Reliability and Economic Effects of Maintenance on TNEP Considering Line Loading and Repair

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Abstract—This paper takes into account the reliability and economic effects of line maintenance on the transmission network expansion planning considering line repairs and the reliability effect of line loadings. For this purpose, a quantitative relationship between line lifetimes and the value of the transmission system is introduced in order to formulate the economic effect of maintenance. Also, the failure rate and mean time to repair coefficients are employed to calculate reliability effects of line maintenance using the cost of load shedding. Furthermore, the effect of line loadings on transmission system reliability is formulated through the line failure rates. The proposed model is tested on the Garver’s network and the IEEE Reliability Test System, which is followed by a discussion of the results.

Index Terms—Line loading, line maintenance and repair, power system reliability, transmission expansion planning.

NOMENCLATURE

Sets:

\( \text{(ij)} \) Line \( j \) of corridor \( i \).

\( \Omega^b, \Omega^c \) Set of all buses and all corridors.

\( \Omega^e, \Omega^g \) Set of existing corridors and corridors, including substations.

\( \Omega^l, \Omega^h \) Set of load buses and generation buses.

Constants:

\( \tilde{a}_{f,n,m}, \tilde{b}_{s,n,m} \) The ratio of the change in power flow on the line connected between buses \( n \) and \( m \) to the change in generation on bus \( f \) and to the change in demand on bus \( q \) when element \( z \) fails.

\( B_z(x) \) Effective age of element \( z \) at the end of the \( x \text{th} \) mission (year).

\( C^h \) Per unit cost of power losses ($/MWh).

\( C_i^r, C_i^s \) Construction cost of a line circuit and a substation 138/230 kV in corridor \( i \) ($).

\( C_{z,s}, C_{z,t}^r \) Fixed maintenance and repair cost of element \( z \) ($).

\( C_{i}^k \) Replacement cost of a line circuit in corridor \( i \) ($).

\( D_n, G_n \) Total demand and generation on bus \( n \) (MW).

\( k^l \) Losses coefficient.

\( l_i, V_i \) Length (km) and voltage level (kV) of corridor \( i \).

\( n_i, n_i^0, n_i^{rl} \) Maximum and initial number of circuits in corridor \( i \).

\( P_{i,f}, P_{nm} \) Initial operation period and regular life of element \( z \) (year).

\( r_i^l, \gamma_i^l \) Maximum permissible active power of corridor \( i \) and maximum permissible active power transmitted from bus \( n \) to \( m \) (MW).

\( \rho_i, (\Omega^{-1}/\text{km}) \) Resistance (\( \Omega/\text{km} \)) and susceptance (\( \Omega^{-1}/\text{km} \)) of each circuit per kilometer of corridor \( i \).

\( T \) Planning horizon (year).

\( VOLL_n \) Value of lost load (VOLL) for bus \( n \) ($/MW$).

\( \lambda_z, \tau_z \) Basic value of failure rate (1/year) and mean time to repair (MTTR) (h) of element \( z \).

\( \mu_z \) Basic value of annual number of repairs for element \( z \).

\( \delta_z \) Salvage value factor of element \( z \).

Variables:

\( I, S, l_n \) Load shedding of bus \( n \) due to outage of element \( z \) (MW).

\( n_i, n_i^n \) Number of new circuits and substations in corridor \( i \).

\( n_i^{lv} \) Life expectancy of element \( z \) (year).

\( \Delta \theta_i \) Difference between voltage phase angle of start and end buses in corridor \( i \) (radian).
I. INTRODUCTION

The main goal of transmission network expansion planning (TNEP) is to determine when and where new transmission lines should be installed in the network in order to help existing lines to reliably meet customer demand for electric power [1]. Nevertheless, some of the existing transmission lines may be old [2] and must be replaced by new ones. However, the replacement of old transmission lines may be costly and uneconomical over the long term. On the other hand, as the failure rates and outage probabilities of electrical components in older networks increase, transmission system reliability is reduced. This is an important challenge for planners because whereas the replacement of transmission lines is costly, retaining the old lines in the network may degrade the system’s reliability, and reliability is the essential factor for long-term planning. A way to tackle this difficulty is to employ maintenance concepts, because maintenance activities could improve system reliability [3]. Recently, extensive research has been conducted on TNEP. Some of these studies have solved this problem by considering various parameters, such as reliability criteria, risk and security indices, and uncertainties. Others have investigated this problem together with generation expansion planning.

