, and hence present quite different models. In this way, the aim of this chapter is to standardize and classify the concept of the integrated lot-sizing and cutting stock problems. The classification is performed by firstly presenting a general integrated model composed of three production levels and multiple time periods. The models in the literature are then classified according to their characteristics with respect to the time horizon and production levels. Other important issues related to the objects, pieces and final products, as well as the consideration of capacity constraints and setups at different levels, are also evaluated. The presentation of a general integrated model and the classification of the various integrated models in the current literature are the main contributions of this chapter. To the best of our knowledge, no general review and analysis of integrated lot-sizing and cutting stock problems have been done so far. Furthermore, this analysis also allows us to point out interesting areas for future research. This research has also the collaboration of professor Raf Jans from University of Montréal, Canada and it is based on the working paper Melega et al. (2017a).

Chapter 3 presents the solution methods proposed to the general integrated lot-sizing and cutting stock problem (see Chapter 2). The cutting process modeled in the general integrated problem is considered as an one-dimensional problem in this chapter. A literature review of the solution methods to integrated problems is also addressed in this chapter, in order to highlight the main strategies used in this field. The solution methods in the literature are classified as exact and heuristic approaches and we consider the same studies previously classified in Chapter 2. Considering the solution methods to the general integrated problem, we are interested in heuristic approaches that overcome the difficulties faced in the cutting stock problem and take advantages of multi-level structures and/or deals wisely with the binary values of the setup variables in the lot-sizing problem. Faced on these, the solution methods proposed in this thesis are based on known solution methods from the literature, which have showed to be the best way to deal with these difficulties. The solution methods consider the column generation procedure as a first step in Level 2 and then integer programming and decomposition heuristics are addressed in an attempt to find a feasible solution to the problem. The decomposition heuristics consider two strategies to select the variables in order to decompose the problem. A hybrid heuristic that combines the relax-and-fix and the column generation procedures is also proposed. A whole set of data is generated to the general integrated problem based on known data set in the literature for each problem individually. The computational study is performed around four analysis which are in terms of size of the problem, the length of pieces, the capacity constraint and costs in the objective function in order to evaluate the impact of these variations in the computational results. This research is based on the working paper Melega et al. (2017b).

Other contributions of this research are in Appendix A and Appendix B of this thesis. We consider these studies in the appendix in order to keep in the chapters the studies which more directly are related to each other. However, the models and solution approaches presented in the papers from appendixes are classified in Chapter 2 and Chapter 3 of this thesis.

In the Appendix A, a study of mathematical models to the integrated lot-sizing and cutting stock problem is presented. Most of the studies in the literature that address the integrated lot-sizing and cutting stock problem study several cases of the problem in practice. In these studies, alternative formulations for the *LSP* and *CSP* are not tested in order to choose the formulation that best fits the problem and the data set. So, this research tries to cover the gap by proposing alternative mathematical formulations, considering different features of the lot-sizing and cutting stock problems. The capacitated lot- sizing problem is modeled using the mathematical model proposed by Trigeiro et al. (1989). We also considered the variable redefinition strategy (Eppen and Martin, 1987) which reformulates the lot-sizing problem as a shortest path problem. To model the one-dimensional cutting stock problem, we consider three mathematical models from the literature. The first model is based on the ideas present in Kantorovich (1960) (to a mathematical formulation see Valério de Carvalho (1999)), which determines the best way to cut objects to meet the demand, minimizing the number of objects used. For this model, an upper bound on the number of objects is considered. The second model dealt with, and perhaps the best known among the academic community, is the one proposed by Gilmore and Gomory (1961, 1963). This model produces good lower bounds when compared to the Kantorivich's model, however it has an large number of variables, which indicates the use of column generation to deal with this difficulty. The third model was proposed by Valério de Carvalho (1999, 2002). The author proposes an alternative mathematical model for the one-dimensional cutting stock problem based on an arc flow problem. This model is strong in the sense that it presents a linear relaxation as good as the Gilmore and Gomory model. The cutting stock models from the literature are extended to consider several time periods and several types of objects in stock. In a second part of this study, these models are integrated into the uncapacitated lot-sizing problem proposed by Wagner and Whitin (1958) and different types of objects are considered. The column generation is used to solve the linear relaxation of the proposed models and the mixed integer problems are solved by an optimization package. An extensive computational study using randomly generated data is performed, in order to evaluate and indicate the impact of these different formulations. This study also has the collaboration of professor Raf Jans and has been published in the journal *Pesquisa Operacional* (see Melega et al. (2016)).

The research presented in Appendix B consists of a practical application of the integrated lot-sizing and cutting stock problem in a furniture industry. An integrated mathematical model is proposed that includes lot-sizing decisions with safety stock level constraints and saw capacity constraints taking into account saw cycles. The solution method proposed to the integrated model is compared to a simulation of the company's decision, which consists of taking the lot size and the cutting stock decisions separately and sequentially. A column generation solution method is proposed to solve the related problem and the mixed integer problem is solved by an optimization package. An extensive computational study is conducted using instances generated based on data collected at a typical small scale Brazilian factory. The integrated approach performs well, both in terms of reducing the total cost of raw materials as well as the inventory costs of pieces. They also indicate that the model can support the main decisions taken and can bring improvements to the factory's production planning. This research extend the ideas present in Vanzela (2012) and in Vanzela et al. (2013) in order to admit that different cutting patterns are not able to be cut in the same saw-cycle, which consists of relaxations in the previous studies. Our contribution in this paper consists of the extension of the model, the adaptations of the solution methods considering the extended model, with saw cycle constraints, and in the performing of the computational study. This paper has the collaboration of professor Socorro Rangel from Universidade Estadual Paulista, São José do Rio Preto - SP, Brazil and has been recently published in the journal *Computers & Operational Research* (see Vanzela et al. (2017)).

Finally in Chapter 4, we highlight the main contributions of this thesis and point out future interesting directions of the studies to integrated lot-sizing and cutting stock problems.

Chapter 2

Classification and Literature Review of Integrated Lot-Sizing and Cutting Stock Problems

In this chapter, a literature review and classification of integrated lot-sizing and cutting stock problems are presented, in which a general mathematical formulation to the integrated lot-sizing and cutting stock (*GILSCS*) problems is proposed and used as a tool in the classification of the studies in the literature. The general integrated model is composed of three levels and takes into account two main aspects of integration. In a planning horizon which consists of multiple periods, the inventory provides a link between the periods. This integration, across time periods, constitutes the first type of integration. The general integrated problem also considers the production of different types of items: objects are fabricated or purchased and then they are cut into pieces which are assembled into final product. The integration between these three production levels constitutes the second type of integration. The characteristics of the general integrated model, such as, the different types of integration, provide an instrument that allows us to use it to classify the various models proposed in the literature, as can be seen in this chapter.

This Chapter is organized as follows. Firstly, in Section 2.1, we present a brief discussion of the problems from the literature which are addressed in this thesis and some models which are used as references in the formulation of the general integrated model. For each problem, it is also highlighted some relevant extensions to the scope of this chapter.

The classification of the literature is presented in Section 2.2, by first showing the classification criteria, which is built from the two types of integration presented in the general integrated model, i. e., the integration across time periods and the integration between production levels. Other important issues related to the objects, pieces and final products, as well as the consideration of capacity constraints and setups at different levels, are also evaluated. In Section 2.3, conclusions and new future research are pointed out based on the literature review of integrated lot-sizing and cutting stock problem and based on the formulation of the general integrated model.

The presentation of a general integrated model and the classification of the various integrated models in the current literature are the main contributions of this chapter. To the best of our knowledge, no general review and analysis of integrated lot-sizing and cutting stock problems have been done so far. Furthermore, this analysis also allows us to point out interesting areas for future research. The scope of the chapter is limited to the modelling aspects related to the integrated models. A detailed discussion of the solution approaches is presented in Chapter 3.

2.1 Mathematical Models

In this section, basic models for the lot-sizing problem and cutting stock problem are presented, as well as a brief discussion of some relevant extensions for each problem. Next, the general integrated model is proposed.

2.1.1 Discussion of the Lot-Sizing Problem (*LSP*)

The lot-sizing problem was firstly introduced by Wagner and Whitin (1958) for a single item and by Manne (1958) for multiple items and with capacity constraints. In this chapter, we consider a capacitated lot-sizing problem with setup times (CL) which was proposed by Trigeiro et al. (1989) and analyzed further in Degraeve and Jans (2007), among others. Consider the following sets, parameters and decision variables:

Sets:
$T = \{1, \ldots, s\}$: set of time periods (index t);
$I = \{1, \ldots, q\}$: set of items (index i).

Parameters:
sc_t^i: setup cost of item i in period t;
vc_t^i: unit production cost of item i in period t;
hc_t^i: unit holding cost of item i in period t;
st_t^i: setup time of item i in period t;
vt_t^i: unit production time of item i in period t;
d_t^i: demand of item i in period t;
$sd_{t\tau}^i$: sum of demands of item i from period t to period τ;

Cap_t: capacity (time limit) available to produce the items in period t.

Decision Variables:
X_t^i: production quantity (in units) of item i in period t;
S_t^i: inventory (in units) for item i at the end of period t,
Y_t^i: binary variable indicating the production or not of item i in period t.

Model *CL*

$$min \sum_{t \in T} \sum_{i \in I} \left(sc_t^i Y_t^i + vc_t^i X_t^i + hc_t^i S_t^i \right) \tag{2.1}$$

Subject to :

$$S_{t-1}^i + X_t^i = d_t^i + S_t^i \qquad \forall i, \ \forall t \tag{2.2}$$

$$X_t^i \leq sd_{ts}^i Y_t^i \qquad \forall i, \ \forall t \tag{2.3}$$

$$\sum_{i \in I} \left(st_t^i Y_t^i + vt_t^i X_t^i \right) \leq Cap_t \qquad \forall t \tag{2.4}$$

$$Y_t^i \in \{0, 1\} \qquad \forall i, \ \forall t \tag{2.5}$$

$$X_t^i, \ S_t^i \in \mathbb{R}_+ \qquad \forall i, \ \forall t \tag{2.6}$$

The objective function (2.1) minimizes the total setup cost, production cost and inventory holding cost. Constraints (2.2) are the demand balance constraints: inventory carried over from the previous period and production in the current period are used to meet the current demand, and build up inventory that can be used in the next periods. Constraints (2.3) force the setup variable to one if any production takes place in the period. The next constraints (2.4) impose that the total production and setup time cannot exceed the available capacity in each period. Finally, constraints (2.5) and (2.6) are the integrality and non-negativity constraints.

This model corresponds to a big bucket problem, in which different items can be produced in the same time period, while in a small bucket problem at most one type of item can be produced within a time period. The *CL* model represents a single level system, which corresponds to a problem in which the final products are obtained directly after processing in a single operation. The corresponding demand for the final products is known as independent demand. In a multi-level system, there is a relationship among the items described in the Bill-of-Material. Production of final products will trigger a demand for components, which represents the dependent demand. In the multi-level model, the demand balance constraint (2.2) is changed to distinguish between the dependent and independent demand. To formulate a multi-level problem we consider the following additional data:

$S(i)$: set of direct successors of item i (index l);

r_l^i: number of items i used in one unit of item l.

Item i can have its own independent demand, as well as, its dependent demand created by the production of its direct successors. The new demand balance constraint is then written as:

$$S_{t-1}^i + X_t^i = d_t^i + \sum_{l \in S(i)} r_l^i X_t^l + S_t^i \qquad \forall i,\ \forall t \tag{2.7}$$

Among the various extensions of the lot-sizing problem we highlight the use of several machines to produce the items (Toledo and Armentano, 2006; Jans, 2009; Fiorotto and de Araujo, 2014; Fiorotto et al., 2015), as well as the possibility of backlogging some part of the demand (Pochet and Wolsey, 1988). The setup structure in the capacity constraint is another consideration that affects the problem complexity (Karimi et al., 2003). The setup of a product can be dependent or independent of the previous product in the production sequence (Guimarães et al., 2014), or the final setup in a period can be carried over to the next period (Gopalakrishnan et al., 2001; Sahling et al., 2009).

Alternative formulations for the lot-sizing problem have been proposed in the literature. We point out the network reformulation proposed by Eppen and Martin (1987), which reformulates the lot-sizing problem as a shortest path problem, and the simple plant location reformulation proposed by Krarup and Bilde (1977). These reformulations are strong formulations, since they provide tighter lower bounds when compared with the original formulations. Extensions of these reformulations have been successfully applied in decomposition algorithms to obtain further improved lower bounds (Jans and Degraeve, 2004; Süral et al., 2009; de Araujo et al., 2015; Fiorotto et al., 2015).

The variable redefinition strategy (Eppen and Martin, 1987), called here as *SP*, is of particular interest of this thesis (see Appendix A) and it is detailed as follows. The original idea was proposed for uncapacitated problems and Jans and Degraeve (2004), Jans (2009), Melega et al. (2013) and Fiorotto and de Araujo (2014) extended it to cases with capacity constraints, related parallel machines, various plants and unrelated parallel machines, respectively.

In order to formulate the lot-sizing problem as a shortest path problem, we need the following definitions of parameters and variables, respectively:

$cv_{it\tau}$: total production and inventory holding cost for producing item i in period t at a quantity that meets the demands for the periods from t until τ, $cv_{it\tau} = vc_{it} sd_{t\tau}^i + \sum_{v=t+1}^{\tau} \sum_{u=t}^{v-1} hc_u^i d_v^i$;

$zv_{it\tau}$: fraction of the production plan for item i to meet demand from period t to period τ.

The lot-sizing problem variables have the following correspondence:

$$X_{it} = \sum_{\tau=t}^{s} sd_{it\tau} zv_{it\tau} \qquad \forall i,\ \forall t \tag{2.8}$$

and the demand constraints (2.2) and setup constraints (2.3) are rewritten in terms of the new decision variables as follows:

$$\sum_{\tau \in T} zv_{i1\tau} = 1 \qquad \forall i \tag{2.9}$$

$$\sum_{\tau=1}^{t-1} zv_{i\tau t-1} = \sum_{\tau=t}^{s} zv_{it\tau} \qquad \forall i,\ \forall t \backslash \{1\} \tag{2.10}$$

$$\sum_{\tau=t}^{s} zv_{it\tau} \leq Y_{it} \qquad \forall i,\ \forall t \tag{2.11}$$

Constraints (2.9) and (2.10) define the flow constraints in the *SP* model. For each item i, a unit flow is sent through the network (constraint (2.9)), imposing that its demand has to be meet without backlogging in each period (2.10). Constraint (2.11) ensures that item i will be produced in period t only if there is a setup prepared to produce that item. To an application of this reformulation, see Appendix A.

2.1.2 Discussion of the Cutting Stock Problem (*CSP*)

To model the cutting stock problem and its main elements, three formulations from the literature are considered in this thesis. The first one is a compact formulation for the one-dimensional case and was proposed by Kantorovich (1960). This formulation is also known as the generalized assignment model for the *CSP* (Degraeve and Peeters, 2003). Here it is called *KT*. Consider the following sets, parameters and decision variables for the *KT* model:

Sets:
P: set of pieces (index p).

Parameters:
UB: upper bound on the number of objects;
L: object length;
l_p: length of piece p;
d^p: demand of piece p.

Decision Variables:

y_o: binary variable that indicates whether object o is used or not;

h_o^p: number of units of piece p cut from object o.

Model *KT*

$$min \sum_{o=1}^{UB} y_o \tag{2.12}$$

Subject to :

$$\sum_{o=1}^{UB} h_o^p \geq d^p \qquad \forall p \tag{2.13}$$

$$\sum_{p \in P} l_p h_o^p \leq L y_o \qquad \forall o \tag{2.14}$$

$$y_o \in \{0,1\} \qquad \forall o \tag{2.15}$$

$$h_o^p \in \mathbb{Z}_+ \qquad \forall p,\ \forall o \tag{2.16}$$

The objective function (2.12) minimizes the number of cut objects. Constraints (2.13) ensure that the demand for each piece is satisfied. Constraints (2.14) are the knapsack constraints and guarantee that if object o is cut, then the combination of the piece sizes that will be cut from it cannot exceed its size. Finally, the set of constraints (2.15) and (2.16) impose integrality and non-negativity conditions.

The second formulation presented was proposed by Valério de Carvalho (1999, 2002), here denoted by *VC*. The author proposed an alternative mathematical model for the one-dimensional cutting stock problem based on an arc flow problem. The formulation uses the idea of cutting patterns, which define the way that the pieces are cut from an object. The problem of finding a valid cutting pattern, is modeled as a problem of finding a path in a directed acyclic graph $G = (V, A)$, with $V = \{0, 1, \ldots, L\}$, where L is the length of the object, i. e., the distance of one vertex to other represents one unit of objects length. The set of arcs in the graph is defined as $A = \{(g, h); 0 \leq g < h < L$ and $h - g = l_p$ for all $p \in P\}$. The losses in the object generated from the cutting process are represented in the graph by additional arcs between the vertices $(g, g+1)$ to $g = 0, \ldots, L-1$. The author also considers additional constraints to guarantee that demand of each piece is meet and the resulting problem consists of a cutting stock problem. As decision variables for the *VC* model considers:

f: flow through the network;

z_{gh}: number of cutting patterns which have a piece of size $(h - g)$ allocated at a distance g from the beginning of the object.

Model *VC*

$$minf \tag{2.17}$$

Subject to :

$$-\sum_{(0,h)\in A} z_{0h} = -f \tag{2.18}$$

$$\sum_{(g,h)\in A} z_{gh} - \sum_{(h,s)\in A} z_{hs} = 0 \qquad l = 1, \ldots, L-1 \tag{2.19}$$

$$\sum_{(h,L)\in A} z_{hL} = f \tag{2.20}$$

$$\sum_{(h,h+l_p)\in A} z_{h,h+l_p} = d_p \qquad \forall p \tag{2.21}$$

$$z_{gh},\ f \in \mathbb{Z}_+ \qquad \forall (g,h) \in A \tag{2.22}$$

The objective function (2.17) minimizes the flow through the network. The flow set for this problem represents the number of used objects (cutting patterns), since one flow unit defines a path, which in turn defines a cutting pattern. The set of constraints (2.18), (2.19) and (2.20) correspond to flow conservation constraints. Constraints (2.21) guarantee that the demand of each piece is meet and finally constraints (2.22) are the integrality and non-negativity constraints.

This model presents many symmetric solutions, or alternatives that correspond to the same cutting patterns. For this reason, Valério de Carvalho (1999) presented some reduction criteria to eliminate some arcs, reducing the number of symmetric solutions without eliminating any valid cutting pattern from set A. One of the criteria consists in allocating the items in order of decreasing length in each cutting pattern, that is, an item of length i_1 can only be placed after another item length i_2 if $i_1 \leq i_2$, or at the beginning of the object. Another criterion does not allow starting a cutting pattern with loss. Thus, the first arc of loss will be inserted in the graph at a distance from the beginning of the object representing the shortest item length. To an application of the *KT* and *VC* formulation see Appendix A.

Next, we introduce the last model addressed in this thesis for the *CSP*. This is probably the most well-known model for the cutting stock problem and was proposed by Gilmore and Gomory (1961, 1963), and here it is referred to as *GG*. This formulation is more flexible when compared with the *KT* and *VC* models in the sense that it can easily be adapted to take into account multi-dimensional problems and other extensions of the *CSP*.

The *GG* formulation makes use of the idea of cutting patterns, which define the way that the pieces are cut from an object. To define a cutting pattern considers a_j^p as the number of pieces p cut in the cutting pattern j. The vector $(a_j^1, a_j^2, \ldots, a_j^P)^t$ represents the jth cutting pattern and must respect the physical limitations of the object to be cut.

In the one-dimensional case, each jth cutting pattern must satisfy the constraints (2.23) - (2.24).

$$\sum_{p \in P} l_p a_j^p \leq L \tag{2.23}$$

$$a_j^p \in \mathbb{Z}_+ \qquad \forall p \tag{2.24}$$

The Figure 2.1 shows an exemple of one-dimensional cutting patterns, i. e., just one dimension is taken into account in the cutting process. In this example, there is available one type of object from stock with length L and three pieces of lengths l_1, l_2 and l_3 are demanded. The pieces can be cut from the objects according to possible cutting patterns, i. e., satisfying (2.23) - (2.24).

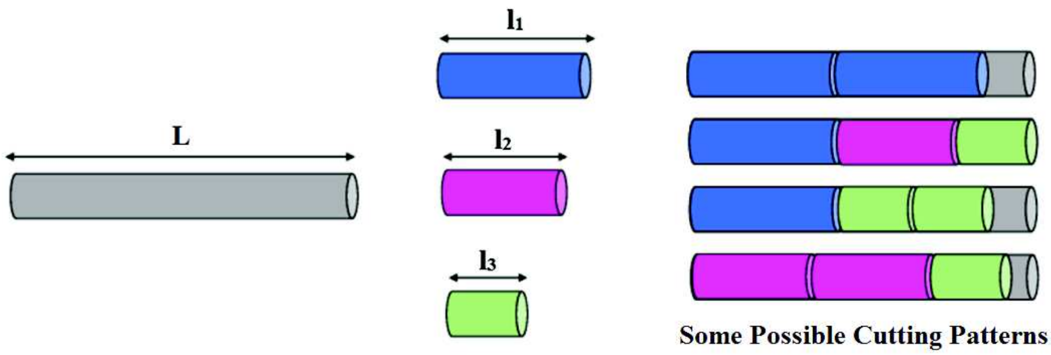


Figure 2.1: One-Dimensional Cutting Patterns.

Considering two-dimensional cutting patterns, the problem consists of geometrically combining pieces along the length and width of objects in stock (rectangle) without overlapping. The Figure 2.2 illustrates this type of problem, in which an object of dimensions $L \times W$ (length and width) has to be cut into smaller rectangular pieces of dimensions $l_1 \times w_1, \ldots, l_5 \times w_5$ according to a cutting pattern.

The built of a two-dimensional cutting patterns can be influenced by the production environment in which it is inserted and can present several characteristics, such as, type of item, type of cut and number of stages, that impact in the evaluation of the cutting pattern quality. For example, the majority of furniture industries, due to the cutting machines limitations, considers that only ortogonal guillotine cuts can be performed (see Figure 2.3).

Another important consideration is related to the number of stages in the cutting process, i. e., the sequence in which the cutting patterns are performed in order to obtain the pieces. The number of stages in a cutting pattern is determined by the number of times that the object must be rotated in 90° in order to cut all the pieces (Yanasse and

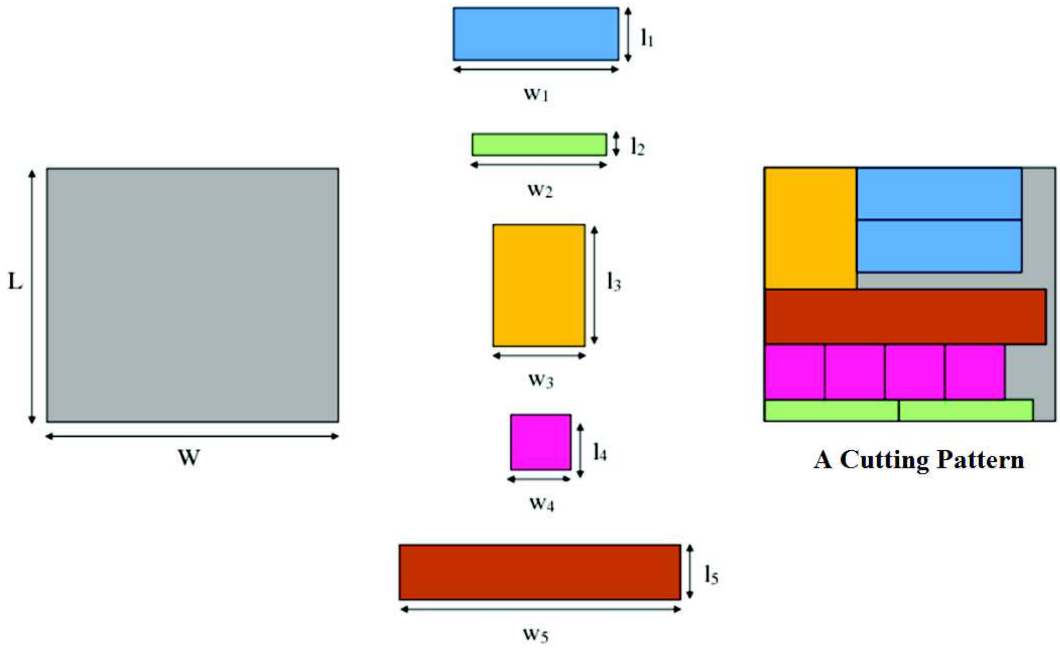


Figure 2.2: Two-Dimensional Cutting Pattern.

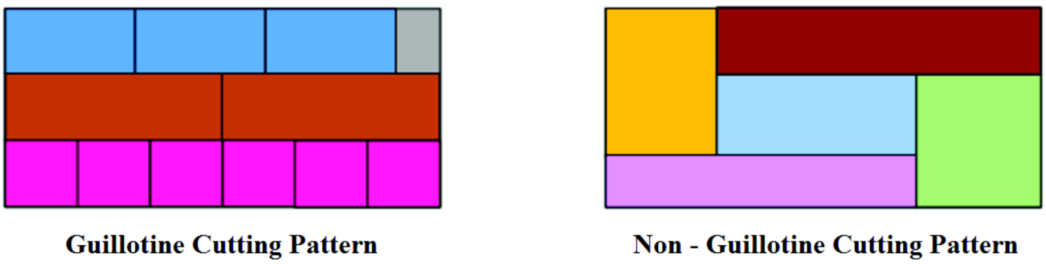


Figure 2.3: Guillotine and Non-Guillotine Cutting Patterns.

Morabito, 2008). The Figure 2.4 presents a cutting pattern in which it is necessary just one rotation of the object. In the first stage, the guillotine cutting results in a set of strips, whereas in the second stage guillotine cuts are made in each strip. At the end of the final stage, if all the items have been obtained, the cutting pattern is said to be exact, otherwise it is said to be non-exact. The trimming in a non-exact cutting pattern is usually done in a secondary cutting machine and therefore it is not counted as an additional stage (Morabito and Arenales, 2000).

Several exact, approximate or heuristic approaches to the generation of two-dimensional cutting patterns can be found in the literature, due to its fundamental importance in the cutting stock problems (Gilmore and Gomory, 1965; Lodi and Monaci, 2003; Rangel and Figueiredo, 2008; Yanasse and Morabito, 2008). For more details of two-dimensional cut-

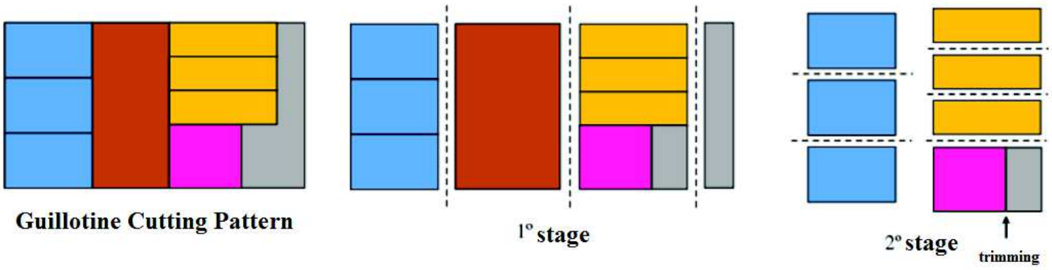


Figure 2.4: Guillotine Two-Stages Cutting Pattern.

ting stock problems see Appendix B, which considers an application in a furniture industry.

After these considerations, the following sets, parameters and decision variables are necessary for the GG model:

Sets:
$J = \{1, \ldots, N\}$: set of all possible cutting patterns (index j).

Parameters:
a_j^p: number of times piece p cut in the cutting pattern j.

Decision Variables:
Z_j: number of objects cut according to the cutting pattern j.

Model *GG*

$$min \sum_{j \in J} Z_j \tag{2.25}$$

Subject to :

$$\sum_{j \in J} a_j^p Z_j \geq d^p \qquad \forall p \tag{2.26}$$

$$Z_j \in \mathbb{Z}_+ \qquad \forall j \tag{2.27}$$

The objective function (2.25) minimizes the number of cut objects. Constraints (2.26) ensure that the demand for each piece is met through the cutting of objects in stock using different cutting patterns. Constraints (2.27) are integrality constraints. Each solution of the GG model consists of a set of cutting patterns, and their corresponding application frequencies (Z_j). Each cutting pattern (a_j^p) must satisfy the physical limitations of the object to be cut and, in the one-dimensional case, the constraints (2.23)-(2.24).

In this thesis, we limit the presentation of mathematical models for cutting stock problems to the *KT*, *VC* and *GG* models. However, other mathematical models can also be found in the literature, such as, the model proposed by Dyckhoff (1981). In the model, called as “one-cut” model, each decision variable corresponds to a single cutting operation performed in a single object. An object of any size is cut in two pieces, and at least one piece has the size of a demanded piece. A cut operation can be made in large objects from stock or in pieces resulting from previous cuts.

The *GG* formulation can be obtained by applying the Dantzig-Wolfe decomposition principle to the *KT* formulation, which gives rise to a very tight linear relaxation (Valério de Carvalho, 2002; Ben Amor and Valério de Carvalho, 2005). It is possible to show that the linear relaxations values of the *VC* and *GG* models are equivalents, i. e., they provide the same lower bound (see Valério de Carvalho (1999)).

Due to the large number of decisions variables, the *GG* formulation becomes difficult to solve. For this reason, Gilmore and Gomory (1961, 1963, 1965) proposed relaxing the integrality constraints and solving the resulting linear programming problem using a column generation technique. The Z_j columns and the associated parameters a_j in (2.26) are generated by solving a subproblem and attractive columns are added to the master problem to improve the current solution. For the one-dimensional cutting stock problem, the subproblem is an integer knapsack problem (Gilmore and Gomory, 1961, 1963; Martello and Toth, 1990). Considering higher dimensions for the subproblems, other solution methods have also been proposed in the literature (Christofides and Whitlock, 1977; Arenales and Morabito, 1995; Yanasse and Katsurayama, 2005).

Typically, the solution of the relaxed problem is fractional and an integer solution can be obtained using heuristics based on the approximate fractional solution and rounding procedures (Stadtler, 1990; Wäscher and Gau, 1996; Poldi and Arenales, 2009) or by a branch-and-price procedure, which embeds a column generation process within a branch-and-bound approach (Vance et al., 1994; Degraeve and Peeters, 2003; Belov and Scheithauer, 2006; Alves and Valério de Carvalho, 2008).

The *CSP* in its standard form consists in determining how large objects from stock can be cut into smaller pieces in order to meet the demand for the pieces. However, industrial cutting problems are often embedded in a production environment which is significantly different from the standard *CSP* model. Consequently, over the years, the standard model has been extended to consider several different aspects found in industrial practice, which results in different features, constraints and objectives. Some extensions which will be of particular interest for the general integrated model are described as follows.

The use of different types of objects (e. g. with different lengths, weights or thicknesses) is an important feature in some industries and can lead to a better overall material utilization. In general, the addition of costs associated with different objects is necessary. This results in other objective functions, such as minimizing the costs of the objects or the waste of material. When the costs of objects is proportional to the object length, a trim loss minimization arises (Belov and Scheithauer, 2002). With the addition of different objects, establishing an optimal solution for this problem becomes more difficult. The objects may be available in an unlimited quantity (Valério de Carvalho, 2002; Furini and Malaguti, 2013) or may be subject to an upper bound (Belov and Scheithauer, 2002; Poldi and Arenales, 2009). In general, after the objects pass through the cutting process, inevitable trim loss is produced, which at the end can become a significant waste of raw material. In some cases, however, the trim loss of a cutting pattern does not necessarily become a waste of material. If its size is long enough, the raw material can be stored in the warehouse to be used later as input in the cutting process to produce the pieces (Arbib et al., 2002; Andrade et al., 2014; Cherri et al., 2014).

In the extensions discussed up to now, the raw material cost has a huge impact in the cutting stock problem. However, in some industrial applications such as in the paper industry, the raw material has a low value per unit, whereas several complex processing operations are required to obtain the final products. In such a case, it is typically inappropriate to define the decision criterion as the minimization of trim loss or objects costs and a more realistic criterion is the minimization of other control costs (Haessler, 1975). An important case is the incorporation of costs associated with the use of a new cutting pattern different from the previous one, since a setup is necessary whenever a new cutting pattern is started and the cutting equipment has to be prepared for this new cutting pattern. Setups of this kind involve the loss of production time capacity, additional costs and consumption of resources. In such cases, the minimization of setup cost is added to the minimization of objects or waste costs (Haessler, 1975).

To model a setup constraint in the GG formulation, we first define W_j as a binary variable indicating if cutting pattern j is used or not and M as a large number. The following constraint is added to the GG model (Diegel et al., 1996; Vanderbeck, 2000):

$$Z_j \leq MW_j \qquad \forall j \tag{2.28}$$

Constraint (2.28) ensures that a cutting pattern setup is done whenever a cutting pattern is used at least once. In some cases, the number of cutting patterns is minimized (Vanderbeck, 2000). In other cases, a limit is imposed on the number of cutting pattern setups, while optimizing some other objective, such as the deviation from the demand (Umetani et al., 2003) or the minimization of the number of objects used (Umetani et al., 2006). Other approaches first optimize a regular objective such as the minimization of the number of objects used or the waste, and next find a solution that minimizes the number of cutting patterns used (Foerster and Wäscher, 2000; Yanasse and Limeira, 2006; Cui et al., 2015). Some papers also consider a multi-objective problem (Golfeto et al., 2009; de Araujo et al., 2014). A recent literature review on the cutting stock problem with setups can be found in Henn and Wäscher (2013).

2.1.3 A General Integrated Problem

A General Integrated Lot-Sizing and Cutting Stock model (*GILSCS*) is proposed in this section, with the purpose of discussing and classifying the papers in the literature that address both problems simultaneously. We consider a production environment composed of three levels (see Figure 2.5), where objects are acquired and next cut into pieces. These pieces form the input for the assembly process, in which final products are made. The general integrated model is based on the *LSP* proposed by Trigeiro et al. (1989) and on the *CSP* proposed by Gilmore and Gomory (1961, 1963).

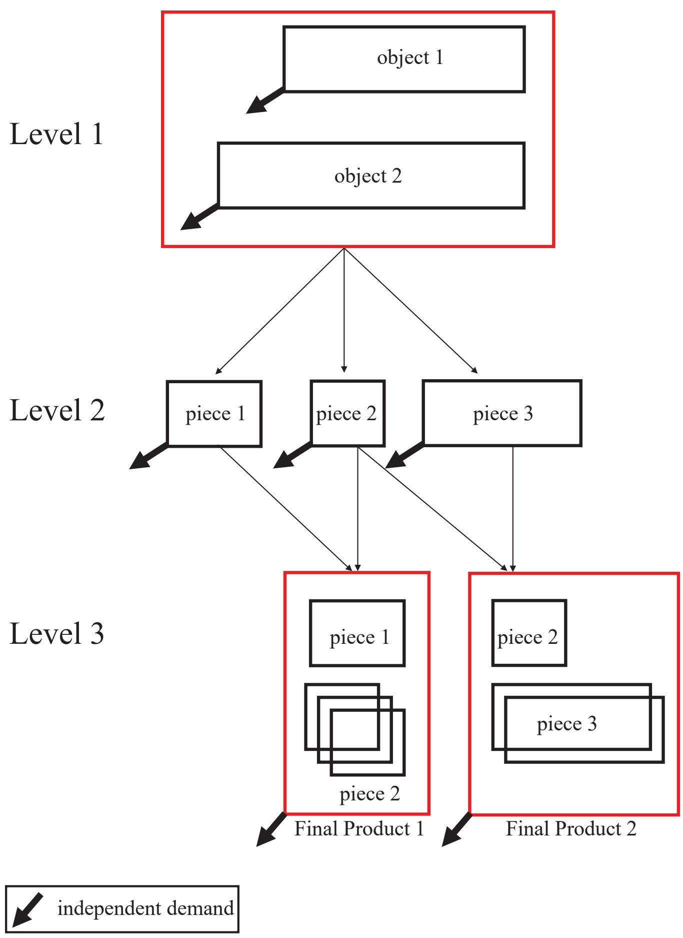


Figure 2.5: Integration Between Production Levels.

Level 1 corresponds to the planning of the acquisition and/or production of the objects which will be cut in pieces. This can be done either by internal production or by ordering from an outside supplier. The objects may differ in size, thickness and other characteristics, as well. Level 2 corresponds to the cutting process, in which the objects are cut into pieces according to cutting patterns. The pieces can be used as components for the assembly of final products or they can be directly considered as the final products which still need to undergo some finishing process. The production of the final products is modeled at Level 3. In most of the cases, a final product requires more than one type of piece as a component. There is an independent demand for the final products, which will trigger a dependent demand for pieces and objects. We assume that, apart from final products, both pieces and objects may have independent demand.

Although Figure 2.5 shows a first type of integration, i. e., among different production levels, it does not yet show the second type of integration, i. e., among different time periods. Indeed, the integrated model is a dynamic model where multiple periods are considered simultaneously, whereas the standard cutting stock model is a one-period model. This dynamic aspect is shown in Figure 2.6, which presents an integrated problem with three levels and two time periods. In each period, we have independent demand for the final products and possibly for the cut pieces and objects as well. The link between the different periods is provided by the inventory which can be carried over from one period to the next. We can keep inventory for the final products, for the cut pieces and for the objects. Demand can be satisfied either from production in the current period or from inventory carried over from the previous period.

In order to formulate the general integrated model, consider the following sets, parameters and decision variables:

Sets:
$T = \{1, \ldots, s\}$: set of time periods (index t);
$O = \{1, \ldots, l\}$: set of different types of objects (index o);
$P = \{l+1, \ldots, l+m\}$: set of pieces (index p);
$F = \{l+m+1, \ldots, l+m+n\}$: set of final products (index f);
$J_o = \{1, \ldots, N_o\}$: set of cutting patterns for object type o (index j).

Parameters:
sc_t^o: setup cost/fix ordered cost for object type o in period t;
vc_t^o: unit production cost/purchase cost of object type o in period t;
hc_t^o: unit holding cost of object type o in period t;
d_t^o: independent demand of object type o in period t;
st_t^o: setup time for object type o in period t;
vt_t^o: unit production time of object type o in period t;

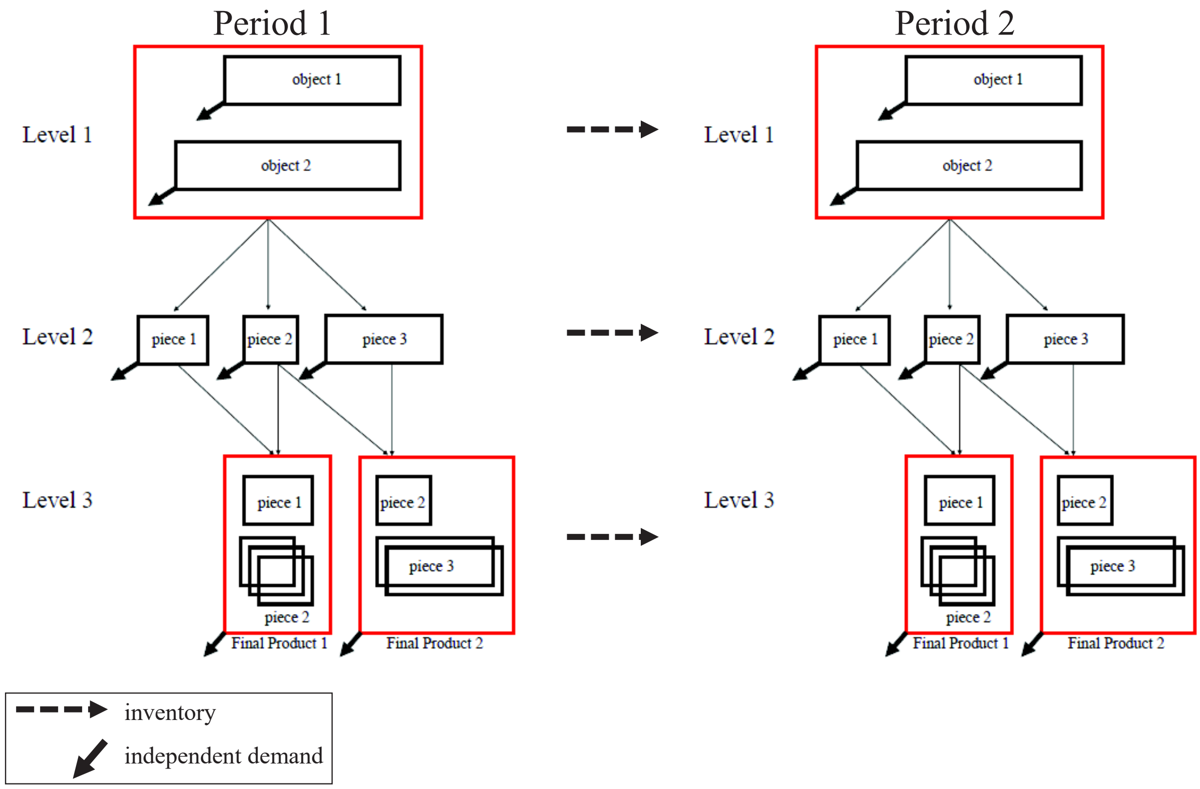


Figure 2.6: Integration Between Production Levels with $|T| = 2$.

$CapO_t$: production capacity (in time units) available to produce the objects in period t;

u_j^o: setup cost for cutting pattern j for object type o;

c_j^o: cost of cutting an object type o according to cutting pattern j;

hc_t^p: unit holding cost of piece p in period t;

d_t^p: independent demand of piece p in period t;

r_f^p: number of pieces of type p required in the final product f

a_{oj}^p: number of pieces p cut from object type o using the cutting pattern j;

st_{jt}^o: setup time of the object type o cut according to cutting pattern j in period t;

vt_{jt}^o: production time to cut object type o according to cutting pattern j in period t;

$CapP_t$: cutting capacity (in time units) available in period t;

sc_t^f: setup cost of final product f in period t;

vc_t^f: unit production cost of final product f in period t;

hc_t^f: unit holding cost of final product f in period t;

st_t^f: setup time of final product f in period t;

vt_t^f: unit production time of final product f in period t;

d_t^f: demand of final product f in period t;

sd_{tr}^{f}: sum of demand of final product f from period t until period r;

$CapF_t$: production capacity (in time units) available to produce the final products in period t;

M: large number.

Decision Variables:

X_t^o: production/purchase quantity (in units) of object o in period t;

S_t^o: inventory (in units) of object o at the end of period t,

Y_t^o: binary variable indicating the production/purchase or not of object o in period t;

X_t^p: production quantity (in units) of piece p in period t;

S_t^p: inventory (in units) of piece p at the end of period t,

Z_{jt}^o: number of objects of type o cut according to cutting pattern j in period t;

W_{jt}^o: binary variable indicating the setup or not of cutting pattern j for object type o in period t;

X_t^f: production quantity (in units) of final product f in period t;

S_t^f: inventory (in units) of final product f at the end of period t,

Y_t^f: binary variable indicating the setup or not of final product f in period t.

The general integrate model is as follows:

Model *GILSCS*

$$
\begin{aligned}
& min \sum_{t\in T}\sum_{o\in O}(sc_t^oY_t^o + vc_t^oX_t^o + hc_t^oS_t^o) + \\
& \sum_{t\in T}\sum_{o\in O}\sum_{j\in J_o} u_j^oW_{jt}^o + \sum_{t\in T}\sum_{o\in O}\sum_{j\in J_o} c_j^oZ_{jt}^o + \sum_{t\in T}\sum_{p\in P} hc_t^pS_t^p + \\
& \sum_{t\in T}\sum_{f\in F}\left(sc_t^fY_t^f + vc_t^fX_t^f + hc_t^fS_t^f\right)
\end{aligned} \tag{2.29}
$$

Subject to:

$$S_{t-1}^f + X_t^f = d_t^f + S_t^f \qquad \forall f,\ \forall t \tag{2.30}$$

$$X_t^f \leq MY_t^f \qquad \forall f,\ \forall t \tag{2.31}$$

$$\sum_{f\in F}\left(st_t^fY_t^f + vt_t^fX_t^f\right) \leq CapF_t \qquad \forall t \tag{2.32}$$

$$S^p_{t-1} + X^p_t = \sum_{f \in F} r^p_f X^f_t + d^p_t + S^p_t \qquad \forall p,\ \forall t \qquad (2.33)$$

$$X^p_t = \sum_{o \in O} \sum_{j \in J_o} a^p_{oj} Z^o_{jt} \qquad \forall p,\ \forall t \qquad (2.34)$$

$$Z^o_{jt} \leq M W^o_{jt} \qquad \forall j,\ \forall o,\ \forall t \qquad (2.35)$$

$$\sum_{o \in O} \sum_{j \in J_o} \left(st^o_{jt} W^o_{jt} + vt^o_{jt} Z^o_{jt} \right) \leq CapP_t \qquad \forall t \qquad (2.36)$$

$$S^o_{t-1} + X^o_t = \sum_{j \in J_o} Z^o_{jt} + d^o_t + S^o_t \qquad \forall o,\ \forall t \qquad (2.37)$$

$$X^o_t \leq M Y^o_t \qquad \forall o,\ \forall t \qquad (2.38)$$

$$\sum_{o \in O} (st^o_t Y^o_t + vt^o_t X^o_t) \leq CapO_t \qquad \forall t \qquad (2.39)$$

$$X^o_t,\ S^o_t \in \mathbb{R}_+,\ Y^o_t \in \{0,1\} \qquad \forall o,\ \forall t \qquad (2.40)$$

$$X^p_t,\ S^p_t \in \mathbb{R}_+,\ Z^o_{jt} \in \mathbb{Z}_+,\ W^o_{jt} \in \{0,1\} \qquad \forall p,\ \forall j,\ \forall o,\ \forall t \qquad (2.41)$$

$$X^f_t,\ S^f_t \in \mathbb{R}_+,\ Y^f_t \in \{0,1\} \qquad \forall f,\ \forall t \qquad (2.42)$$

The objective function (2.29) minimizes the overall costs at each level. At Level 1, the costs are related to the production (or purchase) of objects and consist of a fixed setup (or order) cost, the production (or purchasing) cost and the inventory cost. At Level 2, the costs refer to the cutting process. At this level, we take into account the setup cost of a cutting pattern, the cost of cut of each object according to a cutting pattern and the cost of holding the pieces in inventory. The last terms in the objective function correspond to the setup, production and inventory costs of final products at the Level 3.

Constraints (2.30), (2.31) and (2.32) refer to the final products and jointly with constraints (2.42) can be seen as a lot-sizing problem at Level 3. Constraints (2.30) are the demand balance constraints for the final products. Constraints (2.31) force the setup variable to one if any production takes place in that period. Constraint (2.32) imposes the capacity limits of the production process for final products.

