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Abstract: We present a framework linking axion-like particles (ALPs) to neutrino masses through the minimal inverse seesaw (ISS) mechanism in order to explain the dark matter (DM) puzzle. Specifically, we explore three minimal ISS cases where mass scales are generated through gravity induced operators involving a scalar field hosting ALP. In all of these cases, we find gravity stable models providing the observed DM relic density and, simultaneously, consistent with the phenomenology of neutrinos and ALPs. Remarkably, in one of the ISS cases, the DM can be made of ALPs and sterile neutrinos. Furthermore, other considered ISS cases have ALPs with parameters inside regions to be explored by proposed ALP experiments.

Contents

1 Introduction

The discovery of neutrino oscillations [1, 2] and the fact that baryonic matter only yields a few percent contribution to the energy density of the Universe [3] are two experimental evidences calling for physics beyond the standard model (SM). On the theoretical side, the apparent absence of CP violation in the QCD sector is also a strong motivation for going beyond the SM since it can be dynamically explained by the Peccei-Quinn mechanism [4], which requires to extend the SM gauge group with a global symmetry and the existence of a pseudo-Nambu-Goldstone boson, the axion [5, 6]. Besides elegantly solving the strong CP problem [7–10], the Peccei-Quinn mechanism may be also related to the solution of DM and neutrino puzzles by offering a candidate for cold DM, the axion itself, [11–13] and a connection to the neutrino mass generation [14–20].

In the same vein, ALPs, arising from spontaneous breaking of approximate global symmetries, are also theoretically well motivated since these appear in a variety of ultraviolet extensions of the SM [21–24] and, as in QCD axion models, these can make up all of Universe DM [25], or be a portal connecting the DM particle to the SM sector [26]. Moreover, there are some astrophysical phenomena such as the cosmic γ-ray transparency [27–30], the X-ray excess from the Coma cluster [31, 32] and the X-ray line at 3.55 keV [33, 34] that suggest the presence of ALPs. These hints have led to a plethora of search strategies involving astrophysical observation production and detection in laboratory experiments [21, 22, 35] with the aim of establishing the ALP properties.

In the context of ALP models, the approximate continuous symmetry is typically assumed to be remnant of an exact discrete gauge symmetry as gravity presumably breaks the global symmetries through Planck-scale suppressed operators. This discrete gauge symmetry protects the ALP mass against large gravity-induced corrections and it can also be used to stabilize other mass scales present in the theory. In particular, with the aim of generating neutrino masses the authors in Refs. [23, 36] used these type of discrete gauge symmetries in order to protect the associated lepton number breaking scale. In this work, we go further by building a self-consistent framework of ALP DM and neutrino masses via the ISS mechanism [37, 38]. For this purpose we make use of appropriate discrete gauge symmetries to protect the suitable ALP mass reproducing the correct DM relic abundance as well as to stabilize the mass scales present in the ISS mechanism. It turns out that the ISS mass terms are determined -up to some factor- by $v_\sigma^n/M_{\rm Pl}^{n-1}$, where v_σ is the vacuum expectation value (VEV) of the scalar field σ that spontaneously breaks the global symmetry $\mathrm{U}(1)_A$ and hosts the ALP, a. n is an integer that is determined by the invariance of such terms under the symmetries of the model and some phenomenological constraints.

In order to implement the ISS mechanism we extend the SM matter content by introducing $n_{N_R}(n_{S_R})$ generations of SM-singlet fermions $N_{\rm R}(S_{\rm R})$ as it is usual. In this work, we consider the minimal number of singlet fermionic fields that allows to fit all the neutrino oscillation data: the $(n_{N_R}, n_{S_R}) = (2,2)$, $(2,3)$ and $(3,3)$ cases [39]. In each case, the ALP plays the role of the DM candidate. Moreover, for the (2,3) ISS case there is a possibility of having a second DM candidate: the sterile neutrino (the unpaired singlet fermion) [40–42]. Motivated by that, we also build a multicomponent DM framework where the DM of the Universe is composed by ALPs and sterile neutrinos, with the latter being generated through the active-sterile neutrino mixing [43] and accounting for a fraction of the the DM relic density.

The rest of the paper is organized as follows: in Sec. 2 we discuss phenomenological and theoretical conditions that lead to a successful protection of the ALP mass and the ISS texture against gravity effects. In Sec. 3 we search for viable models simultaneously compatible with DM phenomenology, neutrino oscillation observables and lepton flavor violating processes. Finally, we present our discussion and conclusions in Sec 4.

2 Framework

The goal of this section is to present the main ingredients of a SM extension in order to link axion like particles (ALPs) to neutrino mass generation, and at the same time, to offer an explanation for the current DM relic density reported by Planck collaboration [3]. In order to achieve that, the SM matter content must to be extended with some extra fields. Besides the scalar σ and femionic $S_{\mathrm{R}\alpha}$ and $N_{\mathrm{R}\beta}$ fields, an extra electrically charged fermion E is also added to the SM to make possible the coupling of ALPs to photons, $g_{a\gamma}$. That is necessary because $g_{a\gamma}$ is anomaly induced and there is no any $\mathrm{U}(1)_A$ symmetry anomalous in the electromagnetic group just with the SM charged fermions.

Another key point of the framework is the existence of a Z_N discrete gauge symmetry. In order to understand its role, firstly, note that to impose an anomalous $\mathrm{U}(1)_A$ symmetry to the Lagrangian does not seem sensible in the sense that in the absence of further constraints on very high energy physics we should expect all relevant and marginally relevant operators that are forbidden only by this symmetry to appear in the effective Lagrangian with coefficient of order one. However, if this symmetry follows from some other free anomaly symmetry, in our case from the a Z_N discrete gauge symmetry, all terms which violate it are then irrelevant in the renormalization group sense. Secondly, the Z_N symmetry also protects both the ALP mass and the ISS texture against gravity effects as we will explain in more detail later on. For these reasons, the effective Lagrangian will be invariant under a Z_N discrete gauge symmetry.