Choi et al. [4] optimized the expansion cost of the transmission network so that the total capacity of the branches involved in the minimum cut-set would be greater than or equal to the system’s peak load demand. Also, they [5] minimized the investment budget for constructing new transmission lines by considering two probabilistic reliability criteria as problem constraints, as well as the uncertainties associated with the forced outage rates of the grid elements. Later, they introduced a new methodology for selecting the optimum expansion plan and level of reliability for a transmission system, which minimizes the sum of construction costs and customer outage costs [6]. Braga and Saraiva [7] presented a new formulation for the TNEP problem, but the objective function only includes expansion and generation costs and a reliability criterion: power not supplied (PNS). Akbari[8] minimized the costs of line construction, expected operation, and expected load shedding by considering load uncertainty and the voltage security constraint. Silva et al. [9] proposed a new methodology to solve the TNEP problem by considering reliability worth through the assessment of the interruption costs represented by the loss of load cost (LOLC) index. Gupta et al. [10] added reliability criteria of expected demand not served (EDNS) and expected generation not served (EGNS) to the objective function of the probabilistic transmission expansion planning problem. They showed that when EDNS is minimized, the capacity of the existing lines should be upgraded along with the addition of new transmission lines. Delgado and Claro [11] included investment cost and risk in the objective function of the TNEP problem, considering uncertainty in demand. The proposed model helps planners to balance important concerns such as network under-utilization, load curtailment, and the impossibility of providing power from the cheapest generators. Orfanos et al. [12] solved the probabilistic transmission expansion planning problem considering load uncertainty and wind power generation reliability. Rahmani et al. [13] presented a new methodology based on risk/investment to solve the TNEP problem considering multiple future generation and load scenarios. The proposed model enables planners to determine the necessary funding for installing transmission lines at a permissible risk level. Lopez et al. [14] proposed a new approach for transmission system expansion considering the impact of risk management on transmission investments. In addition, Bulent Tor et al. [15] solved the TNEP problem considering transmission system security and congestion. They showed that the annual evaluation of transmission investments and congestion along with local generation investment costs ensures more realistic assessments of generation and transmission investment decisions. Furthermore, Muñoz et al. [16] modeled long-term power transmission expansion planning considering the operation costs of wind power plants. The authors investigated the variability of wind resources and the effects of wind power operation on system security. Finally, Pozo et al. [17] presented a three-level model for the expansion of an electric network. The model represents the anticipation of transmission expansion planning for the investment in generation capacity.
However, in all of these studies, effects of the line maintenance and repair, as well as line loading on transmission system reliability and expansion costs, have not been considered; in other words, the TNEP problem has not been optimized simultaneously with maintenance and repair. It should be noted that with a low rate of maintenance, line repair cost increases and reliability may be degraded. However, if maintenance is very frequent, the repair cost decreases and reliability may be improved, but maintenance costs will increase sharply. While increasing the maintenance budget may lead to an increase in total system cost, it could forestall construction of some new lines and costly expansion of the transmission system in the future. Furthermore, power flow on the lines affects line failure rates and network reliability [18]. In other words, if the power flow to the lines increases, line failure rates rise, and consequently, transmission system reliability is degraded. Meanwhile, if the magnitude of current to the lines is reduced, line failure rates decrease, thereby improving transmission system reliability. Thus, it is very useful to have a general age-dependent and loading-reliant model available that explicitly considers the economic and reliability effects of maintenance on TNEP. In this paper, a model of the line loading impact on failure rates and the maintenance effects on network reliability and transmission system value is introduced to solve the TNEP problem using the decimal codification genetic algorithm (DCGA) technique. The main aims and contributions of the present study are:

1) To present a mathematical formulation that investigates the approximate effects of line maintenance on transmission network value and repair.

2) To introduce a quantitative relationship between line maintenance, line loading, and transmission reliability.

II. PROBLEM FORMULATION

The TNEP problem is formulated by using the DC power flow model to minimize objective function (1).

\[
\begin{align*}
\min F_1 &= \sum_{i \in \Omega^n} C_{i}^{C} n_i + \sum_{i \in \Omega^e} C_{i}^{S} n_i^e \\
&\quad + \sum_{i \in \Omega^e} C_{i}^{R} + 8760k L P^L \\
&\quad + \sum_{i \in \Omega^e} \sum_{j=1}^{n_i} \left[ C_{i,j}^{M} + C_{i,j}^{r} \right] \\
&\quad + \sum_{n \in \Omega^n} VOLL_n \sum_{i \in \Omega^e} \sum_{j=1}^{n_i+n_j} LS_{n,i,j} \Pr_{i,j} - VTS.
\end{align*}
\]

Where,

\[
\begin{align*}
\gamma_i &= \left(n_i + n_i^e\right) \gamma_i' \\
\rho_i &= \frac{r_i^e}{n_i + n_i^e} \\
P^L &= \sum_{i \in \Omega^e} \frac{\ell_i r_i^2 P^2}{V_i^2} \\
C_{i,j}^{M} &= k_{i,j} C_{i,j}^{M} \\
C_{i,j}^{r} &= k_{i,j}^r C_{i,j}^{r}
\end{align*}
\]
each corridor is limited by maximum permissible active power of the same corridor. Equations (16)–(18) show the right-of-way constraint, maximum number of new substations and life expectancy limitation.

A. Calculation Method of Load Shedding

All terms of objective function (1) are calculated for normal conditions (when no line outage occurs) except for the sixth term. In simple terms, load shedding must be calculated after determining of proposed configuration by DCGA in each iteration. However, power flows to all lines are changed (the power flows to some lines increase, while the power flows to others decrease) when a transmission line fails. In this case, the DC power flow node balance ((12)) is no longer satisfied, because generation and demand on each bus remain fixed. Therefore, new equations are required to calculate the load curtailment of each bus with respect to its VOLL (the curtailment for loads with lower VOLLS is more possible than loads with higher VOLLS) and to balance the power flow on each bus. According to the previous statements and in light of the fact that the goal is to obtain an optimal expansion plan with the lowest cost of load curtailment (maximum reliability), objective function (19) with constraints (20)–(22) are defined in order to calculate the load shedding for each contingency state (single line outage).

\[
\min F_2 - \sum_{n \in \Omega^B} VOLL_n LS_{n(i,j)}.
\]  

(19)

Subject to:

\[
P_{nm}^{(i,j)} = \sum_{f \in \Omega^B} \hat{a}_{f,nm}^{(i,j)} G_f + \sum_{q \in \Omega^B} \hat{b}_{q,nm}^{(i,j)} (D_q - LS_{q(i,j)})
\]

(20)

\[
|P_{nm}^{(i,j)}| \leq P_{nm}^{(i,j)} \quad \forall n, m \in \Omega^B, n \neq m, i \in \Omega^C,
\]

\[
j = 1, \ldots, (n + n_q).
\]

\[
0 \leq LS_{n(i,j)} \leq D_n \quad \forall n \in \Omega^B, i \in \Omega^C,
\]

\[
j = 1, \ldots, (n + n_q).
\]

(21)

Equations (21) and (22) show the power flow limit on transmission lines in contingency states, as well as minimum and maximum load shedding for buses. In (20), \(\hat{a}_{f,nm}^{(i,j)}\) and \(\hat{b}_{q,nm}^{(i,j)}\) represent the ratio of the change of power flow on the line connected between buses \(n\) and \(m\) to the change of generation in bus \(f\) and the change of demand on bus \(q\), respectively, when line \(j\) of corridor \(i\) fails. These factors are determined by the DC power flow for each contingency [19]. It is assumed that the slack bus can compensate for changes in generation and demand in all contingencies.