Constraints (2.33), (2.34), (2.35), (2.36) and (2.41) are related to the production of the cut pieces and can be seen as a multi-period cutting stock problem with capacity constraints at Level 2. Constraint (2.33) ensures that the dependent demand ($\sum_{f \in F} r^p_f X^f_t$) and the independent demand (d^p_t) for pieces is satisfied. This constraint also models the interdependency between the decisions of Level 2 and Level 3 and corresponds to an integration between the levels of the *LSP* and *CSP*. Constraint (2.34) is a definition constraint and defines the number of pieces of type p cut in period t in function of the selected cutting patterns. Note that the variable X^p_t in constraint (2.33) can be replaced by $\sum_{o \in O} \sum_{j \in J_o} a^p_{oj} Z^o_{jt}$ according to the definition constraint (2.34). Constraint (2.35) forces a pattern setup in the cutting machine, whenever an object is cut according to a different cutting pattern. The setup addressed in this formulation is independent from the preceding cutting pattern. The capacity constraint (2.36) considers the use of only one machine in the cutting process and takes into account the time consumed for setting up the cutting patterns, as well as the time for cutting the objects according to a specific cutting pattern.

Constraint (2.37) is the demand balance constraint for objects at Level 1. This constraint also links Level 1 and Level 2 ensuring the production (or purchase) of a sufficient number of objects needed in the cutting process and corresponds to another integration between the levels of the *CSP* and *LSP*. Constraints (2.38) is the setup forcing constraint related to the production of objects. The capacity limit for the production of objects is modeled by constraint (2.39). The time spent to setup the machine for a specific object type and to produce the objects consumes the capacity available. Note that if the objects are purchased from a supplier instead of internally produced, there is no such capacity constraint. Constraints (2.37) - (2.40) model a lot-sizing problem at Level 1 with both dependent and independent demand. Finally, (2.40), (2.41) and (2.42) are the non-negativity and integrality constraints for the *GILSCS* model.

Some remarks related to the general integrated problem are necessary before using this model as a tool for classifying the current literature. The motivation to choose the *GG* formulation to model the cutting stock problem is mainly due to the flexibility of this formulation. More specifically, it allows the consideration of setup costs and setup times related to the cutting pattern, as well as a specific cost and time related to the cutting of an object according to a specific cutting pattern. The models of Dyckhoff (1981) and Valério de Carvalho (1999, 2002) are less suitable for being extended to include setup aspects (Henn and Wäscher, 2013). Another point of flexibility of the *GG* formulation is the generation of the cutting patterns through the solution of a subproblem. The formulation *GG* itself does not need to specify the dimensionality of the problem, which contributes to its use in different industrial applications, which is not the case of the *KT* nor *VC* formulation.

As we mentioned before, stronger reformulations, such as variable redefinition, depend on the sum of the demand of the items thought of periods. In this way, if there is any unknown demand, due to dependent demand of items, for example, the variable redefinition is not possible in this case. Therefore, the variable redefinition, when applied to the general integrated problem, is done just at Level 3, which consists of the production of final products, with known independent demand. To an example of this reformulation, see Appendix A.

In order to analyze the dimensions of the problem, Table 2.1 shows the number of continuous, integer and binary variables present in the *GILSCS* problem, as well as, the number of constraints. We consider values to some parameters, as an example, in order to estimate the size of the problem in this instance. The value for each one of the variables and constraints are present in the last column of the table. The values assign for the parameters are: $T = 20, O = 3, P = 10$ and $F = 50$. Due to the fact that cutting patterns are influenced by other aspects, we consider the number of possible cutting patterns as a parameter (N_o). We can see that at the step that the number of cutting patterns grows the number of integer and binary variables and constraints grows proportionally.

Dimension of the *GILSCS* Model

Number of	***GILSCS***	Example
Continuous Variables	$2s(3l + 2m + 2n)$	720
Integer Variables	lN_os	$60N_o$
Binary Variables	$s(2l + lN_o + m + n)$	$160 + 60N_o$
Constraints	$s(6l + 4l + 2n + lN_o + 3)$	$660 + 60N_o$

Table 2.1: Dimension of the *GILSCS* Model.

2.2 A classification and discussion of the literature

In this section, a literature review of the integrated lot-sizing and cutting stock problem is carried out using classification criteria based on various aspects of the newly proposed general integrated model (*GILSCS*).

2.2.1 Classification Criteria

The models from the literature that address the integration of lot-sizing problems and cutting stock problems are analyzed and classified according to two main aspects. The first is the integration between production levels (see Figure 2.5). A model is classified at a specific level (1, 2 and 3) if there is a decision variable associated with this level. The second main criterion is related to the integration across multiple time periods (see Figure 2.6). The standard lot-sizing problem (*CL*), as defined before, has a discrete time horizon consisting of multiple periods. The integration between periods comes from the possibility to hold items in inventory. Some lot-sizing models assume a continuous time horizon and infinite time periods with a constant demand rate, in which it is also possible to hold inventory. The standard cutting stock problem (*GG*) only considers one period, and hence there is no possibility to keep inventory. The pieces which are left at the end of the single period are considered waste.

Once classified according to these two main axes of integration, i. e., between production levels and across time periods, other features are also analyzed such as capacity and setups. Several aspects with respect to the three types of products (objects, pieces and final products) are also discussed. Since most of the current research in this area is based on practical applications, we also provide information on the type of industry in which the model is based.

In order to define the search criteria for the papers addressed in the literature review of integrated lot-sizing and cutting stock problem, keywords were defined and then used to identify those relevant studies. Two groups of keywords were defined and used as a toll in a search in Google Scholar. The first group contains keywords related to the word ”integrated” (”integrated lot sizing and cutting stock problem”, ”integrated lot-sizing and cutting stock problem”, ”integrated cutting stock and lot sizing problem” and ”integrated cutting stock and lot-sizing problem”). The second group of keywords are related to ”combined” (”combined lot sizing and cutting stock problem”, ”combined lot-sizing and cutting stock problem”, ”combined cutting stock and lot sizing problem” and ”combined cutting stock and lot-sizing problem”). A third group were also considered where the words ”lot-sizing” and ”cutting stock” appears together (”lot-sizing and cutting stock”, ”lot sizing and cutting stock”, ”cutting stock and lot-sizing” and ”cutting stock and lot sizing”). These searches are able to find 20 studies in the literature which are addressed in the classification. We also considers more 12 other papers that are considerably mentioned as integrated/combined problems by most of the authors in these previous papers and they seem to be relevant to improve the quality of literature review.

Tables 2.2, 2.3 and 2.4 show a summary of papers in the literature that address the integration of the lot-sizing problems and cutting stock problems. In this literature review, we restrict our analysis to studies which are publicly available and have been published in English in international journals, technical reports, and conference proceedings. We classified 32 papers in total, of which 24 have been published in the last 10 years, which shows the increasing interest on this topic of the academic community (see Graphic 2.7).

2.2.2 Classification and Discussion

	Time Horizon	Time Periods	Production Levels			Application
			Level 1	Level 2	Level 3	
Farley (1988)	discrete	one period	—	✓	✓	Textile
Reinders (1992)	discrete	multiple periods	✓	✓(2)	—	Wood Processing
Hendry et al. (1996)	discrete	multiple periods	✓	✓	—	Copper
Krichagina et al. (1998)	continuous	infinite	✓	✓	—	Paper
Nonås and Thorstenson (2000, 2008)	continuous	infinite	—	✓	—	Off-road truck
Respício and Captivo (2002)	discrete	multiple periods	—	✓	—	Paper
Correia et al. (2004)	discrete	multiple periods	✓	✓(2)	—	Paper
Arbib and Marinelli (2005)	discrete	multiple periods	✓	✓	✓	Gear Belts
Gramani and França (2006)	discrete	multiple periods	—	✓	—	Furniture
Ghidini et al. (2007)	discrete	multiple periods*	—	✓	✓	Furniture
Trkman and Gradisar (2007)	discrete	multiple periods	—	✓	—	General
Ouhimmou et al. (2008)	discrete	multiple periods	✓	✓	✓	Wood Processing
Poltroniere et al. (2008, 2016)	discrete	multiple periods	✓	✓	—	Paper
Aktin and Özdemir (2009)	discrete	multiple periods	—	✓	—	Medical Apparatus
Gramani et al. (2009)	discrete	multiple periods	—	✓	✓	Furniture
Malik et al. (2009)	discrete	multiple periods	✓	✓	—	Paper
Gramani et al. (2011)	discrete	multiple periods	—	✓	✓	Furniture
Santos et al. (2011)	discrete	multiple periods*	—	✓	✓	Furniture
Alem and Morabito (2012)	discrete	multiple periods	—	✓	✓	Furniture
Suliman (2012)	discrete	multiple periods	—	✓	✓	Aluminium
Alem and Morabito (2013)	discrete	multiple periods	—	✓	✓	Furniture
Silva et al. (2014)	discrete	multiple periods	✓	✓	—	Furniture
de Athayde Prata et al. (2015)	discrete	multiple periods	—	✓	—	Precast Concrete Beams
Silva et al. (2015)	discrete	one period	✓	✓	—	Textile
Agostinho et al. (2016)	discrete	multiple periods	✓	✓	—	General
Leão and Toledo (2016)	discrete	multiple periods	✓	✓	—	Paper
Melega et al. (2016)	discrete	multiple periods	✓	✓	✓	General
Poldi and de Araujo (2016)	discrete	multiple periods	✓	✓	—	Paper
Vanzela et al. (2017)	discrete	multiple periods	—	✓	✓	Furniture
Wu et al. (2017)	discrete	multiple periods	—	✓	✓	General

() Number of sub-levels.

(*) Use of sub-periods

Table 2.2: Classification According to the Time Dimension and Production Levels

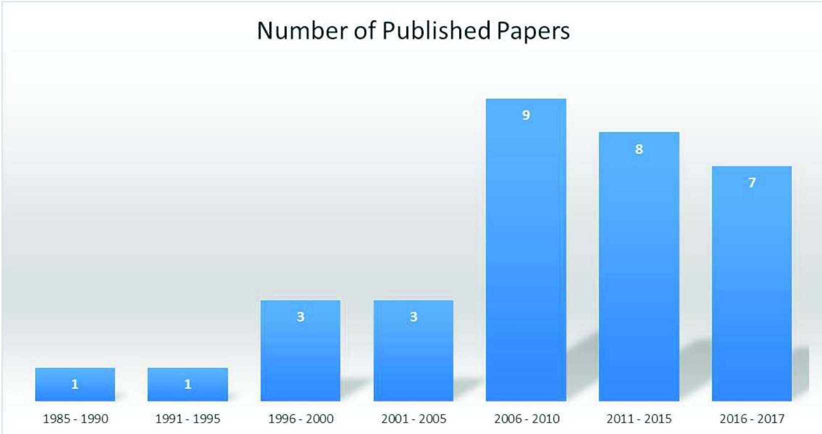


Figure 2.7: Number of Published Papers on Integrated Lot-Sizing and Cutting Stock Problems.

In the literature, the term "integrated lot-sizing and cutting stock model" can refer to many different types and degrees of integration. As discussed, the main types of integration which we observe are the integration between production levels and the integration across time periods. In Table 2.2, with respect to the integration across time periods, we indicate the time horizon (continuous or discrete), the number of time periods (one period, multiple periods or infinite) and the consideration of sub-periods (in case the current time period is split into sub-periods according to some characteristic, such as cutting machine capacity). With respect to the integration between production levels, we indicate which production levels (Level 1, Level 2 or Level 3) are being considered in the problem. In some cases, two sub-levels are considered for the cutting stock problem (Level 2). This is indicated by adding the number of sub-levels between brackets. Finally, we also indicate if the problem originates from an application in a specific industry. From Table 2.2, the variety in the different degrees of integration is immediately clear. Some papers extend the standard cutting stock problem to deal with multiple time periods, while others keep the assumption of one time period but extend the problem to multiple production levels. However, most of the papers deal with an integration of both types, but this is usually restricted to two levels, i. e., either Level 1 and Level 2 are integrated or Level 2 and Level 3 are integrated. Only a limited number of papers deal with multiple time periods and an integration between the three production levels.

Analyzing Table 2.2 in more details, we see that three papers (Krichagina et al., 1998; Nonås and Thorstenson, 2000, 2008) differ from the others due to the type of time horizon used in the model. These three papers consider a continuous time horizon and infinite time periods. Although the models presented in these papers do not entirely fit in the proposed framework, their classification contributes to a more general review of the literature. In

Krichagina et al. (1998), besides dealing with a continuous time horizon, the model also takes into account two levels of integration. The constraints at Level 1 deal with idle processing time in the paper machine, i. e., the decision to turn the paper machine on and off in order to produce the objects, whereas the constraints at Level 2 guarantee that the demand of pieces is met allowing backorder. Nonås and Thorstenson (2000, 2008) propose a model for just one level (Level 2), which models the cutting stock problem using cycle times. It is worth mentioning that the authors classify the model as an integrated lot-sizing and cutting stock problem. However, according to this classification, the model is considered as a cutting stock problem with an integration across time periods, due to the fact that there are neither constraints nor variables related to the production of objects and final products.

Further analyzing the aspect of time periods, two studies (Farley, 1988; Silva et al., 2015) address integrated models which take into account just one time period. The models deal only with the integration between levels. Farley (1988) proposed a model to describe the production process in the textile industry. According to the classification, the model integrates Level 2 and Level 3, where at Level 3 constraints guarantee a minimum and maximum production level in the processes after the cutting and Level 2 consists of a capacitated cutting stock problem with setup. In the model, each cut piece at Level 2 corresponds directly to a final product at Level 3, i. e., there is no assembly process of pieces into final products. In our general model, this implies that there is a one-to-one relationship between the cut pieces and final products, which can be modeled via an appropriate setting of the r_f^p parameter. One of the costs in the objective function is associated with oversupply quantity, which according to the authors represents the idea of holding cost in some form. Also dealing with the production process in a textile industry, Silva et al. (2015) propose a model which integrates Level 1 and Level 2. At Level 1, the planning of the production of objects is done considering a minimum production quantity, a setup for each type of produced object and a maximum quantity of objects that can be used from inventory, guaranteeing a sufficient amount of objects necessary in the cutting process. At Level 2, a cutting stock problem with setup for each cutting pattern is modeled.

Some studies propose models which consider the integration only across multiple periods at Level 2. There is no integration between production levels and the cutting stock problem is extended to multiple periods. Respício and Captivo (2002) proposed a model which is an extension of the formulation presented in Gilmore and Gomory (1961) by considering cumulative demand and initial inventory to meet the demanded pieces. Some aspects are neglected such as holding costs over the whole planning horizon. A capacity constraint is modeled in terms of the total processing time to produce and cut objects in each period. In their model, the capacity is aggregated and belongs to Level 1 and Level 2 of the our classification. However, the production level is inserted at Level 2.

In Gramani and França (2006), a problem from the furniture industry is modeled in which the demand of final products is converted into demand of pieces and the resulting model is a capacitated multi-period cutting stock problem with setups. In their model, a setup is related to the overall production level in a specific period, i. e., if there is some production in a period, a setup must be done in that period, otherwise no setup is done. For a more general problem, Trkman and Gradisar (2007) proposed a model which consists in satisfying order sets in consecutive times periods. The objects which are not cut at the end of a period or which are long enough (within a size limit) to be used later are returned to the warehouse and become available for cutting in future time periods. For each period, a new order must be satisfied without backorder, by either using new objects available in that period or by objects/leftovers from stock. The proposed model consists in a multi-period cutting stock problem with usable leftovers.

In Aktin and Özdemir (2009), a two stage methodology is developed, which is implemented at a medical apparatus manufacturer. In the first stage, a model is used to determine the cutting patterns with minimum waste necessary to meet the demand of pieces. These generated cutting patterns are used in the second stage, which consists in a capacitated cutting stock problem with a setup for each cutting pattern. The whole process is considered for each order and a due date is determined considering the minimum number of days required to fulfill the demand of this order, which is a negotiation between the manufacturer and the customer. If the company cannot fulfill the order completely in time, a penalty cost is incurred for this delay. The total time used to cut the objects according to a cutting pattern and to setup the machine for each cutting pattern cannot exceed the total time available (regular time, over time and excess-of-overtime).

de Athayde Prata et al. (2015) proposed a model for the problem found in the precast concrete beams industry. The authors notice that the problem studied is similar to the multi-period cutting stock problem, where the forms used to model the concrete beams represent the objects and the beams represent the pieces which are demanded by the customers. The loss of production is due to unfilled spaces in the forms. In the model, the number of time periods necessary to meet de demand of pieces (precast beams) is determined by dividing the total demand by the total capacity of the forms and rounded. The multiple time periods are basically used in order to reduce the loss, since the inventory of pieces is not modeled. Therefore, these models do not have an integration between the levels. The integration takes place across periods taking into account the production planning for several periods at the level of the cutting stock problem.

The remainder of the studies propose models that treat both types of integration, i. e., among different time-periods and among different levels. Firstly, we will start with papers that consider the integration between Level 1 and Level 2, i. e., the models have decision variables related to the production (inventory, purchase or setup) and cutting of objects over a planning horizon. The integration between these two levels appears in different applications, such as the copper, paper and furniture industry. Some papers (Reinders, 1992; Correia et al., 2004; Silva et al., 2014; Agostinho et al., 2016; Poldi and de Araujo, 2016) assume that the production of objects is already decided, as a parameter, and the inventory balance constraints at Level 1 just model the planning of objects in stock. In cases where leftovers are allowed (Silva et al., 2014; Agostinho et al., 2016) the same inventory balance constraint is modeled. These models do not take into account a setup for producing objects. Consequently, there are also no setup cost associated with Level 1 production and no capacity constraint at Level 1 is modeled. In our general model, this implies that Level 1 is modeled just using constraint (2.37) via an appropriate setting of the X_t^o variables, which in this case are considered as a parameter.

Hendry et al. (1996), Poltroniere et al. (2008, 2016) and Leão and Toledo (2016) proposed integrated models where at Level 1 a complete capacitated lot-sizing problem with setup is modeled. In Malik et al. (2009), the number of pieces cut from an object according to a cutting pattern is a decision variables, whereas the number of objects cut according to a cutting pattern is an input parameter. A constraint guarantees that the number of objects produced over the whole planning horizon is equal to the number of objects cut in the cutting process and there is no inventory of objects. A capacity constraint limits the number of objects produced in each period at Level 1. In all these papers, a multi-period cutting stock problem at Level 2 is modeled, in which cut pieces can be kept in inventory using an inventory balance constraint. In addition, Reinders (1992) models a capacity constraint in the cutting stock problem and overtime can be used, i. e., if necessary, additional time is available to the cutting process.

It is worth mentioning that Reinders (1992) and Correia et al. (2004) proposed linear programming models composed of three production processes. The first production process corresponds to Level 1 of the classification and the second and third production processes correspond to Level 2 of the classification, i. e., the cutting process is performed twice and consecutively. For example, in Reinders (1992) the production planning of tree trunks is done at Level 1, followed by a crosscutting process to produce the logs and a sawing process to produce the boards, which constitute the final demand in the production planning. The crosscutting process and the sawing process correspond to cutting processes belonging to Level 2 of the classification and are operated sequentially. Correia et al. (2004) do not present a mathematical model of the process described, but only describe the general ideas of the constraints and objective function. In Agostinho et al. (2016) and Poldi and de Araujo (2016), the models described are postulated as a multi-period cutting stock problem. However according to this classification, the models are considered as an integrated lot-sizing and cutting stock problem, due to the fact that there are decision variables related to the inventory and the cutting of the objects.

Considering papers that address integrated problems at Level 2 and Level 3, the main models are based on applications in the furniture industry, with the exceptions of Suliman (2012) which discusses an application in the aluminum industry and Wu et al. (2017) which analyse a model for a general application. The papers differ from each other mainly at Level 3, where in some cases (Gramani et al., 2011; Suliman, 2012; Vanzela et al., 2017), the production planning of the final products takes place using only the demand balance constraint of final products. In others studies, there is the addition of a capacity constraint (Ghidini et al., 2007; Santos et al., 2011; Alem and Morabito, 2013), or a setup cost (Gramani et al., 2009). The capacity constraint at Level 3 represents the capacity of the production process necessary to obtain the final products, such as drilling or assemblying. In Alem and Morabito (2012) and Wu et al. (2017), the standard lot-sizing problem is modeled at Level 3. Wu et al. (2017) present a formulation which is similar to the one proposed by Gramani et al. (2009), and consider a setup time related to the assembly of final products. In some papers, there are further restrictions with respect to the inventory accumulation. A number of studies (Ghidini et al., 2007; Gramani et al., 2009; Alem and Morabito, 2012, 2013; Wu et al., 2017) impose that inventory accumulation is allowed at the level of the final products (Level 3), but not at the level of the cut pieces (Level 2). Consequently, if a piece is used in the production process at Level 3, it must be cut at Level 2 in the same time period. In some studies (Gramani et al., 2009, 2011; Vanzela et al., 2017), a capacity constraint is modeled at Level 2, while in others (Ghidini et al., 2007; Suliman, 2012; Alem and Morabito, 2013) there is also the addition of a setup cost or setup time. Only in Santos et al. (2011) a multi-period capacitated cutting stock problem with setups takes place. In several models, additional constraints and variables are needed to model the peculiarities of the production process. For example, in the furniture industry (Santos et al., 2011; Vanzela et al., 2017), safety inventory, overtime and limitations regarding to the saw cycle on the cutting machines need to be taken into account in order to obtain a practical feasible solution. These additional constraints fall outside of the scope of our general model.

Only a few papers in the literature address the integration between the three production levels. However, these models typically represent simplifications when compared to the *GILSCS* model proposed in this thesis, since they do not include all the features present in the *GILSCS* model. Arbib and Marinelli (2005) presented a case study that arises in the production of gear belts. At Level 1, a lot-sizing problem models the trade-off between objects inventory and objects delivery to the cutting process. The quantity of objects that must be provided to the cutting process cannot be less than what is needed and, once sent to the cutting process, a setup cost associated with the delivery of material is imposed. Level 2 consists basically of a multi-period cutting stock problem with a capacity constraint. The cut pieces can either be transformed directly into a final product, or they can be assembled with other pieces to form a final product. The stock of pieces is limited by an inventory capacity. At Level 3, the production planning ensures that the demand of each final product is satisfied in each period directly from production, without inventory.

Ouhimmou et al. (2008) studied the processes from a furniture company. The whole system observed in the company, which consists of the activities of sawing, drying in a kiln and transportation, is not fully modeled by the authors in the presented model. At Level 1, the planning of the procurement of objects takes place, with a supplier capacity constraint and a flow balancing constraint at each sawmill. Level 2 consists of a capacitated cutting stock problem with a setup for each object processed at the sawmill. The last process is the drying process and it is modeled as a lot-sizing problem. The drying process involves transforming green wood boards (pieces) into dry wood boards (final products). Each piece passes through the drying process and subsequently becomes a final product. Constraints ensure that the capacity in the kiln is not violated by the production of pieces and a setup is necessary for each piece in the drying process. The demand of customers needs to be met either from production or from purchase on the market.

In Melega et al. (2016), several models with a general application that integrate the lot-sizing problem and cutting stock problem at different levels are proposed. One of the models is composed of three levels, where at Level 1, a demand balance constraint of objects is modeled with a parameter that limits the number of objects available in each period for each type of object, i. e., the production of objects is considered as a parameter and the decisions variables are those related only to the planning of the objects in stock. At Level 2, a cutting stock problem takes place, where a constraint ensures that a sufficient amount of pieces is cut to meet the planned production. Cut pieces cannot be kept in inventory and must be processed at Level 3 in the same time period. Each piece at Level 2 corresponds directly to a final product at Level 3 and this level is modeled by a capacitated lot-sizing problem with setup.

In conclusion, we observe that the proposed classification allows to standardize the concept of a multi-period cutting stock problem and integrated lot-sizing and cutting stock problem based on the dimensions of production levels and time periods. We classify as a multi-period cutting stock problem those models that consider more than one time period, i. e., consider the integration across time periods, but which do not consider more than one production level (Respício and Captivo, 2002; Gramani and França, 2006; Trkman and Gradisar, 2007; Aktin and Özdemir, 2009; de Athayde Prata et al., 2015). There are those models that we classify as integrated lot-sizing and cutting stock problems, i. e. which take into account the integration between production levels. They can consider just one time period (Farley, 1988; Silva et al., 2015) or multiple time periods, and in the latter case, they consider both types of integration presented in the *GILSCS* problem.

Regarding to these integrated problems, i. e., which consider both types of integration, some studies integrates Level 1 and Level 2 (Reinders, 1992; Hendry et al., 1996; Correia et al., 2004; Poltroniere et al., 2008, 2016; Malik et al., 2009; Silva et al., 2014; Agostinho et al., 2016; Leão and Toledo, 2016; Poldi and de Araujo, 2016), other Level 2 and Level 3 (Ghidini et al., 2007; Gramani et al., 2009, 2011; Santos et al., 2011; Alem and Morabito, 2012, 2013; Suliman, 2012; Vanzela et al., 2017; Wu et al., 2017), which consists mostly of applications in the furniture industry. Only few of them deal with Level 1, Level 2 and Level 3 (Arbib and Marinelli, 2005; Ouhimmou et al., 2008; Melega et al., 2016). In general, the classification of the literature showed that most of the studies in the literature consists of extensions or simplification of the general integrated lot-sizing and cutting stock problem. This fact indicates some possible applications of the general integrated model in different industrial environments, such as, furniture, paper industry and aluminium, among others.

2.2.3 Discussion of Further Operational Aspects

In this section, we discuss some further particularities of the various models proposed in the literature, such as, the use of inventory, capacity and setups. In Table 2.3, we discuss some features related to the objects which are modeled in the lot-sizing problem at Level 1 (third column). With respect to the cutting stock problem at Level 2, we indicate the dimensionality of the problem and features regarding the pieces (second and fourth columns, respectively). In the last column, we report the features with respect to the final products in the lot-sizing problem at Level 3.

	Dimensionality	Objects	Pieces	Final Products
Farley (1988)	two-dimensional	several types (materials, colors)	oversupply / undersupply	no inventory (directly pieces)
Reinders (1992)	one/two-dimensional	several types (classes) / inventory objects availability	independent demand / inventory out of stock / purchase (limited)	—
Hendry et al. (1996)	one-dimensional	several types (diameters) / inventory (limited)	inventory	—
Krichagina et al. (1998)	two-dimensional	one type / no inventory	inventory / backorder	—
Nonås and Thorstenson (2000, 2008)	two-dimensional	one type	inventory cost	—
Respício and Captivo (2002)	one-dimensional	several types (families)	initial inventory	—
Correia et al. (2004)	one/two-dimensional	several types (grades) / no inventory	inventory / independent demand	—
Arbib and Marinelli (2005)	one-dimensional	one type / inventory	inventory (limited)	no inventory
Gramani and França (2006)	two-dimensional	one type	inventory	—
Ghidini et al. (2007)	two-dimensional	several types (thickness)	no inventory	inventory / extra demand
Trkman and Gradisar (2007)	one-dimensional	several types (lengths) / leftover	no inventory / one order per period	—
Ouhimmou et al. (2008)	two-dimensional	several types (qualities) /inventory transportation / supplier capacity	inventory / transportation	inventory / purchase transportation (directly pieces)
Poltroniere et al. (2008, 2016)	one-dimensional	several types (grades) / inventory	inventory	—
Aktin and Özdemir (2009)	one-dimensional	one type / object availability	no inventory	—
Gramani et al. (2009)	two-dimensional	one type	no inventory	inventory
Malik et al. (2009)	one-dimensional	several types (grades) / no inventory	inventory	—
Gramani et al. (2011)	two-dimensional	one type	inventory	inventory
Santos et al. (2011)	two-dimensional	several types (thickness)	inventory safety inventory level	inventory safety inventory level
Alem and Morabito (2012)	two-dimensional	one type	no inventory	stochastic demand / backlog inventory (limited)
Suliman (2012)	one-dimensional	one type / inventory cost object availability	inventory / purchase cost	inventory / backlog cost
Alem and Morabito (2013)	two-dimensional	one type	no inventory	uncertain demand / backlog inventory
Silva et al. (2014)	two-dimensional	one type / inventory / leftover	inventory	—
de Athayde Prata et al. (2015)	one-dimensional	several types (lengths)	no inventory	—
Silva et al. (2015)	two-dimensional	several types (materials) minimum production limited objects used from stock	no inventory / upper and lower bound for demand	—
Agostinho et al. (2016)	one-dimensional	several types (lengths) / inventory leftover (limited) objects and leftover availability	inventory	—
Leão and Toledo (2016)	one-dimensional	several types (grades) / inventory	inventory	—
Melega et al. (2016)	one-dimensional	several types (lengths) inventory / object availability	no inventory	inventory (directly pieces)
Poldi and de Araujo (2016)	one-dimensional	several types (lengths) inventory / object availability	inventory	—
Vanzela et al. (2017)	two-dimensional	several types (thickness)	inventory	inventory / safety inventory
Wu et al. (2017)	one-dimensional	one type	no inventory	inventory

Table 2.3: Level Features

	Capacity Levels			Capacity	Setup
	Level 1	Level 2	Level 3		
Farley (1988)	—	✓	✓	total of cut material total time (several machines)	setup cost (cut of object)
Reinders (1992)	—	✓	—	total time with over time	—
Hendry et al. (1996)	✓	—	—	total time	setup constraint (object) setup time (object)
Krichagina et al. (1998)	✓	—	—	idle time	setup cost (machine shutdown)
Nonås and Thorstenson (2000, 2008)	—	—	—	—	setup cost (startup / pattern)
Respício and Captivo (2002)	✓	✓	—	total production and cutting time (aggregate)	—
Correia et al. (2004)	—	—	—	—	—
Arbib and Marinelli (2005)	—	✓	—	total of cut material	setup cost (object delivery)
Gramani and França (2006)	—	✓	—	total time	setup cost (cutting machine)
Ghidini et al. (2007)	—	✓	✓	total time	setup cost (pattern)
Trkman and Gradisar (2007)	—	—	—	—	—
Ouhimmou et al. (2008)	—	✓	✓	total time / total amount (volume) (several plants)	setup cost / time (cutting machine) setup cost (machine)
Poltroniere et al. (2008, 2016)	✓	—	—	total amount of material (ton) (several machines)	setup cost / setup in capacity (object)
Aktin and Özdemir (2009)	—	✓	—	total time with overtime and excess of overtime	setup cost / time (pattern)
Gramani et al. (2009)	—	✓	—	total amount of cut material	setup cost (final product)
Malik et al. (2009)	✓	—	—	total time	setup cost / time (object)
Gramani et al. (2011)	—	✓	—	total time	—
Santos et al. (2011)	—	✓	✓	saw cycle /total time (several machines)	setup cost / time (pattern)
Alem and Morabito (2012)	—	—	✓	total time with overtime (limited)	setup cost (final product)
Suliman (2012)	—	✓	—	total number of cuts	setup cost (pattern)
Alem and Morabito (2013)	—	✓	✓	total time with overtime	uncertain setup time (pattern) setup constraint (pattern)
Silva et al. (2014)	—	—	—	—	—
de Athayde Prata et al. (2015)	—	—	—	—	—
Silva et al. (2015)	—	—	—	—	setup constraint (object/pattern)
Agostinho et al. (2016)	—	—	—	—	—
Leão and Toledo (2016)	—	✓	—	total time (several machines)	setup cost / time (object)
Melega et al. (2016)	—	—	✓	total time	setup cost / time (final product)
Poldi and de Araujo (2016)	—	—	—	—	—
Vanzela et al. (2017)	—	✓	—	saw cycles	—
Wu et al. (2017)	—	—	✓	total time	setup cost / time (final product)

Table 2.4: Classifications According to Capacity-Related Features

Analyzing the features of the objects at Level 1, we can see from Table 2.3 that less than half of the studies consider only one type of object, whereas most of the studies address multiple types, which correspond to different materials, lengths, colors, classes, diameters, families, grades, thicknesses and qualities. In some studies, the objects at Level 1 can be available in a limited number (Reinders, 1992; Aktin and Özdemir, 2009; Suliman, 2012; Agostinho et al., 2016; Melega et al., 2016; Poldi and de Araujo, 2016). In other studies, that consider usable leftover, there are also residual objects available in addition to the limited number of standard objects (Trkman and Gradisar, 2007; Silva et al., 2014; Agostinho et al., 2016).

In most of the papers that considers Level 1, i. e., there is a decision variable related to this level, the inventory of objects is taken into account in the demand balance constraint. In some studies (Hendry et al., 1996; Silva et al., 2015) the number of objects that can be stored or used from stock is limited, while in other studies (Krichagina et al., 1998; Correia et al., 2004; Malik et al., 2009) no inventory of objects is allowed at Level 1. Ouhimmou et al. (2008) consider additional constraints related to the transportation of the objects to the cutting process and the suppliers capacity with respect to the acquisition of the objects. In Silva et al. (2015), a minimum production quantity is imposed if there is any production of objects. Suliman (2012) does not model a demand balance constraint of objects. However, a holding cost is related to the difference between the number of objects used in the cutting process and the number of objects available in each period (parameter). In Poldi and de Araujo (2016), the number of objects acquired in each period is considered as a parameter in a first model and also as an additional decision variable in a second model, in order to enable a more realistic decision. As discussed before, in some of the papers that include Level 1 (Reinders, 1992; Agostinho et al., 2016; Melega et al., 2016; Poldi and de Araujo, 2016), it is assumed that the production of objects is already decided up front. As such, the demand balance constraints are only related to the planning of objects in stock.

Table 2.3 indicates that the cutting problems at Level 2 deal with either one or two dimensions, and the latter appears more frequently, due to the type of the applications. For instance, there are many papers that present applications in the furniture industry. Using our search criteria, we are not able to find any study which deal with a three-dimensional cutting stock problem.

In some applications, where the cutting stock problem arises at two sub-levels (Reinders, 1992; Correia et al., 2004), the cutting process changes in dimensionality from one sub-level to the other sub-level. In Wu et al. (2017), although a two-dimensional cutting process arises, one of the piece dimensions (width) is considered fixed as the width of the object, in this way, we classified it as a one-dimensional cutting process. For most of the studies in the literature, the demand balance constraint of pieces is modeled at Level 2. The inventory of pieces is modeled either by oversupply (Farley, 1988), initial inventory (Respício and Captivo, 2002) or inventory variables (Reinders, 1992; Hendry et al., 1996; Krichagina et al., 1998; Correia et al., 2004; Arbib and Marinelli, 2005; Gramani and França, 2006; Ouhimmou et al., 2008; Poltroniere et al., 2008, 2016; Malik et al., 2009; Gramani et al., 2011; Santos et al., 2011; Suliman, 2012; Silva et al., 2014; Agostinho et al., 2016; Leão and Toledo, 2016; Vanzela et al., 2017). Due to the specific environment, some practical applications need additional constraints to model the inventory limits (Arbib and Marinelli, 2005), safety inventory levels (Santos et al., 2011), upper and lower bound for pieces demand (Silva et al., 2015) or the transportation of the pieces (Ouhimmou et al., 2008).

In order to meet the demand of pieces, some studies consider the possibility of externally purchasing the pieces in the demand balance constraints (Reinders, 1992) or an additional cost in the objective function. Other strategies, such as undersupply (Farley, 1988), out of stock (Reinders, 1992) or backorders (Krichagina et al., 1998) are also considered in the models to meet the pieces demand. Only few papers consider independent demand of pieces at Level 2 (Reinders, 1992; Correia et al., 2004). In the studies where the inventory of pieces is not incorporated at Level 2 (Ghidini et al., 2007; Trkman and Gradisar, 2007; Aktin and Özdemir, 2009; Gramani et al., 2009; Alem and Morabito, 2012, 2013; Silva et al., 2015; Melega et al., 2016; Wu et al., 2017), a piece necessary in the assembly or further processing must be cut in the same time period as the production of the corresponding final product.

As mentioned before, in some applications (textile, wood processing furniture), there is a one-to-one relationship between a cut piece and a final product. After being cut, the pieces undergo some transformation processes to become final products, but there is no assembly process (Farley, 1988; Ouhimmou et al., 2008; Melega et al., 2016). However, most of the papers that model both Level 2 and Level 3 consider an assembly structure, in which cut pieces correspond to components which are assembled into a final product. In most of the studies that consider Level 3, the demand balance constraints contain only the decisions related to the production and stocking of final products (Ghidini et al., 2007; Ouhimmou et al., 2008; Gramani et al., 2009, 2011; Santos et al., 2011; Alem and Morabito, 2012; Suliman, 2012; Alem and Morabito, 2013; Melega et al., 2016; Vanzela et al., 2017; Wu et al., 2017). In a few other papers (Ouhimmou et al., 2008; Alem and Morabito, 2012, 2013), purchase and backlog variables are added in the balance constraints in order to meet the demand of final products.

In Suliman (2012), there is a cost in the objective function related to the non-fulfilled demand of final products. Similar to the extensions at Level 1 and Level 2, safety inventory and inventory limits can be added at Level 3 (Santos et al., 2011; Alem and Morabito, 2012; Vanzela et al., 2017). The complexity at Level 3 is increased if a stochastic environment (with respect to demand and production costs) is taken into account (Alem and Morabito, 2012, 2013).

In Table 2.4, we provide a further analysis in terms of capacity constraints and setups. Concerning the capacity, we indicate the presence or absence of a capacity constraint at each of the three levels and report the features of this capacity in terms of resource consumption. Regarding to the setup, we evaluate the type of the setup considered (cost and/or time) and the type of product the setup refers to (i. e., a setup related to the object, cutting machine, pattern or final product).

We observe that capacity constraints are most frequently imposed at Level 2, followed by Level 3 and then by Level 1. In the studies which consider more than one level, the capacity constraint is generally imposed at just one level. However, some papers (Farley, 1988; Respício and Captivo, 2002; Ghidini et al., 2007; Ouhimmou et al., 2008; Santos et al., 2011; Alem and Morabito, 2013) consider a capacity constraint at more than one level, mostly in models which integrate Level 2 and Level 3. None of the papers has a capacity constraint at each level, as it is the case of the *GILSCS* model.

The resource consumption in the capacity constraint is stated mostly in terms of total time availability. However, the capacity constraint can be imposed in different ways according to the specific application and the level that it is related to, such as, the total amount of produced material (Poltroniere et al., 2008, 2016), total amount of cut material (Farley, 1988; Arbib and Marinelli, 2005; Gramani et al., 2009), saw cycles (Santos et al., 2011; Vanzela et al., 2017) and number of cuts (Suliman, 2012). Krichagina et al. (1998) consider a capacity constraint that calculates the idleness of the machine with respect to the production of objects at Level 1 and guarantees that it is always positive. In Respício and Captivo (2002), the capacity constraint is modeled in terms of the total processing time of produced and cut objects in each period. In this way, an aggregate capacity is modeled considering Level 1 and Level 2 simultaneously. In papers where a capacity is considered at more than one level, the resource consumption is not necessarily modeled in the same way at each level (Farley, 1988; Ouhimmou et al., 2008; Santos et al., 2011). In some studies (Reinders, 1992; Aktin and Özdemir, 2009; Alem and Morabito, 2012, 2013), overtime is allowed, whereas in other (Farley, 1988; Poltroniere et al., 2008, 2016; Santos et al., 2011; Leão and Toledo, 2016), several machines are used to produce the items.

It is worth to mention that the addition of setups in a model considerably increases its complexity. Faced with this challenge, some studies consider the occurrence of setups in the model, but only few of them consider both setup costs and setup times (Ouhimmou et al., 2008; Aktin and Özdemir, 2009; Malik et al., 2009; Santos et al., 2011; Leão and Toledo, 2016; Melega et al., 2016; Wu et al., 2017). In some papers, the setup is incorporated in the problem through the consideration of a setup time in the capacity constraint and/or a setup constraint, but no setup cost in the objective function is addressed (Hendry et al., 1996; Alem and Morabito, 2013; Silva et al., 2015). Different classes of setup can be found, such as setups related to the cutting of objects (Farley, 1988), a machine shutdown in objects production (Krichagina et al., 1998), a startup of pieces production (Nonås and Thorstenson, 2000, 2008), the delivery of objects to the cutting process (Arbib and Marinelli, 2005) and the use of the cutting machine (Gramani and França, 2006; Ouhimmou et al., 2008). In Poltroniere et al. (2008, 2016), each machine on which the objects are produced has a capacity constraint in terms of tons of objects, which takes into account the quantity of objects produced and the waste of objects due to the changes in the type of object (setup). A stochastic setup time related to the use of a cutting pattern further increases the complexity of the model (Alem and Morabito, 2013).

2.3 Conclusions and New Research Directions

In this thesis, we are interested in the integration of two known problems from the literature, which are the lot-sizing problem (*LSP*) and the cutting stock problem (*CSP*). A general integrated lot-sizing and cutting stock problem (*GILSCS*), which considers two types of integration, is proposed. The *GILSCS* model incorporates several aspects found in practice and enables us to classify the current literature and give directions for future research that addresses integrated problems.

The general integrated model is composed of three levels. At the final level (Level 3) we have a lot-sizing problem for the production of final products. At the intermediate level (Level 2) we have a cutting stock problem based on the idea of cutting patterns and at the first level (Level 1), we have a lot-sizing problem related to the production of objects. The model incorporates some features which are inspired by general practical observations, and enables us to classify the current literature in this field. The classification is based on two aspects: the integration across multiple time periods and the integration between production levels. The integration across time periods comes from the possibility to hold items in inventory. Other features are also evaluated, such as the dimensionality in the cutting process, and the capacity and setup structure.

The classification of the literature shows that most of the studies consider the integration across time periods and the integration between production Level 2 and Level 3. The large number of papers which integrate Level 2 and Level 3 is due to the practical applicability of this type of model in some industries such as the furniture industry. Another relevant feature inherited from the focus on practical applications is the dimensionality of the cutting problem, which is predominantly a two-dimensional cutting stock problem. The capacity constraint is often employed at just one level and is typically computed in terms of total time consumption. Only a few studies consider both setup cost and setup time in their models. Setups can relate to various aspects such as the cutting of objects, a machine shutdown in the objects production, the startup of the production, the delivery of objects to the cutting process or the use of the cutting machine. An uncertain environment related to the demand, setup time and production costs is rarely considered in the literature of integrated problems.

As a conclusion, the classification indicates that, even though many papers in the current literature analyse an integrated lot-sizing and cutting stock problem, they vary widely with respect to the level of integration on the time and production level dimensions. Furthermore, our analysis indicates that the current models also consider varying assumptions with respect to the inventory, the production capacities and the setups. Therefore, we are able to say that the models studied so far correspond mainly to simplifications or simple extensions of the general integrated lot-sizing and cutting stock problem (*GILSCSL*) and this highlights the value of a comprehensive model, as formulated in this thesis.

After analyzing and classifying the literature of integrated lot-sizing and cutting stock problems, some insights and opportunities for future research are observed and discussed next.

The bulk of the research on this topic has been done fairly recently, i. e., in the past decade. There is still a lot of work to be done on the integrated problems. Several extensions can be considered. In the multi-level problems, there might be different characteristics of the production environment that need to be incorporated for each level specifically, such as, supplier capacity, backlog, out of stock, safety inventory, and others that we present as follows. In addition to the *GILSCS* model, there might be an integration with other processes, such as the supplier selection, in which the choice of different suppliers may be based on the quality, price and speed of the orders, or the routing and packing/loading of the final products to the customers.

A direct extension of the *GILSCS* model which is common in practice, is the use of multiple machines to produce the customers' order. This can arise at Level 1 and Level 3 of the model considering multiple machines used to produce different objects and final products. At Level 2, multiple machines may also arise with the problem of assigning orders to parallel or sequential cutting machines (Menon and Schrage, 2002) or the allocation of cutting patterns to specific machines (Giannelos and Georgiadis, 2001). Some of the extensions discussed for the *CSP*, such as reusable leftovers (Cherri et al., 2014), may be of interest for insertion in the integrated model in order to better describe specific industry practices.

The optimization of two or more time periods, i. e., the integration across time periods, is typically done in production planning problems, such as the capacitated lot-sizing problem. However, the standard cutting stock problem only considers one time period and its extension to multiple periods is little explored in the literature (Trkman and Gradisar, 2007; Poldi and de Araujo, 2016). With the use of multiple time periods, the demand and supply of materials arise in all time periods and pieces produced in excess as well as material unused at the end of a time period may be used at a later period.

Capacity limitations are important in real life problems and should be taken into account in the models. Henn and Wäscher (2013) notice that in the cutting stock literature no models with setups exist which consider capacity constraints, i. e., there is no model that takes into account the limitations of the capacity in the cutting stage, considering the cutting and the setup processes. Therefore, a multi-period cutting stock problem with capacity constraint as addressed at Level 2 in the *GILSCS* model is not explored in the literature to model cutting processes and provides an interesting and relevant avenue for future research.

The capacity constraint may not only be related to the production time of the cutting process, but may also be related to other aspects, such as the total amount of area to produce the final products (in two-dimensional problems) and saw cycles (Santos et al., 2011; Vanzela et al., 2017). In some industrial applications, delivering the orders on time can be far more important than reducing the resulting waste and the cost of cut objects. Models that consider due dates in the formulation better describe the need of the industry in such a case (Arbib and Marinelli, 2014; Reinertsen and Vossen, 2010; Arbib and Marinelli, 2017).

Beyond the various objectives discussed previously for both problems, an alternative approach is a multi-criteria optimization (Wäscher, 1990). In a multi-criteria optimization approach, a good solution is not the result of the optimization of one criterion (such as total cost), but constitutes a good compromise between several criteria. The need for such an alternative approach can arise from the difficulty to obtain real values for the costs associated in the objective function.

The cutting plan described by the current models provides a set of cutting patterns and the corresponding frequencies of the patterns. However, in some settings, it becomes necessary to determine a production plan that also indicates the optimal sequence of the cutting patterns. The inclusion of the pattern sequence in the model may be related to a specific objective function, usually related to a practical application, such as, the minimization of the knives changes, where each insertion and removal of knives takes time to be processed; the minimization of open stacks (i. e. the number of mounting compartments around the cutting machine), in which a stack remains open until the last cutting pattern that contains the piece of the stack is cut; the minimization of the order spread, which refers to the number of open stacks during the cutting process (Foerster and Wäscher, 1998; Rinaldi and Franz, 2007; Garraffa et al., 2016; Yanasse and Lamosa, 2007). It is worth to mention that this sequencing problem which emerges as an extension at Level 2 can also be relevant at Level 1 and Level 3 in the production of objects and final products. As the lot-sizing problems at Level 1 and Level 3 also consider the production of several items, the sequence in which these items are produced can influence the quality, total cost, and even the feasibility of the solution. In such a case, an integrated lot-sizing and scheduling problem with sequence-dependent setups arises at Level 1 and Level 3

(Drexl and Kimms, 1997; Copil et al., 2016).

As said before, the integrated problems so far discussed in this literature review take into account one or two-dimensional cutting stock problems. Another extension for the general integrated problem proposed is the consideration of a three dimensional cutting stock problem, or even its extension to consider packing problems instead of the cutting stock problem (Molina et al., 2016).