2.1 Lagrangian

The effective Lagrangian that we consider to relate the ISS mechanism to ALP DM reads

$$\mathcal{L} \supset \mathcal{L}_{\rm SM}^{\rm Yuk} + \mathcal{L}_\sigma + \mathcal{L}_{\rm ISS} + \mathcal{L}_E, \tag{2.1}$$

where $\mathcal{L}_{\rm SM}^{\rm Yuk}$ is nothing more than the Yukawa Lagrangian of the SM

$$\mathcal{L}_{\rm SM}^{\rm Yuk} = Y_{ij}^{(u)} \overline{Q_{{\rm L}i}} \widetilde{H} u_{{\rm R}j} + Y_{ij}^{(d)} \overline{Q_{{\rm L}i}} H d_{{\rm R}j} + Y_{ij}^{(l)} \overline{L_i} H l_{{\rm R}j} + {\rm H.c.}, \tag{2.2}$$

with the usual $Q_{{\rm L}i}, u_{{\rm R}i}, d_{{\rm R}i}$ and $L_i, l_{{\rm R}i}$ fields denoting the quarks and leptons of the SM, respectively. H is the Higgs $\mathrm{SU}(2)_L$ doublet with $\widetilde{H} = i\tau_2 H^*$ (τ_2 is the second Pauli matrix). The term in $\mathcal{L}_\sigma$ (Lagrangian involving the σ field) which is relevant in our discussion is

$$\mathcal{L}_\sigma \supset g \frac{\sigma^D}{M_{\rm Pl}^{D-4}} + {\rm H.c.}, \tag{2.3}$$

with $g = e^{i\delta} |g|$ and D being an integer. The σ field is parametrized as $\sigma(x) = \frac{1}{\sqrt{2}}[v_\sigma + \rho(x)] e^{i\frac{a(x)}{v_\sigma}}$, with $a(x)$ being the ALP field and $\rho(x)$ the radial part that will gain a mass of order of the vacuum expectation value [23, 44, 45]

$$10^9 \lesssim \sqrt{2}\,\langle\sigma\rangle \equiv v_\sigma \lesssim 10^{14}\ {\rm GeV}. \tag{2.4}$$

The term in eq. (2.3) gives an ALP mass written as follows

$$m_a = |g|^{\frac{1}{2}} D M_{\rm Pl} \lambda^{\frac{D}{2}-1}, \tag{2.5}$$

where $10^{-10} \lesssim \lambda \equiv \frac{v_\sigma}{\sqrt{2} M_{\rm Pl}} \lesssim 10^{-5}$ and $M_{\rm Pl} = 2.44 \times 10^{18}$ GeV is the reduced Planck scale.

Now, we turn our attention to the coupling of ALPs to photons which is anomaly induced. Specifically, ALP couples to photons via $\frac{g_{a\gamma}}{4} a\, F_{\mu\nu} \widetilde{F}^{\mu\nu}$ where $F_{\mu\nu}$ and $\widetilde{F}^{\mu\nu}$ are the electromagnetic field strength and its dual, respectively. For this reason, the total Lagrangian in eq. (2.1) must be invariant under an anomalous $\mathrm{U}(1)_A$ global symmetry. Nevertheless, only with SM model fermions and the neutral $S_{{\rm R}\alpha}$ and $N_{{\rm R}\beta}$ fermions is not possible to have an anomalous $\mathrm{U}(1)_A$ symmetry in the electromagnetic group. Therefore, we include the $\mathrm{SU}(2)_L$ singlet fermion, E, with an unit of electric charge. Once done that, ALPs couple to photons with $g_{a\gamma}$

$$g_{a\gamma} = \frac{\alpha}{2\pi} \frac{C_{a\gamma}}{v_\sigma}, \tag{2.6}$$

where $\alpha \approx 1/137$ and $C_{a\gamma}$ is the electromagnetic anomaly coefficient, usually of order one in our models [45, 46].

The dimension D of the gravity induced mass operator must be, in general, larger than 4 because the astrophysical and cosmological constraints on ALPs. To be more specific, we show, in Figure 1, some regions of the ALP space of parameters $-g_{a\gamma}$ vs m_a- where ALPs give an explanation for some astrophysical anomalies and others forbidden regions [22, 47–49]. Note that $C_{a\gamma}$ directly determines the width of the red band in Figure 1 where ALPs are DM candidates (we have used $1 \leq C_{a\gamma} \leq 2$ for illustration).

Regarding the neutrino mass generation, we have that, once introduced the $N_{\mathrm{R}\beta}$ and $S_{\mathrm{R}\alpha}$ fields, the $\mathcal{L}_{\mathrm{ISS}}$ Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\mathrm{ISS}} = y_{i\beta}\overline{L_i}\widetilde{H}N_{\mathrm{R}\beta} + \zeta_{\alpha\beta}\frac{\sigma^p}{M_{\mathrm{Pl}}^{p-1}}\overline{S_{\mathrm{R}\alpha}}(N_{\mathrm{R}\beta})^{\mathrm{C}} + \eta_{\alpha\alpha'}\frac{\sigma^q}{2M_{\mathrm{Pl}}^{q-1}}\overline{S_{\mathrm{R}\alpha}}(S_{\mathrm{R}\alpha'})^{\mathrm{C}} \\ + \theta_{\beta\beta'}\frac{\sigma^r}{2M_{\mathrm{Pl}}^{r-1}}\overline{N_{\mathrm{R}\beta}}(N_{\mathrm{R}\beta'})^{\mathrm{C}} + \text{ H.c.},\end{aligned} \tag{2.7}$$

where the $y_{i\beta}$, $\zeta_{\alpha\beta}$, $\eta_{\alpha\alpha'}$, $\theta_{\beta\beta'}$, coupling constants, with $i, j = 1, 2, 3,\ \alpha, \alpha' = 1, 2, (\text{or } 3)$ and $\beta, \beta' = 1, 2, (\text{or } 3)$, are generically assumed of order one. The exponents p, q, r are integer numbers chosen for satisfying some phenomenological constraints discussed below. Negative values for these exponents will mean that the term is $\sim \sigma^{*n}$ instead of $\sim \sigma^n$. Note that, without loss of generality, the exponent p can be assumed to be positive. We will only consider the minimal number of neutral fermionic fields, $S_{\mathrm{R}\alpha}$ and $N_{\mathrm{R}\beta}$, that allow to fit all the neutrino oscillation data [39]. Specifically, we study the $(2, 2)$, $(2, 3)$ and $(3, 3)$ cases. As the σ field gets a VEV the gravity induced terms in eq. (2.7) give the mass matrix for light (active) and heavy neutrinos [36]. Specifically, we can write the mass matrix in the $(\nu_{\mathrm{L}}, N_{\mathrm{R}}^{\mathrm{C}}, S_{\mathrm{R}}^{\mathrm{C}})$ basis as