B. Effect of the Maintenance Cost Coefficient \(k_z\) On the Life Coefficient \(\delta_z\)

Transmission equipment has a regular lifetime under normal operational conditions if the required maintenance activities are performed. Predefined maintenance expenditures are required to carry out these activities. If the maintenance budget is more or less than this cost, the component age (life expectancy) becomes longer or shorter than its regular life, respectively [20]. This fact can be described mathematically using the age factor of element \(z\) as follows [20]:

\[
a_z = \frac{A_z \{x_i\}}{B_z \{x_i\}} = \left( \frac{C^M_z - C^M_z}{C^R_z} \right)^{1/m_z},
\]

(23)

\(m_z\) is a feature of element \(z\) that determines the relationship between maintenance cost and age factor. Larger and smaller \(m_z\) correspond to younger and older elements, respectively. Rewriting (23) for the transmission lines in TNEP results in (24) as follows:

\[
\alpha_{(i,j)} = \frac{A_{(i,j)} \{T\}}{B_{(i,j)} \{T\}} = \left( \frac{C^M_{(i,j)} - C^M_{(j,i)}}{C^R_{(i,j)}} \right)^{1/m_{(i,j)}},
\]

(24)

The following algebraic equation is obtained by replacing \(A_{(i,j)} \{T\} - n_{(i,j)}^{(i)} - n_{(i,j)}^{(j)} - T\) and \(H_{(i,j)} \{T\} - n_{(i,j)}^{(i)} - n_{(i,j)}^{(j)}\) in (24):

\[
\delta_{(i,j)} = (1 - \alpha_{(i,j)}) \left( \beta_{(i,j)} \right)^{1/m_{(i,j)}} \left( k_{(i,j)} - 1 \right)^{1/m_{(i,j)}} + \left( \alpha_{(i,j)} + \frac{T}{n_{(i,j)}^{(i,j)}} \right).
\]

(25)

This equation, called “curve of life coefficient,” indicates the relationship between \(\delta_{(i,j)}\) and \(k_{(i,j)}\), where \(\delta_{(i,j)} = n_{(i,j)}^{(i)} / n_{(i,j)}^{(j)}\), \(\alpha_{(i,j)} = n_{(i,j)}^{(j)} / n_{(i,j)}^{(i)}\), and \(\beta_{(i,j)} = (C^M_{(i,j)} / C^R_{(i,j)})\). The curve is depicted in Fig. 1 for various characteristic constants \((m_{(i,j)})\), \(\alpha_{(i,j)} = 0.6 (n_{(i,j)}^{(i)} - 18)\) and \(n_{(i,j)}^{(j)} = 30\), \(\beta_{(i,j)} = 0.7\), and \(T = 12\) compared to the curve of life coefficient for a new transformer adopted from the results presented in [21].

However, \(\alpha_{(i,j)}\) cannot be assumed to be constant when \(m_{(i,j)}\) is variable, because it depends on the initial line age. It should be noted that \(m_{(i,j)}\) varies in the interval [1, \(M_{(i,j)}\)] because of \(\alpha_{(i,j)}\) limitation \((0 < \alpha_{(i,j)} < 1)\), where \(M_{(i,j)}\) depends on line characteristics. Accordingly, for new lines, \(\alpha_{(i,j)} = 0\) and \(m_{(i,j)} = M_{(i,j)}\), and for lines which are quite old, \(\alpha_{(i,j)} = m_{(i,j)} = 1\). Therefore, from Fig. 1, the aforementioned results, and the non-linear coherence between \(m_{(i,j)}\) and \(\alpha_{(i,j)}\) ((25)), the following equation is deduced:

\[
m_{(i,j)} = M_{(i,j)} - (M_{(i,j)} - 1) \alpha_{(i,j)}^{(1/m_z)}.
\]

(26)
Fig. 2. Curve of life coefficient for different initial line ages.

Fig. 3. Curve of failure rate.

Fig. 2 shows the results when $m_{(i,j)}$ is replaced in (25). The planning horizon and regular lifetimes are considered to be 30 years, $\beta_{(i,j)} = 0.1$, $M_{(i,j)} = 1.5$ and $s = 2$.

Fig. 2 shows that the life expectancy of lines that have been operated for longer periods increases beyond that of newer lines for the same maintenance cost coefficients. This indicates the importance of maintaining older transmission lines.

C. Effect of $k_z$ on the Failure Rate Coefficient ($\zeta_z$)

The transmission lines have basic failure rates under normal operational conditions if the required maintenance activities are carried out. If the maintenance costs are more or less than the cost of carrying out these activities, the failure rate becomes less or more than its basic value, respectively (Fig. 3). This can be described mathematically using curve of failure rate as follows, where $\zeta_{(i,j)} = \lambda_{(i,j)}^{M_{(i,j)}} / \lambda_{(i,j)}$ (See Appendix A).

\[ \zeta_{(i,j)} = (1 - \alpha_{(i,j)}) \left( \frac{T}{n_{(i,j)}} \right)^{1 - \alpha_{(i,j)}} \left( 1 - \eta (1 - \alpha_{(i,j)}) \right)^{1/m_{(i,j)}} \left( k_{(i,j)} - 1 \right)^{1/m_{(i,j)}}. \]  

The curve is depicted in Fig. 3 for various $m_{(i,j)}$, $\alpha_{(i,j)} = 0.6$, $\beta_{(i,j)} = 0.7$, and $T = 75$ compared to the curve of the failure rate for a new transformer [21]. This curve is shown in Fig. 4 for different coefficients $\alpha_{(i,j)}$, $T = 30$, $\beta_{(i,j)} = 0.1$, and $M_{(i,j)} = 1.5$. Fig. 3 demonstrates the importance of line maintenance to the reliability of older transmission systems.