A final important aspect for future research is the consideration of uncertainty. Few papers in the literature of integrated lot-sizing and cutting stock problems address optimization problems with uncertain parameters. Alem and Morabito (2012) employed robust optimization tools to derive robust models for production planning in the furniture industry, when production costs and products demands are uncertain parameters. Alem and Morabito (2013) proposed a two-stage stochastic optimization model under stochastic demand and setup times. Beraldi et al. (2009) consider the case of demand uncertainty for a cutting stock problem.

In conclusion, we see that there is no shortage of challenging and relevant avenues for future research. The resolution of industrial problems will continue to be an important source of inspiration to further refine the models.

Chapter 3

The General Integrated Lot-Sizing and Cutting Stock Problems: Solution Methods

In this chapter, we present solution methods to the general integrated lot-sizing and cutting stock (*GILSCS*) problem proposed in Chapter 2. Due to the fact that integrated lot-sizing and cutting stock problems are derivative from two problems in the literature which are classified as NP-hard (Maes et al., 1991; McDiarmid, 1999; Yanasse and Limeira, 2006), the approaches proposed in this chapter, in order to search for a feasible solution for the general integrated problem, are based on solution methods which offer a satisfactory solution quality with reasonable computational effort, such as, easily approximation algorithms and/or faster heuristic procedures. As mentioned before, we are interested in heuristic approaches that overcome the difficulties faced in the cutting stock problem and takes advantages of multi-level structures presents in the lot-sizing problem. The solution methods are also proposed as generic as possible in order to be able to be extended to other integrated problems.

This chapter is organized as follows. In Section 3.1, we present a review and classification of the solution methods in the literature to the integrated lot-sizing and cutting stock problems. We consider the same studies previously classified, according to their models in Chapter 2, and we point out the main strategies addressed in these studies in order to solve the integrated problem.

The solution methods proposed for the general integrated model are presented in Section 3.2. We consider three solution strategies based on column generation and relax-and-fix procedures. In order to obtain the matrix of cutting patterns at Level 2 of the general integrated model, the column generation is applied as a first step in all the procedures and then, integer programming and relax-and-fix procedure are addressed in an attempt to find a feasible solution to the general integrated problem. We also consider two selection strategies of the variables to decompose the problem, which are according to the time horizon and the final products.

A computational study is presented in Section 3.3, which consists of data generation, setting of the parameters to the relax-and-fix procedure and computational results. A whole set of data is generated to the general integrated problem based on known data set in the literature. The computational study is performed around four analysis, which are in terms of the size of the problem, the length of pieces, the capacity constraints and the costs in the objective function. Finally, conclusions and future research are presented in Section 3.4.

The main contributions of this chapter are a literature review and classification of the solution methods to integrated lot-sizing and cutting stock problems, the development of solution methods based on column generation and relax-and-fix procedures and the complete set of data to this type of the problem, with three levels and multiple time periods.

3.1 Solution Methods: a Literature Review

In this section, a literature review and classification of the solution methods to the integrated lot-sizing and cutting stock problems are discussed in order to point out the main strategies used in this field.

In the cutting stock problems, the difficulty to obtain an optimal solution comes from the large number of possible cutting patterns and the cutting patterns frequency be an integer number, which considerably increases the difficulty in solving the problem. One of the strategies largely used and probably the best known to overcome large number of variables is the column generation procedure. Gilmore and Gomory (1961, 1963, 1965) propose relaxing the integrality of the variables and solving the resulting linear programming problem with column generation technique. The columns (cutting patterns) are generated by solving a subproblem and attractive columns are added to the master problem interactively to improve the current solution. For the one-dimensional cutting stock problem, the subproblem is an integer knapsack problem (Gilmore and Gomory, 1961, 1963; Soma and Toth, 2002). Considering higher dimensions for subproblems, other strategies have also been proposed in the literature (Christofides and Whitlock, 1977; Arenales and Morabito, 1995; Yanasse and Katsurayama, 2005). Typically, the solution of the relaxed master problem is fractional and an integer solution can be obtained using heuristics based on approximate fractional solution and rounding procedures (Stadtler, 1990; Wäscher and Gau, 1996; Poldi and Arenales, 2009) or by a branch-and-price procedure, which embeds the column generation procedure within a branch-and-bound approach (Vance et al., 1994; Degraeve and Peeters, 2003; Belov and Scheithauer, 2006; Alves and Valério de Carvalho, 2008)

The main difficulties in solving lot-sizing problems are related to the integrality of the setup variables. In the literature, many solution approaches to solve the lot-sizing problem are proposed. They can be generally divided into two areas: exact and heuristic solution approaches. The exact solution approaches are related to mathematical programming, which includes valid inequalities and strong reformulation, whereas the heuristic solution approaches involve neighborhood search and decomposition strategies. According to Akartunali and Miller (2009), heuristic approaches regarding to decomposition ideas can be grouped as: (i) Lagrangian-based decomposition; (ii) Coefficient modification; (iii) Forward/Backward schemes and Relax-and-Fix; (iv) Local search.

Considering the integrated lot-sizing and the cutting stock problem, a literature review and classification of the solution approaches to solve the problems is presented. The solution methods are classified according to two main aspects, which are related to the difficulties from both problems embedded in the integrate problem, i. e., the high number of cutting patterns and the integrality of the decision variables (cutting patterns and setups). Firstly, the solution methods are analyzed takeing into account the manners in which the cutting patterns are addressed in the solution methods and in a second analysis, the solution methods are investigated over the strategies employed to find a feasible solution to the mixed-integer problem.

Table 3.1 shows a summary of all the papers considered in this literature review and their classification according to the solution methods addressed to solve the integrated lot-sizing and cutting stock problem. The classification, in terms of cutting patterns generation, considers a priori and iteratively cutting patterns generation. The strategies used in an attempt to find a feasible solution to the integrated problem are classified as exact and heuristic solution approaches. The benchmark used to compare the approaches and the indication if the problem originates from an application in a specific industry are also presented. For a further review and classification, in terms of mathematical models, of the literature to integrated problems, see Chapter 2.

	Application	Cutting Pattern Generation		Solution Methods		Benchmarks
		A Priori	Iteratively	Exact	Heuristic	
Farley (1988)	Textile (clothing)	✓			✓	Integer Programming
Reinders (1992)	Wood Processing		✓	✓		—
Hendry et al. (1996)	Copper	✓			✓	Other Approaches; Practical Results
Krichagina et al. (1998)	Paper	✓			✓	Other Approaches
Nonås and Thorstenson (2000, 2008)	Off-road truck	✓	✓		✓	Other Approaches
Respício and Captivo (2002)	Paper		✓	✓		—
Correia et al. (2004)	Paper	✓			✓	Integer Programming
Arbib and Marinelli (2005)	Gear Belts	✓		✓		Models/Heuristics based on practice
Gramani and França (2006)	Furniture		✓	✓	✓	Models/Heuristics based on practice Integer Programming
Ghidini et al. (2007)	Furniture		✓	✓		—
Trkman and Gradisar (2007)	General		✓	✓		Other Approaches
Ouhimmou et al. (2008)	Wood Processing Furniture	✓			✓	Practical Results; Integer Programming
Poltroniere et al. (2008)	Paper		✓		✓	Other Approaches
Aktin and Özdemir (2009)	Medical Apparatus	✓			✓	Other Approaches
Gramani et al. (2009)	Furniture		✓		✓	Models/Heuristics based on practice
Malik et al. (2009)	Paper		✓		✓	Models/Heuristics based on practice
Gramani et al. (2011)	Furniture		✓	✓	✓	Models/Heuristics based on practice
Santos et al. (2011)	Furniture	✓			✓	Other Approaches
Alem and Morabito (2012)	Furniture	✓			✓	Integer Programming
Suliman (2012)	Aluminium		✓		✓	Practical Results
Alem and Morabito (2013)	Furniture	✓			✓	Integer Programming
Silva et al. (2014)	Furniture		✓	✓	✓	Models/Heuristics based on practice
de Athayde Prata et al. (2015)	Precast Concrete Beams		✓		✓	Other Approaches
Silva et al. (2015)	Textile		✓		✓	Other Approaches
Agostinho et al. (2016)	General		✓		✓	Other Approaches
Leão and Toledo (2016)	Paper		✓	✓	✓	Other Approaches
Melega et al. (2016)	General		✓	✓	✓	Integer Programming
Poldi and de Araujo (2016)	Paper		✓	✓	✓	Integer Programming
Vanzela et al. (2017)	Furniture		✓		✓	Models/Heuristics based on practice

Table 3.1: Classification of the Solution Methods for Integrated Problems.

From Table 3.1, we can observe that most of the studies consider a iteratively cutting pattern generation and as expected, most of the studies address heuristic methods in order to solve the integrated problem, since the two problems, as we previously said, are classified as NP-Hard. In some studies (Gramani and França, 2006; Gramani et al., 2011; Silva et al., 2014; Leão and Toledo, 2016; Melega et al., 2016; Poldi and de Araujo, 2016) both types of strategies, exact and heuristic, are employed to solve the integrated problem and compared in the computational results.

The studies are mostly inspired by practical applications, hence, the computational results of the solution methods proposed in these studies are compared in different ways, in order to point out the efficiency and quality of the solution approaches. In some cases, the results are compared with the practical results from the company (Hendry et al., 1996; Ouhimmou et al., 2008; Suliman, 2012) or with models and heuristics that simulate the practical production process found in the plant (Arbib and Marinelli, 2005; Gramani and França, 2006; Gramani et al., 2009, 2011; Malik et al., 2009; Silva et al., 2014; Vanzela et al., 2017). Another comparison strategy is based on integer programming, which consists of a mixed-integer model solved by an optimization package (Farley, 1988; Correia et al., 2004; Gramani and França, 2006; Ouhimmou et al., 2008; Alem and Morabito, 2012, 2013; Melega et al., 2016; Poldi and de Araujo, 2016). Due to some aspects and difficulties of the models, the comparison of the results is performed by comparing with other approaches also proposed by the authors in order to compare the efficiency and quality of the solution approaches (Hendry et al., 1996; Krichagina et al., 1998; Nonås and Thorstenson, 2000, 2008; Trkman and Gradisar, 2007; Poltroniere et al., 2008; Aktin and Özdemir, 2009; Santos et al., 2011; de Athayde Prata et al., 2015; Silva et al., 2015; Agostinho et al., 2016; Leão and Toledo, 2016). The details about the solution methods, their similarities and particularities are given in the following sections.

3.1.1 A Priori Cutting Pattern Generation

As one of the problems embedded in integrated problems is the cutting stock problem, the solution approaches must take into account manners in which the cutting patterns are addressed in the solution methods.

In this section, we are interested in the studies, in which the cutting patterns are provided a priori to the solution methods, i. e., before applying the solution methods to obtain an integer solution to the integrated problem, the cutting patterns are generated and then inserted in the mathematical model, which can then be solved by the corresponding method. An example of this is the two-step procedure with a priori cutting pattern generation. In such solution approach, the priori cutting patterns generation is considered as a first step of the solution method and the search for a feasible solution to the integrate problem is addressed in the second step of the procedure.

In Table 3.2, the studies in the literature are classified considering firstly the strategies employed to a priori generation of the cutting patterns and then the solution approach used in an attempt to find a feasible solution to the integrated problem. We also remark (with *) those studies, in which the solution methods consist of a two-step procedure.

	Ways of Priori Cutting Patterns Generation				
	Heuristic Approach	Linear Programming	Lexicographic Search	Provided by the Company	**Approach to Feasible Solution**
Farley (1988)				✓	Integer Programming
Hendry et al. (1996)*	✓				Integer Programming; First Fit Decreasing; Hybrid Heuristic
Krichagina et al. (1998)*		✓			Brownian Analysis
Nonås and Thorstenson (2000, 2008)	✓				Local and Global Solution Procedures
Correia et al. (2004)*			✓		Simplex Method with Rounding Heuristic; Integer Programming
Arbib and Marinelli (2005)				✓	Branch-and-Bound Approach
Ouhimmou et al. (2008)				✓	Time Decomposition Heuristic; Integer Programming
Aktin and Özdemir (2009)*	✓				Integer Programming
Santos et al. (2011)				✓	Rolling Horizon Strategy
Alem and Morabito (2012)				✓	Robust Optimization and Integer Programming
Alem and Morabito (2013)				✓	risk-neural stochastic approach and Integer Programming; risk-averse two-stage stochastic approach and Integer Programming

(*) Two-Step Procedure

Table 3.2: Solution Methods: A Priori Cutting Pattern Generation and Feasible Solutions.

In the most of the studies with a priori cutting pattern generation, the cutting pattern used in the solution methods are those provided by the company (Farley, 1988; Arbib and Marinelli, 2005; Ouhimmou et al., 2008; Santos et al., 2011; Alem and Morabito, 2012, 2013), i. e., these cutting patterns are usually used by the company and also considered in these studies. Some specific approaches such as, linear programming (Krichagina et al., 1998), lexicography search (Correia et al., 2004), heuristic approaches (Hendry et al., 1996; Nonås and Thorstenson, 2000, 2008; Aktin and Özdemir, 2009) are also addressed to generate a priori cutting patterns. The procedures based on linear programming consist of linear models, in which the output solutions correspond to the matrix of cutting patterns, whereas heuristic approaches generate all possible cutting patterns or select those that provide a waste within a certain limit or even respect some physical restrictions.

In this literature review, the solution approaches based on two-step procedures, as mentioned before, consider the generation of the cutting patterns as a first step of the solution methods, and these cutting patterns are used to build the mixed-integer models addressed in the second step. The studies vary in relation to the procedure used in the first step, such as, heuristics and lexicographic search, whereas in the second step are used either integer programming, i. e., the mixed-integer model is solved by an optimization package (Hendry et al., 1996; Correia et al., 2004; Aktin and Özdemir, 2009) or the simplex method with a rounding procedure (Correia et al., 2004). Hendry et al. (1996) also consider integer programming to the first step of the procedure, in which the subset of possible cutting patterns are generated by a heuristic approach. The results obtained with the proposed approaches are compared each other and with the current production in the industry using real-world data. The strategies show better results than those provided by the industry.

Although Krichagina et al. (1998) propose a suboptimal two-step procedure, due to the characteristics of the proposed model and the practical environment in which it is embedded, the authors consider a continuous time horizon. The two-step procedure consists in solving a linear programming model at first step to select a small subset of cutting patterns and the resulting model is approximated as a control problem for Brownian motion in the second step. The results show that the proposed approach outperforms others from the literature regarding to the total costs, whereas it is simpler to employ when compared with others. Nonås and Thorstenson (2000) also consider a continuous time horizon and obtain a model with concave objective function and linear constraints. As solution methods, two global search procedures and three local search procedures are considered. For the global search procedures the authors propose a tailor-made version of Murthy's extreme point raking method (Murty, 1968). Other approaches (sequential heuristic, successive partitioning, cutting planes) are also compared to the Murty's procedure without success.

In the remaining of the studies, the set of cutting patterns is provided a priori by the company (see Table 3.2) and interesting solution approaches have been proposed in the literature to solve the integrated problem. Arbib and Marinelli (2005) propose a branch-and-bound approach to solve two proposed models, being one of the models based on the policy employed in the company. In the first analysis, both models are considered in a daily basis, in which they differ each other by the used cutting pattern. The models provides a significant trim loss reduction and the gains are rather high when considering week planning. In Ouhimmou et al. (2008), an heuristic based on a time decomposition approach is proposed, where just binary variables that are equal to 1 are fixed in a subproblem. The results show that the heuristic performs better when compared with an integer linear programming for the same computational time and with a reduction of 22% of the total operations cost, when compared with industry results.

Santos et al. (2011) solve the problem making use of the rolling horizon planning strategy with an optimization package. Instances are generate with real data from industry for two different types of cutting patterns generated a priori. The results show that the instance with the n-group cutting patterns obtained an average objective value slight smaller than the one given with the cutting patterns from the factory. Alem and Morabito (2012) propose models considering an environment with uncertain costs and demand. The models are analyzed on real and simulated instances and solved by an optimization package. The results show that the optimal objective function value increases as robustness is enforced. In Alem and Morabito (2013), a risk-neutral stochastic approach and risk-averse two-stage stochastic approach are presented to deal with the uncertain data (setup times in cutting processes and demand). The resulting proposed models are solved by an optimization package using real-life data. In all proposed strategies, elapsed times increase as the number of scenarios increased.

Due to the derivation from practical applications, which can results into difficult models, Farley (1988) presents some reductions in the model proposed in order to enable a currently available software to find a feasible solution. The author does not show the results obtained with the proposed models, however he has tested in the operational environment and run together with the present manual planning system of the factory.

3.1.2 Iteratively Cutting Pattern Generation

In this section, we are interested in solution approaches that take into account iteratively the generation of cutting patterns, i. e., the solution methods are responsible to generate the cutting patterns and also to find a feasible solution to the mixed-integer model. Table 3.3 shows the papers which apply this type of approach. Firstly, we present the way in which the cutting patterns are generated and then the approach addressed in order to find a feasible solution to the integrated problem.

The column generation procedure is probably the most well-known approach to generate the cutting patterns in cutting stock problems, and this is not different in the integrated problems, in which it is the main approach used to interactively generate the cutting patterns in the solution methods (Reinders, 1992; Nonås and Thorstenson, 2000, 2008; Respício and Captivo, 2002; Gramani and França, 2006; Ghidini et al., 2007; Gramani et al., 2009, 2011; Silva et al., 2015; Agostinho et al., 2016; Leão and Toledo, 2016; Melega et al., 2016; Poldi and de Araujo, 2016; Vanzela et al., 2017). Heuristic approaches are addressed iteratively as well (Poltroniere et al., 2008; Suliman, 2012). The use of models that treat the cutting pattern as a decision variable can be seen in the literature (Trkman and Gradisar, 2007; Malik et al., 2009; Silva et al., 2014; de Athayde Prata et al., 2015; Leão and Toledo, 2016; Melega et al., 2016; Poldi and de Araujo, 2016), in which in most cases, the formulations to the cutting stock problem are basically based on the formulation attributed to Kantorovich (1960) (see Valério de Carvalho (1999)), i. e.,

	Ways of a Iteratively Cutting Pattern Generation			
	Variables	Heuristic Approach	Column Generation	**Approach to Feasible Solution**
Reinders (1992)			✓	—
Nonås and Thorstenson (2000, 2008)			✓	Local and Global Solution Procedures
Respício and Captivo (2002)			✓	Branch-and-Price Approach
Gramani and França (2006)			✓	Rounding with procedures to Minimum Path Problem; Decomposition Heuristics
Ghidini et al. (2007)			✓	—
Trkman and Gradisar (2007)	✓			Exact Methods with leftovers
Poltroniere et al. (2008)		✓		Lagrangian Relaxation with Feasibility Heuristics
Gramani et al. (2009)			✓	Lagrangian Relaxation with Feasibility Heuristics; Decomposition Heuristics
Malik et al. (2009)	✓			Excel Based Genetic Procedure
Gramani et al. (2011)			✓	—
Suliman (2012)		✓		Backward Lot-Sizing Approaches
Silva et al. (2014)	✓			Integer Programming; Practical Heuristics
de Athayde Prata et al. (2015)	✓			Integer Programming
Silva et al. (2015)			✓	Integrality Constraints with Optimization Package
Agostinho et al. (2016)			✓	Integrality Constraints with Optimization Package
Leão and Toledo (2016)	✓		✓	Integer Programming; Integrality Constraints with Optimization Package
Melega et al. (2016)	✓		✓	Integer Programming; Integrality Constraints with Optimization Package
Poldi and de Araujo (2016)	✓		✓	Integer Programming; Rolling Horizon Planning Strategy
Vanzela et al. (2017)			✓	Integrality Constraints with Optimization Package; Decomposition Heuristics

Table 3.3: Solution Methods: Iteratively Cutting Pattern Generation and Feasible Solution.

constraints that represent the physical restrictions of the object are added in the model.

We can see from Table 3.3 that three studies consider linear models, i. e., no solution approaches are addressed to obtain a feasible solution (with $-$ in place) and, in these cases, the authors analyze the linear solution found by the column generation procedure. In Reinders (1992), the model is validated in a real-word industry in Germany and incorporated in a prototype for a decision support system. The authors observe that the cost of stock out is useful to obtain a balance between production efficiency and profit. Ghidini et al. (2007) propose a model based on a practical application in the furniture industry and, after simplifications in the model, the simplex method with column generation is used. The authors observe that just with very tight machine capacities the objective function is perturbed. In Gramani et al. (2011), the column generation procedure is applied in the linear restricted master problem, which is solved by an optimization package. The results show a significant overall gain when compared to a decomposition approach.

In order to obtain a feasible solution to the problem, some authors embedded the column generation procedure into other approaches. Nonås and Thorstenson (2000) propose local and global search procedures that generate the cutting patterns during the search for a solution by column generation procedure, in order to avoid memory allocation problems. The cutting patterns are generate in the way that a large production requirement is obtained, without excess of production and with a small amount of waste. In general, this solution method performes better than sequential heuristic. Nonås and Thorstenson (2008) propose improvements in the column generation procedure presented in Nonås and Thorstenson (2000), which includes characteristics from different heuristics (three-search heuristic and sequential heuristic), in an attempt to solve the model more quickly and with large size. The results obtained show that the new procedure obtain better solutions in terms of objective function and solution time. Respício and Captivo (2002) solve the proposed model using a branch-and-price approach with an optimization package. Due to the strategies used to branch the variables, the master problem structure changes and the new values of the dual variables associated with these new constraints are also taken into account in the objective function of the column generation subproblem. The results show that computing time depends on the instance characteristics and not on its size.

In Gramani and França (2006), the authors propose a heuristic method using an analogy with a network shortest path problem. In the network shortest path, each node represents a period and each arc corresponds to an associated capacitated cutting stock problem. The capacitated cutting stock problem is solved by the simplex method with column generation and the solution is round up to integer values. In this way, the network is assembled, which consists in solving a minimum path problem, solvable by algorithms in the literature. The authors compare the proposed method with a decomposition approach. The gains show to be considerable in terms of total costs and it shows also to be fast and capable of solving industrial-size problems. The authors also compared the solution obtained by the proposed method with the optimal solution obtained by solving the integrated problem with a commercial package. For this analysis, just small examples are generated, since it is necessary to generate all possible cutting patterns for the optimization package. In these instances, the proposed approach finds the optimal solution for half of the instances and very good results for the other instances. Poldi and de Araujo (2016) propose a heuristic, which consists in a simplex method without commercial package, with column generation and rolling horizon strategies. The heuristic approach shows to be much faster than the integer programming. The authors also consider the number of objects available in each period as a decision variable and the obtained results show to be even better than the previous one.

In some studies (Silva et al., 2015; Agostinho et al., 2016; Leão and Toledo, 2016; Melega et al., 2016; Vanzela et al., 2017), after the column generation procedure has been finished, the integrality constraints of the variables are added in the master problem and the resulting problem is solved by an optimization package. The studies differ each other basically by the proposed models, which consider specific constraints due to the practical application embedded, for more details about the mathematical models see Melega et al. (2017a). In Vanzela et al. (2017), the company practice is reproduced by a decomposition heuristic. The authors observe a reduction in the total cost, as well as, a light reduction in terms of loss of raw material when compared to the company's decision. In Silva et al. (2015), the authors conclude that the proposed approach is highly dependent on the different costs considered in the objective function.

Agostinho et al. (2016) solve the proposed model considering each period separately, in which the retails generated in the previous period are included into the stock and there is no inventory of items. The results show that, in terms of waste, generating retails is better than considering situations that do not allow the generation of retails. In Leão and Arenales (2012), the authors propose alternative models to the integrated problem based on different models for the cutting stock problem and approaches to the lot-sizing problem. In terms of lower bound and computational time, the models based on decomposition approaches to lot-sizing problem, show better results, whereas the compact model shows to be more eficiente in terms of quality of the feasible solution when compared with the decomposition models. Melega et al. (2016) propose several models for the integrated problem using known models for each problem separately. The number of feasible solutions and the quality of these solutions, found by the proposed models, are influenced by the tightness of the capacity and the data set.

Solution methods based on Lagrangian relaxation are also explored in the literature of integrated problems. Poltroniere et al. (2008) propose two heuristics based on the Lagrangian relaxation of the linking constraints, in which the cutting patterns are generated by heuristic approaches. The resulting problem in turn can be decomposed into two separable problems: the lot-sizing problem with capacity and setup and, the cutting stock problem, which are solved by specific heuristics. The heuristics differ each other by the order in which the problems are solved. In general, the best results for the objective function, gap and running time are found by cutting-lot heuristic. In Gramani et al. (2009), the Lagrangian relaxation is applied to the linking and capacity constraints and as in Poltroniere et al. (2008) and the Lagrangian problem can be decomposed in the lot-sizing problem and cutting stock problem. The lot-sizing problem is decomposed and solved by dynamic programming, whereas the cutting stock problem is solved by column generation. A smoothing heuristic is applied in each iteration of the Lagrangian heuristic in order to obtain a feasible solution. The results showed that the gap obtained with the heuristic is always smaller than or equal to the results from decomposition heuristic and, in some cases, it is very close or even equal to zero.

In some studies (Trkman and Gradisar, 2007; Malik et al., 2009; Silva et al., 2014; de Athayde Prata et al., 2015; Leão and Toledo, 2016; Melega et al., 2016; Poldi and de Araujo, 2016), as we previously said, the integrated problem addresses the cutting patterns as decision variables, in which restrictions are added in the model in order to limite the physical restrictions of the objects. Malik et al. (2009) develop an Excel-based Genetic procedure to search for the optimal solution for the proposed model. The strategy has found solutions which are likely close the global optimum and compared with a decomposition approach, a high reduction in the total costs and improvements in the customer service levels is observed. In some studies (de Athayde Prata et al., 2015; Leão and Toledo, 2016; Melega et al., 2016; Poldi and de Araujo, 2016) the proposed mathematical model is basically solved by an optimization package. In de Athayde Prata et al. (2015), to solve the proposed model it is necessary firstly to determine the required production time, that is calculated by dividing the total demand of products by the total daily capacity. According to the authors, the feasibility in the application of the model to find an optimal solution to practical cases is observed with small computational time. An extension of the model is presented, which allows the control of maximum admissible losses with an increase in the computational time.

An exception of the studies that considers the cutting patterns as a decision variable is Silva et al. (2014). The authors propose two mathematical models based on the "one-cut" from Dyckhoff (1981), which is solved by an optimization package. The authors also present two heuristics based in industrial practice to evaluate and analyze the proposed model using real-world instances. The Heuristic I consists in the anticipation of the production of all items to the period one and Heuristic II solves in each period a cutting stock problem. The heuristics are compared each other and with the best (optimal in some cases) solutions found by the models. The Heuristic II is able to find values very close or even equal to the optimal.

A non-linear integer model is proposed in Suliman (2012), which is solved using a strategy based on backward lot-sizing solution approaches with a pattern generation selection procedure to generate the cutting patterns. In this way, the approach proceeds in the backward direction from the last planning period establishing for each period, the final products to be produced, their quantities and the cutting patterns to be used. The proposed approach is compared with the industry policy, which shows to be quite better with respect to the trim loss.

3.2 Solution Methods for *GILSCS* Problem

In this section, the solution methods proposed to the general integrated lot-sizing and cutting stock problem are presented. The solution methods are strongly related to two known strategies from the literature, which has been successfully used to solve the problems separately. The cutting process considered in the general integrated problem is the one-dimensional case, i. e., just one dimension is taken into account in the cutting process.

One of the approaches addressed in this chapter is based on the column generation procedure, which has been largely used as solution approach itself or embedded in other solution methods in order to solve cutting stock problems (Hifi, 2002; Wang and Wäscher, 2002; Oliveira and Wäscher, 2007; Wäscher et al., 2007; Morabito et al., 2009) or even to solve the integrated problems. In this section, the column generation procedure is considered as common step for all the solution approaches in order to generate the matrix of cutting patterns at Level 2 of the general integrated model from Chapter 2.

The other proposed approaches are based on decomposition strategies, more precisely, in relax-and-fix procedures. The relax-and-fix procedure is a relatively simple and straightforward approach, which has been used successfully to solve lot-sizing problems, in particular multi-levels lot-zing problems. Stadtler (2003) propose a time-oriented decomposition heuristic to solve the multi-item multilevel lot-sizing problem with setup times. For each subproblem a formulation based on the simple plant location formulation is developed (Krarup and Bilde, 1977). These mixed-integer subproblems are solved by a mathematical programming software. The computational tests show that the proposed heuristic provides a better solution quality than a well-known special purpose heuristic. Akartunali and Miller (2009) propose a heuristic framework which generates both good solutions and competitive lower bounds using strong formulations to multi-levels lot-sizing problems. The heuristic uses the relax-and-fix idea considering a time-oriented decomposition. The computational results demonstrate the efficiency of the heuristic, particularly for challenging problems. Mohammadi et al. (2010) consider the multi-level capacitated lot-sizing problem with sequence-dependent setups. To solve the problem, MIP-based heuristics all relied on rolling-horizon heuristics and relax-and-fix procedure are provided. Toledo et al. (2015) propose a heuristic that is based on constructive and improvement heuristics to solve multi-level capacitated lot-sizing problem with backlogging. A relax-and-Fix heuristic is firstly used to build an initial solution, and this is further improved by applying a Fix-and-optimize heuristic. The computational results show that our combined heuristic approach is very efficient and competitive, outperforming benchmark methods for most of the tests. In this chapter, the relax-and-fix ideas, considering different decomposition strategies, are used to search a feasible solution to the general integrated lot-sizing and cutting stock problem.

3.2.1 Column Generation Procedure - common step

In this section, the column generation procedure is described and addressed as a common, and first, step in all the heuristic solution approaches. The column generation procedure is used to overcome difficulties present in the *GILSCS* problem, which consists of the high number of variables Z^o_{jt}, i. e., the high number of possible cutting patterns at Level 2 of the *GILSCS* model.

The column generation procedure starts relaxing the integrality constraints of the variables in the *GILSCS* problem and just the columns related to the homogeneous cutting patterns, i. e., columns of type: $(0, \cdots, a^p_{op}, \cdots, 0)$, where $a^p_{op} = \lfloor \frac{L^o}{l^p} \rfloor$, $\forall p \in P$, are considered in the sub-matrix associated to the Z^o_{jt} variables (restricted master problem). The cutting patterns and the corresponding Z^o_{jt} variables are generated as the column generation procedure evolves and new columns become necessary. The columns related to the other variables are already included in the restricted master problem, except W^o_{jt} setup variables, which are created as the corresponding Z^o_{jt} are generated (constraints (2.35) and (2.36)).

The current restricted master problem is solved using the optimization package and the dual variables associated to the constraints (2.33), (2.36) and (2.37) are recovered. Note that, in the linear relaxation of the *GILSCS* problem, the constraint (2.35) becomes an equality and the variables W^o_{jt} can be replaced by Z^o_{jt}/M in the constraint (2.36) and consequently the constraint (2.35) is eliminated from the model. In this way, the dual variables associated to this constraints are equal to 0 and they are not considered in the column generation procedure.

Let $\begin{bmatrix} \sum_{p \in P} \pi^p_t & \gamma_t & \tau_t \end{bmatrix}^T$ be the dual variables associated to constraints (2.33), (2.36) and (2.37), respectively. For each period t and object type o, a subproblem of the type (3.1)-(3.3), which consists of a knapsack problem, is solved in order to find if there is an attractive cutting pattern for the restricted master problem. The subproblem is given by:

$$OF_{SUB} = min \quad c^o_j \left(L^o - \sum_{p \in P} l^p a^p_{oj} \right) - \sum_{p \in P} \pi^p_t a^p_{oj} - vt^o_{jt}\gamma_t + \tau_t \tag{3.1}$$

Subject to:

$$\sum_{p \in P} l^p a^p_{oj} \leq L^o \tag{3.2}$$

$$a^p_{oj} \in \mathbb{Z}_+ \qquad \forall p \tag{3.3}$$

For each period t and type of object o a new cutting pattern is included in the restricted master problem if $OF_{SUB} < 0$ and the new restricted master problem is solved. The optimization package is also used to solve the subproblem. At the point of the generated columns no longer price out attractively to the restricted master problem in the subproblem (3.1)-(3.3), that is, $OF_{SUB} \geq 0$, the column generation procedure stops.

After the column generation procedure (common step), the matrix of cutting patterns at Level 2 of the *GILSCS* problem is generated and then different heuristic approaches are addressed in order to search for a feasible solution for the resulting mixed-integer *GILSCS* problem. The heuristics are described in more the details in the sections as follows.

3.2.2 Column Generation Based Heuristic - CGH

The Column Generation Based Heuristic (*CGH*) consists of a solution approach which uses a commercial optimization package in order to search for a feasible solution for the *GILSCS* problem, considering the mixed-integer problem descendent, after apply the column generation procedure. In this way, the resulting *GILSCS* problem, considering all the generated column in the column generation procedure and the integrality constraints of the variables, is solved in order to obtain a feasible integer solution to the *GILSCS* problem.

3.2.3 Relax-and-Fix Based Heuristic - RFH

The relax-and-fix is a constructive heuristic approach that determines a solution from solving several mixed-integer problems. The original model is decomposed according to a selected strategy of the variables and relaxed with respect to integrality constraints for some variables and then it is solved as a linear programming problem. The solution information from the linear programming problem is taken to fix decisions according to a selected freezing strategy. Given these fixed decisions, the resulting problem is solved by an optimization package. The iterative procedure of fixing and solving the resulting problem is continued until a feasible integer solution is generated or the resulting problem is infeasible. At each step of the relax-and-fix strategy, integrality constraints for a subset of variables are included according to the selected strategy, while decisions made in previous steps are kept fixed and remaining variables stay relaxed. In this chapter, the set of variables considered in the decomposition of the problem are the binary setup variables.

The Relax-and-Fix Based Heuristic (*RFH*) proposed in this research is used to overcome the difficulties faced with the binary variables of setup and takes advantages of multi-level structures, in order to find a feasible solution for the *GILSCS* problem. The proposed approach is based on the internally rolling schedule heuristic proposed by Stadtler

(2003). In the *RFH*, the column generation procedure is applied as a first step in order to generated the matrix of cutting patterns at Level 2 of the general integrated problem, thenceforward, the relax-and-fix is applied in the resulting problem.

The Relax-and-Fix Based Heuristic is a relatively simple and straightforward approach with a disadvantage that setup decisions are optimized only in a subset of the variables in each iteration and setup decisions fixed in earlier iterations might adversely affect setup decisions in later iteractions.

We propose in this chapter, two selected strategies to the setup variables in order to decompose the problem, which are: time-oriented decomposition and product-oriented decomposition. The decompositions are described in more details as follows.

- **Time-Oriented Decomposition: *RFH_T***

  In the time-oriented decomposition, the entire planning horizon is divided into three parts according to setup decisions, which are: fixed, integral and relaxed decisions. In the planning horizon composed of Δ periods, called time-window, all the interrelations between final products, pieces and objects on different levels are considered as the same as in the *GILSCS* and integral setup decisions are made only within the time-window. For periods preceding the time-window, setup decisions have already been made (fixed) in previous step and for later periods, binary variables are relaxed. Inside of the time-window, two subsets are considered (ψ and ϕ , with $\psi + \phi = \Delta$), which correspond to the number of periods in which the setup variables have been fixed at their binary values in each time-window and the number of overlapping periods of two consecutive time-windows, respectively. Setup decisions belonging to overlapping periods are reconsidered in the next time-window.

  The binary solution obtained for the setup variables in the first few periods within the time-window (ψ periods) are fixed at their binary values. A new mixed-integer problem derived from the *GILSCS* problem is obtained by adding the following constraints:

$$Y^o_{t'} = \overline{Y}^o_{t'} \qquad \forall o,\ \forall t' \in T^\psi \tag{3.4}$$

$$W^o_{jt'} = \overline{W}^o_{jt'} \qquad \forall j,\ \forall o,\ \forall t' \in T^\psi \tag{3.5}$$

$$Y^f_{t'} = \overline{Y}^f_{t'} \qquad \forall f,\ \forall t' \in T^\psi \tag{3.6}$$

  where $\overline{Y}^o_{t'}, \overline{W}^o_{jt'}$ and $\overline{Y}^f_{t'}$ are the values of the fixed setup variables and T^ψ is the set of periods for which the setup variables are fixed in the problem. Although this is not the most compact form to add these constraints, modern solvers automatically detect and resolve the redundancies of the formulation.

  The next time-window moves forward for the next Δ period and the resulting model is then processed in the same manner until all the setup variables are fixed. The last time-window in the relax-and-fix procedure is reached once the number of periods remaining for setup decisions is less then or equal to $3/2(\Delta - \phi)$ (Stadtler, 2003).

In the last step of the Relax-and-Fix Based Heuristic, no relaxation takes place and the integrality constraints for all the remaining variables, binary or not, are added in the model. In this way, the solution in the last time-window, if it exists, corresponds to a feasible solution for the *GILSCS* problem.

- **Product-Oriented Decomposition: *RFH_F***
  In the product-oriented decomposition, the set of final products is split into three parts according to setup decisions, which are: fixed, integral and relaxed decisions. In the set of final products composed of Δ products, called product-window, the entire planning horizon is considered, as well as, the set of cutting patterns related to these final products, that is, for those cutting patterns which have a piece belonging to a final product in the product-window, the integral constraint of setup variables are added to the subproblem, if it has not been added in previous steps. The remaining of the heuristic proceeds as in the time-oriented decomposition.

Figures 3.1 and 3.2 shows the ideia of the rolling schedule with time-oriented and product-oriented decompositions, respectively. The size of the window is considered as 4, the number of overlapping decisions as 2 and the number of fixed decisions as 2, as well. As we can see, the next window moves forward for the next products/periods until all the decisions variables are fixed.

The Relax-and-Fix Based Heuristic can be fully controlled by three parameters $\Delta/\psi/\phi$, hence the assigned values are directly related to the final result of the heuristic. The size of a time/product-window, Δ, reflects in the time spent to run the resulting problem, the quality of the solution obtained and the feasibility of the heuristic. Clearly, the shorter the time/product-window, the easier the resulting problem is to solve. However, this can deteriorate the solution quality since decisions become more myopic and the number of time/product-windows grows, hence the time allocated to solve the resulting problem decreases. In this way, we seek for a value to Δ which neither takes too much time nor finds too poor solutions, in order to move for the next time/product-window.

The consideration of capacity constraints is also an aggravating factor to obtain a feasible solution, due to the fact that there is no way to lookahead in order to notice future bottlenecks. To deal with this issue, the use of overlapping strategies can be considered, which makes better setup decisions to early periods/products of a time/product-window in view of the setup decisions expected in the overlapping time/product-window. However, overlapping strategies increase the number of time/product-windows to be solved, as

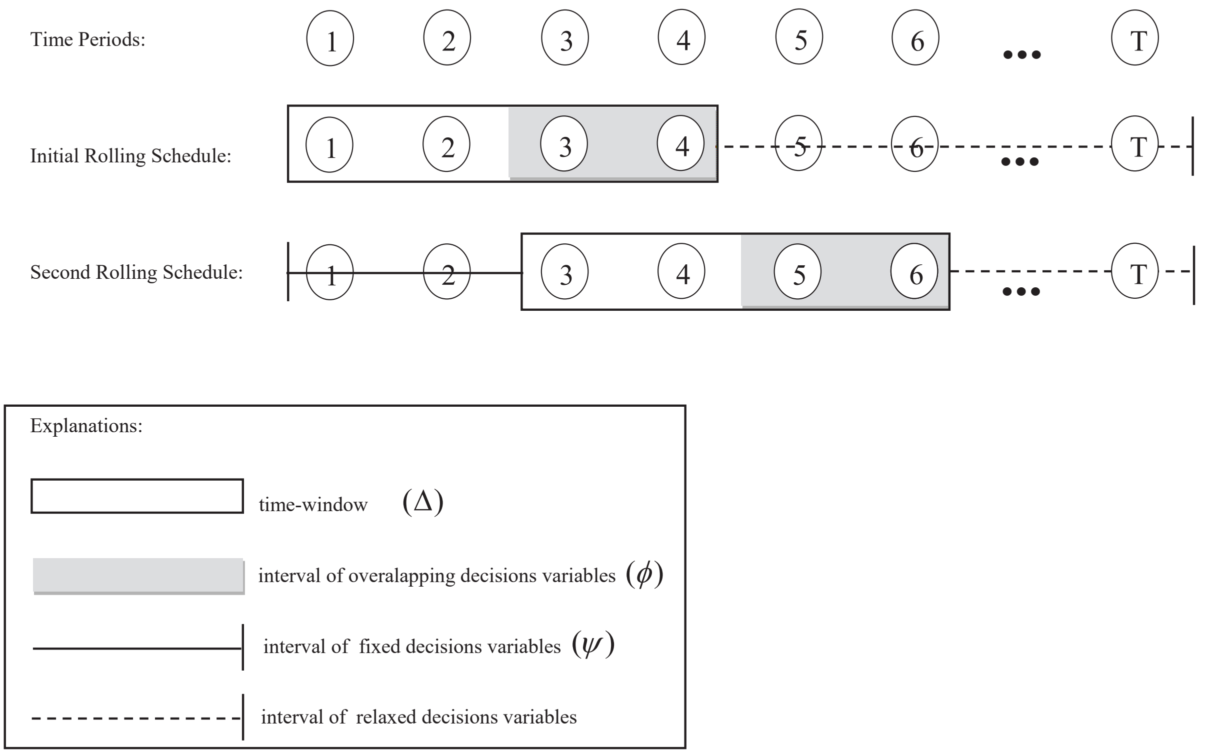


Figure 3.1: Example of Rolling Schedule with Time-Oriented Decomposition.

previously said, they decrease the time allocated to solve the resulting problem and it can also result in poor solutions.

Therefore, the assigned values to the parameters $\Delta/\psi/\phi$ must be well managed to obtain a quality solution to the *GILSCS* problem within the time limit available. An analysis of the assigned values to the parameters $\Delta/\psi/\phi$ in this chapter are described in Section 3.3. The final solution given by the Relax-and-Fix Based Heuristic, if it exists, corresponds to a feasible solution to the *GILSCS* problem.

3.2.4 Hybrid Heuristic - HH

The Hybrid Heuristic proposed in this thesis interacts simultaneously two known procedures already used in this research separately, which are the column generation procedure and the relax-and fix procedure. As in previous approaches, the column generation is applied as a first step in order to generated the matrix of cutting patterns at Level 2 of the general integrated problem, followed by the Hybrid Heuristic, in order to find a feasible solution to the general integrated problem.

The Hybrid Heuristic consists of applying the column generation procedure in each step of the relax-and-fix procedure, i.e., after fixing the setup decisions according to a

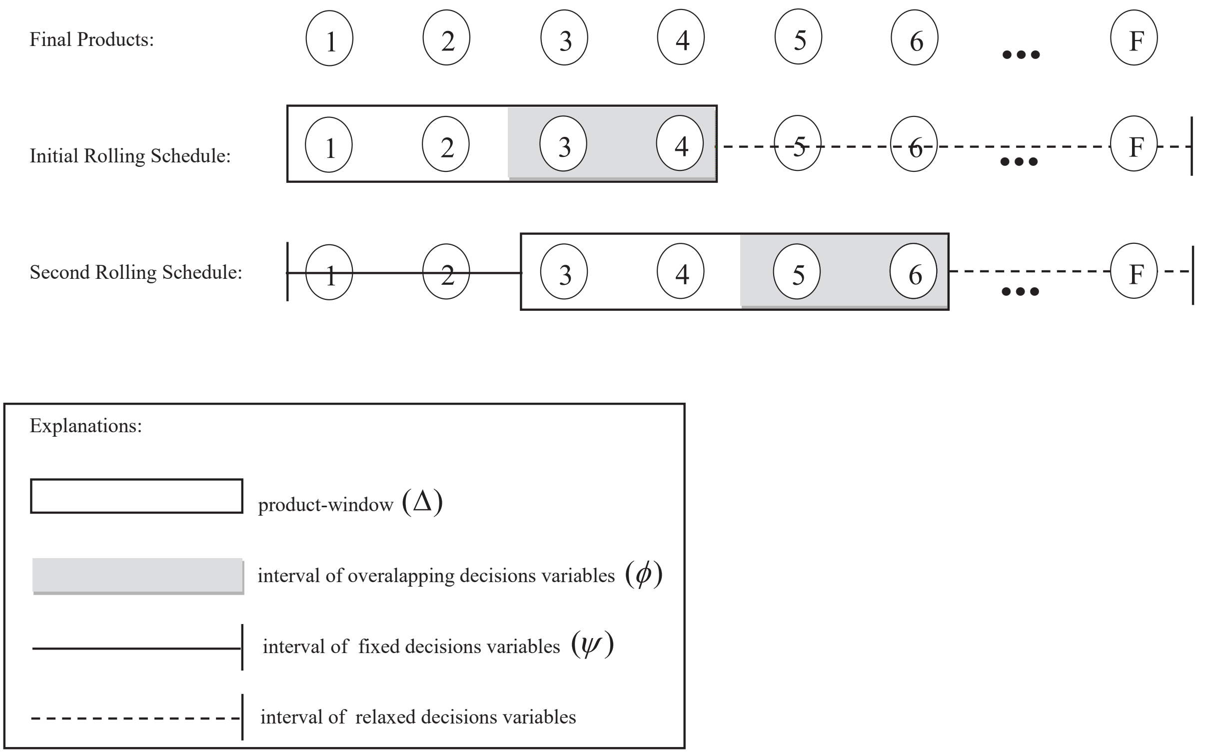


Figure 3.2: Example of Rolling Schedule with Product-Oriented Decomposition.

selected freezing strategy in the relax-and-fix procedure, the column generation procedure is applied in the resulting problem. The Hybrid Heuristic arises due to the fact that in the relax-and-fix procedure several different mixed-integer problems are solved in order to find a feasible solution to the original problem. However, a mixed-integer problem in a step is different from the previous one due to the fixing of the setup variables. In this way, when applying the column generation in each step of the relax-and-fix procedure after fixing the variables, new columns can be attractive to the current problem and can help to improve the solution. These new columns are added to the matrix of the cutting patterns at Level 2 of the general integrated problem. As we consider overlapping strategies, before applying the column generation, all the integrality constraints of the setup variables are removed in order to obtain a linear programming problem.

3.3 Computational Study

This section presents the data generation and the computational results obtained by applying the solution methods described in Section 3.2 to the general integrated problem (*GILSCS*). The building of the proposed model and heuristic approaches were implemented in C++ using the C callable library of the IBM ILOG Cplex 12.6.1 (1 thread) solver. All the computational testes were conducted in a computational time limit of 1800 seconds for each instance and on a 2 x Intel Xeon X5675 @ 3.07 GHz (6 cores each) with 96 Go RAM. We also used the Cplex version with default parameters.

Figure 3.3 presents a summary of all the four solution methods proposed in this chapter. The solution methods initiated at a common step, which consists of the column generation procedure at Level 2 of the general integrated problem. Then, in order to find a feasible solution to the general integrated lot-sizing and cutting stock problem the heuristics are applied. In the *CGH* an optimization package is used to solve the integer problem and in the *RFH*, the relax-and-fix procedure with product-oriented and time-oriented decomposition is addressed. The Hybrid Heuristic considers both procedures, i. e., the relax-and-fix procedure and the column generation procedure interact in the heuristic.