$$M_\nu = \begin{bmatrix} 0 & M_D^\intercal \\ M_D & M_R \end{bmatrix}, \text{ with} \tag{2.8}$$

$$M_D \equiv \begin{bmatrix} m_D \\ 0 \end{bmatrix} \text{ and } M_R \equiv \begin{bmatrix} \mu_N & M^\intercal \\ M & \mu_S \end{bmatrix}. \tag{2.9}$$

where m_D, M, μ_N and μ_S are matrices with dimension equal to $n_{N_R} \times 3$, $n_{N_R} \times n_{S_R}$, $n_{N_R} \times n_{N_R}$, $n_{S_R} \times n_{S_R}$, respectively. The energy scales of the entries in these matrices are determined essentially by $\sqrt{2}\,\langle H\rangle \equiv v_{\mathrm{SM}} \simeq 246$ GeV, λ (or v_σ) and M_{Pl} GeV as follows

$$m_{D\,i\beta} = y_{i\beta}\frac{v_{\mathrm{SM}}}{\sqrt{2}}, \quad M_{\alpha\beta} = \zeta_{\alpha\beta}M_{\mathrm{Pl}}\lambda^p, \tag{2.10}$$

$$\mu_{S\,\alpha\alpha'} = \eta_{\alpha\alpha'}M_{\mathrm{Pl}}\lambda^{|q|}, \quad \mu_{N\,\beta\beta'} = \theta_{\beta\beta'}M_{\mathrm{Pl}}\lambda^{|r|}. \tag{2.11}$$

The mass matrix in eq. (2.8) allows light active neutrino masses at order of sub-eV without resorting very large energy scales in contrast to the type I seesaw mechanism [50–53]. In more detail, assuming the hierarchy $\mu_N \approx \mu_S \ll m_D < M$ (note that making μ_S and μ_N small is technically natural) and taking a matrix expansion in powers of M^{-1}, the light active neutrino masses, at leading order, are approximately given by the eigenvalues of the matrix [54, 55]

$$m_{\nu\mathrm{light}} \simeq m_D^\intercal M^{-1}\mu_S(M^\intercal)^{-1}m_D. \tag{2.12}$$

On the other hand, the heavy neutrino masses are given by the eigenvalues of $m_{\nu\text{heavy}} \simeq M_R$. Note that μ_N does not contribute to the light active neutrino masses at the leading order [54, 55]. Actually, the presence of μ_N term gives a sub-leading contribution to $m_{\nu\text{light}}$, written as $\Delta m_{\nu\text{light}} = -m_D^{\intercal} M^{-1}\mu_S (M^{\intercal})^{-1}\mu_N M^{-1}\mu_S(M^{\intercal})^{-1} m_D$ which is of order $\sim M^{-4}$ [36].

Very well motivated scales for M and μ_S, μ_N are TeV and keV scales, respectively. These choices allow, for instance, new physics in TeV scale and in some scenarios, such as the $(2,3)$ ISS case, the existence of a keV sterile neutrino as a warm dark matter (WDM) candidate [56]. In addition, M has to satisfy

$$M \gtrsim \sqrt{10\frac{\mu_S}{\text{keV}}}\ \text{TeV}, \tag{2.13}$$

because light active neutrino masses are in sub-eV scale and m_D is of order of $\mathcal{O}(v_{\text{SM}})$. Another constraint on the M scale comes from violation of unitary which is of order of ϵ^2 with $\epsilon \equiv m_D M^{-1}$ being approximately the mixing between light active and heavy neutrinos. Roughly speaking, ϵ^2 at the percent level is not excluded experimentally [57–59]. Taking into account the previous considerations, the ranges chosen for M and μ_S are

$$1 \leq M \leq 25\ \text{TeV},\ \ 0.1 \leq \mu_S \leq 50\ \text{keV}. \tag{2.14}$$

Once established the scales of the mass matrices and using eqs. (2.10) and (2.11), the integers p and q in eq. (2.7) can only take the values

$$(p, |q|) = (2,3)\ \ \text{for}\ \ 6\times 10^{10} \lesssim v_\sigma \lesssim 1\times 10^{11}\ \text{GeV}, \tag{2.15}$$

$$(p, |q|) = (3,5)\ \ \text{for}\ \ 2\times 10^{13} \lesssim v_\sigma \lesssim 8\times 10^{13}\ \text{GeV}. \tag{2.16}$$

That happens because the same VEV simultaneously provides M and μ_S scales. For both possibilities in eqs. (2.15) and (2.16) the light active neutrino mass matrix is simplified to

$$m_{\nu\text{light}} = \left[y^{\intercal}\zeta^{-1}\eta\,(\zeta^{\intercal})^{-1}\,y\right]\frac{v_{\text{SM}}^2}{\sqrt{2}v_\sigma}. \tag{2.17}$$

Moreover, the exponent r of the term that generates μ_N is also constrained to be $r \geq 3$ because μ_N must be $\approx \mu_S$.