D. Effect of $k_z$ on the Mean Time to Repair Coefficient ($\chi_z$)

One of the aims of maintenance is to extend the mean time to repair (MTTR) because repairs may be costly [3]. In other words, if maintenance cost increases, the number of repairs may decrease. Usually, repairs to equipment are more expensive than maintenance. So, it is found that low maintenance costs do not have much effect on the number of repairs (MTTR). Conversely, if the maintenance budget is large, it may affect the MTTR considerably. So, the MTTR may increase with increases in maintenance costs until it approaches the level of repair expenditure. In this situation, the MTTR is fixed and not extended with an increase in maintenance costs (Fig. 5). Equation (28) expresses this fact mathematically (see Appendix B for more details). This algebraic equation is known as “curve of MTTR,” and it explains the relationship between $\chi_{(i,j)}$ and $k_{(i,j)}$, where $\chi_{(i,j)} = T_{(i,j)} / \tau_{(i,j)}$. The curve is exhibited in Fig. 5 for different coefficients $\alpha_{(i,j)}$, $\epsilon = 1$, $\omega_1 = 10.36$, $\omega_2 = 2.216 \beta_{(i,j)} = 0.1$, $T / m_{(i,j)} = 1$, $a = 2$, $1 < k_{(i,j)} < b$, and $b = 4$ (a $< b < 4$) compared to the curve of MTTR for a new transformer [21].

\[ \chi_{(i,j)} = \begin{cases} \omega_1 \left( 1 - \alpha_{(i,j)} \right) \left( \beta_{(i,j)} \right)^{1/m_{(i,j)}} \left( a - 1 \right)^{1/2m_{(i,j)}} & 1 < k_{(i,j)} < a \\ \omega_2 \left( 1 - \alpha_{(i,j)} \right)^2 \left( \frac{T}{n_{(i,j)}} \right)^{1/m_{(i,j)}} + \frac{\alpha_{(i,j)}}{x} & a < k_{(i,j)} < b \\ \omega_2 \left( 1 - \alpha_{(i,j)} \right)^2 \left( \frac{T}{n_{(i,j)}} \right)^{1/m_{(i,j)}} \left( k_{(i,j)} - 1 \right)^{1/2m_{(i,j)}} & k_{(i,j)} > b \end{cases} \]  

According to Fig. 5, the curve of MTTR for a new transformer is nearly flat when $k_{(i,j)}$ is less than 2 and greater than 4. In other words, maintenance action does not have much effect on the MTTR curve of a transformer in this area. A power transformer is more expensive and needs more repairs and inspections than a kilometer of transmission line, so this flat area is larger for a line. Thus, constants $a$ and $b$ should be selected to be more than 2 and less than 4, respectively. However, in this paper, 2
Fig. 5. Curve of MTTR for different \( \sigma_{(ij)} \).

and 4 are assumed in order to compare the curves of MTTR for existing lines with the curve of MTTR for a new transformer.

E. Effect of the Repair Cost Coefficient (\( k_f^r \)) on \( \chi_x \)

Along with maintenance efforts, predefined repair activities are required to provide regular lifetimes for existing transmission lines during the operation period. A specific expenditure, known as the “fixed repair cost,” is necessary to perform these activities. The number of repairs, and therefore the total repair cost, decreases by increasing the maintenance cost. Also, the total repair cost is reduced as the fixed repair cost diminishes. This fact can be described analytically as follows:

\[
C_r^{(ij)} = \frac{C_r^{(ij)}}{\mu^{(ij)}} \mu^{(ij)}.
\]

Replacing \( \mu^{(ij)} = 8760/\tau^{(ij)} \) and \( \mu^{(ij)} = 8760/\tau^{(ij)} \) in (29) yields:

\[
C_r^{(ij)} = \frac{C_r^{(ij)}}{\tau^{(ij)}} = \frac{C_r^{(ij)}}{\chi^{(ij)}}.
\]

Equation (31) is obtained by comparing (30) to (6).

\[
\chi^{(ij)} = \frac{1}{k_r^{(ij)}}.
\]

F. Effect of Line Loading on the Line Failure Rate (\( \lambda^{(ij)} \))

The magnitude of line currents affects network reliability through line failure rates [18]. In other words, line failure rates are reduced, and consequently, transmission system reliability is improved by decreasing the power flow to the lines. It is assumed that when the power transmitted through a line is zero, any transmission line \( j \) in corridor \( i \) has the lowest failure rate of \( \lambda^{(ij)} \). If the power transmitted through a line reaches its maximum amount (\( P^{(ij)} \)), its failure rate increases to the basic value (\( \lambda^{(ij)} \)). If the active power of a branch is between its minimum and maximum value, the failure rate is defined through a linear relationship to the percentage of line loading. Thus, the line loading coefficient of the \( j \)th branch in corridor \( i \) is defined as (32).

\[
\rho^{(ij)} = \frac{P^{(ij)}}{P^{(ij)}}.
\]