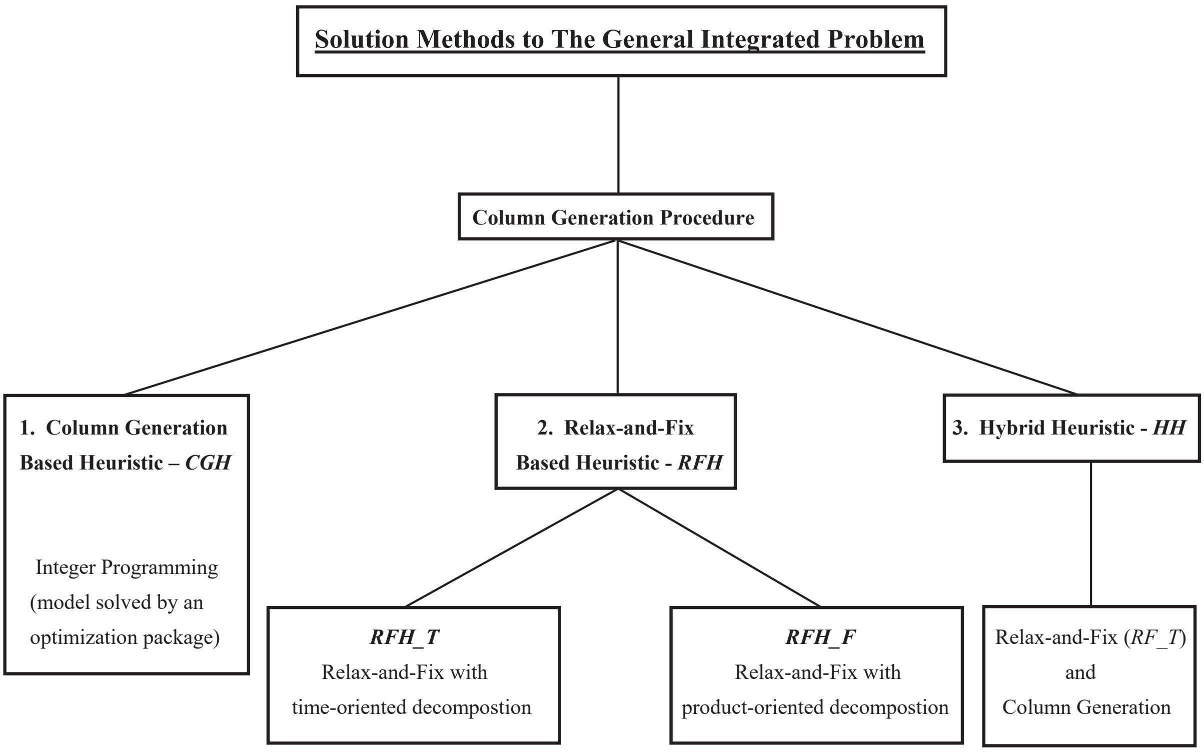


Figure 3.3: Solution Methods to the General Integrated Problem.

Considering the classification of the solution methods presented in Section 3.1, the approaches proposed in this thesis can be classified as heuristic solution methods. The Column Generation Based Heuristic (*CGH*) and Relax-and-Fix Based Heuristic (*RFH*) consider the column generation applied as a first step in order to generate the cutting patterns, in this way, they are classified with a priori cutting pattern generation. On the other hand, as the Hybrid Based Heuristic (*HH*) interacts the column generation in the heuristic approach, we classify it with a interactively cutting pattern generation.

In the Column Generation Based Heuristic (*CGH*) the resulting problem considering all the generated cutting patterns in the column generation procedure and the integrality constraints of the variables is solved by an optimization package with a time limit of 1800 seconds and an optimality gap of 0.1%, in order to search for a feasible solution to the *GILSCS* problem.

In the Relax-and-Fix Based Heuristic (*RFH*) and in the Hybrid Heuristic (*HH*), the column generation procedure is applied as a first step, with the objective of obtain the matrix of cutting patterns at Level 2 of the general integrated problem. After this, the relax-and-fix procedure or the hybrid heuristic is applied in order to find a feasible solution to the *GILSCS* problem. The heuristics have the support of an optimization package to solve the mixed-integer problems with total time limit of 1800 seconds and an optimality gap of 0.1%. The computational time of 1800 seconds is split equally between all time/product-windows. The time left in a time/product-window is added to the next time/product-window time in order to solve the corresponding *GILSCS* problem. At the end of the heuristics, a feasible solution to the *GILSCS* problem is found or the *GILSCS* problem is infeasible using the corresponding decomposition strategy.

The *CGH*, *RFH_T* and *RFH_F* heuristics consider in each step of the column generation procedure, the master problem and the subproblem solved by the optimization package and, the time spent in this initial column generation is not considered in the computational time of 1800 seconds. The time spent in the column generation inside of the Hybrid Heuristic is removed from the total time available to solve the window in each step of the relax-and-fix procedure.

In the column generation procedure no time limit is imposed to stops the procedure, in fact, the column generation stops by optimality or 5 iterations of the procedure without improvements in the objective function value of the restricted master problem. In the last case, the procedure stops and no lower bound is generated. After the column generation procedure, the heuristics are applied in an attempt to find a feasible solution to the *GILSCS* problem. To the instances in which no lower bound is provided by the column generation procedure, the gap is not calculated.

The gap is calculated according to the equation (3.7), where Z_H is the objective function value of the corresponding heuristic and Z_{LB} is the objective function value from column generation procedure.

$$GAP = \frac{100(Z_H - Z_{LB})}{Z_{LB}} \tag{3.7}$$

3.3.1 Data Set

The data set used to generate instances for the *GILSCS* problem is directly related to the well-known data from the literature of the lot-sizing and cutting stock problems, which are the instances from Trigeiro et al. (1989) (lot-sizing problem) and the CUTGEN1 generator proposed by Gau and Wäscher (1995) (cutting stock problem). Consider the data generated in intervals $[a, b]$ with a uniform distribution as follows:

This first part of the data is based on Trigeiro et al. (1989).

- number of time periods (days): $T = 20$;
- number of final products: $F = \begin{cases} 5, & \text{small;} \\ 7, & \text{medium;} \\ 10, & \text{high.} \end{cases}$

  In the original instances of Trigeiro et al. (1989), sets of $10, 20$ and 30 products are considered . The choice of a smaller number of final products is due to the size of the *GILSCS* problem compared to the Trigeiro's model. To $F = 5$ and $F = 7$, the values corresponds respectively, to the first 5 and 7 final products of $F = 10$.
- demand of final products: $d_t^f \in \begin{cases} [0{,}125], & \text{medium;} \\ [0{,}200], & \text{high.} \end{cases}$
- setup cost of final product: $sc_t^f \in \begin{cases} [25{,}75], & \text{low;} \\ [400{,}1200], & \text{high.} \end{cases}$
- production cost of final product: $vc_t^f = 0$;
- inventory cost of final product: $hc_t^f \in [0.8, 1.2]$;
- setup time of final product: $st_t^f \in \begin{cases} [5{,}17], & \text{low;} \\ [21{,}65], & \text{high.} \end{cases}$
- production time of final product: $vt_t^f = 1$;
- capacity of final products production: $CapF_t = \begin{cases} cap_t/0.75, & \text{loose;} \\ cap_t, & \text{normal;} \\ cap_t/1.1, & \text{tight.} \end{cases}$

  The capacity cap_t is generated by the average of lot-by-lot policy: for every period t, it is calculated the amount of resources needed to produce exactly the demands of the final products, sum up this amount for all periods and divide by the number of periods T, i.e., $cap_t = \frac{\sum_{t=1}^{T} \sum_{f \in F} (vt_t^f d_t^f + st_t^f)}{T}$.

For other parameters, some relationship with final products parameters are considered, for which the values are generated according to the intervals $[a, b]$ with a uniform distribution as follows:

- types of objects: $O = 1$;
- number of pieces: $\begin{cases} P = F, & \text{small (each final product corresponds directly to a piece);} \\ P = F, & \text{medium (each final product needs 2 pieces, with the number of different pieces equal the number of final products);} \\ P = 2 * F, & \text{high (each final product needs 2 different pieces, with the number of different pieces equal to twice the number of final products).} \end{cases}$
- number of pieces in each final product: $\sum_{p \in P} r_f^p = \begin{cases} 1, & \text{if } P = F^*; \\ 2, & \text{if } P = F^{**} \text{ or } P = 2 * F. \end{cases}$

  (*) each final product corresponds directly to a piece;

  (**) each final product needs 2 pieces, with the number of different pieces equal the number of final products;
- inventory cost of pieces: $hc_t^p = hc_t^f / r_f^p$;
- independent demand of pieces: $d_t^p = 0$;
- setup cost of cutting pattern: $u_j^o = \begin{cases} 2 * sc_t^f & \text{small;} \\ 5 * sc_t^f & \text{high.} \end{cases}$
- production cost of cutting pattern: $c_j^o = \begin{cases} 1, & \text{small;} \\ 5, & \text{medium;} \\ 10, & \text{high.} \end{cases}$
- setup time of a cutting pattern: $st_{jt}^o = st_t^f$;
- production time of cutting pattern: $vt_{jt}^o = vt_t^f$;
- independent demand of objects: $d_t^o = 0$;
- setup cost of objects: $sc_t^o = \begin{cases} 5 * sc_t^f, & \text{small;} \\ 10 * sc_t^f, & \text{high.} \end{cases}$
- production cost of objects: $vc_t^o = 0$;
- inventory cost of objects: $hc_t^o = 0.02 c_j^o$;
- setup time of objects: $st_t^o = st_t^f$;
- production time of objects: $vt_t^o = vt_t^f$;

- capacity of cutting machine: $CapC_t = \{1, 2\} * CapF_t$;
- capacity of objects production: $CapO_t = \{1, 2\} * CapF_t$;
  The capacity of the cutting machine and objects production is proportional the number of pieces that compose a final product, i.e., $CapO_t = CapF_t$ and $CapC_t = CapF_t$, when each piece corresponds directly to a final product; $CapO_t = 2 * CapF_t$ and $CapC_t = 2 * CapF_t$, when are necessary 2 pieces to compose a final product.

The data as follows are based on CUTGEN1 (Gau and Wäscher, 1995):

- object length: $L^o = 10,000$;
- pieces length: $l^p \in \begin{cases} [0.01, 0.2] * L^o, & \text{small;} \\ [0.01,\ 0.8]{*}L^o, & \text{medium;} \\ [0.2,\ 0.8]{*}L^o, & \text{high.} \end{cases}$

To some of the parameters used in this chapter (final products) are assigned the values from the original instances present in Trigeiro et al. (1989), to the remain parameters, we consider the generation of the data according to the intervals previous described.

In this computational study, we are interested in evaluate short-term decision, which are related to the trade-off present in setup and inventory, as well as, the trade-off in the integration between the different levels. For this, four analysis relating the different classes present in these data are explored. The analysis are in terms of the size of the problem, length of pieces, capacity constraints and costs in the objective function.

3.3.2 Setting the Parameters in the Relax-and-fix Procedure

In this section, the parameters used in the relax-and-fix procedure ($\Delta/\psi/\phi$, which correspond to the size of the window, the number of fixed decision variables and the number of overlapping decisions variables, respectively) are evaluated and selected, which either provide valuable results in terms of feasible solutions and/or showed superior solution quality in a reasonable computational time. In the Table 3.4, the tested values for the parameters, in each one of the decomposition approaches, are presented.

Time-Oriented Decomposition

Tests	Δ	ψ	ϕ	**Tests**	Δ	ψ	ϕ
Test 1	5	1	4	Test 3	10	3	7
Test 2	7	2	5	Test 4	14	4	10

In order to choose the appropriated parameters in the relax-and-fix procedure, the average of 20% of the total number of instances generated in this computational study is

Product-Oriented Decomposition

	5 final products			**7 final products**			**10 final products**		
Tests	Δ	ψ	ϕ	Δ	ψ	ϕ	Δ	ψ	ϕ
Test 1	2	1	1	3	1	2	4	1	3
Test 2	3	1	2	4	1	3	5	1	4
Test 3	4	1	3	5	1	4	7	2	5

Table 3.4: Parameters in the Relax-and-Fix Procedure.

used to evaluate the tested parameters, which corresponds to 84 instances. The results are presented in Table 3.5. The details of the classes in each one of the analysis is presented in the following sections.

Table 3.5 presents, for each one of the tested parameters, the number of instances in which the solution methods are not able to find a feasible solution. The average values for the gap and computational time are also shown in Table 3.5. We consider in this analysis just those instances that are able to find a feasible solution in all the solution approaches and in all the Tests, for each decomposition strategy. A feasible solution is not found by the heuristics either because there is not enough computational time or because the instance is infeasible. The last case only occurs, in this computational tests, to the decomposition heuristics, which is a known fact in the relax-and-fix procedures. The number of instances with no feasible solution is represented by the sum of the infeasible instances (number in bracket) plus the instances that the decomposition approach could not find a feasible solution within the available computational time. For example, considering Test 3 with product-oriented decomposition, of those four instances with no feasible solution, one of them is due to infeasibility of the instance according to the corresponding decomposition.

The results show that the time-oriented decomposition (RFH_T) is able to find a feasible solution for all the tested instances, whereas the product-oriented decomposition (RFH_F) has some difficulties. We can see that, as the size of the window in the product-oriented decomposition increase, as expected, the number of infeasibility decreases (number in bracket), however, the number of instances with no feasible solution, due to no available computational time to search for a feasible solution increases, i. e., with large size to the window the resulting mixed-integer problem is bigger and although the instance became feasible, the optimization package is not able to find a feasible solution in the available computational time. In general, the product-oriented decomposition found the best results to gap at the price of up to 84% bigger computational time (Test 1 in the time-oriented decomposition).

In a further analysis of these values, in order to choose which parameters provide better results and must be fixed in the relax-and-fix procedure, consider firstly the product-oriented decomposition. Comparing Test 2 and Test 3 with Test 1, we can see that there is an improvement of 0.67% and 4.86% in the gap at the price of 26.28% and 41.79% of increase in the computational time, respectively. Therefore, the Test 3 showed to be the best choice to the product-oriented decomposition. Performing the same analysis with the time-oriented decomposition and comparing Test 2, Test 3 and Test 4 with Test 1, the Test 2 is able to find quite good solutions in a reasonable computational time. In this way, the Test 2 is considered to the time-oriented decomposition.

	RFH_F			**RFH_T**			
	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3	Test 4
No Feasible Solution	(3) 3	(2) 2	(1) 4	0	0	0	0
Gap	45.61	45.30	43.40	46.07	45.67	45.00	44.63
Computational Time	860.89	1087.13	1220.66	195.98	371.58	926.96	1309.31

( ) number of infeasible instances

Table 3.5: Number of Instances with No Feasible Solution and Average for Gap and Time.

Therefore, the tests which provides valuable insights in terms of number of feasible solutions and a reasonable solution quality are the Test 3 to the *RFH_F* heuristic and the Test 2 to the *RFH_T*.

The results from these tests are also useful to select the decomposition strategy addressed in the Hybrid Heuristic, in which according to these results, we select the time-oriented decomposition, which presented similar results, in terms of gap, compared to the product-oriented decomposition in a quite shorter computational time. In this way, it is considered in the hybrid heuristic.

3.3.3 Computational Results

This section presents the computational results obtained with the solution methods proposed in Section 3.2 applied to the general integrated problem (*GILSCS*). In this chapter, the computational study considers different analysis, which are in terms of the size of the problem, length of pieces, capacity constraints and costs in the objective function. In each of the analysis, some aspects such as, objective function value, gap, computational time and number of feasible/infeasible solutions are evaluated. For each class in each analysis, 10 instances are considered, which consists in the variation of demand (medium/high) and in the generation of 5 instances for each variation. For some parameter that do not belong to the classes variations, their values are fixed in order to analyze the performance of the model in such type of instances. At the end of this section, we present Tables 3.15

3.16 that shows all the parameters that are common to all the analysis and their corresponding values and a summary of all the classes and their variations in the parameters, respectively. Each of of the classes are described in more details as follows.

We use the default settings of Cplex parameters except a tolerance from the optimal integer solution that is fixed at 0.1%. As we said before, the column generation stops by optimality or 5 iterations of the procedure without improvements in the objective function value, in the last case, the procedure stops and no lower bound is generated. After the column generation procedure, the heuristics are applied in an attempt to find a feasible solution to the *GILSCS* problem.

3.3.3.1 Size of the Problem

In this analysis, the classes consider variations in terms of the size of the problem, i.e, variation related to the number of final products (small / medium / high) and the number of pieces (small / medium / high). The lot-sizing problems have experienced that the higher the number of items, the better is the quality of the solution (Trigeiro et al., 1989). We are interested in seeing what is the impact of the size of the problem, based on the variation of the quantity of final products and pieces, in the solution of the general integrated problem.

In Table 3.6, the 9 classes obtained with the data variation, totalling 90 instances, are shown. For example, the Class 8 (HM) corresponds to a high number of final products, i. e., $F = 10$ and a medium number of pieces, i. e., $P = F$, in which each final products needs 2 pieces. The remain of the data variations are fixed as follows:

- setup cost of final product: $sc_t^f \in [400, 1200]$;
- setup time of final product: $st_t^f \in [21, 65]$;
- capacity of final products production: $CapF_t = cap_t/1.1$;
- setup cost of cutting pattern: $u_j^o = 5 * sc_t^f$;
- production cost of cutting pattern: $c_j^o = 1$;
- setup cost of objects: $sc_t^o = 10 * sc_t^f$;
- capacity of cutting machine: $CapO_t = CapC_t = 2 * CapF_t$;
- pieces length: $l^p \in [0.01, 0.8] * L^o$.

Considering the variations in the number of final products and pieces, the classes with the high number of final products (Classes 7, 8 and 9) have a considerable impact in

Size of the Problem

Classes	Description F/P	**Classes**	Description F/P	**Classes**	Description F/P
Class 1	SS	Class 4	MS	Class 7	HS
Class 2	SM	Class 5	MM	Class 8	HM
Class 3	SH	Class 6	MH	Class 9	HH

Table 3.6: Classes of Instances: Size of the Problems.

the number of instances with no feasible solution. The Relax-and-Fix Based Heuristics with time-oriented decomposition (*RFH_T*) and the Hybrid Heuristic (*HH*) are not able to find a feasible solution to just 2 instances (Class 8), of which one is by infeasibility, whereas the Column Generation Based Heuristic (*CGH*) and the Relax-and-Fix Based Heuristics with product-oriented decomposition (*RFH_F*) obtain the same 13 instances with no feasible solutions, of which 3 are by infeasibility considering the decomposition heuristic.

Table 3.7 shows, for each class, the number of the instances in which the column generation is not able to find a lower bound, i. e., the column generation stopped by 5 iterations without improvements in the objective function value of the restricted master problem. The average for the gap and computational time in each class and heuristic approach are presented as well. The average values for the gap and computational time is considering those instances which are solved by all the heuristics. Only in Class 9, it is not possible to obtain an average for the gap and the computational time, due to difficulty in finding a feasible solution for the instances in this class, in which just one instance is commonly solved by all the heuristics.

In this analysis, the instances with no lower bound is shared by almost all the classes, where Class 9 has a slightly larger number of instances. The column generation procedure spent around 12 seconds to generate the matrix of cutting patterns at Level 2. These 11 instances are the same instances for all the solution approaches and they were removed from the gap calculations.

We can see from Table 3.7 a little improvement, on overall average, in the results obtained for the gap by the *HH* when compared to the *RFH_T*, at the price of a slight increase in the computational time. The *CGH* finds, for the most of classes, the best results for the gap. However, in general, the solution approaches have a quite similar behavior related to the gap, in which, comparing to the worst case, the difference is less than 2% (Class 6). The highest values for the gap are in the classes with medium number of final products (Classes 4, 5 and 6). For high number of final products (Classes 7, 8 and 9), the decomposition heuristics show an increase in the solution quality when compared to the integer programming approach. As we expected, the computational time spent in the decomposition heuristics are shorter when compared to the *CGH* and considering the *RFH_T*, the computational time is, on overall average, 85% shorter.

Classes	No Lower Bound	Gap				Computational Time			
		CGH	*RFH_F*	*RFH_T*	*HH*	*CGH*	*RFH_F*	*RFH_T*	*HH*
Class 1	0	**23.62**	23.67	23.88	23.94	743.47	415.35	**6.18**	13.22
Class 2	2	**11.29**	11.33	11.44	11.48	1222.75	548.27	**8.50**	13.10
Class 3	1	**21.64**	21.67	22.37	22.28	1445.54	1020.16	**20.38**	28.41
Class 4	1	**28.46**	28.48	28.91	28.96	1424.27	1009.66	**26.40**	30.89
Class 5	0	**30.06**	30.28	30.31	30.39	1613.51	823.11	**88.05**	195.38
Class 6	2	**64.54**	65.43	66.48	66.05	1793.05	1550.24	**325.21**	427.81
Class 7	1	23.70	**23.63**	24.40	24.16	1792.19	1729.79	**605.93**	803.50
Class 8	0	20.17	19.76	**19.57**	**19.57**	1792.37	1332.06	**676.59**	941.17
Class 9	4	—	—	—	—	—	—	—	—
Sum/Average	11	27.94	28.03	28.42	28.35	1478.39	1053.58	**219.65**	306.69

Table 3.7: Number of Instances with No Lower Bound and Average for Gap and Computational Time: Size of the Problem.

Based on this analysis, the number of final products considered in the remaining of the computational study, for all the analysis and all the classes, is $F = 7$, which corresponds to relatively difficult instances compared to the 5 final products. Moreover, with this value for the number of final products, the solution methods find feasible solutions for all the instances.

3.3.3.2 Length of Pieces

The variations related to the length of pieces (small / medium / high) are analyzed in this section. Cutting stock problems have shown difficulties in the column generation procedure when the length of pieces are shorter when compared to the object. In this analysis, we are interested in seeing the behavior of the general integrated problem for different length of pieces. Melega et al. (2016) observed that length of pieces have a main influence on the results considering different mathematical models for the integrated lot-sizing and cutting stock problem.

Table 3.8 shows the classes obtained with the data variation, totalling 30 instances. The remain of the data variations are fixed in the following values:

- number of final products: $F = 7$;
- number of pieces: $P = 2 * F$ (each final product needs 2 different pieces, with the number of different piece equal to twice the number of final products);

- setup cost of final product: $sc_t^f \in [400, 1200]$;
- setup time of final product: $st_t^f \in [21, 65]$;
- capacity of final products production: $CapF_t = cap_t/1.1$;
- setup cost of cutting pattern: $u_j^o = 5 * sc_t^f$;
- production cost of cutting pattern: $c_j^o = 1$;
- setup cost of objects: $sc_t^o = 10 * sc_t^f$;
- capacity of cutting machine: $CapO_t = CapC_t = 2 * CapF_t$.

Length of Pieces

Classes	Description l^p
Class 10	S
Class 11	M
Class 12	H

Table 3.8: Classes of Instances: Length of Pieces.

In Table 3.9, we present, for each class and solution approach, the average for the gap and the computational time. The number of instances in which the column generation is not able to find a feasible solution is presented as well. We can see that for the Class 10, which consists of the smallest length of pieces compared to the length of object, the column generation procedure is not able to stop the procedure by optimality for all the instances, in this way, the gap in Table 3.9 for Class 10, it is not presented.

The *CGH* found the best values for the gap in each one of the classes, hence the best average, however, the difference between the solution approaches is less than 0.71%, whereas to computational time, the *CGH* spent around 60% more time compared to the *RFH_T*.

Although the lower bound is not found by all the approaches considering Class 10, an analysis is possible in terms of objective function value, since feasible solutions are found for all the instances and solution approaches. To Class 10, the *RFH_T* obtains gains of 3% compared to the *CGH*, which contribute to a gain of 0.9%, over all the classes.

The length of pieces have also a huge impact in the gap and computational time. As the length of pieces increase compared to the length of object, the gap and computational time decrease. This is due to the high quantity of different cutting patterns generated in the column generation procedure in each class, which makes difficult the search for a feasible solution, since smaller the length of pieces higher is the number of cutting patterns. In the class where the length of pieces is bigger compared to object (Class 12), the number of cutting patterns generated is smaller, hence the search of a feasible solution is facilitated, however, the percentage of waste is increased due to lower combinations of the pieces in the cutting patterns. An interesting point is that, in the Hybrid Heuristic, the number of new generated cutting patterns is around 565 in Class 10 compared to the 144 at Class 12.

Classes	No Lower Bound	Gap				Computational Time			
		CGH	*RFH_F*	*RFH_T*	*HH*	*CGH*	*RFH_F*	*RFH_T*	*HH*
Class 10	10	—	—	—	—	1793.06	**1744.81**	1796.12	1799.18
Class 11	5	**45.52**	45.53	46.83	46.25	1792.64	1629.27	**266.03**	388.65
Class 12	6	**2.58**	2.61	2.70	2.73	1625.81	1136.01	**45.61**	58.52
Sum/Average	21	**24.05**	24.07	24.76	24.49	1737.17	1503.36	**702.59**	748.78

Table 3.9: Number of Instances with No Lower Bound and Average for Gap and Computational Time: Length of Pieces.

3.3.3.3 Capacity Constraint

In this analysis, the classes consider variations in terms of the the capacity constraint, i. e, variations related to the setup time of final products (small / high) and tightness of capacity (loose / normal / tight). The same variations are also considered in the capacity constraints related to objects and pieces, due to relationship in the parameters.

The capacities are of crucial importance in industrial environments, due to limitations caused in the production or even to a better consumption of the resources, in this way, we consider evaluating the variations related to the capacity constraint. In lot-sizing problems, variations in the capacity and setup time have a considerable effect in the solution quality (Trigeiro et al., 1989).

In Table 3.10, it is shown the 6 classes obtained with the data variation, totalling 60 instances. The remain of the data variations are fixed in the following values:

- number of final products: $F = 7$;
- number of pieces: $P = 2 * F$ (each final product needs 2 different pieces, with the number of different piece equal to twice the number of final products);
- setup cost of final product: $sc_t^f \in [400, 1200]$;
- setup cost of cutting pattern: $u_j^o = 5 * sc_t^f$;

- production cost of cutting pattern: $c_j^o = 1$;
- setup cost of objects: $sc_t^o = 10 * sc_t^f$;
- pieces length: $l^p \in [0.01, 0.8] * L^o$;

Capacity Constraint

Classes	Description $st_t^f/CapF_t$	**Classes**	Description $st_t^f/CapF_t$
Class 13	SL	Class 16	HL
Class 14	SN	Class 17	HN
Class 15	ST	Class 18	HT

Table 3.10: Classes of Instances: Capacity Constraint.

Table 3.11 shows the sum of instances with no lower bound found by the column generation procedure, the average for the gap and computational time for each class and heuristic. In order to generate the matrix of cutting patterns at Level 2 of the general integrated problem, the heuristic approaches spent around 0.19 seconds in the column generation procedure, in which for 21 instances it is not possible to find a lower bound, i. e., the column generation stops by 5 iterations without improvements in the objective function value of the restricted master problem. These 21 instances are the same instances for all the heuristic approaches and are removed from the gap calculations.

In this analysis, a feasible solution is found by all the instances. On overall average, the *RFH_F* obtain better results for the gap followed by the *CGH*, however, the solution approaches, also in this analysis, present quite similar behavior, in which the difference between the best and the worst results is not bigger than 3%. We can see a slight improvement of 0.3% in the gap by the *HH* compared to the *RFH_T*. The decomposition approaches, for all the classes have a time consumption smaller than the *CGH*, in which the difference is up to 85% considering the *RFH_T*. There is a slight increase in the computational time of the *HH* compared to the *RFH_T*.

The impact of the data in the computational results is given by two sides. Considering small setup time (Classes 13, 14 and 15), the difficulties in the instances arise to loose and tight capacity, obtaining high values for the gap. Considering high setup time (Classes 16, 17 and 18), there is an increase in the quality of the solution to loose and tight capacity, in this way, the normal capacity presents the high values for the gap.

Classes	No Lower Bound	Gap				Computational Time			
		CGH	*RFH_F*	*RFH_T*	*HH*	*CGH*	*RFH_F*	*RFH_T*	*HH*
Class 13	3	**73.62**	73.66	75.88	75.35	1791.84	1604.79	**128.87**	262.70
Class 14	5	42.75	**42.56**	43.25	43.29	1792.21	1500.22	**229.58**	411.72
Class 15	3	83.26	**82.53**	85.18	84.66	1792.48	1655.68	**497.42**	611.22
Class 16	1	**51.78**	52.07	52.82	52.56	1792.83	1495.40	**120.30**	230.19
Class 17	4	**70.59**	70.81	72.67	72.72	1792.32	1321.38	**138.06**	276.46
Class 18	5	53.69	**53.08**	54.32	54.38	1792.67	1564.77	**287.41**	419.87
Sum/Average	21	62.61	**62.45**	64.02	63.83	1792.39	1523.71	**233.60**	368.69

Table 3.11: Number of Instances with No Lower Bound and Average for Gap and Computational Time: Capacity Constraint.

3.3.3.4 Costs in the Objective Function

The variations related to the costs in the objective function considering the different levels are explored in this analysis, which consist of the setup cost of final products (small / high), setup cost of cutting patterns (small / high), setup costs of objects (small / high) and waste cost of cutting patterns (small / medium / high). The aim of this analysis is to evaluate the impact of the data variation related to the costs present in the objective function in the quality of the solution and time consumption. For lot-sizing problems, the setup cost has a slight effect in the solution quality and, high values to setup cost lead to high values for gaps. Table 3.12 presents the 24 classes obtained with the data variation with a total of 240 instances. The remain of the data variation are fixed in the following values:

- number of final products: $F = 7$;
- number of pieces: $P = 2 * F$ (each final product needs 2 different pieces, with the number of different piece equal to twice the number of final products);
- setup time of final product: $st_t^f \in [21, 65]$;
- capacity of final products production: $CapF_t = cap_t/1.1$;
- pieces length: $l^p \in [0.01, 0.8] * L^o$.

In Table 3.13, the average for the gap and computational time for each class and each heuristic approach are presented, as well as, the sum in each class of the instances with no lower bound. The solution approaches spent around 0.17 seconds to generate the matrix of cutting patterns by the column generation procedure, with 54 instances of which the

Costs in the Objective Function

Classes	Description $sc^f_t/u^o_j/sc^o_t/c^o_j$	**Classes**	Description $sc^f_t/u^o_j/sc^o_t/c^o_j$
Class 19	SSSS	Class 31	HSSS
Class 20	SSSM	Class 32	HSSM
Class 21	SSSH	Class 33	HSSH
Class 22	SSHS	Class 34	HSHS
Class 23	SSHM	Class 35	HSHM
Class 24	SSHH	Class 36	HSHH
Class 25	SHSS	Class 37	HHSS
Class 26	SHSM	Class 38	HHSM
Class 27	SHSH	Class 39	HHSH
Class 28	SHHS	Class 40	HHHS
Class 29	SHHM	Class 41	HHHM
Class 30	SHHH	Class 42	HHHH

Table 3.12: Classes of Instances: Costs in the Objective Function.

procedure is not able to find a lower bound and these instances are removed from the calculations. A feasible solution is found for all the instances in all the solution methods.

The heuristic approaches again have an analogous behavior in each one of the classes, which is more similar in this analysis compared to the other, with a slight gain of the *CGH*. However, the computational time spent is on overall average up to 90% higher than the *RFH_T*.

Considering all the variations in the costs, we can notice a considerable increase in the gap, when the setup cost of final products is high. The other variations in the costs do not present significant variations in the computational results.

Table 3.14 shows the average of the costs in the objective function related to objects (setup and stock), cutting patterns (setup and waste), pieces (stock) and final products (setup and stock). We can see that a huge percentage of the costs in the objective function is related to waste of the cutting patterns followed by the stock of pieces and setup of cutting patterns. The decomposition heuristics considering the time-oriented strategy look at some point only locally related to the time horizon and the heuristics make more setups in order to satisfy the demand of final products and reduce the costs related to the waste, hence reduces the stock of final products.

Classes	No Lower Bound	Gap				Computational Time			
		CGH	*RFH_F*	*RFH_T*	*HH*	*CGH*	*RFH_F*	*RFH_T*	*HH*
Class 19	4	**6.70**	6.75	6.77	6.77	555.13	249.26	**28.00**	46.25
Class 20	6	**1.62**	1.64	1.65	1.64	181.22	72.24	**4.04**	5.98
Class 21	1	**3.10**	3.12	3.15	3.15	359.46	122.14	**54.67**	82.56
Class 22	1	**5.09**	5.12	5.16	5.15	198.59	72.87	**4.29**	7.44
Class 23	1	**2.68**	2.69	2.71	2.73	12.66	5.76	**1.23**	2.14
Class 24	3	**1.14**	**1.14**	1.17	1.16	4.63	3.34	**0.84**	1.46
Class 25	3	**5.38**	5.41	5.46	5.44	586.11	306.53	**10.83**	15.80
Class 26	2	**1.84**	1.87	1.90	1.90	11.99	7.60	**1.81**	2.94
Class 27	0	**3.13**	3.14	3.17	3.18	540.37	276.03	**59.95**	118.33
Class 28	3	**7.05**	7.07	7.15	7.12	125.61	131.13	**10.50**	13.66
Class 29	1	**6.28**	6.32	6.37	6.36	443.35	120.95	**7.71**	11.66
Class 30	3	**1.30**	1.32	1.35	1.36	283.70	121.70	**13.31**	28.54
Class 31	3	17.50	17.55	17.51	**17.45**	1792.33	1446.60	**348.39**	548.55
Class 32	2	**21.20**	21.41	21.46	21.57	1373.94	971.93	**199.33**	231.72
Class 33	3	13.30	**13.28**	13.56	13.45	680.27	497.72	**24.82**	31.44
Class 34	1	**62.25**	62.41	62.74	62.83	1792.10	1503.76	**537.68**	636.56
Class 35	1	**11.80**	11.88	11.94	11.96	1400.22	987.11	**66.91**	83.98
Class 36	0	**15.65**	15.78	15.80	15.82	1217.62	693.44	**36.58**	47.21
Class 37	7	**62.24**	**62.24**	62.93	63.55	1792.90	1411.11	**182.93**	298.80
Class 38	1	12.74	**12.72**	13.01	12.92	1742.24	1033.53	**73.08**	134.26
Class 39	1	**23.97**	25.29	24.26	24.26	1471.70	983.09	**48.87**	70.64
Class 40	4	**32.14**	32.18	32.80	32.70	1792.65	1659.40	**201.71**	322.06
Class 41	2	42.00	**41.87**	42.55	42.54	1527.73	1067.51	**212.57**	296.01
Class 42	1	**26.93**	27.02	27.83	27.85	1450.00	926.54	**52.68**	111.66
Sum/Average	54	**16.13**	16.22	16.35	16.37	889.02	611.30	**90.95**	131.23

Table 3.13: Number of Instances with No Lower Bound and Average for Gap and Computational Time: Costs in the Objective Function.

3.3.3.5 Performance Profile

This section presents an overall overview of the computational study performed in this chapter with the use of the performance profile (Dolan and Moré, 2002), which provides a tool to facilitate the exhibition and the interpretation of comparisons.

Consider $\mathbb{I}$ as the set of $\mathfrak{n}_i$ instances and $\mathbb{M}$ as the set of $\mathfrak{n}_M$ solution methods described in Section 3.2. The values obtained for each instance (gap and computational time) $i \in \mathbb{I}$ using the $m \in \mathbb{M}$ solution methods is denoted by $v_{i,m}$. For each solution method $m \in \mathbb{M}$ a comparison of its performance on the instance $i \in \mathbb{I}$ relative to the performance of the

Costs in the Objective Function	Percentage of Costs			
	CGH	*RFH_F*	*RFH_T*	*HH*
Objects Costs - Setup	0.55	0.55	0.56	0.56
Objects Costs - Stock	0.03	0.03	0.03	0.03
Cutting Patterns - Setup	1.13	1.15	1.20	1.21
Cutting Patterns - Waste	92.73	92.71	92.63	92.64
Pieces - Stock	4.78	4.79	4.81	4.79
Final Products - Setup	0.31	0.33	0.34	0.34
Final Products - Stock	0.46	0.45	0.42	0.42

Table 3.14: Percentage of Costs in the Objective Function.

Parameters	Values	Parameters	Values
T	20	d_t^p	0
O	1	st_{jt}^o	st_t^f
d_t^f	[0,125];[0,200]	vt_{jt}^o	vt_t^f
vc_t^f	0	d_t^o	0
hc_t^f	[0.8, 1.2]	vc_t^o	0
vt_t^f	1	hc_t^o	$0.02c_j^o$
r_f^p	{1,2}	st_{jt}^o	st_t^f
hc_t^p	hc_t^f/r_f^p	vt_{jt}^o	vt_t^f

Table 3.15: Fixed Values of the Parameters for all the Classes.

best solution method is given by the following performance ratio:

$$r_{i,m} = \frac{v_{i,m}}{\min\limits_{m \in \mathbb{M}} \{v_{i,m}\}}$$

.

If the solution method m does not find the corresponding value for an instance i then $r_{i,m}$ is defined as r_M, which is set at one unit more than the worst value of the performance ratio found for all the instances in all the solution methods. The performance of the solution method m compared to other methods is given by the performance profile:

$$\rho_m(\tau) = \frac{1}{\mathfrak{n}_i} |\{i \in \mathbb{I} : \ r_{i,m} \leq \tau\}|$$

with $|.|$ representing the number of elements in the set. The performance profile $\rho_m(\tau)$ is a function that is associated to a given value $\tau \in \mathbb{R}$, and indicates the fraction of instances solved by the solutio method m with a performance within a factor τ of the best performance found. With this, each model has a curve that shows its performance

Classes	**Parameters**									
	F	P	sc_t^f	st_t^f	$CapF_t$	u_j^o	c_j^o	sc_t^o	$CapC_t$ or $CapO_t$	l^p
Class 1	5	F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 2	5	F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 3	5	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 4	7	F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 5	7	F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 6	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 7	10	F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 8	10	F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 9	10	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 10	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.2]L^o$
Class 11	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 12	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.2, 0.8]L^o$
Class 13	7	2F	[400,1200]	[5,17]	$cap_t/0.75$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 14	7	2F	[400,1200]	[5,17]	cap_t	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 15	7	2F	[400,1200]	[5,17]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 16	7	2F	[400,1200]	[21,65]	$cap_t/0.75$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 17	7	2F	[400,1200]	[21,65]	cap_t	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 18	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 19	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$2sc_t^f$	1	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 20	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$2sc_t^f$	5	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 21	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$2sc_t^f$	10	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 22	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$2sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 23	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$2sc_t^f$	5	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 24	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$2sc_t^f$	10	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 25	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 26	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$5sc_t^f$	5	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 27	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$5sc_t^f$	10	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 28	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 29	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$5sc_t^f$	5	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 30	7	2F	[25,75]	[21,65]	$cap_t/1.1$	$5sc_t^f$	10	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 31	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$2sc_t^f$	1	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 32	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$2sc_t^f$	5	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 33	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$2sc_t^f$	10	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 34	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$2sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 35	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$2sc_t^f$	5	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 36	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$2sc_t^f$	10	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 37	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 38	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	5	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 39	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	10	$5sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 40	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	1	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 41	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	5	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$
Class 42	7	2F	[400,1200]	[21,65]	$cap_t/1.1$	$5sc_t^f$	10	$10sc_t^f$	$2CapF_t$	$[0.01, 0.8]L^o$

Table 3.16: Summary of all the Classes and the Variations of the Parameters in the Computational Study.

for each level τ. Due to the fact that r_M can be considerably large, the logarithm scale is used to represent the performance profile. It is done as follows (Dolan and Moré, 2002):

$$\tau \mapsto \frac{1}{\mathfrak{n}_i}|\{i \in \mathbb{I} : \log_2(r_{i,m}) \leq \tau\}|$$

.

The τ factor varies in $[0, r_M)$, with $r_M = 1 + max\{\log_2(r_{\bar{i},\overline{m}}) : \bar{i} \in \mathbb{I} \text{ and } \overline{m} \in \mathbb{M}\}$.

Figures 3.4 and 3.5 show performance profiles obtained with the gap and computational time for all the instances and heuristics in this computational study. As we observed in the previous analysis, the *CGH* showed the best results for most of the analysis and this is also confirmed by performance profile, more precisely in almost 50% of the instances, the *CGH* obtained the best results for gap. The *HH* and *RFH_T* obtained quite similar results in terms of gap.

In this analysis, we are able to see that, although the *RFH_T* finds better results for the gap in just 6% of the instances, the *RFH_T* is from the best results a value of τ small than 0.5, more precisely $\tau = 0.19$. In other words, to value of τ bigger than 0.19, the *RFH_T* presents the better performance than any one of the heuristics, hence, its performance curve dominates the other (see Figure 3.4).

Considering the computational time (see Figure 3.5), the *RFH_T* showed to be faster in 85% of the instances and its performance curve dominates the others in all the analysis, followed by the *HH*. An approximated value for each heuristic related to the best performance to gap and computational time are presented in Table 3.17.

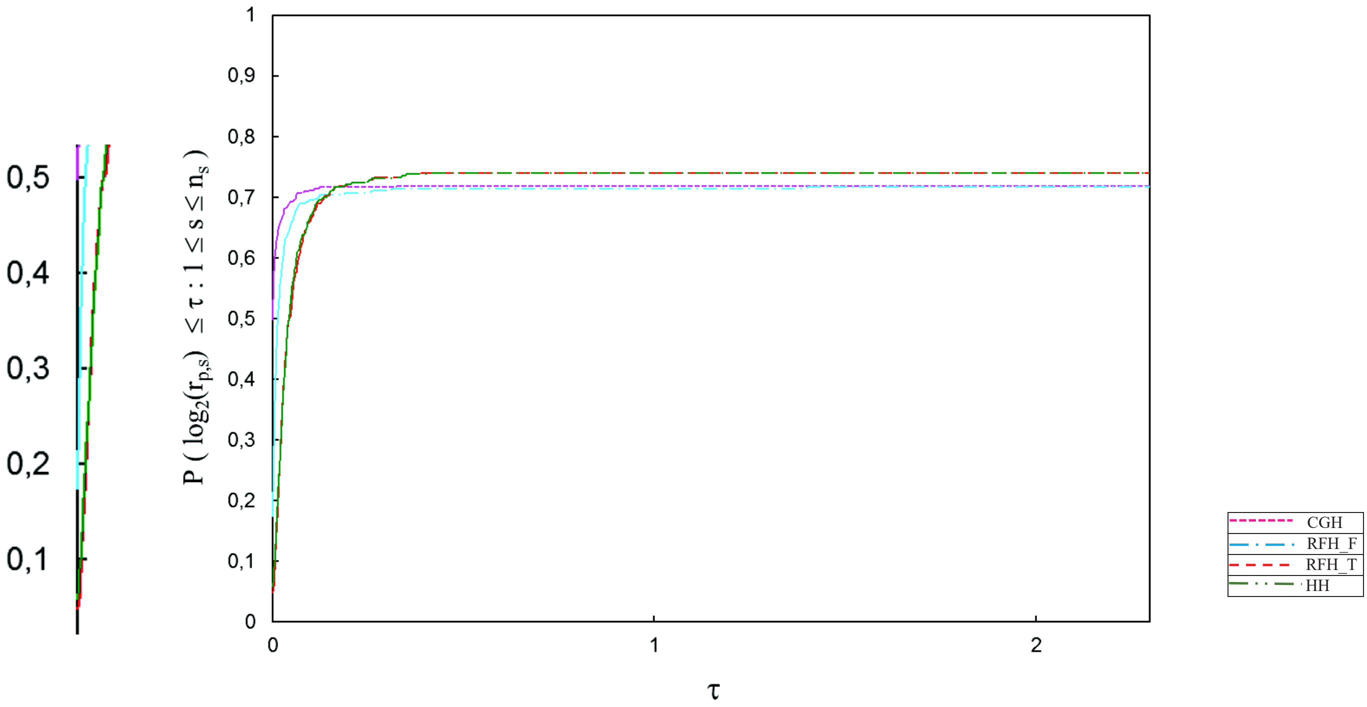


Figure 3.4: Performance Profile: Gap.

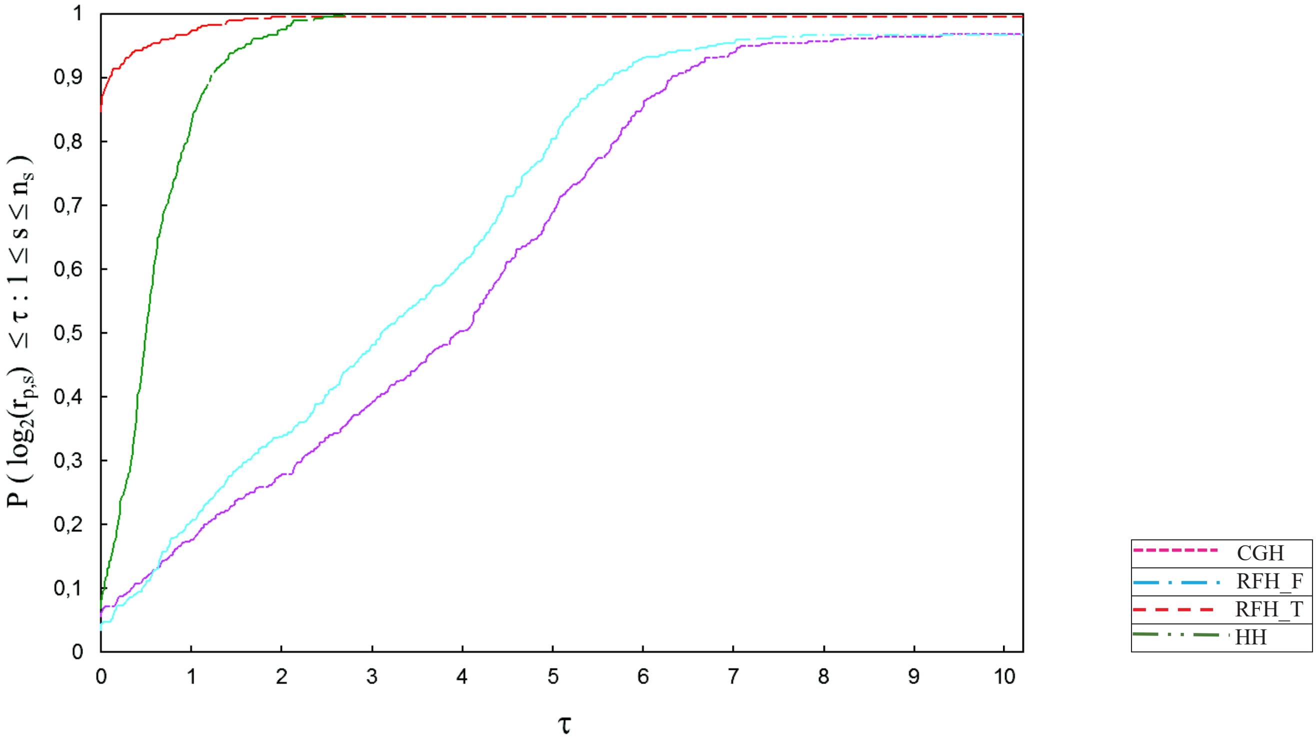


Figure 3.5: Performance Profile: Computational Time.