Finally, we have that $\mathcal{L}_E$, the Lagrangian involving the E charged fermion, is written as

$$\mathcal{L}_E \supset \vartheta_i \frac{\sigma^s}{M_{\text{Pl}}^s}\overline{L_i} H E_{\text{R}} + \kappa\frac{\sigma^t}{M_{\text{Pl}}^{t-1}}\overline{E_{\text{L}}}E_{\text{R}} + \text{H.c.}, \tag{2.18}$$

where ϑ_i and κ are, in principle, assumed of order one. These two terms are also subjected to phenomenological and theoretical constraints. Because the term $\sim \sigma^t \overline{E_{\text{L}}}E_{\text{R}}$ must give a mass large enough for the E fermion to satisfy the experimental constrains (for stable charged heavy Lepton, $m_E > 102.6$ GeV at 95% C.L. [60], or for charged long-lived heavy Lepton, $m_E > 574$ GeV at 95% C.L. assuming mean life above $7\times 10^{-10} - 3\times 10^{-8}$ s [61, 62]), t must be less than 3 but different from zero because the electromagnetic anomaly. On the

	$Q_{\mathrm{L}i}$	$d_{\mathrm{R}i}$	$u_{\mathrm{R}i}$	H_i	L_i	$l_{\mathrm{R}i}$	$N_{\mathrm{R}\beta}$	$S_{\mathrm{R}\alpha}$	E_{L}	E_{R}	σ
B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	0	0	0	0
$\mathbb{L}$	0	0	0	0	1	1	1	a	b	c	d

Table 1. Two of the continuous symmetries of the Lagrangian in eq. (2.1). B and $\mathbb{L}$ are the baryon number and the generalized lepton number, respectively. The charges a, b, c and d are given by $a = qd/2$, $b = sd + c$, $c = 1 - rd$ and $d = (p - q/2)^{-1}$.

other hand, s can take the values 1 or 2 because $\sim \sigma^s \overline{L} H E_{\mathrm{R}}$ determines the interaction of the E fermion with the SM leptons. Whether s is larger than 2, the charged E fermion becomes stable enough to bring cosmological problems, unless its mass is $\lesssim$ TeV. Another constraint comes from searches for long-lived particles in pp collisions [61, 62].

Now an important discussion about the stability of both the ISS mechanism and the ALP mass is in order. In general, the gravitational effects must be controlled to give a suitable ALP mass. With this aim, we introduced a gauge discrete Z_N symmetry assumed as a remnant of a gauge symmetry valid at very high energies [63]. To truly protect the ALP mass against those effects, Z_N must at least be free anomaly [64–67], i.e.,

$$A_2(Z_N) = A_3(Z_N) = A_{\mathrm{grav}}(Z_N) = 0 \quad \mathrm{Mod}\ \frac{N}{2}, \tag{2.19}$$

where A_2, A_3 and A_{grav} are the $[\mathrm{SU}(2)_{\mathrm{L}}]^2 \times Z_N$, $[\mathrm{SU}(3)_{\mathrm{C}}]^2 \times Z_N$ and $[\mathrm{gravitational}]^2 \times Z_N$ anomalies, respectively. Gravitational effects can also generate terms such as $\frac{\sigma^n}{M_{\mathrm{Pl}}^{n-1}} \overline{S_{\mathrm{R}}} S_{\mathrm{R}}^{\mathrm{C}}$, $\frac{\sigma^n}{M_{\mathrm{Pl}}^{n-1}} \overline{S_{\mathrm{R}}} N_{\mathrm{R}}^{\mathrm{C}}$, $\frac{\sigma^n}{M_{\mathrm{Pl}}^{n-1}} \overline{N_{\mathrm{R}}^{\mathrm{C}}} N_{\mathrm{R}}$ or $\frac{\sigma^n}{M_{\mathrm{Pl}}^{n}} \overline{L} \widetilde{H} S_{\mathrm{R}}$ (with n smaller than those in the Lagrangian (2.7)) that jeopardize both the matrix structure - eqs. (2.8) and (2.9) - and the scales of the ISS mechanism. Thus, Z_N will be chosen such that it also prevents these undesirable terms from appear.

In general, Z_N can be written as a linear combination of the continuous symmetries in the model: the hypercharge Y, the baryon number B and the generalized lepton number $\mathbb{L}$. B and $\mathbb{L}$ are shown in Table 1. Nevertheless, since the hypercharge is free anomaly by construction, the Z_N charges (Z) of the fields can be written as $Z = c_1 \mathrm{B} + c_2 \mathbb{L}$ where $c_{1,2}$ are the rational numbers in order to make the Z_N charges integers [36]. Now, we substitute the charges shown in Table 1 into the general form of the Z_N symmetry to obtain the anomaly coefficients. Doing so, we find that $A_3(Z_N) = 0$ and

$$A_2(Z_N) = \frac{3}{2}\left[c_1 + c_2\right], \tag{2.20}$$

$$A_{\mathrm{grav}}(Z_N) = c_2 \left[3 - n_{N_R} - n_{S_R} \times \left[\frac{qd}{2}\right] + sd\right]. \tag{2.21}$$

Note that $A_2(Z_N)$ and $A_{\mathrm{grav}}(Z_N)$ are not, in general, 0 Mod $N/2$ which implies strong constraints on the choice of the Z_N discrete symmetry.

2.2 ALP and sterile neutrino dark matter

Since the ALPs are very weakly interacting slim particles and cosmologically stable, they can be considered as DM candidates [25]. In fact, ALPs may be non-thermally produced via the misalignment mechanism in the early Universe and survive as a cold dark matter population until today. Specifically, its relic density is determined from the following equation [25, 45, 68–73]

$$\Omega_{a,\mathrm{DM}}h^2 \approx 0.16 \left[\frac{\Theta_i}{\pi}\right]^2 \times \left[\frac{m_a}{\mathrm{eV}}\right]^{1/2} \left[\frac{v_\sigma}{10^{11}\ \mathrm{GeV}}\right]^2, \tag{2.22}$$

where Θ_i is the initial misalignment angle, which is taken as $\frac{\pi}{\sqrt{3}}$, because we are assuming a post-inflationary symmetry breaking scenario, favorable for models with $v_\sigma \lesssim 10^{14}$ GeV.

On the other hand, the fraction of DM abundance in form of sterile neutrino depends on its mass, m_{ν_S}, and its mixing angle with the light active neutrino, θ. Specifically, ν_S as a WDM candidate can be generated through the well-known Dodelson-Widrow (DW) mechanism [43] which is present as long as active-sterile mixing is not zero [40–42]. In the $(2,3)$ ISS case, the DW mechanism can account at maximum for $\approx 43\%$ of the observed relic density without conflict with observational constraints [56]. This DM amount can be slightly increased to $\approx 48\%$ when including effect of the entropy injection of the pseudo-Dirac neutrinos provided the lightest pseudo-Dirac neutrino has mass $1-10$ GeV [56]. We are not going to consider these effects here.