Therefore, new failure rates of the existing transmission lines are computed as follows:

\[
\lambda^{(ij)} = \rho^{(ij)}(\lambda^{(ij)} - \lambda^{M^{(ij)}}) + \lambda^{M^{(ij)}}.
\]

III. SOLUTION METHOD

In the present study, the decimal codification genetic algorithm (DCGA) technique [22] is used to solve the objective function (1). This algorithm generally includes the three fundamental genetic operators of reproduction, crossover, and mutation. These operators conduct the chromosomes toward better fitness (objective function). In the first step, an initial population with \( d \) chromosomes is constructed randomly as (34) when constraints (16)–(18) are satisfied.

\[
X = \begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
\vdots \\
X_d
\end{bmatrix}.
\]

In (34), \( X_d \) is \( d \)th chromosome of the population \( X \). This vector consists of integer numbers; each of them is called a “gene”. These genes describe the problem variables.

\[
X_d = \{n_1, \ldots, n_t, \ldots, n_{11}, n_{11}^{e}, \ldots, n_{11}^{e}, \ldots, n_{11}^{e} \}.
\]

Where, \( n_1, n_t, \) and \( n_e \) indicate the number of new circuits, new substations, and life expectancy of the existing lines in corridor \( i \), respectively.

\[
n_i = \{0, 1, \ldots, n_i - n_i \} \quad \forall i \in \Omega^c
\]

\[
n_i^{e} = \{0, 1, \ldots, n_i - n_i^{e} \} \quad \forall i \in \Omega^c
\]

\[
n_i^{e} = \{n_i^{e}, n_i^{e}, \ldots, n_i^{e}, \ldots, n_i^{e} \} \quad \forall i \in \Omega^c.
\]

In (38), \( n_i^{e} \) is the life expectancy for line \( j \) of corridor \( i \). Equation (39) describes a typical population with 5 chromosomes (\( d = 5 \)) for Garver’s network [22]. This 6-bus system includes 15 corridors and 6 existing transmission lines (corridors 1 (1–2), 3 (1–4), 4 (1–5), 6 (2–3), 7 (2–4), 11 (3–5)).

\[
X_1 = [3, 2, 2, 0, 3, 2, 0, 2, 1, 4, 0, 3, 4, 3, 3, 37, 32, 33, 49, 36]
\]

\[
X_2 = [2, 2, 1, 3, 1, 3, 1, 2, 2, 3, 1, 1, 4, 0, 0, 55, 36, 35, 59, 43, 40]
\]

\[
X_3 = [1, 3, 3, 1, 2, 2, 0, 0, 1, 0, 1, 3, 0, 0, 3, 40, 41, 42, 48, 34, 32]
\]

\[
X_4 = [3, 0, 0, 0, 2, 0, 1, 3, 3, 3, 0, 2, 2, 1, 0, 44, 56, 58, 38, 35, 56]
\]

\[
X_5 = [3, 3, 3, 2, 0, 2, 0, 2, 3, 4, 1, 1, 4, 1, 4, 4, 37, 49, 59, 50, 56, 31].
\]

(39)

\( X_1 \) proposes three new 230 kV transmission circuits for corridors 1, 4, 6, 12 (3–6), and 15 (5–6), two new 230 kV transmission circuits for corridors 2 (1–3), 3, and 8 (2–5), four new 230 kV transmission circuits for corridors 10 (3–4), 13 (4–5), and 14 (4–6), and no new transmission circuits for corridors 5 (1–6), 7, and 11. In addition, the life expectancies of the existing lines in corridors 1, 3, 4, 6, 7, and 11 are 37, 37, 32, 33, 49 and 36 years, respectively.
Equation (4) is calculated considering constraints (12) and (14) using DC power flow. If (15) is satisfied, objective function (19) considering constraints (20)–(22) is solved using the \textit{fmincon} function for contingency states (line outages). \textit{fmincon} is a function in the optimization tool box of MATLAB that can be used for minimizing constrained nonlinear multivariable problems. For Garver's network, the objective function (19) is written as (40) when first line (circuit) of corridor 1 fails.

\[
\min F_2 = \sum_{f=1}^{\text{number of lines}} P_{12}^{(1)} + \sum_{q=1}^{\text{number of substations}} V_{q12}^{(1)}(D_q - L_{S_{q(1)}})
\]

Subject to:

\[
P_{12}^{(1)} = \sum_{f=1}^{\text{number of lines}} a_{f12}^{(1)} G_f + \sum_{q=1}^{\text{number of substations}} i_{q12}^{(1)} (D_q - L_{S_{q(1)}})
\]

\[
P_{12}^{(1)} < P_{f2}
\]

\[
P_{56}^{(1)} < P_{56}
\]

\[
0 \leq L_{S_{q(1)}} \leq D_1, \quad 0 \leq L_{S_{2(1)}} \leq D_2, \quad 0 \leq L_{S_{3(1)}} \leq D_3
\]

Each chromosome resulting from the crossover operation will now be subject to the mutation operator in the final step of forming the new generation. This operator selects a few existing integer numbers (variables) in the chromosome and then changes their values at random according to small probability known as a mutation probability \((P_M)\). It should be mentioned that in this process, (16)–(18) must be satisfied, i.e., the values must not exceed their limits. In this study, mutation is applied with a probability of 0.1 \((P_M = 0.1)\). Equations (46) and (47) exhibit \(X_2\) before and after mutation, respectively.

\[
X_2 = [2, 0, 1, 3, 1, 3, 2, 3, 1, 2, 3, 1, 4, 3, 0, 55, 36, 35, 59, 43, 40]
\]

In this study, a more suitable termination criterion has been established: the production of a predefined number of generations after obtaining best fitness and finding no better solution. In this study, the maximum number of generations considered was 2000 and 10000 for the Garver's and RTS systems, respectively.