$\tau = 0$	*CGH*	*RFH_F*	*RFH_T*	*HH*
Gap	0.5	0.17	0.06	0.04
Computational Time	0.05	0.02	0.85	0.07

Table 3.17: Performance Profile Values to Gap and Computational Time.

3.3.3.6 Additional Computational Results

As we observed in the previous analysis, the Column Generation Based Heuristic (*CGH*) finds considerably good results when compared to the other heuristic approaches at a price of a high computational time. In this section, we performed an analysis with the purpose of comparing the relative quality of these solutions by considering the same computational time. For this, the Column Generation Based Heuristic (*CGH*) is compared with the Relax-and-Fix Based Heuristic with time-oriented decomposition (*RFH_T*) considering all the 420 instances of this computational study and the computational time used by the *RFH_T*, which corresponds to the smallest computational time between all the solution approaches.

Table 3.18 shows the sum for the number of instances in which the heuristics are not able to find a feasible solution and the average for the gap in each of the analysis related to capacity constraints, costs in the objective function, length of pieces and size of the problem. We can see that the number of instances with no feasible solution increases to the *CGH* and the quality of these solutions is not kept for the same computational time used by *RFH_T*. The instances in which the heuristics are not able to find a feasible solution are removed from the calculation. Considering the analysis related to capacity constraints and length of pieces, the *CGH* found a gap slight better than the *RFH_T*, due to a overall overview of the problem compared to the decomposition heuristic. However, to other analysis (costs and size of the problem), the *RFH_T* found gaps up to 34% better. On overall average, the gains considering the *RFH_T* are around 5%, which shows a considerable advance of the decomposition approaches compared to the linear programming models.

	No Feasible Solution		**Gap**	
Analysis	*CGH*	*RFH_T*	*CGH*	*RFH_T*
Size Problem	16	2	35.88	**35.14**
Size Pieces	0	0	**26.76**	27.21
Capacity	0	0	**64.28**	64.78
Costs	4	0	24.18	**15.86**
Sum/Average	20	2	37.77	**35.75**

Table 3.18: Number of Instances with No Feasible Solution and Average for Gap.

3.4 Conclusions and Research Directions

In this chapter, we provide a literature review and classification of solution methods to the integrated lot-sizing and cutting stock problems, in order to highlight the main strategies used in this field. The solution methods are classified according to two aspects, which are the way in which the cutting patterns are addressed in the solution methods and the approach employed to find a feasible solution to the mixed-integer problem. The literature is split into a priori cutting pattern generation and iteratively cutting pattern generation, whereas the solution methods are classified as exact or heuristic approaches. The literature review shows that most of the studies considers the cutting patterns generated iteratively and heuristic methods.

Based on the literature review and considering the *GILSCS* problem, we are interested in approaches that overcome the difficulties from both problems, which are high number of the variables in the cutting stock problem and takes advantages of multi-levels structures in the lot-sizing problem at the same time that deals wisely with the binary values of the setup variables. Two strategies are proposed, which are the well-known column generation procedure and decomposition heuristics, more precisely, relax-and-fix procedure. The column generation is applied at Level 2 of the *GILSCS* problem, whereas the Relax-and-Fix is applied in the three levels of the *GILSCS* problem.

The first solution method, called Column Generation Based Heuristic (*CGH*), consists of applying the column generation procedure at Level 2 in order to find the matrix of cutting patterns and the resulting mixed-integer problem is solved by an optimization package in order to obtain a feasible solution for the *GILSCS* problem. In the second solution method, called Relax-and-Fix Based Heuristic (*RFH*), the column generation is also applied as first step in order to obtain the matrix of cutting patterns and to obtain a feasible solution for the *GILSCS* problem, the relax-and-fix procedure is applied. The decomposition of the problem is performed by a product-oriented decomposition (*RFH_F*) and a time-oriented decomposition (*RFH_T*). Another solution method proposed in this chapter also applies the column generation as a first step in order to obtain the matrix of cutting patterns and also interacts the two previous solution methods addressed, i. e., the column generation and the relax-and-fix. This solution method is called Hybrid Heuristic (*HH*).

In order to analyze the solution methods, a complete set of data is generated based on two data sets from the literature, since no data set for this kind of problem has already been proposed. The computational study is performed around 4 analysis which are related to the capacity constraint, costs in the objective function, length of pieces and size of the problem. The analysis are evaluated over some aspects, such as, number of feasible solutions, gap and computational time.

In the computational study we are able to see that the column generation procedure is considerably fast to find a lower bound for the general integrated problem, however, for some instances, the column generation have some difficulties to find a lower bound, due to the tailing-off effect. The *CGH* and *RFH_F* had some troubles to find a feasible solution considering a high number of final products. The results showed that, in terms of the size of the problem and costs in the objective function analysis, the *CGH* presented better results for upper bound and gap. Considering the analysis related to the length of pieces and capacity constraints, the better results are recovered by the *RFH_T* and *RFH_F*, respectively. The *HH* presented slight better results for gap with a small increase in the computational time compared to the *RFH_T*. We noted that, even considering different analysis, in general, the procedures present quite similar results to upper bound and gap. Based in its performance profile, the *CGH* obtain the best results, in terms of gap, however, from a certain point, the *RFH_T* is able to dominate the remaining of the approaches. The same conclusion is not possible considering the computational time, in which the time consumption to the *CGH* is considerable high compared to the *RFH_T*.

The *CGH* and the *RFH_T* are also compared considering the same computational time used by the *RFH_T*. The results showed that the number of instances without a feasible solution is 10 times more in the *CGH*. The *RFH_T* is able to find gap up to 34% better, in which in a overall average is around 5%.

The impact of the data set in the computational results is observed in each one of the analysis in which the number of final products considerably increase the difficulty of the problem and computational time, i. e., the number of instances without a feasible solution increases with the increases of the number of final products. The quality of the gap is influenced by the length of pieces compared to the size of the object, in other words, as the gap decreases, the size of the pieces increases, at the price of an increase in the percentage of waste. The length of pieces also influences directly in the quality of the column generation, i. e., in the number of instances in which it is not possible to find a lower bound. For small length of pieces, it is more difficult to find a lower bound. The gap is also influenced by the setup costs to final products in objective function, in which the higher the setup costs are, the more difficult the instances.

We can see from the computational results that, in general, the heuristic approaches find gaps considerably high. Therefore, as future research we intend to apply improved heuristics, such as, fix-and-optimize, in order to improve the quality of the solution to the *GILSCS* problem, as well as, heuristics based on decompositions per level of the general integrated problem. It is worth mentioning that, a general calculation to the lower bound and strategies of stabilization in the column generation procedure applied to the *GILSCS* problem are also important points to be researched.

Chapter 4

Conclusions and Future Perspectives

In this thesis, the integration of two problems is the subject of interest, which consists in integrating of the lot-sizing problem and the cutting stock problem. The basic idea of integrated problems is to consider, simultaneously, the decisions related to both problems so as to capture the interdependency between these decisions in order to obtain a better global solution.

We propose a general formulation to the integrated lot-sizing and cutting stock problem which is composed of three levels and several time periods. In each one of the levels there is a different type of production. At Level 1 (first level), we have a lot-sizing problem, which is related to the production of objects. At Level 2 (intermediate level), we have a cutting stock problem based on the idea of cutting patterns, which is responsible for cutting the objects into pieces and theses pieces are used as an input to Level 3 (final level), which consists of a lot-sizing problem for the production of final products. The model incorporates some features which enables us to classify the current literature in this field. The classification is based on two aspects: the integration across multiple time periods and the integration between production levels. Other features are also evaluated, such as the dimensionality in the cutting process, capacity constraint and setups. We are also capable to point out, based on the literature review and on the formulation of the general integrated model, new future research directions to integrated problems.

The solution methods over the years have received support from the improvements of optimization theory, software and hardware, which provide better results and a bigger integration among the production processes. In this thesis, we present a literature review of the solution methods applied to the integrated problem and we propose different solution approaches to solve the general integrated model. The solution methods are based on well-known procedures from the literature and intended to overcome the difficulties from both problems, which are the high number of the variables in the cutting stock problem and the integrality of the variables, at the same time that take advantages of multi-levels structure.

Four strategies are proposed to solve the general integrated problem, in which all of them initiate in a common step that consists of the column generation procedure at Level 2. The solution approaches apply integer programming and relax-and-fix procedure to find a feasible solution to the general problem. Another solution method proposed in this thesis interacts the column generation and the relax-and-fix in a hybrid procedure. In order to analyze the solution methods, a computational study is performed around four analysis which are in terms of the size of the problem, length of pieces, capacity constraints and costs in the objective function. The computational study shows that the column generation have some difficult into finding a lower bound, due to the tailing-off effect. We also noted that, even considering different analysis, in general, the procedures presents quite similar results to upper bound and gap. However, the same conclusion is not possible considering the computational time, in which the time consumption to the approaches based on the relax-and-fix procedures is substantially shorter compared to integer programming. The impact of the data set in the computational results is observed in each one of the analysis.

We also present in this thesis (see Appendix A), a study of mathematical models proposed in the literature for modelling the lot-sizing problem and the one-dimensional cutting stock problem. The aim is to propose and compare new integrated models pointing out the advantages of each model, as well as, the impact of the data set on the solutions. For the lot-sizing problem, we study the model proposed by Wagner and Whitin (1958), the model proposed by Trigeiro et al. (1989) and a reformulation of the lot-sizing problem proposed by Eppen and Martin (1987). For the cutting stock problem, extensions of the models proposed by Kantorovich (1960), Valério de Carvalho (1999, 2002) with reduction criteria and Gilmore and Gomory (1961, 1963) have been proposed to incorporate multiple periods and several types of objects. As solution methods, we present two strategies. The first one uses an optimization package for finding the solution and in the second one, the column generation technique is used in a heuristic strategy to obtain a feasible solution. The use of the reformulated lot-sizing model shows significant improvements in the lower bound compared to the classical lot-zing model. In general, the models integrated with the classical lot-sizing problem and the cutting stock problem based on Valério de Carvalho (1999, 2002) obtained the largest number of feasible solutions compared to other mathematical models in all analysis. It is also possible to generalize the fact that the models that integrate with the model proposed by Kantorovich (1960) have produced poor results in all experiments. Although the integration with the Gilmore and Gomory model does not show the best results for all classes, the proximity to the best results contributes to the achievement of best overall average.

An application of the integrated lot-sizing and cutting stock problem is also presented in this thesis (see Appendix B), in which a mathematical model is proposed in order to analyze the main decisions of the production process of small furniture factories. The proposed model is compared to practice simulation, which consists of solving each problem separately and sequentially. A column generation technique is used to solve the problems in each approach. Good overall results are obtained when comparing the solutions of the integrated problem with the approach of sequentially solving the problems. Further computational tests are carried out in order to evaluated the impact of the different costs in the objective function for each approach, which are variations in the inventory costs of pieces, costs of objects and inventory costs of final products. The results showed that, in general, the integrated approach is better and the variations on the costs of plates and inventory costs of final products do not have a strong impact on the differences among the approaches. As a main conclusion, the computational study shows that the obtained solution can be put into practice, i. e., the models can support the main decisions taken and can bring improvements to the factory's production planning decisions.

Aiming to extend this work, we suggest as future research:

- The consideration of reformulations, such as, shortest path and facility location, applied to the *GILSCS* model.
- The consideration of higher dimensions to the cutting process in the *GILSCS* model.
- An integration of the *GILSCS* model with other processes, such as the supplier selection, in which the choice of different suppliers may be based on the quality, price and speed of the orders, or the routing and packing/loading of the final products to the customers.
- The inclusion of the cutting pattern sequence in the *GILSCS* model that may be related to a specific objective function, such as, the minimization of the knives changes, where each insertion and removal of knives takes time to be processed.
- The application of improved heuristics, such as, fix-and-optimize, as well as, heuristics based on decompositions per level in order to improve the quality of the solution to the *GILSCS* problem.
- A general calculation to the lower bound in the column generation procedure considering the different mathematical models in the literature.
- Strategies of stabilization in the column generation procedure applied to the *GILSCS* problem.

Bibliography

G. T. Agostinho, A. C. Cherri, S. A. de Araujo, and D. N. Nascimento. The multiperiod cutting stock problem with usable leftover. In *Annals of Latin-Iberian-American Conference on Operations Research (CLAIO)*, 2016.

K. Akartunali and A. J. Miller. A heuristic approach for big bucket multi-level production planning problems. *European Journal of Operational Research*, 193(2):396 – 411, 2009.

T. Aktin and R. G. Özdemir. An integrated approach to the one-dimensional cutting stock problem in coronary stent manufacturing. *European Journal of Operational Research*, 196(2): 737 – 743, 2009.

D. Alem and R. Morabito. Risk-averse two-stage stochastic programs in furniture plants. *OR Spectrum*, 35(4):773–806, 2013. ISSN 0171-6468.

D. J. Alem and R. Morabito. Production planning in furniture settings via robust optimization. *Computers & Operations Research*, 39(2):139 – 150, 2012.

C Alves and J. M. Valério de Carvalho. A stabilized branch-and-price-and-cut algorithm for the multiple length cutting stock problem. *Computers & Operations Research*, 35(4):1315 – 1328, 2008.

R. Andrade, E. G. Birgin, R. Morabito, and D. P. Ronconi. Mip models for two-dimensional non-guillotine cutting problems with usable leftovers. *Journal of the Operational Research Society*, 65(11):1649 –1663, 2014.

C. Arbib and F. Marinelli. Integrating process optimization and inventory planning in cutting-stock with skiving option: An optimization model and its application. *European Journal of Operational Research*, 163(3):617 – 630, 2005.

C. Arbib and F. Marinelli. On cutting stock with due dates. *Omega*, 46:11 – 20, 2014.

C. Arbib and F. Marinelli. Maximum lateness minimization in one-dimensional bin packing. *Omega*, 68:76 – 84, 2017.

C. Arbib, F. Marinelli, F. Rossi, and F. Di Iorio. Cutting and reuse: An application from automobile component manufacturing. *Operations Research*, 50(6):923–934, 2002.

M. Arenales, R. Morabito, and H. Yanasse. Special issue: Cutting and packing problems. *Pesquisa Operacional*, 19(2):107 – 299, 1999.

M. N. Arenales and R. Morabito. An and/or-graph approach to the solution of two-dimensional non-guillotine cutting problems. *European Journal of Operational Research*, 84(3):599 – 617, 1995.

G. Belov and G. Scheithauer. A cutting plane algorithm for the one-dimensional cutting stock problem with multiple stock lengths. *European Journal of Operational Research*, 141(2):274 – 294, 2002.

G. Belov and G. Scheithauer. A branch-and-cut-and-price algorithm for one-dimensional stock cutting and two-dimensional two-stage cutting. *European Journal of Operational Research*, 171(1):85 – 106, 2006.

H. Ben Amor and J. Valério de Carvalho. Cutting stock problems. In G. Desaulniers, J. Desrosiers, and M. M. Solomon, editors, *Column Generation*, pages 131–161. Springer US, 2005.

P. Beraldi, M. E. Bruni, and D. Conforti. The stochastic trim-loss problem. *European Journal of Operational Research*, 197:42–49, 2009.

N. Brahimi, S. Dauzere-Peres, N. M. Najid, and A. Nordli. Single item lot sizing problems. *European Journal of Operational Research*, 168(1):1 – 16, 2006.

L. Buschkühl, F. Sahling, S. Helber, and H. Tempelmeier. Dynamic capacitated lot-sizing problems: a classification and review of solution approaches. *OR Spectrum*, 32(2):231–261, 2008.

A. C. Cherri, M. N Arenales, H. H. Yanasse, K. C. Poldi, and A. C. G. Vianna. The one-dimensional cutting stock problem with usable leftovers - a survey. *European Journal of Operational Research*, 236(2):395 – 402, 2014.

N. Christofides and C. Whitlock. An algorithm for two-dimensional cutting problems. *Operations Research*, 25(1):30 – 44, 1977.

K. Copil, M. Wörbelauer, H. Meyr, and H. Tempelmeier. Simultaneous lotsizing and scheduling problems: a classification and review of models. *OR Spectrum*, pages 1 – 64, 2016.

M. H. Correia, J. F. Oliveira, and J. S. Ferreira. Reel and sheet cutting at a paper mill. *Computers & Operations Research*, 31(8):1223 – 1243, 2004.

Y. Cui, C. Zhong, and Y. Yao. Pattern-set generation algorithm for the one-dimensional cutting stock problem with setup cost. *European Journal of Operational Research*, 243(2):540 – 546, 2015.

S. A. de Araujo, K. C. Poldi, and J. Smith. A genetic algorithm for the one-dimensional cutting stock problem with setups. *Pesquisa Operacional*, 34:165 – 187, 2014.

S. A. de Araujo, B. de Reyck, Z. Degraeve, I. Fragkos, and R. Jans. Period decompositions for the capacitated lot sizing problem with setup times. *INFORMS Journal on Computing*, 27 (3):431–448, 2015.

B. de Athayde Prata, A. R. Pitombeira-Neto, and C. J. de Moraes Sales. An integer linear programming model for the multiperiod production planning of precast concrete beams. *Journal of Construction Engineering and Management*, 141(10):04015029, 2015.

Z. Degraeve and R. Jans. A new dantzig-wolfe reformulation and branch-and-price algorithm for the capacitated lot-sizing problem with setup times. *Operations Research*, 55(5):909–920, 2007.

Z. Degraeve and M. Peeters. Optimal integer solutions to industrial cutting-stock problems: Part 2, benchmark results. *INFORMS Journal on Computing*, 15(1):58–81, 2003.

A. Diegel, E. Montocchio, E. Walters, S. van Schalkwyk, and S. Naidoo. Setup minimising conditions in the trim loss problem. *European Journal of Operational Research*, 95(3):631 – 640, 1996.

E. D. Dolan and J. J. Moré. Benchmarking optimization software with performance profiles. *Mathematical Programming*, 91(2):201 – 213, 2002.

A. Drexl and A. Kimms. Lot sizing and scheduling - survey and extensions. *European Journal of Operational Research*, 99(2):221 – 235, 1997.

H. Dyckhoff. A new linear programming approach to the cutting stock problem. *Operations Research*, 29(6):1092 – 1104, 1981.

H. Dyckhoff. Cutting and packing a typology of cutting and packing problems. *European Journal of Operational Research*, 44(2):145 – 159, 1990.

H. Dyckhoff, H-J. Kruse, D. Abel, and T. Gal. Trim loss and related problems. *Omega*, 13(1): 59 – 72, 1985.

H. Dyckhoff, G. Scheithauer, and J. Terno. Cutting and packing: An annotated bibliography. *Annotated Bibliographies in Combinatorial Optimization*, pages 393 – 412, 1997.

G. D. Eppen and R. K. Martin. Solving multi-item capacitated lot-sizing problems using variable redefinition. *Operations Research*, 35(6):832 – 848, 1987.

A. A. Farley. Mathematical programming models for cutting-stock problems in the clothing industry. *The Journal of the Operational Research Society*, 39(1):41 – 53, 1988.

D. J. Fiorotto and S. A. de Araujo. Reformulation and a lagrangian heuristic for lot sizing problem on parallel machines. *Annals of Operations Research*, 217(1):213–231, 2014.

D. J. Fiorotto, S. A. de Araujo, and R. Jans. Hybrid methods for lot sizing on parallel machines. *Computers & Operations Research*, 63:136 – 148, 2015.

H. Foerster and G. Wäscher. Simulated annealing for order spread minimization in sequencing cutting patterns. *European Journal of Operational Research*, 110(2):272 – 281, 1998.

H. Foerster and G. Wäscher. Pattern reduction in one-dimensional cutting stock problems. *International Journal of Production Research*, 38(7):1657 – 1676, 2000.

F. Furini and E. Malaguti. Models for the two-dimensional two-stage cutting stock problem with multiple stock size. *Computers & Operations Research*, 40(8):1953 – 1962, 2013.

M. Garraffa, F. Salassa, W. Vancroonenburg, G. Vanden Berghe, and T. Wauters. The one-dimensional cutting stock problem with sequence-dependent cut losses. *International Transactions in Operational Research*, 23:5 – 24, 2016.

T. Gau and G. Wäscher. Cutgen1: A problem generator for the standard one-dimensional cutting stock problem. *European Journal of Operational Research*, 84(3):572 – 579, 1995.

C. T. L. S. Ghidini, D. Alem, and M. N. Arenales. Solving a combined cutting stock and lot-sizing problem in small furniture industries. In *Proceedings of the 6th International Conference on Operational Research for Development (VI-ICORD)*, 2007.

N. F. Giannelos and M. C. Georgiadis. Scheduling of cutting-stock processes on multiple parallel machines. *Chemical Engineering Research and Design*, 79(7):747 – 753, 2001.

P. C. Gilmore and R. E. Gomory. A linear programming approach to the cutting-stock problem. *Operations Research*, 9(6):849 – 859, 1961.

P. C. Gilmore and R. E. Gomory. A linear programming approach to the cutting stock problema - part ii. *Operations Research*, 11(6):863 – 888, 1963.

P. C. Gilmore and R. E. Gomory. Multistage cutting stock problems of two and more dimensions. *Operations Research*, 13(1):94 – 120, 1965.

R. R. Golfeto, A. C. Moretti, and L. L. de Salles Neto. A genetic symbiotic algorithm applied to the cutting stock problem with multiple objectives. *Advanced Modeling and Optimization*, 11(4):473 – 501, 2009.

M. Gopalakrishnan, K. Ding, J-M. Bourjolly, and S. Mohan. A tabu-search heuristic for the capacitated lot-sizing problem with set-up carryover. *Management Science*, 47(6):851–863, 2001.

M. C. N. Gramani and P. M. França. The combined cutting stock and lot-sizing problem in industrial processes. *European Journal of Operational Research*, 174(1):509 – 521, 2006.

M. C. N. Gramani, P. M. França, and M. N. Arenales. A lagrangian relaxation approach to a coupled lot-sizing and cutting stock problem. *International Journal of Production Economics*, 119(2):219 – 227, 2009.

M. C. N. Gramani, P. M. França, and M. N. Arenales. A linear optimization approach to the combined production planning model. *Journal of the Franklin Institute*, 348(7):1523 – 1536, 2011.

L. Guimarães, D. Klabjan, and B. Almada-Lobo. Modeling lotsizing and scheduling problems with sequence dependent setups. *European Journal of Operational Research*, 239(3):644 – 662, 2014.

R. W. Haessler. Controlling cutting pattern changes in one-dimensional trim problems. *Operations Research*, 23(3):483–493, 1975.

L. C. Hendry, K. K. Fok, and K. W. Shek. A cutting stock and scheduling problem in the copper industry. *The Journal of the Operational Research Society*, 47(1):38 – 47, 1996.

S. Henn and G. Wäscher. Extensions of cutting problems: setups. *Pesquisa Operacional*, 33:133 – 162, 08 2013.

M. Hifi. Special issue: Cutting and packing problems. *Studia Informatica Universalis*, 2(1): 1–161, 2002.

A. I. Hinxman. The trim-loss and assortment problems: A survey. *European Journal of Operational Research*, 5(1):8 – 18, 1980.

R. Jans. Solving lot-sizing problems on parallel identical machines using symmetry-breaking constraints. *INFORMS Journal on Computing*, 21(1):123–136, 2009.

R. Jans and Z. Degraeve. Improved lower bounds for the capacitated lot sizing problem with setup times. *Operations Research Letters*, 32(2):185 – 195, 2004.

R. Jans and Z. Degraeve. Meta-heuristics for dynamic lot sizing: A review and comparison of solution approaches. *European Journal of Operational Research*, 177(3):1855 – 1875, 2007.

R. Jans and Z. Degraeve. Modeling industrial lot sizing problems: a review. *International Journal of Production Research*, 46(6):1619–1643, 2008.

L. V. Kantorovich. Mathematical methods of organizing and planning production. *Management Science*, 6(4):366 – 422, 1960.

B. Karimi, S. M. T. Fatemi Ghomi, and J. M. Wilson. The capacitated lot sizing problem: a review of models and algorithms. *Omega*, 31(5):365 – 378, 2003.

J. Krarup and O. Bilde. Plant location, set covering and economic lot size: An 0 (mn) - algorithm for structured problems. In L. Collatz, G. Meinardus, and W. Wetterling, editors, *Numerische Methoden bei Optimierungsaufgaben Band 3*, volume 36 of *International Series of Numerical Mathematics*, pages 155 – 180. Birkhäuser Basel, 1977.

E. V. Krichagina, R. Rubio, M. I. Taksar, and L. M. Wein. A dynamic stochastic stock-cutting problem. *Operations Research*, 46(5):690–701, 1998.

A. P. S. Leão and F. M. B. Toledo. Decomposition methods for the lotsizing and cutting stock problem in paper industries. Technical report, ICMC-USP, 2016.

A. P. S. Leão and M. N. Arenales. Análise de modelos matemáticos para o problema de corte de estoque unidimensional acoplado ao dimensionamento de lotes. In *Anais do Simpósio Brasileiro de Pesquisa Operacional, Rio de Janeiro-RJ (SBPO)*, 2012.

A. Lodi and M. Monaci. Integer linear programming models for 2-staged two-dimensional knapsack problems. *Mathematical Programming*, 94(2-3):257–278, 2003.

J. Maes, J. O. McClain, and L. N. Van Wassenhove. Multilevel capacitated lotsizing complexity and lp-based heuristics. *European Journal of Operational Research*, 53(2):131 – 148, 1991.

M. M. Malik, M. Qiu, and J. Taplin. An integrated approach to the lot sizing and cutting stock problems. In *International Conference on Industrial Engineering and Engineering Management (IEEE)*, pages 1111–1115, 2009.

A. S. Manne. Programming of economic lot sizes. *Management Science*, 4(2):115 – 135, 1958.

S. Martello and P. Toth. *Knapsack Problems: Algorithms and Computer Implementations*. John Wiley & Sons, Inc., New York, NY, USA, 1990.

C. McDiarmid. Pattern minimisation in cutting stock problems. *Discrete Applied Mathematics*, 98(1-2):121 – 130, 1999.

G. M. Melega, D. J. Fiorotto, and S. A. de Araujo. Formulações fortes para o problema de dimensionamento de lotes com várias plantas. *TEMA (São Carlos)*, 14:305 – 318, 2013. ISSN 2179-8451.

G. M. Melega, S. A. de Araujo, and R. Jans. Comparison of mip models for the integrated lot-sizing and one-dimensional cutting stock problem. *Pesquisa Operacional*, 36:167 – 196, 2016.

G. M. Melega, S. A. de Araujo, and R. Jans. Classification and literature review of integrated lot-sizing and cutting stock problems. Technical report, CIRRELT - Universidade de Montréal, Canadá, 2017a.

G. M. Melega, S. A. de Araujo, and R. Jans. General integrated lot-sizing and cutting stock problems: Solution methods. Technical report, UNESP - IBILCE, 2017b.

S. Menon and L. Schrage. Order allocation for stock cutting in the paper industry. *Operations Research*, 50(2):324–332, 2002.

M. Mohammadi, S. M. T. Fatemi Ghomi, B. Karimi, and S. A. Torabi. Rolling-horizon and fix-and-relax heuristics for the multi-product multi-level capacitated lotsizing problem with sequence-dependent setups. *Journal of Intelligent Manufacturing*, 21(4):501 – 510, 2010.

F. Molina, R. Morabito, and S. A. de Araujo. Mip models for production lot-sizing problems with distribution costs and cargo arrangement. *Journal of the Operational Research Society*, 67(11):1395 – 1407, 2016.

R. Morabito and M. N. Arenales. Optimizing the cutting of stock plates in a furniture company. *International Journal of Production Research*, 38(12):2725–2742, 2000.

R. Morabito, M. N. Arenales, and H. H. Yanasse. Special issue on cutting, packing and related problems. *International Transactions in Operational Research*, 16(6):659 – 659, 2009.

K. G. Murty. Solving the fixed charge problem by ranking the extreme points. *Operations Research*, 16(2):268 – 279, 1968.

S. L. Nonås and A. Thorstenson. A combined cutting-stock and lot-sizing problem. *European Journal of Operational Research*, 120(2):327 – 342, 2000.

S. L. Nonås and A. Thorstenson. Solving a combined cutting-stock and lot-sizing problem with a column generating procedure. *Computers & Operations Research*, 35(10):3371 – 3392, 2008.

J. F. Oliveira and G. Wäscher. Cutting and packing. *European Journal of Operational Research*, 183(3):1106–1108, 2007.

M. Ouhimmou, S. D'Amours, R. Beauregard, D. Ait-Kadi, and S. Singh Chauhan. Furniture supply chain tactical planning optimization using a time decomposition approach. *European Journal of Operational Research*, 189(3):952 – 970, 2008.

Y. Pochet and L. A. Wolsey. Lot-size models with backlogging: Strong reformulations and cutting planes. *Mathematical Programming*, 40(1-3):317 – 335, 1988.

Y. Pochet and L. A. Wolsey. *Production planning by mixed integer programming*. Springer Science & Business Media, 2006.

K. C. Poldi and M. N. Arenales. Heuristics for the one-dimensional cutting stock problem with limited multiple stock lengths. *Computers & Operations Research*, 36(6):2074 – 2081, 2009.

K. C. Poldi and S. A. de Araujo. Mathematical models and a heuristic method for the multiperiod one-dimensional cutting stock problem. *Annals of Operations Research*, 238(1):497 – 520, 2016.

S. C. Poltroniere, K. C. Poldi, F. M. B. Toledo, and M. N. Arenales. A coupling cutting stock-lot sizing problem in the paper industry. *Annals of Operations Research*, 157(1):91–104, 2008.

S. C. Poltroniere, S. A. de Araujo, and K. C. Poldi. Optimization of an integrated lot-sizing and cutting stock problem in the paper industry. *Tendências em Matemática Aplicada e Computacional (TEMA)*, 17(3):305–320, 2016.

S. Rangel and A. G. de Figueiredo. O problema de corte de estoque em indústrias de móveis de pequeno e médio portes. *Pesquisa Operacional*, 28:451 – 472, 2008.

M. P. Reinders. Cutting stock optimization and integral production planning for centralized wood processing. *Mathematical and Computer Modelling*, 16(1):37 – 55, 1992.

H. Reinertsen and T. W.M. Vossen. The one-dimensional cutting stock problem with due dates. *European Journal of Operational Research*, 201(3):701 – 711, 2010.

A. Respício and M. E. Captivo. Integrating the cutting stock problem in capacity planning. *Department of Informatics and Centre of Operational Research. University of Lisbon/Portugal*, 2002.

F. Rinaldi and A. Franz. A two-dimensional strip cutting problem with sequencing constraint. *European Journal of Operational Research*, 183(3):1371 – 1384, 2007.

P. Robinson, A. Narayanan, and F. Sahin. Coordinated deterministic dynamic demand lot-sizing problem: A review of models and algorithms. *Omega*, 37(1):3 – 15, 2009.

F. Sahling, L. Buschkühl, H. Tempelmeier, and S. Helber. Solving a multi-level capacitated lot sizing problem with multi-period setup carry-over via a fix-and-optimize heuristic. *Computers & Operations Research*, 36(9):2546 – 2553, 2009.

S. G. Santos, S. A. de Araujo, and M. S. N. Rangel. Integrated cutting machine programming and lot sizing in furniture industry. *Pesquisa Operacional para o desenvolvimento*, 3(1):1–17, 2011.

E. Silva, F. Alvelos, and J. M. Valério de Carvalho. Integrating two-dimensional cutting stock and lot-sizing problems. *Journal of the Operational Research Society*, 65(1):108–123, 2014.

E. Silva, C. Viães, J. F. Oliveira, and M. A. Carravilla. Integrated cutting and production planning: A case study in a home textile manufacturing company. In A. P. F. D. B. Póvoa and J. L. de Miranda, editors, *Operations Research and Big Data*, volume 15 of *Studies in Big Data*, pages 213–220. Springer International Publishing, 2015.

N. Y. Soma and P. Toth. An exact algorithm for the subset sum problem. *European Journal of Operational Research*, 136(1):57 – 66, 2002.

H. Stadtler. A one-dimensional cutting stock problem in the aluminium industry and its solution. *European Journal of Operational Research*, 44(2):209 – 223, 1990.

H. Stadtler. Multilevel lot sizing with setup times and multiple constrained resources: Internally rolling schedules with lot-sizing windows. *Operations Research*, 51(3):487 – 502, 2003.

S. M. A. Suliman. An algorithm for solving lot sizing and cutting stock problem within aluminum fabrication industry. In *International Conference on Industrial Engineering and Operations Management, 2012*, 2012.

H. Süral, M. Denizel, and L. N. Van Wassenhove. Lagrangean relaxation based heuristics for lot sizing with setup times. *European Journal of Operational Research*, 194(1):51 – 63, 2009.

D. J. Thomas and P. M. Griffin. Coordinated supply chain management. *European Journal of Operational Research*, 94(1):1 – 15, 1996.

C. F. M. Toledo, M. da Silva Arantes, M. Y. B. Hossomi, P. M. França, and K. Akartunalı. A relax-and-fix with fix-and-optimize heuristic applied to multi-level lot-sizing problems. *Journal of Heuristics*, 21(5):687 – 717, 2015.

F. M. B. Toledo and V. A. Armentano. A lagrangian-based heuristic for the capacitated lot-sizing problem in parallel machines. *European Journal of Operational Research*, 175(2):1070 – 1083, 2006.

W. W. Trigeiro, L. J. Thomas, and J. O. McClain. Capacitated lot sizing with setup times. *Management Science*, 35(3):353–366, 1989.

P. Trkman and M. Gradisar. One-dimensional cutting stock optimization in consecutive time periods. *European Journal of Operational Research*, 179(2):291 – 301, 2007.

S. Umetani, M. Yagiura, and T. Ibaraki. One-dimensional cutting stock problem to minimize the number of different patterns. *European Journal of Operational Research*, 146(2):388 – 402, 2003.

S. Umetani, M. Yagiura, and T. Ibaraki. One-dimensional cutting stock problem with a given number of setups: A hybrid approach of metaheuristics and linear programming. *Journal of Mathematical Modelling and Algorithms*, 5(1):43–64, 2006.

J. M. Valério de Carvalho. Exact solution of bin-packing problems using column generation and branch-and-bound. *Annals of Operations Research*, 86(0):629–659, 1999.

J. M. Valério de Carvalho. Lp models for bin packing and cutting stock problems. *European Journal of Operational Research*, 141(2):253 – 273, 2002.

P. H. Vance, C. Barnhart, E. L. Johnson, and G. L. Nemhauser. Solving binary cutting stock problems by column generation and branch-and-bound. *Computational Optimization and Applications*, 3(2):111 – 130, 1994.

F. Vanderbeck. Exact algorithm for minimising the number of setups in the one-dimensional cutting stock problem. *Operations Research*, 48(6):915–926, 2000.

M. Vanzela. Problema integrado de dimensionamento de lotes e corte de estoque na indústria moveleira. Master thesis, Universidade Estadual Paulista "Júlio de Mesquista Filho"(UNESP), 2012.

M. Vanzela, S. Rangel, and S. A. de Araujo. The integrated lot sizing and cutting stock problem in a furniture factory. *IFAC Proceedings Volumes*, 46(7):390 – 395, 2013.

M. Vanzela, G. M. Melega, S. Rangel, and S. A. de Araujo. The integrated lot sizing and cutting stock problem with saw cycle constraints applied to furniture production. *Computers & Operations Research*, 79:148 – 160, 2017.

H. M. Wagner and T. M. Whitin. Dynamic version of the economic lot size model. *Management Science*, 5(1):89–96, 1958.

P. Y. Wang and G. Wäscher. Cutting and packing. *European Journal of Operational Research*, 141(2):239–240, 2002.

G. Wäscher. An lp-based approach to cutting stock problems with multiple objectives. *European Journal of Operational Research*, 44(2):175 – 184, 1990.

G. Wäscher and T. Gau. Heuristics for the integer one-dimensional cutting stock problem: A computational study. *Operations-Research-Spektrum*, 18(3):131–144, 1996.

G. Wäscher, H. Haußner, and H. Schumann. An improved typology of cutting and packing problems. *European Journal of Operational Research*, 183(3):1109 – 1130, 2007.

T. Wu, K. Akartunali, R. Jans, and Z. Liang. Progressive selection method for the coupled lot-sizing and cutting-stock problem. *INFORMS Journal on Computing*, pages 1 – 36, 2017.

H. H. Yanasse and D. M. Katsurayama. Checkerboard pattern: proposals for its generation. *International Transactions in Operational Research*, 12(1):21–45, 2005.

H. H. Yanasse and M. J. P. Lamosa. An integrated cutting stock and sequencing problem. *European Journal of Operational Research*, 183(3):1353 – 1370, 2007.

H. H. Yanasse and M. S. Limeira. A hybrid heuristic to reduce the number of different patterns in cutting stock problems. *Computers & Operations Research*, 33(9):2744 – 2756, 2006.

H. H. Yanasse and R. Morabito. A note on linear models for two-group and three-group two-dimensional guillotine cutting problems. *International Journal of Production Research*, 46 (21):6189–6206, 2008.

Appendix A

Comparison of MIP Models for the Integrated Lot-Sizing and One-Dimensional Cutting Stock Problem

Pesquisa Operacional (2016) 36(1): 167-196

Printed version ISSN 0101-7438 / Online version ISSN 1678-5142
www.scielo.br/pope
doi: 10.1590/0101-7438.2016.036.01.0167

COMPARISON OF MIP MODELS FOR THE INTEGRATED LOT-SIZING AND ONE-DIMENSIONAL CUTTING STOCK PROBLEM

Gislaine Mara Melega[1*], Silvio Alexandre de Araujo[1] and Raf Jans[2]

Received November 30, 2015 / Accepted April 1, 2016

ABSTRACT. Production processes comprising both the lot-sizing problem and the cutting stock problem are frequent in various industrial sectors. However these problems are usually treated separately, which can generates suboptimal overall solution and consequently causes production losses. In this paper, we propose different mathematical models for the integrated problem combining alternative models for the lot-sizing and the cutting stock problem, in order to evaluate and indicate the impact of these changes on the models' performance. An extensive computational study is done using randomly generated data and as a solution strategy we used a commercial optimization package and the application of a column generation technique.

Keywords: Capacitated Lot-Sizing Problems, One-Dimensional Cutting Stock Problem, Column Generation.

1 INTRODUCTION

Technological advances and increasing competition in industries have become worldwide phenomena. Some prominent aspects of these advances are the emphasis on knowledge and the development of new technologies. Inserted on this context, mathematical models that describe and improve the production processes have been studied and used as tools to support decision making in the business environment. Among the various decision processes, this work is part of the tactical/operational planning of production with the lot-sizing problem and cutting stock problem. The Lot-Sizing Problem (*LSP*) considers the tradeoff between the setup and inventory costs to determine at minimal cost the size of production lots to meet the demand of each final product, while the cutting stock problem (*CSP*) involves the cutting of large objects into smaller items, so as to minimize the total loss of material.

The literature mostly deals with these two problems separately. However, in industrial sectors such as furniture, paper and aluminum, the lot-sizing and cutting stock problems are found in

*Corresponding author.
[1]Departamento de Matemática Aplicada, IBILCE, UNESP – Universidade Estadual Paulista, 15054-000 São José do Rio Preto, SP, Brasil. E-mails: gislainemelega@gmail.com; saraujo@ibilce.unesp.br
[2]HEC Montréal and CIRRELT, Canada H3T 2A7 QC, Canada. E-mail: raf.jans@hec.ca

consecutive phases. For these cases, a decoupled vision of the problems usually used by companies, can provide good local solutions but, at the end of planning processes, these solutions may conflict with the global objectives and feasibility of production. This fact was observed in Gramani et al. (2009) in the furniture industry. In this case, solutions generated from the lot-sizing problem can lead to infeasible solutions for the cutting stock problem with regard to the production capacity of the cutting machine. If, on the other hand, for a given period, the production exceeds demand, this circumstance may generate cutting patterns with less loss, and a decrease of setup costs. However, there is an increase of inventory costs and physical space for allocation of items to be stored will be needed, which may not exist in practice. For this reason, the relevance of these issues in the industrial sectors and the benefits in dealing with problems in an integrated way makes this an appropriate research topic today. Motivated by this, the present paper proposes mathematical models and solution methods for solving the integrated lot-sizing and cutting stock problem.

In this work, the capacitated lot-sizing problem is modeled using the mathematical model proposed by Trigeiro et al. (1989), denoted here by *CL*. We also considered the variable redefinition strategy (Eppen & Martin, 1987) which reformulates the lot-sizing problem as a shortest path problem. This reformulation is called *SP*. The idea of this variable redefinition was originally proposed for uncapacitated problems and Jans & Degraeve (2004), Jans (2009), Fiorotto & de Araujo (2014) and Melega et al. (2013) extended this to cases with capacity constraints, related parallel machines, unrelated parallel machines and various plants, respectively. More information on reformulation for lot-sizing problems can be found in Denizel et al. (2008), which shows several theoretical and computational results between some reformulations.

To model the one-dimensional cutting stock problem, we consider three mathematical models from the literature. The first one is the model developed by Kantorovich (1960), here denoted by *KT*. This model is also called the generalized assignment formulation for *CSP* (Degraeve & Peeters, 2003) and determines the best way to cut objects to meet the demand, minimizing the number of objects used. For this model, an upper bound on the number of objects is considered. The second model dealt with, and perhaps the best known among the academic community, is the one proposed by Gilmore & Gomory (1961, 1963) denoted here by *GG*. This model produces good lower bounds when compared to the *KT* model, however it has an large number of variables, which indicates the use of column generation to deal with this difficulty.

The third model was proposed by Valério de Carvalho (1999, 2002), here denoted *VC*. The authors propose an alternative mathematical model for the one-dimensional cutting stock problem based on an arc flow problem. Thus, finding a valid cutting pattern for the cutting stock problem is equivalent to finding a path in a directed acyclic graph. The authors also consider additional constraints to guarantee that demand is satisfied. This model is efficient in the sense that it presents a linear relaxation as good as the Gilmore and Gomory model.

Furthermore, the cutting stock models described above were originally proposed for a single object type so they are extended to consider several types of objects in stock (*MO* from multiobjects). In a second part of this study, these models are integrated into the uncapacitated lot-sizing problem proposed by Wagner & Whitin (1958), here called *WW*.

Next, in Section 2, there is a literature review with the major studies that address the lot-sizing and the cutting stock problem in an integrated way. The Sections 3, 4 and 5 describe the proposed integrated models with capacity constraints, several object types and a reformulation for the lot-sizing, respectively. In Section 6, the solution strategies are described. The computational results are presented in Section 7. And finally, the conclusions are presented in Section 8.

2 LITERATURE REVIEW

The importance of addressing the problems in an integrated way has become increasingly evident in the literature and is closely related to industrial applications. Possibly Farley (1988) had been the first paper that treats these problems in an integrated way. The author addressed the problem in an clothing industry, where the problem involves irregular two-dimensional cuts. Hendry et al. (1996) conducted a case study on a copper components factory. The objective was to investigate alternative methods of generating production plans for casting in order to minimize costs and meet demand. Nonås & Thorstenson (2000, 2008) studied the problem coupled with the addition of setup cost in a Norwegian company that produces off-road trucks. The company needed a production plan that minimizes the total cost in the cutting process and in the production line. Therefore, the authors proposed a model and different solution methods. Gramani & França (2006) formulated a mathematical model for the integrated problem in a furniture industry and proposed a solution method based on an analogy with the shortest path problem. Computational results compared the proposed method to the method used in the industry. Poltroniere et al. (2008) studied the integrated problem applied to the paper industry. The authors analyzed the production process and proposed a mathematical model for machines operating in parallel and presented two heuristics based on Lagrangian relaxation considering the problems in a decoupled way.

Ghidini & Arenales (2009) treated the coupled problem applied to the furniture industry. To solve the problem, which considers the final composition of the products, the authors proposed two heuristics based on the primal simplex method with column generation. A procedure for obtaining an integer solution has been proposed. Gramani et al. (2009) proposed a mathematical model that considers final products and a capacity constraint on the saw machine in a furniture industry. Two solution methods were proposed. One is a decomposition heuristic and the other one is a Lagrangian based heuristic, in which the resulting problem is decomposed into two subproblems. A smoothing heuristic to recover feasibility is used. Gramani et al. (2011) extend their model from 2009 to consider cut items inventory and this model decomposed into two, one for the lot-sizing problem and one for the cutting stock problem in order to portray the solution method that industries practiced. The integrated problem was solved using the column generation technique. The computational results showed that the column generation technique can obtain gains of more than 12.7%, compared with the decomposition model used in practice. Also in the context of a furniture factory Santos et al. (2011) presented a mixed integer programming model for the integrated problem in the context of a furniture factory. The authors considered the cutting patterns used by industry and n-groups cutting pattern to solve the problem with a set of real data provided by the factory. An extension to Gramani et al. (2011) model was proposed in Vanzela et al. (2013) in order to simulate the practice of a small furniture factory.

Alem & Morabito (2012) used robust optimization tools to solve the integrated problem in the furniture industry, where the production costs and demand of products are uncertain parameters. Computational tests were performed using real and simulated data. Leão & Arenales (2012) studied a simplification of the model studied in Poltroniere et al. (2008). Two approaches have been studied: (i) for a single type of object and (ii) for various types of objects. Three mathematical models for each approach have been proposed and they used the column generation as the solution method. To obtain a feasible solution, the columns obtained with the column generation method are used and the resulting integer problem with this limited set of columns is solved.

Silva et al. (2014) presented two integer programming models for the integrated problem. In these models, both the bringing forward of items for production and also the stocking of reusable leftovers for the cutting process in later periods were allowed. The authors proposed two heuristics and evaluated the models through a computational study, checking the quality of the solutions when compared to work described in the literature. In more recent papers, Poldi & de Araujo (2016) studied the multi-period cutting stock problem and Molina et al. (2016) proposed an integrated lot sizing and packing problem that is related to the problem studied in this paper.

Most of the studies in the literature that address the integrated lot-sizing and cutting stock problem study several cases of the problem in practice. In these studies, alternative formulations for the *LSP* and *CSP* are not tested in order to choose the formulation that best fits the problem and the data set. So, extending the ideas of Longhi et al. (2015) this paper tries to cover the gap by proposing alternative mathematical formulations, considering different features of the lot-sizing and cutting stock problem such as capacity constraints and several object types. Furthermore, through the use of extensive computational tests, it intends to evaluate and point out the impact of these changes in the performance of the models.