For $m_{\nu_S} > 0.1$ keV, the relic density produced in the usual DW mechanism is given by [56, 74]

$$\Omega_{\nu_S,\mathrm{DM}}h^2 = 1.1\times 10^7 \sum_\alpha C_\alpha(m_S)|U_{\alpha S}|^2 \left[\frac{m_{\nu_S}}{\mathrm{keV}}\right]^2; \qquad \alpha = e,\mu,\tau, \tag{2.23}$$

where $C_\alpha(m_S)$ are active flavor-dependent coefficients which are calculated solving numerically the Boltzmann equations (an appropriated value in this case is $C_\alpha(m_S) \simeq 0.8$ [74]). We also have that the sum of $U_{\alpha S}$, the elements of the leptonic mixing matrix, is the active-sterile mixing, i.e., $\sum_\alpha |U_{\alpha S}|^2 \sim \sin^2(2\theta)$. For the case $m_{\nu_S} < 0.1$ keV there is a simpler expression written as follows [56, 75]

$$\Omega_{\nu_S,\mathrm{DM}}h^2 = 0.3 \left[\frac{\sin^2 2\theta}{10^{-10}}\right] \left[\frac{m_{\nu_S}}{100\ \mathrm{keV}}\right]^2. \tag{2.24}$$

In ref. [56], after imposing bounds coming from stability, structure formation and indirect detection, in addition to the constraints arising from the neutrinos oscillation experiments, it was found that the sterile neutrino as WDM in the $(2,3)$ ISS provides a sizable contribution to the DM relic density for $2 \lesssim m_{\nu_S} \lesssim 50$ keV and active-sterile mixing angles $10^{-8} \lesssim \sin^2(2\theta) \lesssim 10^{-11}$. The maximal fraction of DM made of ν_S is achieved when $m_{\nu_S} \simeq 7$ keV [76–79].

Now, we are going to search for models satisfying all mentioned conditions in Section 2 as well as

$$\Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2 = \Omega_{\nu_S,\mathrm{DM}}h^2 + \Omega_{a,\mathrm{DM}}h^2, \tag{2.25}$$

where $\Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2 = 0.1197 \pm 0.0066$ (at 3σ) is the current relic density as reported by Planck Collaboration [3].

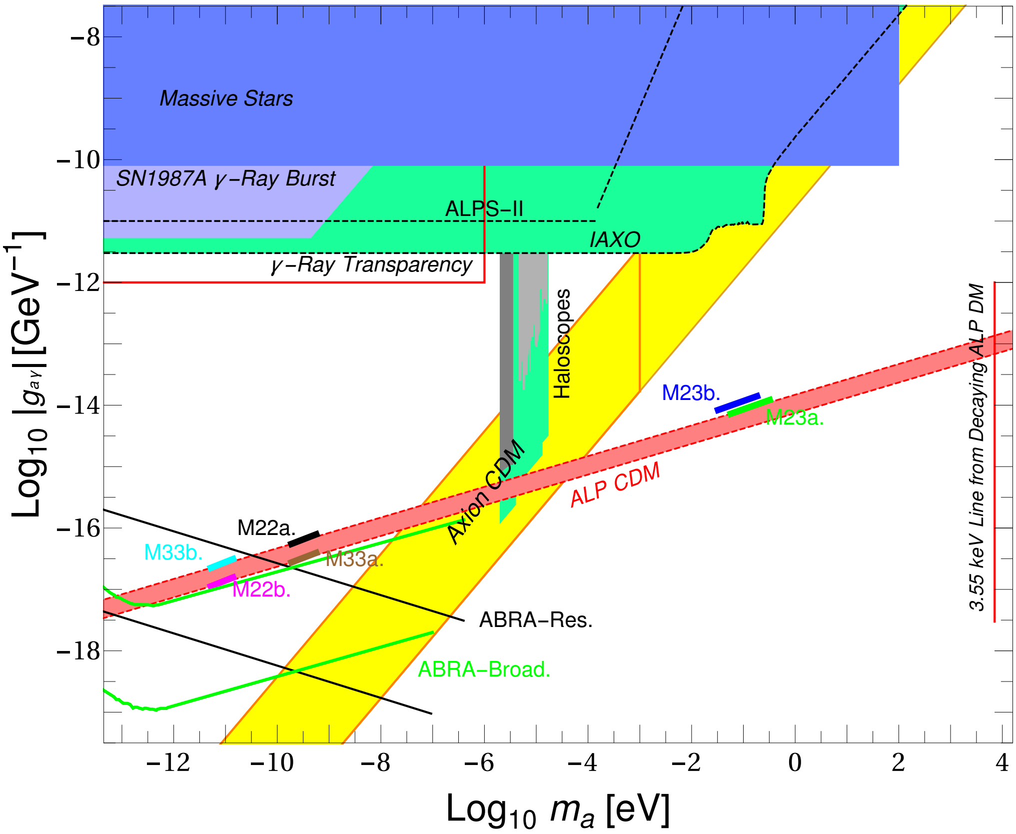

Figure 1. ALP parameter space. This figure shows some excluded regions from the non-observation of an anomalous energy loss of massive stars due to ALP (or axions) emission [80], of a γ-ray burst from SN 1987A due to conversion of an ALP in the galactic magnetic field [81–83] and of dark matter axions or ALPs converted into photons in microwave cavities placed in magnetic fields [84–87]. It is also showed the red band where the ALP may constitute all of cold dark matter (ALP CDM), and the regions where the ALP may explain the cosmic γ-ray transparency and the X-ray line at 3.55 keV [30–32, 88, 89]. The green regions are the projected sensitivities of the light-shining-through-wall experiment ALPS-II, of the helioscope IAXO, of the haloscopes ADMX and ADMX-HF [84, 86]. The black (green) solid line in the lower left corner shows the sensitivity of the proposed ABRACADABRA experiment [90] using a resonant (broadband) circuit. Further, it is showed the region of QCD axion (the yellow band) -which was recently extended to cover the lower right corner [91]- studied in the context of some realistic axion models [60], with the region below the orange solid line corresponding to the axion CDM. The benchmark regions M22a, M22b, M23a, M23b, M33a and M33b corresponding to respective models $(2,2)$ ISS, $(2,3)$ ISS and $(3,3)$ ISS which generate a considerable amount of the current DM relic density are also shown.