IV. SIMULATION RESULTS

The proposed planning technique was applied to Garver's network and the IEEE Reliability Test System (IEEE RTS). This approach can also be applied to large-scale systems. It should be mentioned that the planning horizon was 15 years for both case study networks.
A chromosome of a size equal to $X_k$ is determined.

Initial population is constructed randomly.

If each chromosome is feasible, i.e., satisfying constraints (12), (14), (15)-(18) then calculate the objective function (19) and (13), (20)-(22) as well as objective function (1). The infeasible chromosomes are discarded.

Is termination criterion satisfied?

Selection operator chooses the best chromosomes of sizes equal to the number of chromosomes in initial population.

The best individual is selected.

Reproduction operator reproduces each chromosome in proportion to the value of objective function (1).

Crossover operator is applied with $P_c$ rate.

Mutation operator is applied with $P_m$ rate.

Choose the current population.

Fig. 6. Flowchart of the proposed method.

### TABLE I

<table>
<thead>
<tr>
<th>Corridor</th>
<th>$n_i^{10}$ (year)</th>
<th>$\lambda_i$ (1/year)</th>
<th>Bus</th>
<th>$YOLL_i$ ($/MW$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>5</td>
<td>0.5</td>
<td>1</td>
<td>1100</td>
</tr>
<tr>
<td>1-4</td>
<td>10</td>
<td>0.61</td>
<td>2</td>
<td>1000</td>
</tr>
<tr>
<td>1-5</td>
<td>20</td>
<td>0.42</td>
<td>3</td>
<td>1500</td>
</tr>
<tr>
<td>2-3</td>
<td>25</td>
<td>0.41</td>
<td>4</td>
<td>1200</td>
</tr>
<tr>
<td>2-4</td>
<td>15</td>
<td>0.52</td>
<td>5</td>
<td>1300</td>
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<tr>
<td>3-5</td>
<td>25</td>
<td>0.39</td>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>

### A. Garver’s Network

All data for this 6-bus system containing 230 kV lines are described in [22]. The maximum number of circuits on each corridor ($\pi_k$), the regular life ($n_i^{12}$), and the basic value of MTTR ($\tau_k$) for all transmission lines are considered to be 4, 30 years, and 11 hours, respectively. Moreover, the reliability data and initial operation period ($n_i^{10}$) of the existing lines are listed in Table I.

The proposed method was applied to the case study system in two scenarios. In Scenario 1, the TNEP problem was solved without considering line maintenance, repair, and loading effects, while in Scenario 2, these effects were considered.

1) **Scenario 1:** The goal was to solve the traditional TNEP problem considering fixed maintenance and repair costs ($k_{i(j)} = k'_{i(j)} - \rho_{i(j)} - 1$). The proposed idea was tested on the case study system. The new lines that needed to be added to the network are listed in Table II. Also, the existing corridors 1–5, 2–3 and 3–5 needed to be replaced by new transmission lines because of their age (See Table I). In addition, the expansion, operation, and reliability costs of the network are provided in Table III.

![Convergence curve of the algorithm for Scenario 1 of Garver's network.](image)

### TABLE II

<table>
<thead>
<tr>
<th>Corr.</th>
<th>$n_i$</th>
<th>$V_i$ (kV)</th>
<th>Corr.</th>
<th>$n_i$</th>
<th>$V_i$</th>
<th>Corr.</th>
<th>$n_i$</th>
<th>$V_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-6</td>
<td>4</td>
<td>230</td>
<td>4-6</td>
<td>3</td>
<td>230</td>
<td>5-6</td>
<td>4</td>
<td>230</td>
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</table>

### TABLE III

<table>
<thead>
<tr>
<th>Expansion cost of the transmission system</th>
<th>Construction cost of new lines</th>
<th>55.7</th>
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<tbody>
<tr>
<td>Active losses cost</td>
<td>Replacement cost of existing lines</td>
<td>5.06</td>
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<tr>
<td>Maintenance cost</td>
<td>Repair cost</td>
<td>7.2</td>
</tr>
<tr>
<td>Cost of probable load shedding (Reliability cost)</td>
<td>Total cost of transmission network</td>
<td>1.83</td>
</tr>
</tbody>
</table>

2) **Scenario 2:** In this scenario, the reliability and economic effects of line maintenance, as well as the reliability effects of line loading and repair on the TNEP problem were considered. The proposed idea was applied to the network under study, and results are provided in Tables IV–VIII. Also, the objective function values of both scenarios versus different iterations are illustrated in Figs. 7 and 8 in order to show the convergence process of the algorithm.

![Convergence curve of the algorithm for Scenario 2 of Garver's network.](image)
TABLE IV
PROPOSED EXPANSION PLAN IN SCENARIO 2 FOR GARVER’S NETWORK

<table>
<thead>
<tr>
<th>Corridor</th>
<th>n_i</th>
<th>V_i (kV)</th>
<th>Corridor</th>
<th>n_i</th>
<th>V_i (kV)</th>
</tr>
</thead>
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<tr>
<td>2-3</td>
<td>1</td>
<td>230</td>
<td>4-6</td>
<td>3</td>
<td>230</td>
</tr>
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<td>2-6</td>
<td>4</td>
<td>230</td>
<td>5-6</td>
<td>4</td>
<td>230</td>
</tr>
</tbody>
</table>