3 MATHEMATICAL MODELS WITH CAPACITY CONSTRAINT

In this section the lot-sizing problem proposed by Trigeiro et al. (1989) (*CL* model) is integrated with various cutting stock formulations as described above. The integrated lot-sizing and cutting stock model is a two level production model, with production of final products at the final level, and the cutting of the pieces at the preceding level. The proposed model assumes that final products can be kept in inventory, but pieces cannot be kept in inventory. Furthermore, the Bill of Material indicates that one final product requires exactly one piece, and each type of final product requires a different piece. Therefore, there is a one-to-one relationship between final products and pieces in the model.

This might be a relevant model in various industrial applications. A first relevant setting is present, for instance, in the furniture industry. There is a demand for the final products (wardrobe, for example) which creates demand for subassemblies or components (a door, for example) (Gramani & França, 2006). After the cutting process, the cut items pass through another productive process (e.g. drilling and painting) before being ready to be assembled into a final product. Usually these other productive processes are considered the bottlenecks of the complete production process (Ghidini et al., 2007; Santos et al., 2011; Alem & Morabito, 2013), and their capacity is considered in the model, whereas the final assembly is not considered in the model since it is not a bottleneck process. Ghidini et al. (2007) and Alem & Morabito (2013) considered the drill machine capacity in terms of items. Santos et al. (2011) considered aggregated capacity constraints for all the other machines (except the cutting machine). For the one-dimensional case, considered in this paper, the same type of capacity constraints appear. For instance, in tubular furniture industries, the bending machine is a bottleneck of the production process and its capacity must be considered in the mathematical model.

A second potential practical background appears in industries where the cut items are directly transformed in end items, i.e., there is no need to assemble several items to compose a final product, but there is a production process between the cut items and the end items. The capacity constraint in terms of end items means that the capacity constraints are related to the final production process to transform the cut items into the final items.

Consider the following parameters and decision variables:

Parameters:
T: number of time periods (index t);
I: number of items (index i);
sc_{it}: setup cost of item i in period t;
hc_{it}: unit holding cost of item i in period t;
st_{it}: setup time of item i in period t;
vt_{it}: production time of item i in period t;
cap_t: capacity (in unit of time) in period t;
d_{it}: demand of item i in period t;
sd_{itr}: sum of demand of item i from period t until period r;
L: object length;
l_i: length of item i;
co: fixed cost of an object.

Decision Variables:
X_{it}: production quantity of item i in period t;
S_{it}: inventory for item i at the end of period t; Y_{it}: binary variable indicating the production or not of item i in period t.

The first cutting model used to formulate the integrated problem, is the *KT* model, which gives the model *CLKT* from *CL+KT*. For this, consider the following data and variables specific to this model:

Q: number of objects available in stock (index q);
y_{qt}: binary variable that indicates whether object q is used in period t;
h_{iqt}: variable that indicates the amount of item i cut from object q in period t.

Model *CLKT*

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}(sc_{it}Y_{it} + hc_{it}S_{it}) + \sum_{t=1}^{T}\sum_{q=1}^{Q} coy_{qt} \tag{1}$$

Subject to :

$$X_{it} + S_{i,t-1} = d_{it} + S_{it} \qquad \forall i,\ \forall t \tag{2}$$

$$X_{it} \leq sd_{itT}Y_{it} \qquad \forall i,\ \forall t \tag{3}$$

$$\sum_{i=1}^{I}(st_{it}Y_{it} + vt_{it}X_{it}) \leq Cap_t \qquad \forall t \tag{4}$$

$$\sum_{i=1}^{I} l_i h_{iqt} \leq L y_{qt} \qquad \forall q,\ \forall t \tag{5}$$

$$\sum_{q=1}^{Q} h_{iqt} = X_{it} \qquad \forall i,\ \forall t \tag{6}$$

$$Y_{it},\ y_{qt} \in \{0, 1\} \qquad \forall i,\ \forall q,\ \forall t \tag{7}$$

$$h_{iqt} \in \mathbb{Z}_+ \qquad \forall i,\ \forall q,\ \forall t \tag{8}$$

$$X_{it},\ S_{it} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \tag{9}$$

The objective function (1) minimizes the machine setup costs, inventory holding costs and the cost of cutting objects. Constraints (2) are the demand constraints: inventory carried over from the previous period and production in the current period are available to be used to satisfy current demand and build up inventory. Constraints (3) force the setup variable to one if any production takes place in that period. Next, there is a constraint on the available capacity in each period (4). Constraints (5) are from the cutting stock problem and ensure that if object q is used, the combination of the lengths of items that will be cut from it should not exceed its length. Constraints (6) are responsible for the integration of the lot-sizing and cut stock problem decision, that is, we have to cut a sufficient amount of items to meet the planned production amount. Finally, the set of constraints (7), (8) and (9) define the non-negativity and integrality conditions. It is known that the model *KT* for the cutting stock problem has a weak linear relaxation and presents many symmetric solution (Valério de Carvalho, 2002).

Observe that the variables X_{it} and constraints (6) could be eliminated by replacing X_{it} by $\sum_{q=1}^{Q} h_{iqt}$ in the rest of the model. However to compare this model with other proposed models, the variable X_{it} and the constraints (6) are kept explicitly in the model. This remark is also valid for the other models presented below in the Sections 3 and 4.

The second integrated formulation presented below (*CLVC*) uses the *VC* model for the cutting stock problem, which is modeled as an arc flow problem. Consider a path in a directed acyclic graph $G = (V, A)$, with $V = \{0, 1, \ldots, L\}$ and the set of arcs of the graph defined as $A = \{(j, l); 0 \leq j < l < L$ and $l - j = l_i$ for all $i \leq I\}$. The losses in the object generated from the cutting process are represented in the graph by additional arcs between the vertices $(j, j+1)$ to $j = 0, \ldots, L-1$. As decision variables for the integrated model *CLVC* we have:

f_t: flow through the network in period t;
z_{jlt}: number of cutting patterns which have an item of size $(l - j)$ allocated at a distance j from the beginning of the object in period t.

Model *CLVC*

$$\min \sum_{t=1}^{T} \sum_{i=1}^{I} (sc_{it} Y_{it} + hc_{it} S_{it}) + \sum_{t=1}^{T} cof_t \qquad (10)$$

Subject to :

$$X_{it} + S_{i,t-1} = d_{it} + S_{it} \qquad \forall i,\ \forall t \qquad (11)$$

$$X_{it} \leq sd_{itT} Y_{it} \qquad \forall i,\ \forall t \qquad (12)$$

$$\sum_{i=1}^{I} (st_{it} Y_{it} + vt_{it} X_{it}) \leq Cap_t \qquad \forall t \qquad (13)$$

$$-\sum_{(0,l)\in A} z_{0lt} = -f_t \qquad \forall t \qquad (14)$$

$$\sum_{(g,l)\in A} z_{glt} - \sum_{(l,h)\in A} z_{lht} = 0 \qquad l = 1, \ldots, L-1, \forall t \qquad (15)$$

$$\sum_{(l,L)\in A} z_{lLt} = f_t \qquad \forall t \qquad (16)$$

$$\sum_{(h,h+l_i)\in A} z_{h,h+l_i,t} = X_{it} \qquad \forall i,\ \forall t \qquad (17)$$

$$Y_{it} \in \{0, 1\} \qquad \forall i,\ \forall t \qquad (18)$$

$$z_{lht},\ f_t \in \mathbb{Z}_+ \qquad \forall (l, h) \in A,\ \forall t \qquad (19)$$

$$X_{it},\ S_{it} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \qquad (20)$$

The objective function (10) minimizes the machine setup and inventory costs of the items, as well as the cost of the flow through the network. The flow set for this problem represents the number of used objects (cutting patterns), since one flow unit defines a path, which in turn defines a cutting pattern.

Constraints (11), (12) and (13) refer to the lot-sizing problem and are defined as in *CLKT*. The set of constraints (14), (15) and (16) correspond to flow conservation constraints and are characteristic of the Valério de Carvalho model. Constraints (17) integrate the two problems and constraints (18), (19) and (20) are non-negativity and integrality constraints.

This model also presents many symmetric solutions, or alternatives that correspond to the same cutting patterns. For this reason, Valério de Carvalho (1999) presented some reduction criteria to eliminate some arcs, reducing the number of symmetric solutions without eliminating any valid cutting patterns from set A. This study applies the criterion consisting of allocating the items in order of decreasing length in each cutting pattern, that is, an item of length i_1 can only be placed after another item length i_2 if $i_1 \leq i_2$, or at the beginning of the object. Another criterion also used does not allow starting with a cutting pattern with loss. Thus, the first arc of loss will be inserted in the graph at a distance from the beginning of the object representing the shortest item length.

The third model *CLGG* combines the lot-sizing model *CL* with the cutting stock model *GG* (Gilmore & Gomory, 1961, 1963), which has the following parameters and decision variables:

J: set of cutting patterns (index j);
a_{ij}: parameter indicating the quantity of item i cut according to the cutting pattern j;
x_{jt}: variable indicating the number of objects cut according to cutting pattern j in period t.

Model *CLGG*

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}(sc_{it}Y_{it} + hc_{it}S_{it}) + \sum_{t=1}^{T}\sum_{j\in J} cox_{jt} \tag{21}$$

Subject to :

$$X_{it} + S_{i,t-1} = d_{it} + S_{it} \qquad \forall i,\ \forall t \tag{22}$$

$$X_{it} \leq sd_{itT}Y_{it} \qquad \forall i,\ \forall t \tag{23}$$

$$\sum_{i=1}^{I}(st_{it}Y_{it} + vt_{it}X_{it}) \leq Cap_t \qquad \forall t \tag{24}$$

$$\sum_{j\in J} a_{ij}x_{jt} = X_{it} \qquad \forall i,\ \forall t \tag{25}$$

$$Y_{it} \in \{0, 1\} \qquad \forall i,\ \forall t \tag{26}$$

$$x_{jt} \in \mathbb{Z}_+ \qquad \forall j,\ \forall t \tag{27}$$

$$X_{it},\ S_{it} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \tag{28}$$

The objective function (21) minimizes the sum of setup costs, inventory costs and cost related to the number of objects used in the cutting process. The sets of constraints (22), (23) and (24) refer to the lot-sizing problem and are defined as in the model *CLKT*. Constraints (25) link the cutting variable with the production variable and finally the constraints (26), (27) and (28) are non-negativity and integrality constraints on the variables. The model *GG* is an extended model and can be obtained by applying Dantzig – Wolfe decomposition on the *KT* and *VC* models

(Valério de Carvalho, 2002). As a consequence this model has a large amount of variables and is generally solved by column generation.

4 MATHEMATICAL MODELS WITH SEVERAL TYPES OF OBJECTS

For the mathematical models presented in this section, we integrate the model proposed by Wagner & Whitin (1958), here denoted by *WW* with the cutting stock models extended to consider several types of objects. The *WW* model describes the uncapacitated lot-sizing problem. In this case, consider the following parameters and additional decision variables:

Parameters:
K: number of object types available in stock (index k);
e_{kt}: planned supply of object type k at the beginning of period t;
L_k: object length of type k;
cw: cost per unit of raw material waste.

Decision Variable:
s_{kt}: number of objects of type k stocked at end of period t.

The model described below (*WWKTMO*) models the cutting stock problem based on model *KT*, considering several types of objects *MO*. In the original variables a new index is inserted, which refers to the type of object to be cut.

Parameters:
se_{kt}: sum of planned supply of objects of type k from period 1 to period t $\left(se_{kt} = \sum_{a=1}^{t} e_{ka}\right)$;
$q \in \{1, \ldots, se_{kt}\}$, index for the available objects.

Decision Variables:
y_{qkt}: binary variable that indicates whether object q of type k is used in period t;
h_{iqkt}: quantity of item i cut from object q of type k in period t.

Model *WWKTMO*

$$\min \sum_{t=1}^{T} \sum_{i=1}^{I} (sc_{it} Y_{it} + hc_{it} S_{it}) + cw \left(\sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{q=1}^{se_{kt}} L_k y_{qkt} - \sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{q=1}^{se_{kt}} \sum_{i=1}^{I} l_i h_{iqkt} \right) \tag{29}$$

Subject to:

$$X_{it} + S_{i,t-1} = d_{it} + S_{it} \qquad \forall i,\ \forall t \tag{30}$$

$$X_{it} \leq sd_{itT} Y_{it} \qquad \forall i,\ \forall t \tag{31}$$

$$\sum_{i=1}^{I} l_i h_{iqkt} \leq L_k y_{qkt} \qquad \forall q,\ \forall k,\ \forall t \tag{32}$$

$$\sum_{k=1}^{K} \sum_{q=1}^{se_{kt}} h_{iqkt} = X_{it} \qquad \forall i,\ \forall t \tag{33}$$

$$e_{kt} + s_{k,t-1} = \sum_{q=1}^{se_{kt}} y_{qkt} + s_{kt} \qquad \forall k,\ \forall t \tag{34}$$

$$Y_{it},\ y_{qkt} \in \{0, 1\} \qquad \forall q,\ \forall i,\ \forall k,\ \forall t \tag{35}$$

$$h_{iqkt} \in \mathbb{Z}_+ \qquad \forall q,\ \forall i,\ \forall k,\ \forall t \tag{36}$$

$$X_{it},\ S_{it}, s_{kt} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \tag{37}$$

The objective function (29) minimizes machine setup costs, inventory holding costs and waste cost in the cutting process. Constraints (30) and (31) refer to the lot-sizing problem and are defined as in the *CLKT* model. The set of constraints (32) ensure that if an object q of type k is used, the combination of the items that will be cut from it should not exceed its length. Constraint (33) is responsible for the integration of the decision variables of the lot-sizing and cutting stock problems. It ensures that a sufficient amount of items are cut to meet the demand. The set of constraints (34) ensure that the number of objects cut in each period does not exceed the availability in stock.

The integrated model presented below (*WWVCMO*), is based on the *VC* formulation to model the cutting stock problem. Since several object types are available in stock, we need to define the maximum object length $L_{\max} = \max_{\forall k}\{L_k\}$, corresponding to the numbers of nodes in the network. The following decision variables are also needed:

f_{kt}: flow through the network in the period t for the object type k;

z_{jlt}: number of cutting patterns which have a item of size $(l - j)$ allocated at a distance j from the beginning of the object used in period t.

Model *WWVCMO*

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I} (sc_{it} Y_{it} + hc_{it} S_{it}) + cw \left(\sum_{t=1}^{T}\sum_{k=1}^{K} L_k f_{kt} - \sum_{t=1}^{T}\sum_{i=1}^{I} \sum_{(l,l+l_i)\in A} l_i z_{l,l+l_i,t} \right) \tag{38}$$

Subject to:

$$X_{it} + S_{i,t-1} = d_{it} + S_{it} \qquad \forall i,\ \forall t \tag{39}$$

$$X_{it} \le sd_{itT} Y_{it} \qquad \forall i,\ \forall t \tag{40}$$

$$-\sum_{(0,l)\in A} z_{0lt} = -\sum_{k=1}^{K} f_{kt} \qquad \forall t \tag{41}$$

$$\sum_{(g,l)\in A} z_{glt} - \sum_{(l,h)\in A} z_{lht} = 0 \qquad l = \{1, \ldots, L_{\max} - 1\} \setminus \bigcup_{a=1}^{K} \{L_a\},\ \forall t \tag{42}$$

$$\sum_{(g,L_k)\in A} z_{gL_kt} = f_{kt} \qquad \forall k,\ \forall t \tag{43}$$

$$\sum_{(h,h+l_i)\in A} z_{h,h+l_i,t} = X_{it} \qquad \forall i,\ \forall t \tag{44}$$

$$e_{kt} + s_{k,t-1} = f_{kt} + s_{kt} \qquad \forall k,\ \forall t \qquad (45)$$

$$Y_{it} \in \{0, 1\} \qquad \forall i,\ \forall t \qquad (46)$$

$$z_{glt},\ f_{kt} \in \mathbb{Z}_+ \qquad \forall (g,l) \in A,\ \forall k,\ \forall t \qquad (47)$$

$$X_{it},\ S_{it} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \qquad (48)$$

The objective function (38) minimizes machine setup costs and storage costs of the items, and waste cost of raw material. Constraints (39) and (40) are defined as in the *CLKT* model. The set of constraints (41), (42) and (43) correspond to the flow conservation constraints. Constraints (44) integrate the cutting stock and lot-sizing problem. Constraints (45) ensure that the stock of the different types of available objects is respected and constraints (46), (47) and (48) are non-negativity and integrality constraints.

The following formulation (*WWGGMO*) models the cutting stock problem proposed by Gilmore & Gomory (1961) with an extension for several objects, which has the following parameters and decision variables. Observe that an index that refers to the type of object is also needed.

Parameters:
J_k: set of cutting patterns that use object type k (index j);
a_{ikj}: quantity of item i cut from object type k according to cutting pattern j.

Decision Variable:
x_{kjt}: number of objects of type k cut using cutting pattern j in period t.

Model *WWGGMO*

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}(sc_{it}Y_{it} + hc_{it}S_{it}) + cw\left(\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j\in J_k} L_k x_{kjt} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j\in J_k}\sum_{i=1}^{I} l_i a_{ikj} x_{kjt}\right) \qquad (49)$$

Subject to:

$$X_{it} + S_{i,t-1} = d_{it} + S_{it} \qquad \forall i,\ \forall t \qquad (50)$$

$$X_{it} \le sd_{itT} Y_{it} \qquad \forall i,\ \forall t \qquad (51)$$

$$\sum_{k=1}^{K}\sum_{j\in J_k} a_{ikj} x_{kjt} = X_{it} \qquad \forall i,\ \forall t \qquad (52)$$

$$e_{kt} + s_{k,t-1} = \sum_{j\in J_k} x_{kjt} + s_{kt} \qquad \forall k,\ \forall t \qquad (53)$$

$$Y_{it} \in \{0, 1\} \qquad \forall i,\ \forall t \qquad (54)$$

$$x_{kjt} \in \mathbb{Z}_+ \qquad \forall k,\ \forall j,\ \forall t \qquad (55)$$

$$X_{it},\ S_{it} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \qquad (56)$$

The objective function (49) minimizes machine setup cost and inventory cost of items, and costs of waste due to the cutting of objects. Constraints (50) and (51) refer to the lot-sizing problem. The set of constraints (52) is responsible for the integration of the decision variables of the two problems. The constraints (53) ensure that the number of cut objects in each period does not exceed the availability in stock, and the constraints (54), (55) and (56) are non-negativity and integrality constraints.

5 INTEGRATED MODELS USING THE REFORMULATED LOT-SIZING PROBLEM *SP*

The integrated models presented in Sections 3 and 4 use the classical formulation for the lot-sizing problem. In order to obtain better lower bounds, the lot-sizing problem is reformulated as a shortest path problem (Eppen & Martin, 1987) and then integrated into the respective cutting stock problem. For that, we need the following definitions of parameters and variables, respectively:

cv_{itr}: inventory holding cost of item i in period t at a quantity that meets the demands for the periods from t until r;

zv_{itr}: fraction of the production plan for item i to satisfy demand from period t to period r.

The lot-sizing problem variables have the following correspondence:

$$X_{it} = \sum_{r=t}^{T} sd_{itr} zv_{itr} \qquad \forall i,\ \forall t \tag{57}$$

and the demand constraints (2) and setup constraints (3) are rewritten in terms of the new decision variables as follows:

$$\sum_{r=1}^{T} zv_{i1r} = 1 \qquad \forall i \tag{58}$$

$$\sum_{r=1}^{t-1} zv_{irt-1} = \sum_{r=t}^{T} zv_{itr} \qquad \forall i,\ \forall t \backslash \{1\} \tag{59}$$

$$\sum_{r=t}^{T} zv_{itr} \leq Y_{it} \qquad \forall i,\ \forall t \tag{60}$$

Constraints (58) and (59) define the flow constraints in the *SP* model. For each item i, a unit flow is sent through the network (constraint (58)), imposing that its demand has to be satisfied without backlogging in each period (59)). Constraint (60)) ensures that item i will be produced in period t only if there is a setup prepared to produce that item.

Note that the constraint (58) forces production in the first period, which it is not a problem for the data set used in this work, because all the demand equal 0 in the used data set is replaced to be strictly greater than zero (equal 1 unit). Otherwise, if the demand for some item is zero for the first period, it would be necessary to slightly adapt the formulation (Pochet & Wolsey, 2006; Jans & Degraeve, 2004).

Thus, we propose new mathematical models that integrate the cutting stock problems (*KT, VC, GG, KTMO, VCMO, GGMO*) into the reformulated lot-sizing problem (*SP*). This is done by replacing the decision variables in the models, by the new variables and constraints described above.

As an example, the integrated model *SPKT* is shown next.

Model *SPKT*

$$\min \sum_{t=1}^{T}\sum_{i=1}^{I}\left(sc_{it}Y_{it} + \sum_{r=t}^{T} cv_{itr}zv_{itr}\right) + \sum_{t=1}^{T}\sum_{q=1}^{Q} coy_{qt} \tag{61}$$

Sujeito a :

$$\sum_{r=1}^{T} zv_{i1r} = 1 \qquad \forall i \tag{62}$$

$$\sum_{r=1}^{t-1} zv_{irt-1} = \sum_{r=t}^{T} zv_{itr} \qquad \forall i,\ \forall t\backslash\{1\} \tag{63}$$

$$\sum_{r=t}^{T} zv_{itr} \le Y_{it} \qquad \forall i,\ \forall t \tag{64}$$

$$\sum_{i=1}^{I}\left(st_{it}Y_{it} + \sum_{r=t}^{T} vt_{it}sd_{itr}zv_{itr}\right) \le Cap_t \qquad \forall t \tag{65}$$

$$\sum_{i=1}^{I} l_i h_{iqt} \le L y_{qt} \qquad \forall q,\ \forall t \tag{66}$$

$$\sum_{q=1}^{Q} h_{iqt} = \sum_{r=t}^{T} sd_{itr}zv_{itr} \qquad \forall i,\ \forall t \tag{67}$$

$$Y_{it},\ y_{qt} \in \{0, 1\} \qquad \forall i,\ \forall q,\ \forall t \tag{68}$$

$$h_{iqt} \in \mathbb{Z}_+ \qquad \forall i,\ \forall q,\ \forall t \tag{69}$$

$$zv_{itr},\ S_{it} \in \mathbb{R}_+ \qquad \forall i,\ \forall t \tag{70}$$

The objective function (61) minimizes the setup costs and inventory holding costs according to the *SP* model, as well as, the cost of the number of objects to be cut. The constraints (62) and (63) define the flow constraints of the *SP* model. The set of constraints (64) also refers to the model *SP* and ensures that there will be production for an item i in period t, only if the machine is setup to produce this item. Capacity constraints (65) ensure that the capacity available is not violated. Constraints (66) refers to the knapsack constraints in the *KT* model. The set of constraints (67) integrates the two problems and finally (68), (69) and (70) are non-negativity and integrality constraints.

6 SOLUTION STRATEGIES

Two heuristic strategies are proposed for solving the models described in sections 3, 4 and 5. The first heuristic consists of solving the models integrated with *KT* and *VC* using a commercial optimization package with a stopping criteria. The solver is stopped at the end of 600 seconds or when the gap between the upper bound and lower bound is less than 0.1%. The second heuristic proposes the search for a feasible solution by using the column generation technique for the models integrated with *VC* and *GG*. The solver is used to solve the restricted master problems, the sub-problems and also to solve the resulting integer restricted master problem.

Next, the column generation based heuristic is explained in more detail together with the procedure to obtain a feasible solution.

6.1 Initial Basic Solution

Considering the model with capacity constraint and integrated with the *GG* model, the master problem starts with the columns related to the homogeneous cutting pattern, i.e., columns of type: $(0, \ldots, a_{ii}, \ldots, 0)$, where $a_{ii} = \lfloor \frac{L}{l_i} \rfloor,\ \forall i$. The same columns are inserted for each period, $t = 1, \ldots, T$.

For the problems with several objects, the master problem starts with homogeneous cutting patterns generated for each object type, i e., for each object k columns of type $(0, \ldots, a_{iki}, \ldots, 0)$, where $a_{iki} = \lfloor \frac{L_k}{l_i} \rfloor,\ \forall i$, are inserted in order to avoid the infeasibility of an initial basic solution. The same columns are inserted for all periods. Similar ideas are applied to the models integrated with *VC*.

6.2 Column Generation Procedure

The sub-problems in the column generation consist of finding a new cutting pattern for the master problem. For the models integrated with *GG*, the sub-problem is composed of the knapsack constraint and for those integrated with *VC* the sub-problem is composed of the flow constraints imposing a flow equal to one, which represents a cutting pattern.

Considering the models with one type of object and a capacity constraint, a sub-problem is solved for every period, and the column with the smallest reduced cost over all periods is included in the master problem. As a consequence, the same column is also inserted for each period. For the problems with several objects, a sub-problem is solved for each period and object type, and the column with the smallest reduced cost is inserted into the master problem. The same column is used for all periods according to the corresponding object in the restricted master problem.

6.3 Feasibility Strategy

The column generation part is stopped when the minimum reduced cost is non-negative for each subproblem or when the time limit is reached. The integer restricted master problem including all the columns obtained in the column generation procedure is solved using the optimization package with a time limit of 600 seconds and an optimality gap of 0.1%.

Table 1 presents a summary of all proposed mathematical models and solution strategies.

Table 1 – Mathematical Models and Solution Strategies.

Models	Considered Problems	Solution Strategies
CLKT *SPKT* *CLVC* *SPVC*	*LSP* with capacity constraint *CSP* single object type	Optimization package based heuristic
CLGG_CG *SPGG_CG* *CLVC_CG* *SPVC_CG*	*LSP* with capacity constraint *CSP* single object type	Column generation based heuristic
WWKTMO *SPKTMO* *WWVCMO* *SPVCMO*	*LSP* without capacity constraint *CSP* with several object types	Optimization package based heuristic
WWGGMO_CG *SPGGMO_CG* *WWVCMO_CG* *SPVCMO_CG*	*LSP* without capacity constraint *CSP* with several object types	Column Generation based heuristic

7 COMPUTATIONAL STUDY

This section presents the computational results used to evaluate and compare the performance of the proposed models. The analysis of the results is done through tables containing the average values found by models with respect to different aspects, as well as the use of the performance profile technique (Dolan & Moré, 2002). This technique provides a tool which facilitates the exhibition and the interpretation of comparisons and it is briefly described in the following paragraphs.

Consider $\mathbb{P}$ as the set of n_p instances and $\mathbb{M}$ as the set of n_M models described in sections 3, 4 and 5. The values obtained for each instance (upper bound and gap) $p \in \mathbb{P}$ using the $m \in \mathbb{M}$ model is denoted by $v_{p,m}$. For each model $m \in \mathbb{M}$ a comparison of its performance on the instance $p \in \mathbb{P}$ relative to the performance of the best model is given by the following performance ratio:

$$r_{p,m} = \frac{v_{p,m}}{\min_{m \in \mathbb{M}}\{v_{p,m}\}}.$$

If the model m does not find a feasible solution for an instance p then $r_{p,m}$ is defined as r_M, which is set at one unit more than the worst value of the performance ratio found for all the feasible instances in all the models. The performance of the model m compared to the other models is given by the performance profile:

$$\rho_m(\tau) = \frac{1}{n_p}|\{p \in \mathbb{P} : \; r_{p,m} \leq \tau\}|$$

with $|.|$ representing the number of elements in the set. The performance profile $\rho_m(\tau)$ is a function that is associated to a given value $\tau \in \mathbb{R}$, and indicates the fraction of instances solved by the model m with a performance within a factor τ of the best performance found. With this, each model has a curve that shows its performance for each level τ. Due to the fact that r_M can be considerably large, the logarithm scale is used to represent the performance profile. It is done as follows (Dolan and Moré, 2002):

$$\tau \mapsto \frac{1}{n_p}|\{p \in \mathbb{P} : \log_2(r_{p,m}) \leq \tau\}|.$$

This way, the τ factor varies in $[0, r_M)$, with $r_M = 1 + \max\{\log_2(r_{\overline{p},\overline{m}}) : \; \overline{p} \in \mathbb{P} \text{ and } \overline{m} \in \mathbb{M}\}$.

The models are written in the AMPL syntax (Fourer et al., 1990) and CPLEX 12.5 (IBM, 2009) is used as solver. All the computational tests are conducted on a 2.93GHz Intel Core i7 processor with 8GB of RAM memory.

7.1 Experiment 1

Next, we describe the data generation and computational results for the integrated problem with capacity constraints (models in Section 3 and the corresponding models in Section 5). In this experiment we present two sets of data: the first one called Data 1, is based on a data generator for the cutting stock problem. The second set, referred to as Data 2, is based on some examples widely used in the literature to solve the lot-sizing problem.

7.1.1 Data Sets

For the Data 1 set, the CUTGEN1 generator proposed by Gau & Wäscher (1995) is used and for the lot-sizing problem the data set is based on Trigeiro et al. (1989). The parameters are generated in intervals $[a, b]$ with a uniform distribution as follows:

- number of periods: $T = 15$
- object length: $L = 1000$
- number of items: $I = \{10, 20, 40\}$
- length of items are generated in three intervals according to $l_i \in [v_1, v_2]$, with $v_1 = 0.01L$ or $0.2L$ and $v_2 = 0.02L$ or $0.8L$
- demand for items is generated according to the idea of Gau & Wäscher (1995) for a single period, so that the average demand over all items in a specific period is equals 100.

- raw material cost: $co = 1$;
- setup cost: $sc_{it} \in [100, 500]$
- inventory cost: $hc_{it} \in [1, 5]$
- production time: $vt_{it} = 1$
- setup time: $st_{it} \in [10, 50]$
- capacity (cap_t) is generated by the average of lot-by-lot policies: for every period t calculate the amount of resources needed to produce exactly the demands of the items in this period, sum up this amount for all periods and divide by the number of periods T, this is, $cap_t = \frac{\sum_{t=1}^{T} \sum_{i=1}^{I} (vt_{it} d_{it} + st_{it})}{T}$.

In the cutting stock problem modeled by *KT*, an upper bound on the number of objects in stock (Q) is needed. Poldi & Arenales (2010) is the basis for calculating the upper bound as $Q = 2\lceil\lambda\rceil$ with, $\lambda = \frac{\sum_{t=1}^{T} \sum_{i=1}^{I} l_i d_{it}}{L}$.

Thus, the Data 1 defines 9 classes (see Table 2), and for each class 10 random instances are generated considering the three levels of capacity, totaling 270 instances.

Table 2 – Classes of Data 1.

Classes	Data 1	
	Items	v_1; v_2
Class 1	10	0.01; 0.2
Class 2	10	0.01; 0.8
Class 3	10	0.2 ; 0.8
Class 4	20	0.01; 0.2
Class 5	20	0.01; 0.8
Class 6	20	0.2 ; 0.8
Class 7	40	0.01; 0.2
Class 8	40	0.01; 0.8
Class 9	40	0.2 ; 0.8

The second data set, called Data 2, is based on the lot-sizing problem. This set is created using 7 specific instances from Trigeiro et al. (1989) (see Table 3). These instances are considered difficult and have been used in several studies in the literature (Jans & Degraeve, 2004; Vyve & Wolsey, 2006; Degraeve & Jans, 2007; de Araujo et al., 2015).

To complete the data file the CUTGEN1 is used as generator for the cutting stock parameters and it was described above. Thus, Data 2 defines 3 classes (see Table 3), and for each class 10 random instances are generated for each of the 7 specific instances, totaling 210 instances.

Table 3 – Specific Instances and Classes of Data 2.

Instance	Items	Periods
G30	6	15
G30wol	6	15
G53	12	15
G57	24	30
G62	6	30
G69	12	30
G72	24	30

Classes	Data 2
	v_1; v_2
Class 10	0.01; 0.2
Class 11	0.01; 0.8
Class 12	0.2 ; 0.8

7.1.2 Computational Results

Firstly, we present the computational results using Data 1.

A preliminary test is performed in order to evaluate the impact of the models and the capacity tightness on the number of feasible solutions found. It is considered three different levels of capacity: $cap0$ where we have the integrated problem without capacity constraint; $cap1$ where the value of the capacity is $cap_t/0.3$; $cap2$ where we have tighter capacity given by $cap_t/0.85$. The results shown that the number of feasible solutions decreases as the capacity becomes more scarce. The *CLVC* model is the only formulation able to find feasible solutions for all instances, but only for the uncapacitated case. The *CLVC* model obtained the largest number of feasible solutions for the three capacity levels using the optimization package based heuristic. For the column generation based heuristic the best performance in this analysis is obtained by *CLVC_CG* model. In general, the difficulty to find a feasible solution is in Classes 4 and 7, which correspond to classes in which the ratio between the item length and object length is small.

The following results are evaluated considering the uncapacitated model (column $cap0$) and with $cap_t/0.3$ (column $cap1$).

In Table 4 the lower bound obtained with the linear relaxation for the models is presented. As a general conclusion, regarding the cutting stock problem, the models integrated with *VC*, *VC_CG* and *GG_CG* are equivalent with relation to the value of the lower bound obtained. For this reason just one of them is presented and the lower bounds are better than the lower bounds resulting from the models integrated with *KT*. The value obtained with $cap0$ and $cap1$ are the same for all the models and just one value is presented. Regarding the lot sizing problem the models integrated with *SP* obtain a substantially better lower bound than the models using *CL*.

Figures 1 and 2 show the performance profiles obtained with the upper bound for all the instances in Data 1. Considering the optimization package based heuristic (see Fig. 1), the *CLVC* model shows a good overall performance in both variations of capacity, since its performance profile dominated the rest of them. This model could find the best upper bound for around 70% of the instances, followed by the *SPVC*. For the column generation based heuristic (see Fig. 2), *CLVC_CG* obtained the best performance for around 50% of the instances, followed by *SPGG_CG*. Just

Table 4 – LP Relaxation for Data 1.

Classes	*CLKT*	*SPKT*	*CLVC*	*SPVC*
Class 1	10978.63	34793.33	10978.65	34794.62
Class 2	15735.78	39828.10	16801.00	41005.42
Class 3	17110.47	41202.79	18733.81	42964.73
Class 4	22083.04	70486.11	22083.04	70486.11
Class 5	31493.67	79562.22	33074.99	81292.92
Class 6	34020.87	82345.81	36519.61	85032.61
Class 7	43091.46	137835.25	43091.46	137835.25
Class 8	59926.51	154188.53	61315.45	155838.87
Class 9	65761.36	159574.61	68065.34	162285.77
Average	33355.75	88868.53	34518.15	90170.70

considering the $cap0$ instances it is possible to obtain an upper bound for all the instances with the *CLVC*, *CLVC_CG* and *CLGG_CG* models. The lowest number of feasible solutions, around 10% and 40%, are found respectively by the *SPKT* and *SPVC_CG* models. In general, mathematical models integrated into *VC* obtained the best results for most instances in both the capacity variations and heuristic strategies.

Figure 3 shows the performance profile obtained for the gap for models solved with the optimization package based heuristic. The gap is that provided by the solver for the final solution for both the cases $cap0$ and $cap1$. In both variations of capacity, the *SPKT* could not find a satisfactory results for the gap. The *CLVC* model showed a good overall performance, in which 100% and 90% of the instances obtained a feasible solution for $cap0$ and $cap1$, although it has the best performance in just 30% of the instances. On the other hand, the *SPVC* model showed the best performance in approximately 60% of the instances, but with the disadvantage of not having solved all the instances. Since the column generation procedure is subject to a time limit, we cannot guarantee the optimality of this lower bound, and therefore no results on the gap are presented for the *CG* methods.

The computational results for Data 2 are given as follow.

The same analysis for Data 2 is performed in order to evaluate the impact of the models and the capacity tightness on the number of feasible solutions found. It is noticed a big difficulty of the *SPKT* model to solve these instances. The *SPVC_CG* model also shows great difficulties in finding a feasible solution. Class 12 presents the largest number of feasible solutions (378 from 560), followed by Class 11 (369 from 560). In Class 12, half of the models found a feasible solution for all the instances; this class is related with the longest items when compared to object length.

In the Table 5 the lower bound obtained with the linear relaxation for the models solved with the optimization package are presented. As discussed above the models *VC*, *VC_CG* and *GG_CG* are equivalent with relation to the lower bound obtained and due to this reason just one of them is presented. The use of a reformulated lot-zing model once again, improves the results compared

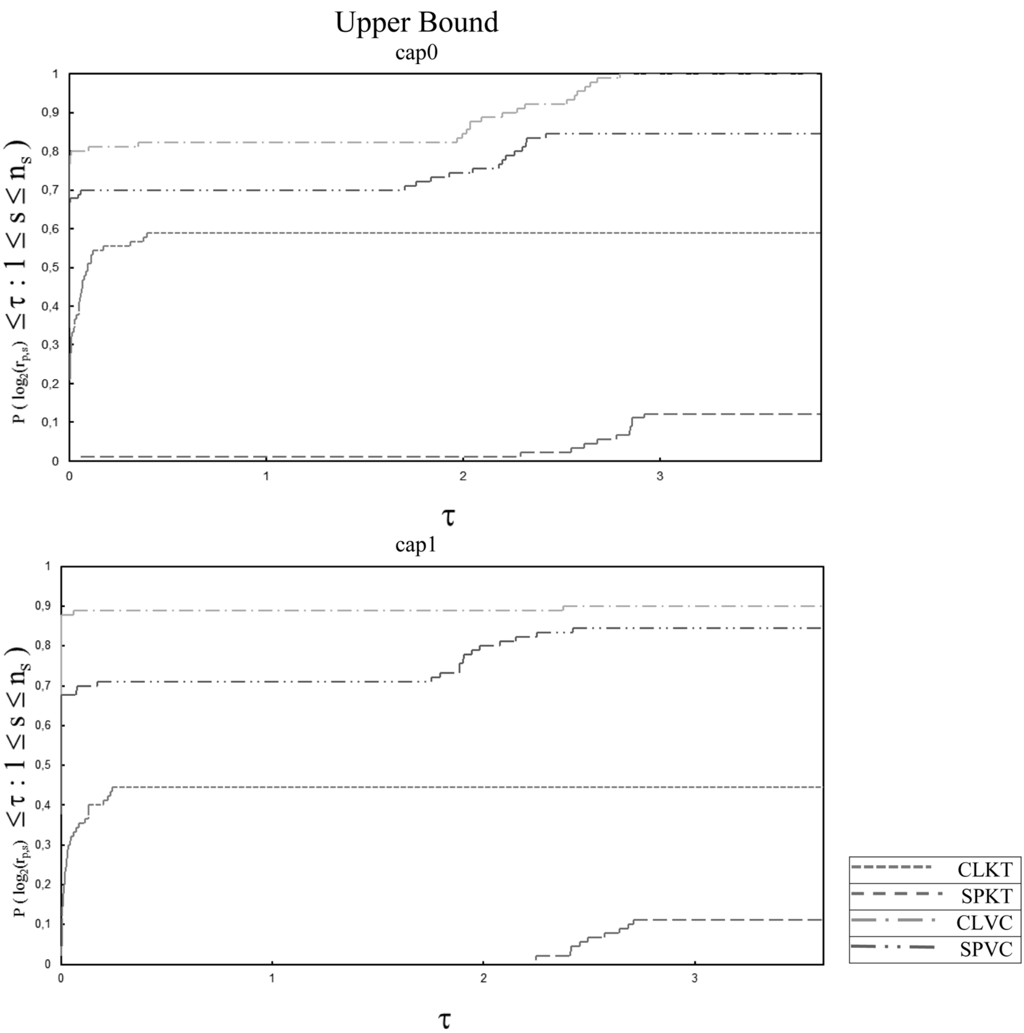


Figure 1 – Performance Profiles: Upper Bound for the Optimization Package Based Heuristic – Data 1.

to the models integrated with the classical lot-zing model. This improvement is 260% on average. The models integrated with *KT* did not achieve a value for the linear relaxation for some instances within a time of 5 hours.

Figure 4 shows the performance profile of the upper bound and gap considering the optimization package based heuristic. The first impact observed in the figure is due to the huge difference between the models integrated with *KT* and *VC* to obtain an upper bound. The *SPKT* and *CLKT* models found an upper bound for around just 5% and 40% of the instances, respectively. On the other hand, the *SPVC* and *CLVC* models found a upper bound for around 65% and 90% of the instances, respectively. The best performance to obtain the upper bounds is found using the *CLVC* model with more than 80% of the instances ($\tau = 0$), compared with the other models,

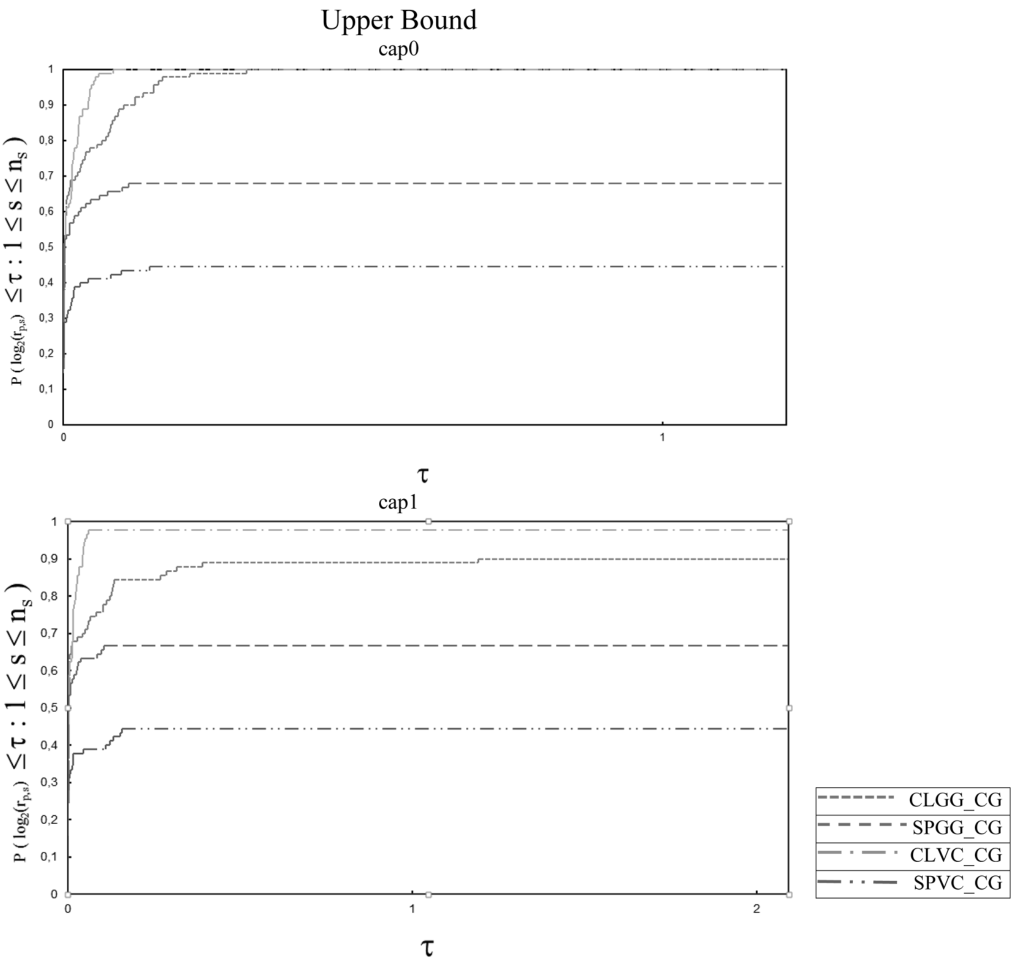


Figure 2 – Performance Profiles: Upper Bound for the Column Generation Based Heuristic – Data 1.

which reached at most 10% of the best results (at $\tau = 0$). The performance obtained with the *SPVC* models changes a little when the gap is analyzed. The performance of the *SPVC* becomes around the same as *CLVC*, 40% of the best results ($\tau = 0$). And one more time, results obtained with *CLVC* model dominate the other models considering the performance values (τ).

Figure 5 presents the performance obtained in the column generation based heuristic for the upper bounds. We can see that the *CLVC_CG* model found an upper bound for around 98% of the instances, although it could reach the best performance for just 20% of the instances. On the other hand, *CLGG_CG* found the best results for the upper bound for almost 75% of the instances solved.

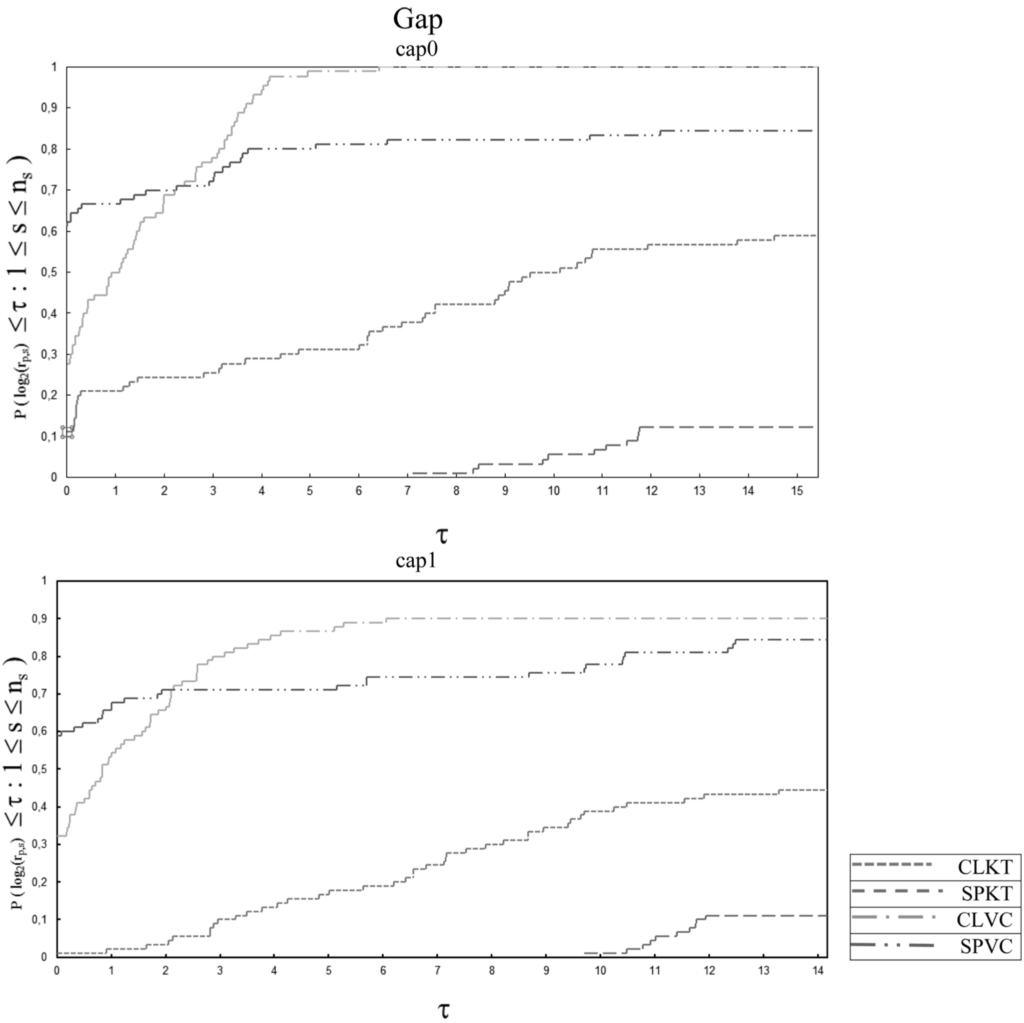


Figure 3 – Performance Profiles: Gap for Optimization Package Based Heuristic – Data 1.