3 Models

In the previous section, we have introduced the general and minimal constraints that models have to satisfy. Now, we proceed to find specific models that give an explanation to the dark matter observed in the Universe. In particular, the $(2,2)$, $(3,3)$ and $(2,3)$ cases of the ISS mechanism are studied in detail.

D	v_σ (GeV) for $\Omega_{a,\mathrm{DM}}h^2 = \Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$	v_σ (GeV) for $\Omega_{a,\mathrm{DM}}h^2 = 0.57 \times \Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$
8	$(1.9 - 3.3) \times 10^{10}$	$(1.5 - 2.8) \times 10^{10}$
9	$(0.6 - 1.1) \times 10^{11}$	$(5.5 - 9.5) \times 10^{10}$
10	$(1.9 - 3.2) \times 10^{11}$	$(1.6 - 2.8) \times 10^{11}$
11	$(5.0 - 8.2) \times 10^{11}$	$(4.4 - 7.2) \times 10^{11}$
12	$(1.2 - 1.9) \times 10^{12}$	$(1.0 - 1.7) \times 10^{12}$
13	$(2.6 - 4.0) \times 10^{12}$	$(2.3 - 3.6) \times 10^{12}$
14	$(5.2 - 7.9) \times 10^{12}$	$(4.6 - 7.0) \times 10^{12}$
15	$(1.0 - 1.4) \times 10^{13}$	$(0.9 - 1.3) \times 10^{13}$
16	$(1.7 - 2.5) \times 10^{13}$	$(1.6 - 2.3) \times 10^{13}$
17	$(2.9 - 4.2) \times 10^{13}$	$(2.7 - 3.8) \times 10^{13}$
18	$(4.8 - 6.7) \times 10^{13}$	$(4.3 - 6.1) \times 10^{13}$
19	$(0.7 - 1.0) \times 10^{14}$	$(6.8 - 9.4) \times 10^{13}$

Table 2. The appropriated $(D,\, v_\sigma)$ values in order to provide $\Omega_{a,\mathrm{DM}}h^2 = \Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$ (first and second columns) and $\Omega_{a,\mathrm{DM}}h^2 = 0.57 \times \Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$ (first and third columns). It has been considered $10^{-3} \leq g \leq 2$ and $\Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$ at 3σ level.

3.1 $(2,2)$ ISS case

Among the minimal configuration of the ISS mechanism consistent with the neutrinos oscillation data and lepton flavor violating (LFV) processes [39, 56], we, firstly, study the $(2,2)$ ISS case because of simplicity. In the neutrino mass spectrum there are two heavy pseudo-Dirac neutrinos with masses $\sim M$ and three light active neutrinos with masses of order of sub-eV coming from the mass matrix in eq. (2.12) [39]. As in this case $n_{N_R} = n_{S_R} = 2$ (similarly for the $(3,3)$ ISS case) there is not a light sterile neutrino ν_S in the mass spectrum because having at least one sterile neutrinos is a general property of the spectrum provided $n_{S_R} > n_{N_R}$. Therefore, all the current DM abundance must be constituted by ALPs, i.e. $\Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2 = \Omega_{a,\mathrm{DM}}h^2$.

In order to find the main features of the model, we find useful to rewrite $\Omega_{a,\mathrm{DM}}h^2$ in terms of D - the exponent of ALP mass, eq. (2.5) - and m_a. Doing that, we find that

$$\Omega_{a,\mathrm{DM}}h^2 \simeq 0.49\, |g|^{\frac{1}{4}}\, \sqrt{D}\, \exp\left[-\frac{D}{4}\ln\frac{\sqrt{2}M_{\mathrm{Pl}}}{1\ \mathrm{GeV}}\right]\left[\frac{v_\sigma}{1\ \mathrm{GeV}}\right]^{\frac{D+6}{4}}, \tag{3.1}$$

where g is assumed to be $10^{-3} \leq |g| \leq 2$. Thus, $\Omega_{a,\mathrm{DM}}h^2$ only depends on $(|g|\,,\, D,\, v_\sigma)$. In Table 2 we show $(D,\, v_\sigma)$ values for two cases: $\Omega_{a,\mathrm{DM}}h^2 = \Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$ and $\Omega_{a,\mathrm{DM}}h^2 = 0.57 \times \Omega_{\mathrm{DM}}^{\mathrm{Planck}}h^2$. The last case applies only for the $(2,3)$ ISS case and will be discussed in Section 3.3.

In order to obtain the Lagrangian in this scenario, we search for discrete symmetries for the two possibilities showed in eqs. (2.15) and (2.16) with the following results: The

$Z_{9,10}$, symmetries allow terms such as $\frac{\sigma^*}{M_{\rm Pl}}\bar{L}\widetilde{H}S_R$, $\frac{\sigma^{*2}}{M_{\rm Pl}}\bar{N}_R N_R^C$, $\bar{L}\widetilde{H}S_R$. As H and σ get VEVs, these terms do not give the appropriate zero-texture of the ISS mechanism shown in eqs. (2.8) and (2.9). We also have searched for all the possible combinations of r, s, t values in the Lagrangian (2.1) without any success. On the other hand, the $Z_{15,16,18}$ symmetries are not free of the gravitational anomaly. In fact, the $Z_{N\leq 20}$ discrete symmetries that satisfy all the anomaly constraints and stabilize the ISS mechanism are $Z_{17,19}$. In the case of Z_{17} symmetry, the Lagrangian, $\mathcal{L}_{Z_{17}}$, is given by eq. (2.1) with parameters equals to $(p, q, r, s, t) = (3, -5, -6, 2, 2)$ in eqs. (2.7) and (2.18). An assignment of the Z_{17} charges and the anomalous $\mathrm{U}(1)_A$ symmetry for this case is shown in Table 3. Note that $Z_{17} = 6\mathrm{B} + 11\mathrm{U}(1)_{\mathbb{L}}$ and that the term $\sim \sigma^{*6}\overline{N_{\mathrm{R}\beta}}(N_{\mathrm{R}\beta'})^{\mathrm{C}}$ gives a negligible contribution for the light active neutrino masses.