TABLE V
NEW LIFETIMES, FAILURE RATES, AND MTTRs FOR GARVER’S NETWORK

<table>
<thead>
<tr>
<th>Corridor</th>
<th>(n^b)</th>
<th>(\lambda^f)</th>
<th>(\lambda_i)</th>
<th>(\tau_i)</th>
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</thead>
<tbody>
<tr>
<td>1-2</td>
<td>36</td>
<td>0.18</td>
<td>0.22</td>
<td>29</td>
</tr>
<tr>
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<td>37</td>
<td>0.18</td>
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<td>26</td>
</tr>
<tr>
<td>1-5</td>
<td>36</td>
<td>0.07</td>
<td>0.22</td>
<td>18.5</td>
</tr>
<tr>
<td>2-3</td>
<td>41</td>
<td>0.03</td>
<td>0.07</td>
<td>17.3</td>
</tr>
<tr>
<td>2-4</td>
<td>38</td>
<td>0.12</td>
<td>0.17</td>
<td>22.6</td>
</tr>
<tr>
<td>3-5</td>
<td>41</td>
<td>0.03</td>
<td>0.35</td>
<td>17.3</td>
</tr>
</tbody>
</table>

TABLE VI
LOADING COEFFICIENTS OF EXISTING LINES IN BOTH SCENARIOS FOR GARVER’S NETWORK

TABLE VII
LOADING COEFFICIENTS OF NEW LINES IN BOTH SCENARIOS FOR GARVER’S NETWORK

TABLE VIII
COSTS IN SCENARIO 2 (MILLION $) FOR GARVER’S NETWORK

B. IEEE RTS

This 24-bus network contains transmission lines at two voltage levels: 138 kV and 230 kV. All data for this test system are presented in [23]. \(\pi^t\) and \(n^t_i\) for all transmission lines are considered to be 2 and 30 years, respectively. Also, \(n^{10}\) and the value of lost load (VOLL) of the existing lines for this network are listed in Tables IX and X. The proposed method was applied to the case study without considering maintenance and repair effects (Scenario 1) and considering them (Scenario 2).

1) Scenario 1: The TNEP problem was solved considering only fixed maintenance and repair costs, network losses, and transmission system reliability. The proposed idea was tested on the IEEE RTS. New lines that needed to be added to the network are listed in Table XI. Also, 22 of the existing corridors needed to be replaced by new transmission lines (Table XII). In addition, a new 138/230 kV substation needed to be constructed in corridor 3–24. The related costs are listed in Table XIII.
2) Scenario 2: The problem was solved considering line maintenance, repair, and loading effects. The proposed idea was applied to the network, and results are provided in Tables XIV–XVIII. In addition, the construction of a new 138/230 kV substation within corridor 3–24 was required. Moreover, the convergence process of the objective functions for both scenarios are shown in Figs. 9 and 10.

The construction costs of new lines for the plans that consider the effects of maintenance are more than those of the other configurations. The reason for this discrepancy is that, in Scenario 2, more new transmission lines must be added to the network (compare Tables IV and XIV with Tables II and XI) in order to decrease the line loadings (see Tables VI and XVI), and consequently, line failure rates (Tables V and XV). These modifications result in network losses that are less than the active losses in Scenario 1. In addition, in Scenario 1, US$7.18 million (US$1.84 million for maintenance and US$5.34 million for repair) is allocated to maintain and repair the lines of existing corridors 1–2, 3–9, 6–10, 8–9, 11–13, 15–21, and 17–18 (lines...
that are not replaced by new ones because their initial operation period plus the planning horizon is smaller than or equal to their regular lifetime) in order to provide regular lifetimes for the RTS network and keep line failure rates and MTTRs at basic values. The results obtained in Scenario 1 for Garver's network confirmed this fact, because US$9.72 million (US$2.52 million for maintenance and US$7.2 million for repair) was allocated to maintain and repair the existing lines of corridors 1–5, 2–3, and 3–5. Nevertheless, the expansion cost of the transmission system in Scenario 2 for the reliability test system was US$22.38 million and for Garver's network US$2.26 million less than the related costs in Scenario 1. According to Table XII, in Scenario 1, the circuits of 22 existing corridors (9 one-circuit 138 kV lines, 10 one-circuit 230 kV lines, and 3 two-circuit 230 kV lines) and the circuits of 3 corridors in Garver’s network (3 one-circuit 230 kV lines) had to be replaced with new transmission lines because of their age (see Tables I and IX for more details). In Scenario 2, however the lifetimes of the lines in all existing corridors were extended (Tables V and XV). This increased the value of the RTS transmission system by US$35.16 million as opposed to incurring US$32.53 million in maintenance and repair costs with the proposed arrangement in Scenario 2. Also, the probable load shedding in Scenario 2 and Scenario 1 of the RTS were 3000 MW and 5120 MW, and for Garver's network 1525 MW and 1681 MW, respectively (i.e., 20 independent variables (unknowns)). Therefore, 719 iterations were necessary before convergence. In Scenario 2, the lifetimes of 6 existing lines, i.e., 6 new variables, were added to unknowns of Scenario 1. This fact caused the number of iterations to increase to 879 (160 more iterations). However, in Scenario 1 of the RTS system, there were 141 corridors and 17 load buses (158 variables). For this reason, the solution took 5550 iterations (7.7 times more than 719) to converge (Fig. 9), because the number of variables was 7.9 times more than number of unknowns in Scenario 1 of Garver’s network. In Scenario 2 of the RTS system, the lifetimes of 29 existing lines (29 new variables) were added to independent variables of Scenario 1. In this case, the number of variables was 7.2 more than that of Scenario 2 in Garver's network. Accordingly, the number of iterations increased to 7713.