7.2 Experiment 2

The following describes the generation of data and computational results for the integrated uncapacitated lot-sizing and the cutting stock problem with several types of objects (the models in Section 4 and its corresponding model in Section 5).

7.2.1 Data Set

The proposed data set is based on Poldi & Arenales (2010) and for the lot-sizing problem the data is based on Trigeiro et al. (1989), but without considering capacity constraints.

Table 5 – LP Relaxation for Data 2.

Class 10	*CLKT*	*SPKT*	*CLVC*	*SPVC*
Example 1	12552.56	38450.41	12552.81	38455.10
Example 2	12534.92	38645.79	12535.19	38649.20
Example 3	25058.61	74650.39	25058.67	74651.33
Example 4	46554.59	143706.15	46554.59	143706.16
Example 5	13311.81	63092.75	13312.38	63096.81
Example 6	30428.71	136060.92	30428.72	136062.38
Example 7	64483.84	299427.41	64483.84	299427.59
Average	29275.01	113433.40	29275.17	113435.51
Class 11				
Example 1	14831.41	40729.26	15257.52	41292.36
Example 2	14813.77	40924.64	15243.59	41476.43
Example 3	29619.41	79211.19	30346.19	80104.87
Example 4	56385.07	153536.63	57257.32	154658.84
Example 5	18060.64	67841.58	18919.93	68835.03
Example 6	40162.36	145794.58	41623.78	147495.46
Example 7	—	—	86738.85	322311.95
Average	—	—	37912.46	122310.71
Class 12				
Example 1	15702.19	41600.04	16508.34	42542.73
Example 2	15684.55	41795.42	16493.35	42720.26
Example 3	31270.20	80861.98	32472.08	82150.89
Example 4	59625.74	156777.30	61617.88	158934.87
Example 5	19827.17	69608.12	21501.31	71404.10
Example 6	43685.89	149318.11	46154.93	152056.45
Example 7	—	—	95674.65	331111.52
Average	—	—	41488.93	125845.83

(−) time limit with no lower bound

- number of periods: $T = \{3, 6\}$
- types of objects: $K = \{3, 5\}$
- number of items: $I = \{10, 20\}$
- object length: $L_k \in [300, 1000]$
- planned supply of object type k in period t: $e_{kt} \in [1.5\lceil\lambda_t\rceil, 2\lceil\lambda_t\rceil]$ with, $\lambda_t = \frac{\sum_{i=1}^{I} l_i d_{it}}{\sum_{k=1}^{K} L_k}$.
- item length: $l_i \in [0, 1\overline{L}, 0, 4\overline{L}]$, whith $\overline{L} = \frac{\sum_{k=1}^{K} L_k}{K}$
- item demand: $d_{it} \in [10, 200]$

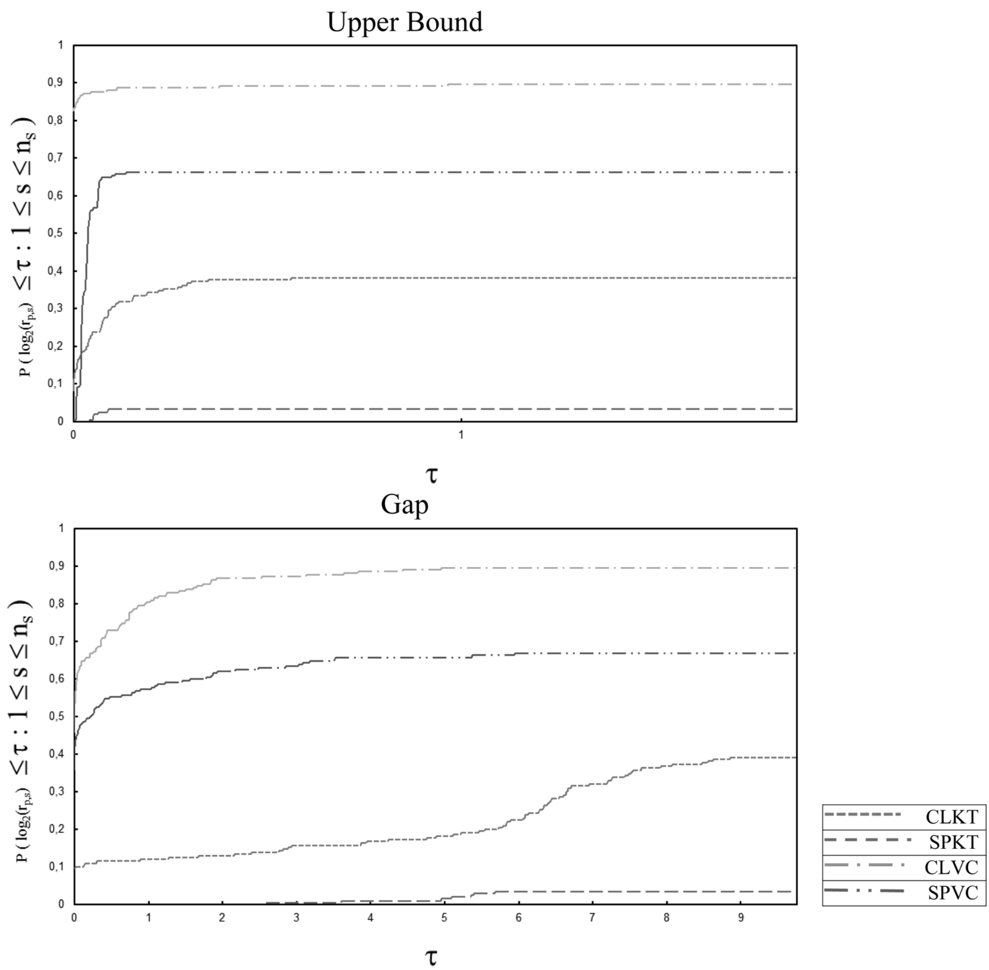


Figure 4 – Performance Profiles: Upper Bound for the Optimization Package Based Heuristic – Data 2.

- raw material cost: $cw = 1$;
- setup cost: $sc_{it} \in [100, 500]$
- inventory cost: $hc_{it} \in [1, 5]$

Thus, the data set defines 8 classes (see Table 6), and for each class 10 random instances are generated, totalling 80 instances.

7.2.2 Computational Results

This data set included 3 infeasible instances (Class 16 instance 10 and Class 20 instances 5 and 10) and these were removed from the analysis.

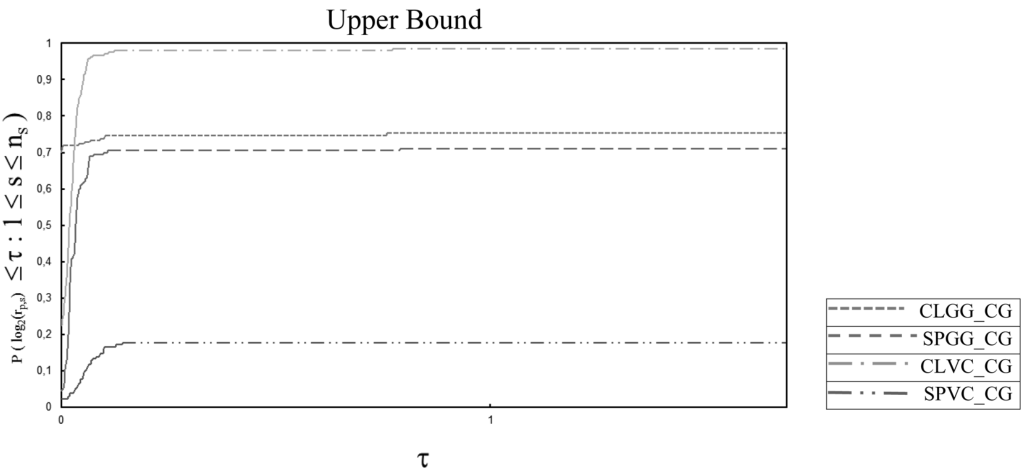


Figure 5 – Performance Profiles: Upper Bound for the Column Generation Based Heuristic – Data 2.

Table 6 – Classes with several types of objects.

Classes	Periods	Objects	Items
Class 13	3	3	10
Class 14	3	3	20
Class 15	3	5	10
Class 16	3	5	20
Class 17	6	3	10
Class 18	6	3	20
Class 19	6	5	10
Class 20	6	5	20

Table 7 presents the computational results obtained for the lower bound. The use of the reformulated lot-sizing model shows significant improvements in the lower bound. A slight improvements can be seen in the use of *VC* model when compared with the *KT* model.

Tables 8 and 9 show the average upper bounds in each class for each mathematical model. The*WWKTMO* and *SPKTMO* models did not find a feasible solution for most classes. Considering the optimization package based heuristic the *WWVCMO* model found the best results for almost all the classes (except Classe 16). For the column generation based heuristic the best results are shared between *WWGGMO_CG* and *SPGGMO_CG* models. As the *WWGGMO_CG* model obtained results which are very close to the best found, this fact contributed to the best global average. Note that *WWVCMO* performs very poorly on Class 20, compared to the column generation methods. This is also confirmed by the largest gap for this class.

Table 10 presents the average for the gap obtained in each class for models solved with the optimization package based heuristic. For this strategy the gap used is that provided by the solver

Table 7 – LP Relaxation.

Classes	*WWKTMO*	*WWVCMO*	*SPKTMO*	*SPVCMO*
Class 13	4861.68	4861.68	5472.80	5472.80
Class 14	9840.43	9840.43	9931.05	9931.05
Class 15	4801.52	4868.99	5201.06	5201.06
Class 16	9872.04	9873.19	9872.04	9873.19
Class 17	6831.39	6831.39	8743.99	8779.76
Class 18	13455.68	13455.68	13510.54	13510.54
Class 19	6584.89	6584.89	7436.32	7436.32
Class 20	13626.16	13637.07	13626.16	13637.07
Average	8734.22	9225.75	8742.66	9230.23

Table 8 – Upper Bounds for the Optimization Package Based Heuristic.

Classes	*WWKTMO*	*WWVCMO*	*SPKTMO*	*SPVCMO*
Class 13	18014.40	**8081.20**	40179.80	8081.70
Class 14	(*5)	**15188.20**	—	15214.40
Class 15	17307.50	**7629.40**	30391.60	7629.90
Class 16	(*6)	14634.00	—	**14627.67**
Class 17	(*4)	**16447.10**	—	16507.30
Class 18	(*2)	**28801.30**	—	29546.30
Class 19	(*5)	**15560.80**	—	15721.90
Class 20	—	**219048.63**	—	(*1)
Average	—	**40673.83**	—	—

(−) time limit with no integer solution

(∗) number of instances with feasible solution

Table 9 – Upper Bounds for the Column Generation Based Heuristic.

Classes	*WWGGMO_CG*	*WWVCMO_CG*	*SPGGMO_CG*	*SPVCMO_CG*
Class 13	8171.40	8509.50	**8161.40**	8456.80
Class 14	**15440.90**	20884.90	15442.40	21045.90
Class 15	7586.40	9196.00	**7577.20**	9373.10
Class 16	**14941.00**	22470.22	14999.89	22707.78
Class 17	**16616.10**	19200.20	16675.70	19736.20
Class 18	**29996.20**	47068.00	33889.80	51973.40
Class 19	15335.90	19712.30	**15334.60**	20369.80
Class 20	**32340.50**	45904.88	52861.25	47941.50
Average	**17553.55**	24118.25	20617.78	25200.56

for the final solution. As can be seen, most of the best gaps are found by *WWVCMO* model. In general, the influence of data on the results relates to the increase in the number of items (in the even-numbered classes), where the gaps are significantly higher compared to the odd-numbered classes.

Table 10 – Final Gap.

Classes	*WWKTMO*	*WWVCMO*	*SPKTMO*	*SPVCMO*
Classe 13	58.79	**0.10**	80.41	0.11
Classe 14	(*5)	**0.29**	—	0.46
Classe 15	58.94	**0.08**	74.78	0.08
Classe 16	(*6)	0.39	—	**0.35**
Classe 17	(*4)	0.24	—	**0.22**
Classe 18	(*2)	**2.78**	—	3.96
Classe 19	(*5)	**0.50**	—	1.55
Classe 20	—	**48.16**	—	(*1)
Average	—	**6.57**	—	—

8 CONCLUSIONS

In this paper, we present a study of mathematical models from the literature for modeling the lot-sizing problem and the one-dimensional cutting stock problem, in order to propose and compare new integrated problems. For the lot-sizing problem, we study the model proposed by Wagner & Whitin (1958), the model proposed by Trigeiro et al. (1989) and a reformulation of the lot-sizing problem proposed by Eppen & Martin (1987). For the cutting stock problem, extensions of the models proposed by Kantorovich (1960), Valério de Carvalho (1999, 2002) with reduction criteria and Gilmore & Gomory (1961) have been proposed to incorporate multiple periods. These models have been extended also to consider various types of objects.

In the literature, there are several studies of practical cases, which use these formulations to model the problems found, as simple or integrated problems. This study aims to present alternative mathematical models for the integrated problem in order to compare and point out the advantages of each model, as well as, the impact of the data set on the obtained solutions. As solution methods, we present two strategies. The first one uses a solver for finding the solution. In the second one, the column generation technique is used in a heuristic strategy to get a feasible solution. An extensive computational study is conducted with different data sets.

The difficulty of the models to obtain a feasible solution becomes clear when considering the capacity constraint in the lot-sizing problem, and this impact is even greater when the capacity constraint is tight. The main influence of the data set on the results is in instances that have an item length considerably smaller when compared to the object length. For the models that consider various object types in stock, the main impact of the data in the results comes from the increase in the number of items.

In general, models integrated with the classical lot-sizing problem and the cutting stock problem model based on Valério de Carvalho (1999, 2002) with column generation obtained the largest number of feasible solutions compared to other mathematical models in all analyses. It is also possible to generalize the fact that the models that integrate with the model proposed by Kantorovich (1960) have produced bad results in all experiments. For models with capacity constraints, the classical lot-sizing model integrated with the Valério de Carvalho model and Gilmore and Gomory in both capacity variations (uncapacitated model and $cap_t = cap_t/0,3$) obtained a lot of good upper bounds. However the use of the shortest path model (Eppen & Martin, 1987) significantly improved the *LP* lower bound and obtained gaps smaller when compared to other models, for most classes. In an analysis for the mathematical models with various object types, although the integration with the Gilmore and Gomory model does not show the best results for all classes for the upper bound, the proximity to the best results contributes to the achievement of best overall average.

A possible direction for future research is to study other extensions of the models proposed and develop more elaborate solution methods, in order to address a greater variety of instances.

ACKNOWLEDGEMENTS

We would like to thank the anonymous referees for their constructive comments, which led to a clearer presentation of the material. This research was funded by the Conselho Nacional de Desenvolvimento Científico e Tecnológico – CNPq and the Fundação de Amparo a Pesquisa do Estado de São Paulo – FAPESP (process no. 474782/2013-1, 2012/20631-2, 2014/17273-2 and 2014/01203-5).

REFERENCES

[1] ALEM D & MORABITO R. 2013. Risk-averse two-stage stochastic programs in furniture plants. *OR Spectrum*, **35**(4): 773–806, 2013. ISSN 0171-6468.

[2] ALEM DJ & MORABITO R. 2012. Production planning in furniture settings via robust optimization. *Computers & Operations Research*, **39**(2): 139–150.

[3] DE ARAUJO SA, DE REYCK B, DEGRAEVE Z, FRAGKOS I & JANS R. 2015. Period decompositions for the capacitated lot sizing problem with setup times. *INFORMS Journal on Computing*, **27**(3): 431–448.

[4] DEGRAEVE Z & JANS R. 2007. A new dantzig-wolfe reformulation and branch-and-price algorithm for the capacitated lot-sizing problem with setup times. *Operations Research*, **55**(5): 909–920.

[5] DEGRAEVE Z & PEETERS M. 2003. Optimal integer solutions to industrial cutting-stock problems: Part 2, benchmark results. *INFORMS Journal on Computing*, **15**(1): 58–81.

[6] DENIZEL M, ALTEKIN FT, SÜRAL H & STADTLER H. 2008. Equivalence of the lp relaxations of two strong formulations for the capacitated lot-sizing problem with setup times. *OR Spectrum*, **30**(4): 773–785.

[7] DOLAN ED & MORÉ JJ. 2002. Benchmarking optimization software with performance profiles. *Mathematical Programming*, **91**(2): 201–213.

[8] EPPEN GD & MARTIN RK. 1987. Solving multi-item capacitated lot-sizing problems using variable redefinition. *Operations Research*, **35**(6): 832–848.

[9] FARLEY AA. 1988. Mathematical programming models for cutting-stock problems in the clothing industry. *The Journal of the Operational Research Society*, **39**(1): 41–53.

[10] FIOROTTO DJ & DE ARAUJO SA. 2014. Reformulation and a lagrangian heuristic for lot sizing problem on parallel machines. *Annals of Operations Research*, **217**(1): 213–231.

[11] FOURER R, GAY DM & KERNIGHAN BW. 1990. A modeling language for mathematical programming. *Management Science*, **36**(5): 519–554.

[12] GAU T & WÄSCHER G. 1995. Cutgen1: A problem generator for the standard one-dimensional cutting stock problem. *European Journal of Operational Research*, **84**(3): 572–579.

[13] GHIDINI CTLS & ARENALES MN. 2009. Otimização de processos acoplados na indústria de móveis: dimensionamento de lotes e corte de estoque. In *Anais do CNMAC-Congresso Nacional de Matemática Aplicada e Computacional*.

[14] GHIDINI CTLS, ALEM D & ARENALES MN. 2007. Solving a combined cutting stock and lot-sizing problem in small furniture industries. In *Proceedings of the 6th International Conference on Operational Research for Development (VI-ICORD)*.

[15] GILMORE PC & GOMORY RE. 1961. A linear programming approach to the cutting-stock problem. *Operations Research*, **9**(6): 849–859.

[16] GILMORE PC & GOMORY RE. 1963. A linear programming approach to the cutting stock problema – part ii. *Operations Research*, **11**(6): 863–888.

[17] GRAMANI MCN & FRANÇA PM. 2006. The combined cutting stock and lot-sizing problem in industrial processes. *European Journal of Operational Research*, **174**(1): 509–521.

[18] GRAMANI MCN, FRANÇA PM & ARENALES MN. 2009. A lagrangian relaxation approach to a coupled lot-sizing and cutting stock problem. *International Journal of Production Economics*, **119**(2): 219–227.

[19] GRAMANI MCN, FRANÇA PM & ARENALES MN. 2011. A linear optimization approach to the combined production planning model. *Journal of the Franklin Institute*, **348**(7): 1523–1536.

[20] HENDRY LC, FOK KK & SHEK KW. 1996. A cutting stock and scheduling problem in the copper industry. *The Journal of the Operational Research Society*, **47**(1): 38–47.

[21] IBM CORPORATION. 2009. *CPLEX*. http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/, 2009. visitada pela última vez em 12/10/2012.

[22] JANS R. 2009. Solving lot-sizing problems on parallel identical machines using symmetry-breaking constraints. *INFORMS Journal on Computing*, **21**(1): 123–136.

[23] JANS R & DEGRAEVE Z. 2004. Improved lower bounds for the capacitated lot sizing problem with setup times. *Operations Research Letters*, **32**(2): 185–195.

[24] KANTOROVICH LV. 1960. Mathematical methods of organizing and planning production. *Management Science*, **6**(4): 366–422.

[25] LEÃO APS & ARENALES MN. 2012. Análise de modelos matemáticos para o problema de corte de estoque unidimensional acoplado ao dimensionamento de lotes. In *Anais do XLIV SBPO-Simpósio Brasileiro de Pesquisa Operacional, Rio de Janeiro-RJ*.

[26] LONGHI AL, MELEGA GM & DE ARAUJO SA. 2015. Modelos matemáticos para o problema integrado de dimensionamento de lotes e corte de estoque unidimensional. *Pesquisa Operacional para o Desenvolvimento*, **7**(1): 82–104.

[27] Melega GM, Fiorotto DJ & de Araujo SA. 2013. Formulações fortes para o problema de dimensionamento de lotes com várias plantas. *TEMA (São Carlos)*, **14**: 305–318. ISSN 2179-8451.

[28] Molina F, Morabito R & de Araujo SA. 2016. Mip models for production lot-sizing problems with distribution costs and cargo arrangement. *Journal of the Operational Research Society. Accept to Publication*.

[29] Nonås SL & Thorstenson A. 2000. A combined cutting-stock and lot-sizing problem. *European Journal of Operational Research*, **120**(2): 327–342.

[30] Nonås SL & Thorstenson A. 2008. Solving a combined cutting-stock and lot-sizing problem with a column generating procedure. *Computers & Operations Research*, **35**(10): 3371–3392.

[31] Pochet Y & Wolsey LA. 2006. *Production planning by mixed integer programming*. Springer Science & Business Media.

[32] Poldi KC & Arenales MN. 2010. O problema de corte de estoque unidimensional multiperíodo. *Pesquisa Operacional*, **30**: 153–174.

[33] Poldi KC & de Araujo SA. 2016. Mathematical models and a heuristic method for the multi-period one-dimensional cutting stock problem. *Annals of Operations Research*, **238**(1): 497–520.

[34] Poltroniere SC, Poldi KC, Toledo FMB & Arenales MN. 2008. A coupling cutting stock-lot sizing problem in the paper industry. *Annals of Operations Research*, **157**(1): 91–104.

[35] Santos SG, de Araujo SA & Rangel MSN. 2011. Integrated cutting machine programming and lot sizing in furniture industry. *Pesquisa Operacional para o desenvolvimento*, **3**(1): 1–17.

[36] Silva E, Alvelos F & Valério de Carvalho JM. 2014. Integrating two-dimensional cutting stock and lot-sizing problems. *Journal of the Operational Research Society*, **65**(1): 108–123.

[37] Trigeiro WW, Thomas LJ & McClain JO. 1989. Capacitated lot sizing with setup times. *Management Science*, **35**(3): 353–366.

[38] Valério de Carvalho JM. 1999. Exact solution of bin-packing problems using column generation and branch-and-bound. *Annals of Operations Research*, **86**: 629–659.

[39] Valério de Carvalho JM. 2002. Lp models for bin packing and cutting stock problems. *European Journal of Operational Research*, **141**(2): 253–273.

[40] Vanzela M, Rangel S & de Araujo SA. 2013. The integrated lot sizing and cutting stock problem in a furniture factory. In *Intelligent Manufacturing Systems*, volume 11, pages 390–395.

[41] Vyve MV & Wolsey LA. 2006. Approximate extended formulations. *Mathematical Programming*, **105**(2-3): 501–522.

[42] Wagner HM & Whitin TM. 1958. Dynamic version of the economic lot size model. *Management Science*, **5**(1): 89–96.

Appendix B

The Integrated Lot-Sizing and Cutting Stock Problem with Saw Cycle Constraints Applied to Furniture Production

Computers & Operations Research 79 (2017) 148–160

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

The integrated lot sizing and cutting stock problem with saw cycle constraints applied to furniture production

Matheus Vanzela, Gislaine Mara Melega, Socorro Rangel*, Silvio Alexandre de Araujo

Departamento de Matemática Aplicada, UNESP-Univ. Estadual Paulista, 15054-000 São José do Rio Preto, SP, Brazil

ARTICLE INFO

Keywords:
Two-dimensional cutting stock
Lot sizing
Integrated problems
Saw cycles constraints
Furniture factory

ABSTRACT

The integrated lot sizing and cutting stock problem is studied in the context of furniture production. The goal is to capture the interdependencies between the determination of the lot size and of the cutting process in order to reduce raw material waste and production and inventory costs. An integrated mathematical model is proposed that includes lot sizing decisions with safety stock level constraints and saw capacity constraints taking into account saw cycles. The model solution is compared to a simulation of the common practice of taking the lot size and the cutting stock decisions separately and sequentially. Given the large number of variables in the model, a column-generation solution method is proposed to solve the problem. An extensive computational study is conducted using instances generated based on data collected at a typical small scale Brazilian factory. It includes an analysis of the performance of the integrated approach against sequential approaches, when varying the costs in the objective function. The integrated approach performs well, both in terms of reducing the total cost of raw materials as well as the inventory costs of pieces. They also indicate that the model can support the main decisions taken and can bring improvements to the factory's production planning.

1. Introduction

The Brazilian furniture industry is concentrated in regional centers, mostly situated in the southern and southeastern regions of the country. The state of São Paulo, situated in the southeast, is responsible for 20% of the national production. The cities of Mirassol and Votuporanga together with their surrounding towns form Local Productive Arrangements (APL – the Portuguese term for *Arranjo Produtivo Local*), when referring to the production of furniture. These two APL represent about 10% of the furniture production in the state of São Paulo [8,33]. An overview of this sector shows a predominance of micro and small businesses and these are responsible for 61.9% of the jobs created in the sector. The furniture demand depends on the behavior of different economic factors such as the residential building market, consumer budgets and the stability of the economy, which explain the growth of the Brazilian furniture industry in recent decades.

The sector in Brazil is divided into segments according to the raw materials (*e.g.* wood, metal and plastic) and the use for which the product is intended (*e.g.* residential, commercial and institutional). Due to organizational and marketing factors, the companies specialize in one or two types of furniture, such as kitchen and bathroom fittings, bedroom furniture, sofas and armchairs among other groups. Some companies manufacture furniture on a large scale and others are specialized in the production of customized furniture according to specific individualized projects.

The competition among the companies in the sector is directly related to the technology and management tools involved. Also, competition with international markets lead to a series of challenges for the Brazilian furniture industry. One approach to these challenges is to invest large amounts of capital in more sophisticated machines and acquiring new technologies. Making improvements focusing on reducing managerial and manufacturing problems is a less common approach but could be the best solution for survival in a highly competitive market, in particular in the case of small-scale factories that are not able to invest much in their equipment. In fact, the modernization of machinery in these types of factories occurs in stages; it is common to find both new modern machines and outdated ones in the same factory. The visits and interviews conducted in the factories located in the Votuporanga APL showed that good, specialized Decision Support Systems are needed to speed up and improve the production planning decision process.

The production planning in these factories basically involves two main decisions: lot sizing and cutting stock (*e.g.* [28,2]). The lot sizing decisions provide the quantity of furniture to be produced in each

* Corresponding author.
E-mail addresses: matheusvanzela@gmail.com (M. Vanzela), gislainemelega@gmail.com (G.M. Melega), socorro@ibilce.unesp.br (S. Rangel), saraujo@ibilce.unesp.br (S.A.d. Araujo).

http://dx.doi.org/10.1016/j.cor.2016.10.015
Received 26 June 2015; Received in revised form 4 October 2016; Accepted 30 October 2016
Available online 02 November 2016

period of the planning horizon and the cutting decisions generate the best possible cutting patterns to obtain the pieces that make up a final product. When these decisions are taken separately, a large amount of raw material could be used and/or more pieces could be kept in storage. As a result, higher production costs might be incurred.

Taking the lot sizing and the cutting stock decisions simultaneously, there is a possibility of bringing forward the production of some final products and to have better combinations of pieces in cutting patterns, which might decrease the total use of raw material and the total number of pieces in storage. However, this bringing-forward might result in additional holding costs associated to final products and therefore a trade-off between bringing forward and postponing production needs to be considered.

In this paper, a study of the production process of a typical small scale Brazilian furniture factory (Factory L) is conducted and a mathematical model that captures the production process of Factory L is proposed and tested using instances based on real data. The remainder of this paper is organized as follows. Section 2 presents a review of some papers that are relevant to this work and highlights its main contributions. Section 3 contains a description of the production process in a typical small-scale furniture factory. In Section 4, a mathematical model that considers the integrated lot sizing and the cutting stock decisions is presented as well as a proposal for the simulation of common practice in the factory at the time of the interviews. The solution procedure based on the column-generation technique is presented in Section 5. In Section 6, results of a computational study using instances based on real data conducted to compare the effectiveness of the integrated and the non-integrated solution approaches are presented. This section also presents results that show the impact on the models when the costs in the objective function are changed. Section 7 presents concluding remarks.

2. Review of related papers

Lot sizing and cutting stock are two well-known problems that have attracted the attention of the production and operations research scientific community in terms of case studies, models and efficient solution methods [20,18,38].

However, only a few papers have considered their integration, also known in the literature as the combined lot sizing and cutting stock problem. Among them, Farley [6] is the first author to publish an integrated cutting stock and production planning problem. The problem is considered in the context of clothing production. Hendry et al. [16] present a two stage solution procedure for the integrated cutting stock and lot sizing problem in the context of copper production. In the first stage, the cutting stock problem with capacity constraints is solved heuristically. The solution of stage one is then used in an integer programming model to determine the lot sizes. Arbib and Marinelli [3] consider a mixed integer programming model to solve the integrated problem. The proposed model allows for the cutting of non-ordered pieces that may be later grouped to meet future demand (cut-and-reuse). The model also includes inventory and transportation costs. Nonas and Thorstenson [21,22] consider a one-dimensional cutting stock problem with holding costs and setup costs associated to cutting patterns. In Nonas and Thorstenson [21] a column generation procedure is proposed with good results for small scale problems. They extend their paper and, in Nonas and Thorstenson [22], the ideas presented in Haessler [15] are considered and the column generation procedure is improved with good results for small and large scale problems. Applications of the integrated problem in the paper industry can be found in Respício and Captivo [27] and Poltroniere et al. [25].

The integrated problem applied in the context of furniture production is presented in Gramani and França [12], Gramani et al. [13], Silva et al. [30], Gramani et al. [14], Alem and Morabito [1,2] and Silva et al. [32]. In Gramani and França [12] the authors analyze the multi-period cutting stock problem where the goal is to minimize the total number of plates used in the cutting process, the inventory costs of pieces and the setup costs, but they do not consider the production and inventory costs of final products. They propose a solution method based on an analogy with the shortest path problem. Gramani et al. [13] extend the model proposed in Gramani and França [12] by including the decisions about the final products. They propose a heuristic method based on Lagrangian relaxation applied to the integrated lot sizing and cutting stock problem. The difficulty faced by the Lagrangian solution approach is that the resulting Lagrangian subproblems are NP-hard capacitated lot sizing problems. Gramani et al. [14] address the model proposed in Gramani et al. [13] by relaxing setups and maintaining the storage of pieces. They consider a trade-off between pieces inventory and raw material waste. For solving this integrated model, they use the CPLEX package with a column generation technique. Silva et al. [30] consider the capacity of the cutting machine and of the drilling machines. They relax the integrality of the setup variables, and use the Simplex method with column generation to deal with the enormous quantity of cutting patterns. Alem and Morabito [1,2] apply robust optimization tools to the integrated lot sizing and cutting stock models considering production costs and product demands as stochastic parameters. Silva et al. [32] proposed two integer programming models to optimize a production process in a furniture industry. The proposed models allow the inventory of items and leftovers, which can be used in subsequent periods. The first model is an extension of the model proposed in a previous research [31]. The second model is based on the model proposed by Dyckhoff [5] for the one-dimensional cutting stock problem (called one-cut), where each decision variables corresponds to a single cutting operation in a single object. Computational results are presented using real data from a furniture industry.

Still related with the present work, we mention that Wagner [37] discusses the cutting stock problem in which lumber is cut in bundles; Henn and Wäscher [17] and Cui et al. [4] study the cutting stock problems with reduction on the number of different cutting patterns; and Poldi and de Araujo [23] consider the multiperiod cutting stock problem.

In this paper, we present a new mathematical model to integrate the lot sizing and the cutting stock decisions in the context of furniture production of small factories. The main difference from other models proposed in the literature is the consideration of safety stock level of final products and capacity constraints taking into account saw cycles. This last characteristic is important because the cutting machine allow the simultaneous cutting of several plates. That is, the plates can be stacked in the machine so that they can be cut simultaneously according to the same cutting pattern. The total number of plates that can be stacked depends on the machine maximum load and on the thickness of the plates. The time necessary to adjust the cutting machine and to cut a stack of objects according to a given cutting pattern is named *saw cycle* [39,34]. So, the total number of saw cycles is an important aspect to be considered when solving the cutting stock problem.

In summary the paper has the following contributions. First, the proposal of an innovative integrated model. Second, the simulation of the practice of small-scale furniture factories through mathematical models that considers the sequential decisions, *i.e.*, first taking the lot sizing decision and afterwards the cutting stock decision. Third, the proposal of a solution method based on column-generation for solving the mathematical models. Finally, the presentation of computational results that show the quality of the proposed integrated approach when compared to a simulation of the factory practice, as well as a study of the impact of cost variations on the different approaches.

This paper extends the initial research published in Santos et al. [28] and in Vanzela et al. [36]. In the latter a relaxed version of the saw cycle constraint is considered and limited computational results are presented. Santos et al. [28] consider a detailed cutting machine capacity and an approximated capacity of the remaining production process. The operational details considered are the setup time for cutting patterns changeover and saw machine capacity in terms of the number of plates that can be simultaneously cut. They solve the problem using a rolling horizon strategy and present results for just two instances considering only a set of *a priori* defined cutting patterns.

3. The furniture production process in small-scale factories

The focus of this work is on factories that produce rectilinear furniture using as its main raw material rectangular wooden plates such as MDF (Medium-Density Fiberboard), OSB (Oriented Strand Board) and other similar materials. The furniture production process described below is based on the information collected at Factory L situated at the Votuporanga APL, a typical small scale Brazilian factory according to a classification based on the number of employees [29]. The furniture production process involves several stages and equipment, the different types of equipment are grouped in sectors according to their function and the production stage. The main differences in the production process of Factory L and of the other companies visited are in specific stages of the production process or in the modernity of certain equipment.

The production floor at Factory L is divided into four main sectors: cutting, woodwork, painting and dispatching. At the first stage (cutting), the required number of plates are cut to produce the pieces that compose each final product. At the time the data was collected, the cutting sector had one automatic cutting machine (main) and one semi-automatic one (secondary) which is used only at peak periods or during maintenance of the main machine.

After cutting the plates, the pieces move on to the woodworking sector. Several operations are conducted in this sector. Some rectangular pieces are processed to form non rectangular shapes according to the product design. All pieces pass through some type of finishing in order to take out any irregularities, and some of them receive edge finishing. The drilling operations are also done is this sector. Due to the precision needed in these operations, the woodworking area is crucial and the quality control must be rigorous. Once the pieces go through all the necessary woodwork operations they are ready to be painted.

The painting sector houses two types of painting (i) Polyurethane (PU) painting, which is a manual process and (ii) Ultraviolet (UV) painting, which is semi-automatic. This area also includes the sanding and cleaning operations. To end the production process, the painted pieces go to the dispatching sector where they are packed according to the final product specifications, labeled and stored for future delivery. Fig. 1 shows the production flow at Factory L. More information about the furniture production process can be found in Vanzela [35] and the references therein.

The Factory L catalogue considered in this study contains eight types of furniture: a multi-cabinet, a dressing table and six different models of wardrobe, from now on named as MC, DT and W1 to W6, respectively. The furniture can be produced in seven different single colors or in a combination of two colors, resulting in over 150 different final products. At the time the data was collected, two products (W4,W6) represented 50% of total sales and thus they received more attention in the production process.

Each final product is composed of a number of rectangular pieces cut from rectangular plates of different thicknesses. The type and number of pieces necessary to obtain one final product represents a major concern for the production manager. Besides the associated cutting stock decision, the total number of pieces will influence other decisions such as total number of drilling operations, final product cost, packaging and transportation. The number of pieces that compose each final product ranges from six to twenty. Some of these products share the same pieces (in terms of size and thickness). This information must be taken into account when considering a combination of final products in the same lot.

From the above description, it is possible to infer that the production manager has a difficult task when taking the lot sizing and the cutting stock decisions. These decisions are taken weekly based on the forecast demand, the sales of similar periods in previous years and on the results of marketing campaigns already in progress. Also, the factory has previous commitments to meet and therefore efficient production planning and control is mandatory. The main concern of the production managers of the factories visited is the final product cost. According to the Factory L data, the main raw material (wooden plates) costs represent 50% of the final product cost.

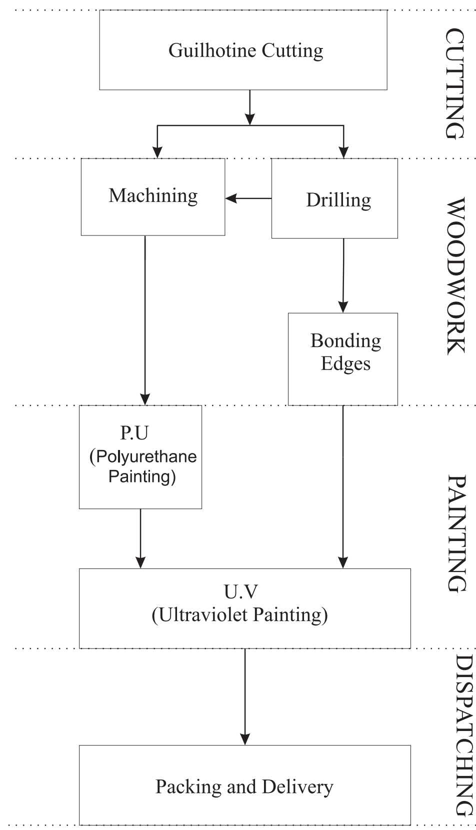


Fig. 1. Furniture production flow diagram.

4. Mathematical model for the Integrated Lot Sizing and Cutting Stock Problem for Furniture Production (*ILSCSP*)

In this section, we present a new mathematical model for the integrated lot sizing and cutting stock problem. The model is based on the description of the production process of small scale furniture factories presented in Section 3. Before stating the problem and presenting the mathematical model, some simplifications to the production process are considered to obtain a computer solvable model.

Simplifications:

1. Only the cutting sector capacity is considered and it is assumed that the other sectors can handle the decisions taken for the cutting stock problem. This is a common assumption in the literature because in most of the furniture plants the cutting sector is the bottleneck of the production process. We have found nine papers in the literature that consider the integrated lot sizing and cutting stock problem applied to furniture industry. Five of them (Gramani and França [12], Gramani et al. [13,14], Alem and Morabito [1] and Vanzela et al. [36]) consider capacity constraints only in the cutting sector, as we do. Three of them (Silva et al. [30], Santos et al. [28] and Alem and Morabito [2]) consider capacity constraints in the cutting sector and also in the drilling sector. Silva et al. [32] do not consider capacity

constraints. Furthermore, Toscano et al. [34] focus specifically on the optimization of the cutting sector of Factory L, which shows the relevance of the cutting sector in furniture industries and that this is a realistic simplification.
2. Setup time for the cutting machine was not taken into account, nor was the cutting patterns changeover time. The estimated impact of these simplifications is taken into account indirectly on the cutting machine capacity.
3. Late delivery and overtime working to meet demand are not allowed.
4. The colors of the final products are not considered.

Considering the simplifications (1)–(4) and a planning horizon divided into periods, the integrated problem considered in this study can be stated as: determine the lot size of final products, the total number of plates and cutting patterns necessary to obtain the pieces that compose the final products, taking into account the demand of each final product in each period, the cutting machine capacity and safety stock levels for final products. The decisions are taken aiming to minimize the overall costs computed in terms of production and inventory costs of the final products, plates cost and pieces inventory costs.

The usual practice of the majority of the factories visited is to divide this problem in two sequential problems. First they solve the lot sizing problem to define the quantities of final products. After that they obtain the total number of pieces necessary to compose the final products and then solve the cutting stock problem to decide the total number of plates to be cut and the associated cutting patterns.

The process of cutting the plates may involve waste of material. The factory is interested in reducing this waste given that it has a strong impact on the cost of the final product. One way of reducing this waste is by increasing the number of types of ordered pieces. A wider variety of pieces may allow for a better arrangement of the pieces on the plate (cutting pattern). Moreover, increasing the demand for given pieces might help to reduce the number of saw cycles due to the fact that more plates may be cut simultaneously with the same cutting pattern. All this can be achieved if the factory brings forward the production of some final products. However, this early production may incur additional inventory costs. To best capture all these cutting stock and lot sizing elements in the decision process, an integrated decision should be taken.

4.1. The model description

To define the integrated mathematical model, let T be the number of periods in the planning horizon and F be the total number of ordered final products. As stated in Section 3, one final product is composed of rectangular pieces of different thicknesses cut from rectangular wooden plates. So, let E be the total number of different plates or pieces thicknesses, and P the total number of pieces. We consider that the stock of rectangular plates of each different thickness is enough to meet all the pieces demand, and that they are all of the same length (L) and width (W). To simplify the model description, we will consider that J is the number of all possible cutting patterns (pre-supposing that these cutting patterns have been generated *a priori* considering all the pieces necessary to obtain the final products). Since two dimensions are relevant in the cutting processes, the cutting patterns in this context are classified as two-dimensional.

The following indices are used to define the parameters, constraints and variables used in the model.

Indices:

$t = 1,\ldots,T$: periods;
$f = 1,\ldots,F$ final products (or simply products below);
$p = 1,\ldots,P$ pieces;
$e = 1,\ldots,E$ plate thickness;
$j = 1,\ldots,J$ cutting patterns.

The following parameters are presumed to be known.

Parameters:

c_f	production cost for product f;
h_f	inventory cost for product f;
D_{ft}	demand for product f in period t;
C_t	maximum production capacity in period t, computed in number of saw cycles.
o^e	thickness of plate e;
co^e	cost of the plate with thickness e;
S	height of the saw;
cap^e	maximum number of plates of thickness e that can be simultaneously cut ($cap^e = \lfloor \frac{S}{o^e} \rfloor$);
$\hat{h}_p^e$	inventory cost of piece p with thickness e;
q_{pf}^e	number of pieces p of thickness e necessary to produce one unit of product f;
L:	length of the plates;
W:	width of the plates;
l_p^e	length of piece p with thickness e;
w_p^e	width of piece p with thickness e;
a_{pj}^e	number of pieces p with thickness e in the cutting pattern j;
I_{f0}	initial inventory of product f;
IP_{p0}^e	initial inventory of piece p with thickness e;
ts	demand percentage used to impose safety stock level of products;
tx	parameter used to adjust the approximated capacity.

The following variables are used to model the decisions associated with the mathematical model.

Variables:

X_{ft}	number of product f produced in period t;
I_{ft}	number of product f stored at the end of period t;
IP_{pt}^e	number of pieces p of thickness e stored in period t;
y_{jt}^e	number of plates of thickness e, cut according to cutting pattern j in period t;
z_{jt}^e	number of saw cycles necessary to cut plates of thickness e according to cutting pattern j in period t.

The proposed integrated lot sizing and cutting stock model consists of coupling the characteristics and considerations for both the lot sizing and the cutting stock decisions in a single model that is defined by the expressions (1)–(10).

The ILSCSP model

$$\text{Min} \quad Z = \sum_{f=1}^{F}\sum_{t=1}^{T}(c_f X_{ft} + h_f I_{ft}) + \sum_{e=1}^{E}\sum_{j=1}^{J}\sum_{t=1}^{T} co^e y_{jt}^e + \sum_{e=1}^{E}\sum_{p=1}^{P}\sum_{t=1}^{T} \hat{h}_p^e IP_{pt}^e \tag{1}$$

Subject to:

$$X_{ft} + I_{f,t-1} - I_{ft} = D_{ft} \quad f = 1,\ldots,F; \; t = 1,\ldots,T \tag{2}$$

$$I_{ft} \geq tsD_{ft} \quad f = 1,\ldots,F; \; t = 1,\ldots,T-1 \tag{3}$$

$$I_{fT} \geq ts\left(\sum_{t=1}^{T} D_{ft}\right) \quad f = 1,\ldots,F \tag{4}$$

$$\sum_{j=1}^{J} a_{pj}^e y_{jt}^e + IP_{p,t-1}^e - IP_{pt}^e = \sum_{f=1}^{F} q_{pf}^e X_{ft} \quad p = 1,\ldots,P; \; t = 1,\ldots,T; \; e = 1,\ldots,E \tag{5}$$

$$\sum_{e=1}^{E}\sum_{j=1}^{J} z_{jt}^{e} \le C_t \quad t = 1, \ldots, T \tag{6}$$

$$z_{jt}^{e} \ge \frac{y_{jt}^{e}}{cap^{e}} \quad j = 1, \ldots, J; \;\; t = 1, \ldots, T; \;\; e = 1, \ldots, E \tag{7}$$

$$X_{ft}, I_{ft} \in \mathbb{R}_{+} \quad f = 1, \ldots, F; \;\; t = 1, \ldots, T \tag{8}$$

$$z_{jt}^{e} \in \mathbb{Z}_{+}, \, y_{jt}^{e} \in \mathbb{Z}_{+} \quad j = 1, \ldots, J; \;\; t = 1, \ldots, T; \;\; e = 1, \ldots, E \tag{9}$$

$$IP_{pt}^{e} \in \mathbb{R}_{+} \quad p = 1, \ldots, P; \;\; t = 1, \ldots, T; \;\; e = 1, \ldots, E \tag{10}$$

- *Objective function* (1): the optimization criterion is the minimization of the total cost calculated by the sum of the production costs (c_f), inventory costs of products (h_f), raw material costs (co^e) and inventory costs of pieces ($\hat{h}_p^e$). This expression translates the tradeoff that should be achieved considering the costs of production, inventory of products and of pieces as well as the cost of the plates.
- *Meeting demand constraints* (2): these constraints guarantee that the demand for products f (D_{ft}) is met by balancing the production in period t, the inventory from the previous period ($t-1$), and the unshipped products that remain in inventory in period t for later use.
- *Safety stock level constraints* (3) *and* (4): these constraints impose safety stock levels for the product f as a percentage of the demand for each product. For the first ($T-1$) periods, the safety levels are stated in terms of the individual demands (tsD_{ft}), and for the final period, the safety stock levels are stated in terms of the total demands ($ts(\sum_{t=1}^{T} D_{ft})$).
- *Coupling constraints* (5): these constraints model the interdependence between the decisions. They take into account the decisions relative to lot sizing (X_{ft} variables) to determine the pieces demand and thus the decisions relative to the cutting of raw material (y_{jt}^{e} variables). It allows for the possibility of storing pieces (IP^{e}_{pt} variables).
- *Saw cycles capacity constraints* (6): these constraints guarantee that no more than C_t saw cycles are used in each period t.
- *Minimum number of saw cycles constraints* (7): these constraints impose a lower bound to the number of cycles necessary to cut the y_{jt}^{e} plates of thickness e according to the cutting pattern j in period t taking into account the cutting machine maximum load (cap^e).
- *Variable domain constraints* (8)–(10): these constraints determine the domains of the variables. It is usual that the lot size decisions X_{ft} are defined as continuous variables.