Model	Symmetry	$Q_{\mathrm{L}i}$	$d_{\mathrm{R}i}$	$u_{\mathrm{R}i}$	H	L_i	$l_{\mathrm{R}i}$	$N_{\mathrm{R}\beta}$	$S_{\mathrm{R}\alpha}$	E_{L}	E_{R}	σ
(2,2)	Z_{17}	1	2	0	16	14	15	13	8	15	12	7
	$\mathrm{U}(1)_A$	0	0	0	0	11/2	11/2	11/2	−5/2	11/2	15/2	1
	Z_{19}	1	14	7	6	16	10	3	9	17	2	4
	$\mathrm{U}(1)_A$	0	0	0	0	11/2	11/2	11/2	−5/2	5/2	7/2	1
(3,3)	Z_{17}	1	10	9	8	14	6	5	7	2	15	4
	$\mathrm{U}(1)_A$	0	0	0	0	11/2	11/2	11/2	−5/2	9/2	7/2	1
	Z_{19}	1	13	8	7	16	9	4	12	9	11	18
	$\mathrm{U}(1)_A$	0	0	0	0	11/2	11/2	11/2	−5/2	11/2	7/2	1
(2,3)	Z_{10}	1	2	0	9	7	8	6	6	8	6	6
	Z_4	0	1	3	3	0	1	3	1	1	1	2
	$\mathrm{U}(1)_A$	0	0	0	0	7/2	7/2	7/2	−3/2	7/2	3/2	1

Table 3. Discrete and continuous charge assignments of the fields in the different models.

The corresponding $g_{a\gamma}$ and m_a values in this case are

$$g_{a\gamma} \cong 7.54 \times 10^{-17} \left[\frac{3.08 \times 10^{13}\ \mathrm{GeV}}{v_\sigma}\right]\ \mathrm{GeV}^{-1},$$
$$m_a \cong 5.59 \times 10^{-10} |g|^{\frac{1}{2}} \left[\frac{v_\sigma}{3.08 \times 10^{13}\ \mathrm{GeV}}\right]^{15/2}\ \mathrm{eV}\ . \tag{3.2}$$

The benchmark region is denoted as M22a in the Figure 1 where we have considered $10^{-3} \leq |g| \leq 2$ and $2.9 \times 10^{13} \lesssim v_\sigma \lesssim 4.2 \times 10^{13}$ GeV. These values for $g_{a\gamma}$ and m_a allow that the ALP explain 100% of the DM relic density.

Sharp predictions for neutrinos masses are not possible with just the knowledge of the p, q, r, s, t values and v_σ. However, the order of magnitude of the mass matrices can be estimated from eqs. (2.10) and (2.11) to be

$$M \cong \zeta \times 1.73\ \mathrm{TeV},\ \mu_S \cong \eta \times 0.13\ \mathrm{keV},\ m_{\nu\mathrm{light}} \simeq \left[y^\intercal \zeta^{-1} \eta \left(\zeta^\intercal\right)^{-1} y\right] \times 1.38\ \mathrm{eV}, \tag{3.3}$$

which is appropriate to satisfy the constraints coming from neutrino oscillation data and unitarity without resorting a fine tuning in couplings. Nevertheless, we have to admit that some care must be taken in order to generate the benchmark region M22a in agreement with bounds coming from LFV processes such as $\mu \to e + \gamma$. It is important comment that Z_{19} also provides a suitable effective Lagrangian with results, roughly speaking, quite similar to the Z_{17}, although with lower values for $g_{a\gamma}$ and m_a. We are not going into the details for shortness, however, we show the benchmark region corresponding to that case in Figure 1 denoted as M22b. We also show the values for $g_{a\gamma}$, m_a and the neutrino mass spectrum in Table 4.

3.2 $(3,3)$ ISS case

Regarding the neutrino mass spectrum, the $(3,3)$ ISS case is quite similar to the previous one in the sense that there is not a light sterile neutrino in the mass spectrum [39] because $n_{N_R} = n_{S_R} = 3$. Therefore, all the DM abundance in this model has to be made of ALPs.

Proceeding in a similar manner to the previous case and taking into account that $A_{\rm grav}(Z_N)$ is now different, we have searched for all anomaly-free Z_N discrete symmetries, with $N \leq 20$, that allow a suitable ISS mechanism. Some commentaries about our results are in order. The Z_9 symmetry is not free of gravitational anomalies, while the Z_{10} symmetry allows dangerous terms such as $\bar{L}\widetilde{H}S_R$, $\frac{\sigma^*}{M_{\rm Pl}}\bar{L}\widetilde{H}S_R$, $\sigma\bar{N}_R N_R^C$, and others, therefore the possibility of building a model for the solution in the eq. (2.15) is not realized. On the other hand, the $Z_{15,16,18}$ symmetries corresponding to the solution in the eq. (2.16), are not free of gravitational anomalies, therefore these are not suitable symmetries. The $Z_{17,19}$ symmetries forbid the dangerous terms and allow an effective Lagrangian. In the case of Z_{17} symmetry, the Lagrangian is characterized by $(p,\, q,\, r,\, s,\, t) = (3,\, -5,\, 7,\, 2,\, 1)$. In other words, it is very similar to Lagrangian $\mathcal{L}_{Z_{17}}$ of the $(2,2)$ ISS model, however, in this case, the mass term for the exotic fermion E has the exponent equal to one and the term associated with μ_N in not allowed with dimension less than seven. Possible assignments for the Z_{17} and $\mathrm{U}(1)_A$ symmetries are also shown in Table 3. Note that $Z_{17} = 9\mathrm{B} + 11\mathrm{U}(1)_{\mathbb{L}}$ and that the term $\sim \sigma^7 \overline{N_{\mathrm{R}\beta}}(N_{\mathrm{R}\beta'})^{\mathrm{C}}$ gives a negligible contribution for the light active neutrino masses.

The corresponding v_σ and m_a values are the same that in the $(2,2)$ ISS case. Nevertheless, the $g_{a\gamma}$ turn to be

$$g_{a\gamma} \cong 3.77 \times 10^{-17} \left[\frac{3.08 \times 10^{13}\ \mathrm{GeV}}{v_\sigma}\right]\ \mathrm{GeV}^{-1}. \tag{3.4}$$

The benchmark region is denoted as M33a in the Figure 1. These values for $g_{a\gamma}$ and m_a allow that the ALP explains 100% of the DM relic density. The scale of the M, μ_S, and $m_{\nu\rm light}$ matrices are equals to those given in eq. (3.3) because the exponents $(p, |q|)$ and v_σ are the same that in the $(2,2)$ ISS case. The Z_{19} case brings similar conclusions and we do not discuss it by shortness. Its benchmark region is denoted as M33b in Figure 1. The numerical values for $g_{a\gamma}$, m_a and for the neutrino mass spectrum are shown in Table 4.