V. CONCLUSION

This paper presented a reliability-based model for transmission expansion planning considering the effects of line maintenance and repair, as well as line loading, on transmission system arrangement. The economic effect of line maintenance on TNEP was formulated using the value of the transmission network and the curve of life coefficient. Its reliability effect was modeled by the cost of load shedding via the curves of failure rate and MTTR. In addition, a quantitative relationship between line loading, system reliability, and maintenance was
introduced using failure rates and line loading coefficient. The simulation results revealed the importance of the proposed TNEP model; lines that seem old and ready to be replaced by new ones can still be economical and reliable in the long run if the required maintenance and repair actions are carried out. Although the maintenance of old lines is costly and may seem uneconomical over the short term, maintenance results in a decrease in the total cost of the transmission network over the long term because of the reduction in transmission system expansion and reliability costs and the increase in transmission system value.

APPENDIX A
CALCULATION METHOD OF $\zeta_{ij}$ VERSUS $k_{ij}$

From Fig. 2 and (25), the mathematical formulation of the life coefficient for a new transformer and a new line can be expressed as (48) and (49), respectively.

\[
\hat{\vartheta}(\text{Transformer}) = \beta (k_{ij} - 1)^{2/3} + 1 \quad (48)
\]

\[
\vartheta_{ij} = (\beta_{ij})^{2/3} \cdot (k_{ij} - 1)^{2/3} + \frac{T}{n_{ij}} \quad (49)
\]

where, $\beta = 0.323$ and $\beta_{ij} = 0.1$ (i.e., $\beta = (3/2) [\beta_{ij}]^{2/3}$). Also, the mathematical description of the failure rate coefficient for a new transformer is adopted from Fig. 4 as follows:

\[
\hat{\zeta}(\text{Transformer}) = 1 - 0.151(k_{ij} - 1)^{2/3} \quad (50)
\]

Equation (51) shows (50) in term of $\beta$, where $\Gamma = 0.468$. In simple terms, if $\Gamma$ is multiplied by $\beta$, the result is 0.151.

\[
\zeta(\text{Transformer}) = 1 - \Gamma \beta (k_{ij} - 1)^{2/3} \quad (51)
\]

By comparing (51) with (48), it can be found out that the failure rate coefficient for a new transformer can be obtained when a negative coefficient such as $\Gamma$ is multiplied by $\beta$. Accordingly, the following mathematical definition can be deduced from (49), when a negative coefficient such as $\eta$ is multiplied by $(\beta_{ij})^{2/3}$.

\[
\zeta_{ij} = \frac{T}{n_{ij}^{2/3}} - \eta \cdot (\beta_{ij})^{2/3} \cdot (k_{ij} - 1)^{2/3} \quad (52)
\]

As shown in Figs. 3 and 4, the slope of the curves for transmission lines is lower than the slope of the curve for a new transformer, therefore, the value of $\eta$ should be considered lower than $\Gamma$, i.e., $\eta = \beta_{ij} \Gamma$, where $0.5 < \eta < 1$. According to (52), and by analyzing the curves of Figs. 2 and 4, the general equation of (27) can be presented. Equation (52) is a special case of (27), where $\alpha_{ij} = 0$ and $M_{ij} = 1.5$.

APPENDIX B
CALCULATION METHOD OF $\chi_{ij}$ VERSUS $k_{ij}$

Equation (53) approximately describes the MTTR curve of a new transformer (see Fig. 5).

\[
\chi_{\text{Transformer}}(k_{ij}) = \begin{cases} 
1 & 1 \leq k_{ij} < 2 \\
3.346(k_{ij} - 1)^{1/3} - 2.216 & 2 \leq k_{ij} < 4 \\
2.63 & k_{ij} > 4 
\end{cases} \quad (53)
\]

Equation (54) expresses (53) in term of $\beta$. It should be noted that (54) is a continuous function for all maintenance cost coefficients. In simple terms, the value of (54) is 1 for $k_{ij} = 2$ and is 2.63 for $k_{ij} = 4$ if $\sigma_1 = 10.36$, $\sigma_2 = 2.216$ and $\beta = 0.323$ ($\beta_{ij} = 0.1$).

\[
\chi_{\text{Transformer}}(k_{ij}) = \begin{cases} 
1 & 1 \leq k_{ij} < 2 \\
\sigma_1 \beta(k_{ij} - 1)^{1/3} - \sigma_2 & 2 \leq k_{ij} < 4 \\
2.63 & k_{ij} > 4 
\end{cases} \quad (54)
\]

In (54), by replacing $\beta = (3/2) [\beta_{ij}]^{2/3}$, $(3/2)\sigma_1 - \omega_1$, $\sigma_2 = \omega_2(T/n_{ij}^{1/3}) (T/n_{ij}^{2/3}) = 1$, $a = 2$, and $b = 4$, the continuous function (55) yields for a new line.

\[
\chi_{ij} = \begin{cases} 
\omega_1 (\beta_{ij})^{2/3} (a - 1)^{1/3} - \omega_2 \frac{a}{n_{ij}} & 1 \leq k_{ij} < a \\
\omega_1 (\beta_{ij})^{2/3} (k_{ij} - 1)^{1/3} - \omega_2 \frac{a}{n_{ij}} & a \leq k_{ij} \leq b \\
\omega_1 (\beta_{ij})^{2/3} (b - 1)^{1/3} - \omega_2 \frac{b}{n_{ij}} & k_{ij} > b 
\end{cases} \quad (55)
\]

where, $\omega_1$ and $\omega_2$ are coefficients of $\sigma_1$ and $\sigma_2$ ($\omega_1 = \beta_1 \cdot \sigma_1$ and $\omega_2 = \beta_2 \cdot \sigma_2$), and $M_{ij} = 1.5$. Usually, values of $\omega_1$ and $\omega_2$ are considered to be lower than $\sigma_1$ and $\sigma_2$ because, as shown in Fig. 5, the curves of the lines have lower slopes than the ones of a new transformer ($0.5 < \beta_1 < \beta_2 < 1$). According to (55), and by analyzing the curves of Figs. 2, 4 and 5, the general equation of (28) can be achieved.

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