It is important to highlight that, in general, when stating lot sizing constraints it is assumed that the safety stock levels are implicit in the demand. The safety stock level constraints (3) and (4) differ from this standard practice because the imposition of safety stock levels is smaller in the first ($T-1$) periods, thus allowing more freedom to allocate the initial inventory to meet demand in any of these periods. The total number of constraints present in the *ILSCSP* model is given by $(FT + F(T-1) + F + PTE + T + JTE)$ and the total number of variables is given by $(2JTE + 2FT + PTE)$.

4.2. Simulation of the factory production planning process

The coupling of the lot sizing and the cutting stock decisions in the *ILSCSP* model are achieved by imposing constraints (5). If these constraints are removed, the model decomposes into two independent models that can be used to simulate the usual factory practice. The remaining constraints will be used to define the Lot sizing problem and the Cutting Stock problems that are solved sequentially in practice. Still, to be realistic, the resulting models have to take into account some elements of each other. In what follows, we will describe the models that will be used to simulate the practice in Factory L.

4.2.1. Model for the Capacitated Lot Sizing Problem for furniture production (CLSP)

The mathematical model (11)–(16) is used to simulate the lot sizing decisions in the context of furniture production. It determines the lot sizes for the products (furniture) as well as the inventory levels in each period of the planning horizon aiming to minimize the total cost of production and inventory of the products. It uses the same indices, parameters and variables as the *ILSCSP* model.

The CLSP model:

$$\text{Min } Z = \sum_{f=1}^{F}\sum_{t=1}^{T} (c_f X_{ft} + h_f I_{ft}) \tag{11}$$

Subject to:

$$X_{ft} + I_{f,t-1} - I_{ft} = D_{ft} \quad f = 1, \ldots, F; \;\; t = 1, \ldots, T \tag{12}$$

$$tx\left(\sum_{f=1}^{F}\sum_{e=1}^{E} \frac{(l_p^e \cdot w_p^e) q_{pf}^{e} X_{ft}}{(L \cdot W) cap^{e}}\right) \le C_t \quad t = 1, \ldots, T \tag{13}$$

$$I_{ft} \ge tsD_{ft} \quad f = 1, \ldots, F; \;\; t = 1, \ldots, T-1 \tag{14}$$

$$I_{fT} \ge ts\left(\sum_{t=1}^{T} D_{ft}\right) \quad f = 1, \ldots, F \tag{15}$$

$$X_{ft}, I_{ft} \in \mathbb{R}_{+} \quad f = 1, \ldots, F; \;\; t = 1, \ldots, T \tag{16}$$

- *Objective function* (11): the optimization criterion is the minimization of total costs considering production (c_f) and inventory costs (h_f).
- *Saw capacity constraints* (13): to obtain a realistic decision, an estimate of the cutting machine capacity is considered. This constraint is stated considering that the pieces can be cut from an imaginary single plate with enough area to cut all the necessary pieces. Then the approximated total number of plates necessary to produce (X_{ft}) products is obtained by dividing the total area used for this imaginary plate divided by the real plate area: $\left(\sum_{f=1}^{F}\sum_{e=1}^{E} \frac{(l_p^e \cdot w_p^e) q_{pf}^{e} X_{ft}}{(L \cdot W)}\right)$. The parameter $cap^e = \lfloor \frac{S}{o^e} \rfloor$ allows the transformation of the used capacity into number of saw cycles. In this way, the definition of the furniture lot sizes takes into account an approximation of the total number of saw cycles necessary to cut all the pieces.
- The constraints (12), (14), (15), (16) have the same purpose as the constraints (2), (3), (4), and (8) in the *ILSCSP* model, respectively.

4.2.2. Model for the multi-period Cutting Stock Problem for a furniture factory (CSP)

The cutting stock model (*CSP*) described by (17)–(22) uses a feasible solution of the lot sizing model (*CLSP*) ($\widetilde{X}_{ft}$) to compute the pieces demand. The *CSP* model then determines the number of plates to be cut and the associated cutting patterns to meet the pieces demand aiming to minimize the total cost of plates and pieces inventory. It uses the same indices, parameters and variables as the *ILSCSP* model.

The CSP model

$$\text{Min} \quad Z = \sum_{e=1}^{E}\sum_{j=1}^{J}\sum_{t=1}^{T} co^{e} y_{jt}^{e} + \sum_{e=1}^{E}\sum_{p=1}^{P}\sum_{t=1}^{T} \hat{h}_p^{e} IP_{pt}^{e} \tag{17}$$

Subject to:

$$\sum_{j=1}^{J} a_{pj}^{e} y_{jt}^{e} + IP_{p,t-1}^{e} - IP_{pt}^{e} = \sum_{f=1}^{F} q_{pf}^{e} \widetilde{X}_{ft} \quad p = 1, \ldots, P; \;\; t = 1, \ldots, T; \;\; e = 1, \ldots, E \tag{18}$$

$$\sum_{e=1}^{E}\sum_{j=1}^{J} z_{jt}^{e} \le C_t \quad t = 1,\ldots,T \tag{19}$$

$$z_{jt}^{e} \ge \frac{y_{jt}^{e}}{cap^{e}} \quad j = 1,\ldots,J;\; t = 1,\ldots,T;\; e = 1,\ldots,E \tag{20}$$

$$z_{jt}^{e} \in \mathbb{Z}_{+},\; y_{jt}^{e} \in \mathbb{Z}_{+} \quad t = 1,\ldots,T;\; j = 1,\ldots,J;\; e = 1,\ldots,E \tag{21}$$

$$IP_{pt}^{e} \in \mathbb{Z}_{+} p = 1,\ldots,P;\; t = 1,\ldots,T;\; e = 1,\ldots,E \tag{22}$$

- *Objective function* (17): minimize the total cost considering the plates cost and the inventory cost of pieces.
- *Meeting pieces demand constraints* (18): these constraints guarantee that, in each period, the pieces demand ($\sum_{f=1}^{F} q_{pf}^{e}\widetilde{X}_{ft}$) is met by balancing the number of pieces produced in period t ($\sum_{j=1}^{J} a_{pj}^{e} y_{jt}^{e}$,), the pieces stored in the previous period ($IP_{p,t-1}^{e}$), and the unused pieces that are stored in period t ($IP_{pt}{}^{e}$) for later use.
- The constraints (19), (20), (21), (22) have the same purpose as the constraints (6), (7), (9) and (10) in the *ILSCSP* model, respectively.

The problem (17)–(22) extends, by considering saw cycle constraints and two dimensions, the multiperiod cutting stock problems that have been considered in the literature, for example, in Poldi and Arenales [24].

5. Solution method

Instances of the *CLSP* model can be solved easily by the available solvers. However, the instances of the *ILSCSP* and *CSP* models can not be solved easily due to the high number variables y_{jt}^{e} (possible cutting patterns) and their integral nature. To get round these difficulties, a column-generation method based on Gilmore and Gomory [9,10] is applied to a relaxed linear model that is obtained by substituting the constraint $y_{jt}^{e} \in \mathbb{Z}_{+}$ by $y_{jt}^{e} \in \mathbb{R}_{+}$.

We will describe the column generation procedure considering the *ILSCSP*. An equivalent procedure is applied to the *CSP*. The Restricted Master Problem (*RMP*) for the *ILSCSP* is defined taking only a subset of ($T{*}P{*}E$) cutting patterns in the sub-matrix associated to the variables $y_{jt}{}^{e}$. The remaining ($J - T{*}P{*}E$) cutting patterns (and the associated variables $y_{jt}{}^{e}$) are removed from the problem and will be generated as necessary. All the columns related to the other variables are included in the RMP, except the variables $z_{jt}{}^{e}$ which are created as the associated variables $y_{jt}{}^{e}$ are generated.

The current *RMP* is solved and the dual variables π_{t}^{e} associated to constraints (5) are recovered. For each thickness e and period t the pricing sub-problem (23) and (24) is solved to identify if there are cutting patterns ($A^{e}{}_{j}$) that can improve on the current *RMP* solution. To simplify the notation, the index t is omitted in the dual variables $\pi^{e}{}_{t}$:

$$Z_{SUB} = \max \quad \pi^{e} A_{j}^{e} \tag{23}$$

$$\text{s. t.} \quad A_{j}^{e} \text{ is a two-dimensional cutting pattern} \tag{24}$$

The columns that satisfy the criterion determined by the reduced cost (25) are included in the *RMP* and the new *RMP* is solved. This iterative process is repeated until no more new columns that satisfy this criterion are generated:

$$\hat{c}_{jt}^{e} = co^{e} - Z_{SUB} < 0, \tag{25}$$

When the generated columns no longer price out, a reduced version of the model (1)–(10) is built considering a subset of cutting patterns. Only the y_{jt}^{e} and the $z_{jt}{}^{e}$variables associated to the initial cutting patterns and the cutting patterns generated for the *RMP* are included in constraints (5)–(7). This reduced mixed integer model is then solved using a commercial optimization software package to obtain a feasible mixed integer solution for the integrated problem.

Several aspects should be considered in the generation of cutting patterns [40]. The majority of cutting machines observed in the furniture factories impose that only orthogonal guillotine cuts can be made. A cut is of orthogonal guillotine type if, when applied to a rectangle, it produces two other rectangles. Another important consideration is the number of times the plate must be rotated in 90° in order to cut all the pieces. This is called the number of stages. If, at the end of the final stage, all the items have been obtained, the cutting pattern is exact, otherwise it is said to be non-exact. The trimming in a non-exact cutting pattern is usually done in a secondary cutting machine and therefore it is not counted as an additional stage [19]. An important class of orthogonal guillotine cutting pattern with high productivity of the cutting machine is the n-group cutting pattern. An n-group cutting pattern is formed by n parts of 1-group patterns. A 1-group pattern is a two-stage cutting pattern formed by a set of strips that can be simultaneously cut in the second stage [11].

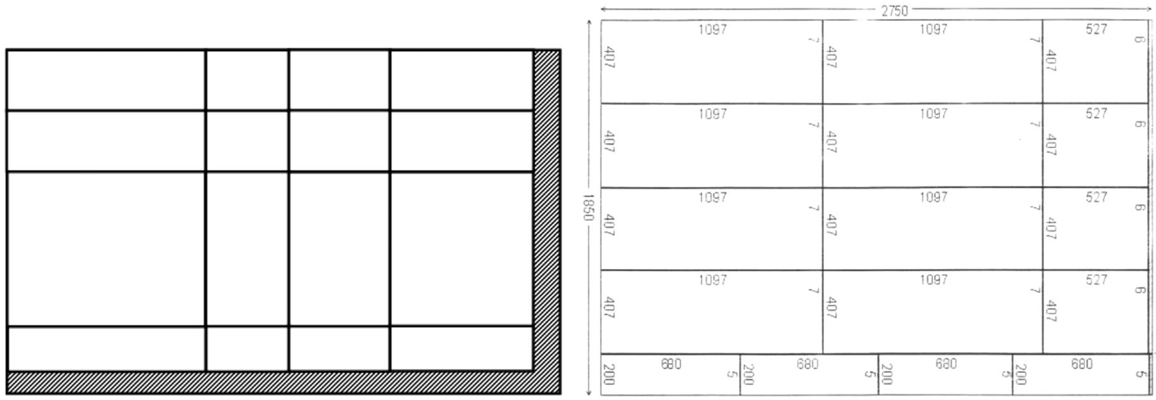
A special case of the 1-group cutting pattern is the maximal homogeneous cutting pattern, *i.e.* a cutting pattern that contains only one type of piece, the maximum possible number. The maximal homogeneous cutting pattern j associated to piece p of thickness e can be represented by the column vector $(A_{j}^{e})^{t} = (0,\ldots,a_{pp}^{e},\ldots,0)$, $a_{pp}^{e} = \left\lfloor \frac{L}{l_{p}^{e}} \right\rfloor \left\lfloor \frac{W}{w_{p}^{e}} \right\rfloor$. The set of P maximal homogeneous cutting patterns is used to initialize the *RMP* and guarantees an initial solution that attend the pieces demand.

Other methods can be used to obtain feasible two-dimensional cutting patterns in the pricing sub-problem (23) and (24) (*e.g.* [10,40]). Regarding the context of furniture production, there are some papers in the literature that consider the stand alone cutting stock problem and propose different approaches to generate orthogonal guillotine two-dimensional cutting patterns [26,40,41,19]. In particular, Rangel and Figueiredo [26] analyze the cutting patterns used in Factory L and present a heuristic procedure to generate cutting patterns based on n-group patterns that simulates the ones used in the factory practice. A comparison of the heuristic solution with the solution given by the factory indicates that the proposed heuristic can generate cutting patterns similar to the ones used in the factory with equal or less waste. Besides the homogenous cutting patterns used to initialize the *RMP*, in this paper we considered only 2-group cutting patterns. Fig. 2 shows an example of a 1-group and of a 2-group cutting patterns, the latter was generated and used in Factory L.

6. Computational study

The objective of the computational study described in this section is to analyze the behavior of the proposed model. The three mathematical models (*ILSCSP*, *CLSP*, and *CSP*) and the column-generation algorithm (described in Sections 4 and 5 respectively) were written in the syntax of the Mosel modeling language and the associated optimization problems were solved using the solver X–PRESS^{MP} [7]. The runs were executed on a machine with 8.0 GB of RAM and an Intel(R) Core(TM) i-7 chip at 3 GHZ.

For the sake of comparison to the integrated model *ILSCSP*, and following the company's decision making process, first the *CLSP* is solved and its solution is used to determine the pieces demand. Then the multi-period cutting stock problem *CSP* is solved. The maximum execution time for solving the integrated model (*ILSCSP*) was set to 3600 s. The same amount of time was given for solving the separated models (*CLSP* + *CSP*), being 60 s for the *CLSP* model and 3540 s for the *CSP* model. The time given for solving each pricing sub-problem (in models *ILSCSP* and *CSP*) was 60 s. In order to evaluate the impact of the capacity constraint on the lot-sizing decisions, we also considered a fourth model built removing the capacity constraint (13) from the *CLSP* model, denoted by *LSP*. The computational study is divided in two parts. The results presented in Section 6.2 consider the instances



(a) A 1-group cutting pattern (b) A 2-group cutting pattern from Factory L

Fig. 2. Examples of 1-group and 2-groups cutting patterns.

generated with the data set described in Section 6.1. The results presented in Section 6.3 consider new instances generated by varying the costs in the objective function.

6.1. Data set

In the data set we consider that one period of time is equivalent to one week and, for each instance, we considered a planning horizon of 4 weeks (1 month). Fifteen instances were generated based on data collected at Factory L and from the furniture market considering the months of March (instances 1–5), August (instances 6–10) and November (instances 11–15). These months represent, low, medium and high demand respectively according to the seasonality of the sector. The main characteristics of the real data are described below. More details of the real data instances can be found in Vanzela [35].

- Problem size: F=8 products; T=4 periods (weeks); P=72 pieces grouped into e=4 thickness (o^e=9 mm, 15 mm, 18 mm, 25 mm for $e = 1,\ldots,4$ respectively).
- Table 1 shows the production costs (c_f) for each product (the plates costs are not included).
- The inventory costs for products (h_f) shown in Table 1 are obtained based on the return rate considering the value c_f applied on the financial market.
- Initial inventory of products (I_{f0}): 60% of total demand of product f.
- Percentage of minimum inventory levels (ts): 60% of the demand for a given period.
- Plates dimension ($L \times W$): (2750 mm × 1850 mm).
- Plates Costs (co^e): the costs were determined according to market prices as 45.79, 65.63, 36.37, and 97.17 for $e = 1,\ldots,4$ respectively.
- Pieces Costs (cp_P^e): computed as $cp_p^e = \left(\frac{l_p^e \cdot w_p^e}{L \cdot W}\right) \cdot co^e$. This cost is also used to compute the inventory costs of pieces $\hat{h}_p^e$.
- Inventory costs of pieces ($\hat{h}_p^e$): based on the return rate considering the value cp_P^e applied on the financial market.
- Initial inventory of pieces (IP_{p0}^e): zero.
- Saw height (S): 105 mm.
- Estimated cutting capacity for a given period (saw cycles) (C_t): In order to calculate (C_t) we observed that the velocity of the saw is on average 8.5 min per cycle. Considering that each week has 5 working days, and that a single day has 8-h shift, we get that the saw capacity in each period t is C_t = 340 saw cycles.
- Capacity adjustment (tx): 0.85.
- Demands for product (D_{ft}) are shown in Table 2. For the sake of simplicity we only show the first instance for each month. Observe that for each month we have 4 periods representing the 4 weeks.
- Product structure: Table A1 (Appendix) show the number of pieces of 15 mm that compose each product.

Table 1
Production and inventory costs of products.

Product	Production cost (c_f)	Inventory cost (h_f)
(1) MC	42.00	0.21
(2) DT	75.00	0.39
(3) W1	175.00	0.84
(4) W2	152.00	0.70
(5) W3	113.00	0.57
(6) W4	188.00	0.85
(7) W5	141.00	0.70
(8) W6	175.00	0.84

6.2. Computational results – Part I

In this section, the computational results obtained with the mathematical models described in Section 4 are presented. Basically, three models or combination of models, are analyzed: the first one, *LSP+CSP*, is typically used in the industry and solves sequentially the non capacitated lot sizing problem followed by the cutting stock problem; the second one, *CLSP+CSP*, requires an estimation of the capacity and also simulate the industry decision process by solving sequentially the capacitated lot sizing problem followed by the cutting stock problem; the third one, the *ILSCSP*, models the proposal of taking an integrated decision.

6.2.1. Lot sizing results

We begin by analyzing the cost associated with the solution obtained by each model considering only the lot sizing costs (objective function (11)). As the production costs are constant over the periods and the demand must be met, there is no reason to compare the production cost related to the integrated decision (*ILSCSP*) and the lot sizing (*LSP* and *CLSP*) decisions, because they will be the same. The difference appears in the Inventory Costs related to the products which are shown in Table 3 with the best results marked in bold. It can be seen that the *LSP* model obtains better results than the *CLSP* and *ILSCSP* for all but two instances, instances 1 and 4, for which the value are the same for all models. These results were expected since the *LSP* has only to meet the demand and satisfy the safety stock. On the other hand, the models that consider capacity constraint (the *CLSP+CSP* and the *ILSCSP*) are forced to keep in stock a greater number of final products in order to meet the demand without exceeding the capacity of the cutting machine. Note that the *ILSCSP* model gives the second best results (in 12 out of 15 instances when compared only to the *CLSP+CSP*), which shows that the integrated approach has a better overview

Table 2
Weekly demand for the first instances of March (low demand), August (medium demand) and November (high demand).

March (low demand)

Instance 1

Product	**Periods**				**SumF**
	1	**2**	**3**	**4**	
(1) MC	40	40	40	40	160
(2) DT		120			120
(3) W1		80	80		160
(4) W2				70	70
(5) W3		25	25		50
(6) W4	115	115			230
(7) W5			160		160
(8) W6	170	170			340
SumT	325	550	305	110	**1290**
% Prod	25.19	42.64	23.64	8.53	**100.00**

August (medium demand)

Instance 6

Product	**Periods**				**SumF**
	1	**2**	**3**	**4**	
(1) MC	60		40	100	200
(2) DT		85	100		185
(3) W1		190			190
(4) W2			66		66
(5) W3	33			33	66
(6) W4		100		150	250
(7) W5	50			50	100
(8) W6	200			240	440
SumT	343	375	206	573	**1497**
%Prod	22.91	25.05	13.76	38.28	**100.00**

November (high demand)

Instance 11

Product	**Periods**				**SumF**
	1	**2**	**3**	**4**	
(1) MC				310	310
(2) DT	70	50	50		170
(3) W1	50		100	100	250
(4) W2		105			105
(5) W3					0
(6) W4		200	100	120	420
(7) W5	45		100		145
(8) W6	100	200	120	100	520
SumT	265	555	470	630	**1920**
% Prod	13.80	28.91	24.48	32.81	**100.00**

of the available capacity then the estimation given in *CLSP*.

6.2.2. Cutting stock results

The following considers the results obtained with the *LSP+CSP*, *CLSP+CSP* and *ILSCSP* models for the costs regarding to the multi-period cutting stock problem. Some factors can influence the solution in each model. The possibility of storing pieces and the characteristics of each instance can affect the solutions of the models. To analyze this influence, Table 4 shows the raw material costs and the total inventory costs of pieces (over all the four periods) obtained for each model, with the best results marked in bold. The models *CLSP+CSP* and *ILSCSP* resulted in better solutions than the model *LSP+CSP*. Due to the capacity constraints of the models *CLSP+CSP* and *ILSCSP*, some lots of products were brought forward which allowed better combinations of pieces and consequently the generation of more efficient cutting patterns without bringing forward the cut of too many pieces. On the other hand, the model *LSP+CSP* brought forward the cut of some pieces in order to generate efficient cutting patterns, this resulted in an increase in the pieces inventory costs (for all but one instance). Moreover, the total number of plates cut for the *LSP+CSP* instances is higher than the total number of plates necessary for the instances of the other two models. See the raw material costs in Table 4, which are higher for all but two instances.

Table 3
Inventory costs of final products.

Inventory costs ($)			
Inst	**LSP**	**CLSP**	**ILSCSP**
1	1566.37	1566.37	1566.37
2	**2004.52**	2140.62	2149.98
3	**1452.28**	1585.05	1576.41
4	1982.80	1982.80	1982.80
5	**1575.74**	1613.73	1595.43
6	**1724.16**	1967.78	1909.06
7	**1698.61**	1802.89	1754.22
8	**1548.27**	1909.61	1825.00
9	**1765.04**	1867.91	1824.83
10	**1541.89**	1745.22	1656.07
11	**2104.12**	2693.19	2649.07
12	**2315.23**	3075.60	2911.92
13	**2057.67**	2700.09	2611.55
14	**1836.48**	2544.23	2542.85
15	**2088.09**	3035.82	2901.50

Table 5 presents the costs from the cutting stock problem (objective function (17)). As a consequence of the results shown in Table 4, in general, the *CLSP+CSP* and *ILSCSP* models perform better than the *LSP+CSP* model. Observe that these results are contrary to the results regarding the lot sizing costs, the *CLSP+CSP* and the *ILSCSP* models together gave better results for 13 out of the 15 instances.

6.2.3. Lot sizing and cutting stock results

Table 6 shows the total cost values for each instance of the *ILSCSP*, *LSP+CSP*, and *CLSP+CSP* models (objective function (1) or (11) plus (17)). These results show that there is a reduction in the total cost for almost all the instances (13 out of 15) when considering the capacity constraint in the production planning (the solutions to the *ILSCSP* and *CLSP+CSP* models). This improvement is mainly due to the estimation given to the capacity constraint, which generates a planning of products that can reduce the costs of the cutting stock problem. The reduction of the cutting cost is due to the earlier production of some pieces that will soon be used to produce some products and therefore results in better solutions to the cutting stock problem as seen in the previous tables.

Comparing the results obtained by *CLSP+CSP* and *ILSCSP* we can see that the integrated approach does not always provide the best solution. This allows us to conclude that, for these instances, a good estimation of the cutting machine capacity in the lot sizing level can be as effective as the integrated approach. We have not found this kind of behavior for the scenarios described in the literature. Since the three solution approaches are heuristic methods, the column generation procedure is halted after a pre-defined amount of time and an optimization package is used to find a feasible integer solution, it is acceptable to have this kind of results. The results presented in Section 6.3 allow a better understanding of the behavior of these solution approaches and the impact of the costs in the associated decisions.

Table 4
Raw-material cost and inventory costs of pieces.

Inst	Raw-material cost ($)			Inventory cost of pieces ($)		
	LSP+CSP	CLSP+CSP	ILSCSP	LSP+CSP	CLSP+CSP	ILSCSP
1	242,613.00	242,547.00	**241,574.85**	**30.13**	37.91	35.87
2	243,058.00	244,596.00	**241,875.01**	253.84	155.70	**104.63**
3	242,378.00	**235,621.00**	245,466.79	275.50	165.81	**106.50**
4	231,037.00	231,635.00	**229,935.41**	96.26	**72.38**	95.30
5	238,667.00	240,109.00	**237,688.51**	102.12	99.42	**89.68**
6	268,618.00	**265,030.00**	265,385.11	327.61	**107.74**	121.53
7	279,239.00	**272,860.00**	279,524.04	181.92	176.24	**143.54**
8	279,295.00	**275,829.00**	279,258.73	546.17	**205.08**	231.65
9	267,463.00	268,495.00	**258,873.88**	178.88	140.79	**109.60**
10	267,740.00	269,953.00	**267,174.90**	333.75	**155.26**	201.82
11	**342,065.00**	363,572.00	362,031.59	862.00	425.31	**335.63**
12	351,825.00	**339,963.00**	341,956.82	1161.06	**317.52**	382.19
13	360,878.00	**354,469.00**	359,316.39	924.16	**343.73**	407.19
14	363,107.00	**361,155.00**	369,011.13	1112.07	277.33	**265.03**
15	**341,006.00**	360,464.00	360,932.69	1409.60	**328.78**	427.15

Table 5
Cutting stock costs.

Cutting stock cost ($)			
Ins	LSP+CSP	CLSP+CSP	ILSCSP
1	242,643.13	242,584.91	**241,610.72**
2	243,311.84	244,751.70	**241,979.65**
3	242,653.50	**235,786.81**	245,573.28
4	231,133.26	231,707.38	**230,030.71**
5	238,769.12	240,208.42	**237,778.19**
6	268,945.61	**265,137.74**	265,506.65
7	279,420.92	**273,036.24**	279,667.58
8	279,841.17	**276,034.08**	279,490.38
9	267,641.88	268,635.79	**258,983.48**
10	268,073.75	270,108.26	**267,376.72**
11	**342,927.00**	363,997.31	362,367.22
12	352,986.06	**340,280.52**	342,339.01
13	361,802.16	**354,812.73**	359,723.58
14	364,219.07	**361,432.33**	369,276.16
15	**342,415.60**	360,792.78	361,359.84

Table 6
Total costs.

Total costs ($)			
Ins	LSP+CSP	CLSP+CSP	ILSCSP
1	430,779.50	430,721.28	**429,747.09**
2	430,626.36	432,202.31	**429,439.62**
3	429,415.78	**422,681.85**	432,459.70
4	418,426.05	419,000.17	**417,323.51**
5	425,654.86	427,132.15	**424,683.62**
6	481,890.77	**478,326.52**	478,636.71
7	494,440.54	**488,160.14**	494,742.79
8	492,610.44	**489,164.69**	492,536.38
9	480,627.92	481,724.71	**472,029.31**
10	476,799.04	479,036.88	**476,216.19**
11	**620,916.11**	642,575.50	640,901.29
12	631,186.29	**619,241.12**	621,135.93
13	639,744.83	**633,397.82**	638,220.13
14	641,940.55	**639,861.57**	647,704.01
15	**620,388.69**	639,713.60	640,146.34

6.3. Computational results – Part II

In this section, two further tests were performed in order to evaluate the impact of the costs in the proposed approaches. The first one involves variations in the inventory costs of pieces and the second analysis considers variations in the costs of plates and in the inventory costs of final products.

6.3.1. Varying the inventory costs of pieces

The costs associated with the inventory of pieces in the 15 instances used in the tests described in Section 6.2 were based on the return rate considering the value of the pieces costs applied on the financial market. However, it might be important to increase these costs by considering the effort necessary to manage these pieces in stock. In what follows, we study the behavior of the solutions approaches when the inventory costs of a piece (h_p^e) is increased by a factor of 10, 50 and 100.

Table 7 shows the total inventory costs of pieces for each instance, considering the three different variations ($10 \times h_p^e$, $50 \times h_p^e$ and $100 \times h_p^e$) for each model. In this analysis it is possible to see the increase in the inventory costs of pieces whereas the parameter values increase, but this increase is much smaller for the integrated approach *ILSCSP*. Analyzing the quantity of pieces in stock, we observed that, when the factor 100 is considered, this quantity is on average up to 67.7% and 78.6% reduced considering *CLSP+CSP* and *ILSCSP* models, respectively, when compared with the *LSP+CSP* model. The large difference in the quantity of pieces in stock for the instances of the *ILSCSP* model is due to its feature that takes into account the inventory costs of pieces simultaneously with the other decisions costs. As the inventory costs of pieces increase there is a tendency to increase the inventory of final products in order to obtain a better overall solution. Hence, by anticipating the final products it is possible to obtain good combinations of items and cutting patterns which allows the reduction of the total costs of plates without the need to anticipate pieces production and thus reducing the total inventory costs of pieces. The other two models do not have this feature, and an attempt to minimize the inventory costs of final products is done when taking the lot sizing decision. Afterwards, on the multi-period cutting stock decision, there is a tendency to anticipate pieces in order to have better combinations on producing the cutting patterns. Obviously this reduces the costs of plates, but increases the inventory costs of pieces.

Table 8 shows the difference (in percentage) of the total costs of the *CLSP+CSP* and *ILSCSP* models compared with the *LSP+CSP* model considering the 3 variations of the inventory costs of pieces. The values

Table 7
Varying inventory costs of pieces.

Costs of pieces inventory									
Ins	**LSP+CSP**			**CLSP+CSP**			**ILSCSP**		
	$10 \times \hat{h}_p^e$	$50 \times \hat{h}_p^e$	$100 \times \hat{h}_p^e$	$10 \times \hat{h}_p^e$	$50 \times \hat{h}_p^e$	$100 \times \hat{h}_p^e$	$10 \times \hat{h}_p^e$	$50 \times \hat{h}_p^e$	$100 \times \hat{h}_p^e$
1	267.45	1207.43	1637.34	267.45	1362.56	1495.09	409.87	987.17	1928.10
2	2547.96	12,148.83	21,629.35	684.88	2434.61	4994.72	366.09	1530.16	2417.88
3	2260.74	8948.94	17,895.32	905.12	2826.90	2791.12	577.20	1214.25	2159.45
4	286.57	681.69	1336.66	445.67	1193.77	1237.82	270.95	1133.66	1385.97
5	1093.02	3 403.05	6588.78	1073.98	2098.28	4393.24	699.43	2069.49	3886.38
6	3250.66	14,593.79	28,107.57	648.45	3569.12	2644.91	632.10	1760.64	3824.92
7	2601.82	8 708.72	12,197.20	876.98	2172.70	8568.81	705.61	2915.34	5794.96
8	4978.39	25,153.90	49,648.37	1342.31	6714.42	8407.02	974.47	5526.55	3727.37
9	1470.46	7 776.54	13,647.60	1102.00	1898.36	2413.66	646.27	1354.78	2612.71
10	2937.28	14,282.85	27,209.89	1149.77	3369.38	7885.44	1238.17	2965.76	3501.05
11	8663.20	41,625.07	79,602.22	2668.31	11,744.94	24,407.14	1205.94	3547.81	7259.96
12	10,190.08	50,319.76	99,253.49	1729.19	5452.17	4906.95	1082.46	4748.83	6696.40
13	9449.54	43,085.95	83,831.88	2135.35	6179.99	4906.95	1186.53	4020.59	4727.31
14	10,657.36	52,944.56	91,395.27	3085.81	7392.32	8153.27	1476.92	6060.84	9056.85
15	12,829.72	65,980.41	136,410.00	2540.14	11,685.74	25,530.96	1723.17	188.79	7150.06

Table 8
Total costs difference.

Difference in total cost									
Ins	**LSP+CSP**			**CLSP+CSP**			**ILSCSP**		
	$10 \times \hat{h}_p^e$	$50 \times \hat{h}_p^e$	$100 \times \hat{h}_p^e$	$10 \times \hat{h}_p^e$	$50 \times \hat{h}_p^e$	$100 \times \hat{h}_p^e$	$10 \times \hat{h}_p^e$	$50 \times \hat{h}_p^e$	$100 \times \hat{h}_p^e$
1	100	100	100	0.000	0.372	−0.492	−2.434	1.809	0.361
2	100	100	100	0.099	1.299	3.140	0.953	2.143	4.004
3	100	100	100	0.507	2.729	4.941	−1.247	4.825	2.916
4	100	100	100	0.793	−0.148	0.073	−0.886	1.255	0.273
5	100	100	100	−0.085	−0.353	−0.979	−0.216	−0.256	−2.285
6	100	100	100	0.767	2.142	5.278	0.969	2.297	5.603
7	100	100	100	1.060	1.601	0.559	−2.078	−1.269	−0.694
8	100	100	100	1.302	3.554	7.883	−0.190	3.632	8.321
9	100	100	100	−0.296	0.964	1.983	1.045	0.904	1.335
10	100	100	100	0.244	1.853	3.466	0.621	1.945	6.204
11	100	100	100	−1.564	1.652	5.234	−0.694	2.733	7.143
12	100	100	100	−1.638	5.333	12.111	−2.282	2.753	8.995
13	100	100	100	0.613	4.078	12.603	0.331	4.476	9.825
14	100	100	100	2.797	6.762	10.656	0.747	8.281	7.878
15	100	100	100	−1.368	5.505	14.195	−1.624	6.526	15.579

Table 9
Classes.

Classes	co^e	h_f
1	L	L
2	L	H
3	H	L
4	H	H

associated to the *LSP+CSP* model represents 100% of the total costs and for the other approaches the values correspond to the percentage of gains (positive values) or losses (negative values), when compared to the respective *LSP+CSP* values. The results shows that as the inventory costs of pieces increase, the gains for both models also increase, specially for instances with high demand (November). Moreover, the gains obtained by the *ILSCSP* model is substantially higher than the gains obtained by the *CLSP+CSP* model. Therefore, it is possible to affirm that the integrated approach can handle more effectively the

Table 10
Total costs: Improvements.

Month	Class	**LSP+CSP**	**CLSP+CSP**	**ILSCSP**
March	1	100	0.6	−0.05
	2	100	−0.22	−0.58
	3	100	0.72	0.03
	4	100	−0.03	−0.53
August	1	100	2.69	3.75
	2	100	2.76	2.63
	3	100	2.33	3.62
	4	100	2.46	3.52
November	1	100	2.65	29.4
	2	100	3.3	29.07
	3	100	1.35	28.56
	4	100	1.62	28.79

Table A1
Product structure – thickness 15 mm.

Product structure – thickness 15 mm										
p	l_p	w_p	**(1) MC**	**(2) DT**	**(3) W1**	**(4) W2**	**(5) W3**	**(6) W4**	**(7) W5**	**(8) W6**
1	290	80		2						
2	385	140		16						
3	405	145				4				
4	416	140		8						
5	433	145			18	12	2	18	18	24
6	450	164		8						
7	459	145							6	12
8	470	430			2					
9	470	355						2		
10	494	60					2		5	3
11	495	198							3	6
12	500	396				2				
13	515	60								2
14	525	440			5		3		4	5
15	527	425				2				
16	530	60			6	6	3		3	6
17	530	90			2	2		4		
18	530	530					2		2	2
19	530	110				2				
20	530	527				2				
21	530	410				5				
22	535	530			2			2		
23	574	60		1						
24	600	100		2						
25	610	142			4		6	4		
26	610	530			2		2		2	2
27	610	565						2		
28	610	100		1						
29	615	485		1						
30	632	445					6		6	6
31	646	145				6	1		6	6
32	665	415		4						
33	680	198				6	6		6	6
34	701	145			9			9		
35	710	248		2						
36	726	480						1		
37	726	60						2		
38	728	480						1		
39	736	198			9			9		
40	760	60	6							
41	760	450	2							
42	770	522						3		
43	780	420	4							
44	820	480	1							
45	1090	770						3		
46	1095	440			5		3		4	5
47	1100	60			2	4	3		3	6
48	1100	530					2		2	2
49	1100	535			4			2		
50	1100	90			2					
51	1100	480						2		
52	1100	527				5				
53	1100	407				5				
54	1120	410				4				
55	1470	565					6			
56	1775	388	2							
57	1840	450	2							
58	1930	565							6	
59	2300	565			6			6		
60	2100	565				6				
61	2440	565								6

decisions when costs for managing the inventory of pieces are taken into account.

6.3.2. Varying the costs of plates and the inventory costs of final products

Based on the practical data, described in Section 6.1, we vary the costs of plate (co^e) and inventory costs of final products (h_f) in order to estimate the impact of these variations in the objective function of the proposed approaches. The changes in the costs are between 10% and 30% compared with the original value. The new sets of costs are generated in the intervals $[cost - cost \times 0.3, cost - cost \times 0.1]$ for low costs (L) and $[cost + cost \times 0.1, cost + cost \times 0.3]$ for high costs (H). The parameter *cost* refers to the two analyzed costs (co^e and h_f). The inventory costs of pieces ($\widehat{h_p^e}$) are fixed to the values of the second variation considered in Section 6.3.1 (*i.e.* 50 times the original costs). The possible combinations for low (L) costs and high (H) costs form 4 classes of instances that are presented in Table 9.

Considering the data sets from Section 6.1 we took the first instance for each month (March, August and November) which represent instances with Low, Medium and High demand, respectively. Observe that these are exactly the three instances for which the demands are presented in Table 2. Based on each one of these 3 instances we randomly generated five new instances varying the costs according to the interval previously described and considering 4 classes from Table 9. Therefore, a total of 3 (practical instances) × 5 (random instances) × 4 (classes)=60 new instances were generated.

Table 10 shows, for each class, the average difference (in percentage) of the total costs to the *CLSP+CSP* and *ILSCSP* models when compared with the *LSP+CSP* model. Considering the month with low demand, the three models are almost equivalent. However, when the level of demand increase (August and November), the integrated approach provides better results for all the classes and the gains are even greater for the instances with high demands reaching a gain of 29.4% compared to the *LSP+CSP* model. So, it is possible to conclude that the integrated approach can handle more effectively the decisions in high demand sceneries, *i.e.*, when the capacity utilization is more intensive. Considering the four different classes which represent different costs scenarios, it is possible to see that for the majority of the cases the impact in the gains is slightly reduced as the costs of plates increase (classes 3 and 4). It is not possible to draw a conclusion about the variation of the inventory costs of final products.

7. Conclusions

A mathematical model was proposed and implemented to analyze the main decisions of the production process of small scale furniture factories. The integrated model (*ILSCSP*) was proposed to capture the interdependencies of the lot sizing and the cutting stock decisions and thus promote a new approach to the decision making process. To simulate the *modus operandi* of Factory L, the *ILSCSP* model was decomposed in two models: a Lot sizing model (capacitated-*CLSP* and non capacitated-*LSP*) and a multi-period cutting stock model (*CSP*). The tests to validate the models were based on the product list of Factory L and on parameters taken from the market. A column generation technique was used to solve the Restricted Master Problem related to the *ILSCSP* and *CSP* models. Good overall results were obtained when comparing the *ILSCSP* solutions to the solutions of the approach of sequentially solving the non capacitated model *LSP+CSP*. Compared to the capacitated model *CLSP+CSP*, the results of the integrated model *ILSCSP* were competitive with the advantage that it is not necessary to estimate the used capacity, which is a difficult task in practice.

Further computational tests were executed in order to evaluated the impact of the different costs in the objective function for each approach. At first, three variations in the inventory costs of pieces were considered by taking into account the effort necessary to handle and administrate the pieces in stock. Then a variation in the costs of plates and in the inventory costs of final products are studied. The results showed that, in general, the integrated approach is better, mainly when the inventory costs of pieces are high and/or the demands are high. The variations on the costs of plates and inventory costs of final products do not have a strong impact on the differences among the three models.

We conclude from the computational study that the solution obtained can be put into practice, the models can support the main decisions taken and can bring improvements to the factory's production planning decisions.

Acknowledgments

This research was partially funded by the Brazilian agencies: Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES and Fundação de Amparo a Pesquisa do Estado de São Paulo - FAPESP (2012/20631-2, 2014/17273-2 and 2013/07375-0).

Appendix A. Products structure

See Table A1.

References

[1] Alem D, Jr., Morabito R. Production planning in furniture settings via robust optimization. Comput Oper Res 2012;39:139–50.

[2] Alem D, Jr., Morabito R. Risk-averse two-stage stochastic programs in furniture plants. OR Spectr 2013;35:773–806.

[3] Arbib C, Marinelli F. Integrating process optimization and inventory planning in cutting-stock with skiving option: an optimization model and its application. Eur J Oper Res 2005;163:617–30.

[4] Cui Y, Zhong C, Yao Y. Pattern-set generation algorithm for the one-dimensional cutting stock problem with setup cost. Eur J Oper Res 2015;243:540–6.

[5] Dyckhoff H. A new linear programming approach to the cutting stock problem. Oper Res 1981;29:1092–104.

[6] Farley AA. Mathematical programming models for cutting-stock problems in the clothing industry. J Oper Res Soc 1988;39:41–53.

[7] Fico. Fico Xpress 7.2: X-Press Mosel Language Reference Manual. (⟨http://www.fico.com/account/resourcelookup.aspx?theID=725⟩Access: 24 June 2015); 2013.

[8] FIESP. Fundação das Indústrias do Estado de São Paulo. Análise setorial de mercado: Setor de móveis. Technical report. São Paulo-SP, Brazil; 2009.

[9] Gilmore PC, Gomory RE. A linear programming approach to the cutting-stock problem. Oper Res 1961;9:849–59.

[10] Gilmore PC, Gomory RE. A linear programming approach to the cutting-stock problem - part ii. Oper Res 1963;11:863–88.

[11] Gilmore PC, Gomory RE. Multistage cutting stock problems of two and more dimensional. Oper Res 1965;14:1045–74.

[12] Gramani MCN, França PM. The combined cutting stock and lot sizing problem in industrial process. Eur J Oper Res 2006;74:509–21.

[13] Gramani MCN, França PM, Arenales MN. A Lagrangian relaxation approach to a coupled lot-sizing and cutting stock problem. Int J Prod Econ 2009;119:219–27.

[14] Gramani MCN, França PM, Arenales MN. A linear optimization approach to the combined production planning model. J Frankl Inst 2011;348:1523–36.

[15] Haessler RW. A heuristic programming solution to a nonlinear cutting stock problem. Manage Sci 1971;17:793–802.

[16] Hendry LC, Fok KK, Shek KW. A cutting stock and scheduling problem in the copper industry. J Res Dev 1996;16:553–64.

[17] Henn S, Wäscher G. Extensions of cutting stock problems: setups. Pesqui Oper 2013;33:133–62.

[18] Jans R, Degraeve Z. Modeling industrial lot sizing problems: a review. Int J Prod Res 2008;46:1619–43.

[19] Morabito R, Arenales M. Optimising the cutting of stock plates in a furniture company. Int J Prod Res 2000;38:2725–42.

[20] Morabito R, Arenales MN, Yanasse HH. Special issue on cutting, packing and related problems. Int Trans Oper Res 2009;16:659.

[21] Nonas SL, Thorstenson A. A Combined cutting-stock and lot-sizing problem. Eur J Oper Res 2000;120:327–42.

[22] Nonas SL, Thorstenson A. Solving a combined cutting-stock and lot-sizing problem with a column generating procedure. Comput Oper Res 2008;35:3371–92.

[23] Poldi KC, de Araujo SA. Mathematical models and a heuristic method for the multiperiod one-dimensional cutting stock problem. Ann Oper Res 2016;238:497–520.

[24] Poldi KC, Arenales MN. O problema de corte de estoque unidimensional multi-período. Pesqui Oper 2010;30:153–74.

[25] Poltroniere SC, Poldi KC, Toledo FMB, Arenales MN. A coupling cutting stock lot sizing in the paper industry. Ann Oper Res 2008;157:91–104.

[26] Rangel S, Figueiredo A. O problema de corte de estoque em indústria de móveis de pequeno e médio portes. Pesq Oper 2008;28:451–72.

[27] Respício A, Captivo ME. Integrating the cutting stock problem in capacity planning. Department of Informatics and Centre of Operational Research, University of Lisbon-Portugal; 2002. p. 1–11.

[28] Santos SMPG, Araujo SA, Rangel S. Integrated cutting machine programming and lot sizing in furniture industry. Pesq Oper desenvolvimento 2011;3:1–17.

[29] SEBRAE. Serviço Brasileiro de Apoio á Micro e Pequena Empresa. (⟨http://www.sebrae.com.br/uf/goias/indicadoresdas-mpe/classificacao-empresarial⟩. Access: 24 June 2015); 2013.

[30] Silva CTL, Alem Junior DJ, Arenales MN. A combined cutting stock and lot-sizing problem in the small furniture industry. In: International Conference on Operational Research for Development-ICORD VI; 2007.

[31] Silva E, Alvelos F, Valério de Carvalho JM. An integer programming model for two- and three-stage two-dimensional cutting stock problems. Eur J Oper Res Soc 2010;205:699–708.

[32] Silva E, Alvelos F, Valério de Carvalho JM. Integrating two-dimensional cutting stock and lot-sizing problems. J Oper Res Soc 2014;65:108–23.

[33] SPDR. Secretaria de Planejamento e Desenvolvimento Regional, 2012. Caracterização Socio Economica de São Paulo Região Administrativa de São José do Rio Preto, outubro. 76p. ⟨http://www.planejamento.sp.gov.br/noti_anexo/files/

uam/trabalhos/SJ%20Rio%20Preto.pdf⟩; 2012 [Access: 14 June 2015].

[34] Toscano A, Rangel S, Yanasse HH. A heuristic approach to minimize the number of saw cycles in small-scale furniture factories. Ann Oper Res. [online first: 2015]. http://dx.doi.org/10.1007/s10479-015-1955-9.

[35] Vanzela AM. Problema Integrado de Dimensionamento de Lotes e Corte de Estoque na Indústria Moveleira. (Master dissertation), Unesp, São José do Rio Preto-SP, Brazil. ⟨http://www.dcce.ibilce.unesp.br/pos/webfacil/publico/File/mdl_alunos_diss_633_0.pdf⟩; 2012 [Access: 14 June 2015].

[36] Vanzela M, Rangel S, Araujo SA. The integrated lot sizing and cutting stock problem in a furniture factory. In: 11th IFAC workshop on intelligent manufacturing systems, São Paulo. Intelligent Manufacturing Systems, vol. 11; 2013. p. 390–5 [received the award "Best Industrial Paper"].

[37] Wagner BJ. A genetic algorithm solution for one-dimensional bundled stock cutting. Eur J Oper Res 1999;117:368–81.

[38] Wäscher G, HauBner H, Schumann H. An improved typology of cutting and packing problems. Eur J Oper Res 2007;183:1109–30.

[39] Yanasse HH, Harris RG, Zinober ASI. Uma heurística para a redução do número de ciclos de serras no corte de chapas. In XIII ENEGEP - Congresso Latino Americano de Engenharia Industrial, São Carlos-SP, Brazil; 1993.

[40] Yanasse HH, Morabito R. Linear models for 1-group two-dimensional guillotine cutting problems. Int J Prod Res 2006;44:3471–91.

[41] Yanasse HH, Morabito R. A note on linear models for two-group and three-group two-dimensional guillotine cutting problems. Int J Prod Res 2008;46:6189–206.

Autorizo a reprodução xerográfica da presente Tese, na íntegra ou em partes para fins de pesquisa.

São José do Rio Preto, 21 de Março de 2017.

Gislaine Mara Melega