We remark that a similar effective Lagrangian for the $(3,3)$ ISS case was worked in the Ref. [36] with the aim of explaining some astrophysical phenomena. However, in that case, the DM abundance via ALPs was not considered.

3.3 $(2,3)$ ISS case

Because there are $n_{N_R} = 2$ and $n_{S_R} = 3$ neutral fermions, the neutrino mass spectrum contains two heavy pseudo-Dirac neutrinos with masses $\sim M$ and three light active neutrinos with masses of order of sub-eV. In addition, there is a sterile neutrino, ν_S, with mass of order $\sim \mu_S$. The presence of both the ν_S and the ALP, a, brings the possibility of having two DM candidates in the $(2,3)$ scenario.

First, let's consider the case $\Omega_{\rm DM}^{\rm Planck} h^2 = \Omega_{a,\rm DM} h^2$, i.e., when the DM abundance is totally constituted by ALPs. Now, from eqs. (2.15) and (2.16) and Table 2, we can see that $(D,\, v_\sigma) = \big(9,\, (0.6-1.1)\times 10^{11}\ \text{GeV}\big)$ and $(D,\, v_\sigma) = \big(10,\, (1.9-3.2)\times 10^{11}\ \text{GeV}\big)$ corresponds to the $(p, |q|) = (2,3)$ solution in eq. (2.15) (note that v_σ corresponding to $D = 10$ is slightly out of allowed range in eq. (2.15)). Moreover, the values $D = 9, 10$ restrict the symmetry to be $Z_{9,10}$. For these discrete symmetries we find solutions for anomaly free $Z_{9 \text{ and } 10}$ charges, i.e, solutions to eqs. (2.20) and (2.21) with $(p, |q|) = (2,3)$. Nevertheless, all the solutions for the $Z_{9 \text{ and } 10}$ charges allow terms such as $\sim \sigma \overline{N_{{\rm R}\beta}} (N_{{\rm R}\beta'})^{\rm C}$, $\sim \frac{\sigma^{*2}}{M_{\rm Pl}} \overline{N_{{\rm R}\beta}} (N_{{\rm R}\beta'})^{\rm C}$, $\sim \frac{\sigma^*}{M_{\rm Pl}} \overline{L_i} \widetilde{H} S_{{\rm R}\alpha}$ and other terms in the Lagrangian that do not give the correct texture to the mass matrix in the ISS mechanism. We also have searched for all the possible combinations of r, s, t values in the Lagrangian (2.1) with $(p, |q|) = (2,3)$ without any success. Therefore, the $(p, |q|) = (2,3)$ case can not offer a realization for an effective model providing all the observed DM abundance via ALPs when all the constraints in Section 2 are considered. However, from Table 2 we see that for $D = 15, \ldots, 19$ with a larger value of v_σ the second solution $(p, |q|) = (3,5)$, cf. 2.16, can, in principle, offer a model (note that, strictly speaking, the v_σ value corresponding to $D = 15$ is slightly out of allowed range in eq. (2.16)). Moreover, the cases of Z_{17} and Z_{19} are excluded because the condition for the gravitational anomaly is never satisfied, while in the $Z_{16,18}$ cases terms as $\sim \overline{L_i} \widetilde{H} S_{{\rm R}\alpha}$ and $\sim \sigma \overline{N_{{\rm R}\beta}} (N_{{\rm R}\beta'})^{\rm C}$ give an incorrect texture for the ISS mass matrix. In fact, after imposing all the constraints, we find that the only symmetry that provides a solution is Z_{15}. In more detail, we find that the discrete symmetry can be written as $Z_{15} = 9{\rm B} + 11{\rm U}(1)_{\mathbb{L}}$ (other combinations for Z_{15} are possible). This model has the effective Lagrangian, $\mathcal{L}_{Z_{15}}$, given by eqs. (2.1), (2.7) and (2.18) with $(p, q, r, s, t) = (3, -5, -4,\ 2,\ 2)$. Note that the term $\sim \sigma^{*4} \overline{N_{{\rm R}\beta}} (N_{{\rm R}\beta'})^{\rm C}$ gives a negligible contribution for the light active neutrino masses.

The $\mathcal{L}_{Z_{15}}$ is also invariant under a ${\rm U}(1)_A$ symmetry which is anomalous in the electromagnetic group as must be to generate a coupling between photons and ALPs, $g_{a\gamma}$. Specifically, from the Z_{15} and ${\rm U}(1)_A$ charges we have that the ALP has $g_{a\gamma}$ and m_a given by

$$g_{a\gamma} \simeq 2.25 \times 10^{-16} \left[\frac{1.03 \times 10^{13}\ \text{GeV}}{v_\sigma}\right] \text{GeV}^{-1}; \tag{3.5}$$

$$m_a \simeq 4.47 \times 10^{-8} |g|^{\frac{1}{2}} \left[\frac{v_\sigma}{1.03 \times 10^{13}\ \text{GeV}}\right]^{13/2} \text{eV}. \tag{3.6}$$

We also check that the neutrino mass spectrum is

$$M \simeq \zeta \times 6.5 \times 10^{-2}\ \text{TeV};\ \mu_S \simeq \eta \times 5.8 \times 10^{-4}\ \text{keV};\ m_{\nu\text{light}} \simeq \left[y^\intercal \zeta^{-1} \eta \left(\zeta^\intercal\right)^{-1} y\right] \times 4.15\ \text{eV}, \tag{3.7}$$

where we have used $v_\sigma \simeq 1.03 \times 10^{13}$ GeV which is one of the suitable values given in Table 2 for Z_{15} which give the 100% of the current DM abundance. From values of M, μ_S, $m_{\nu\text{light}}$ in eq. (3.7) we note that in this scenario there is a some tension to satisfy the unitarity constraint. In more detail, $\left|\frac{y}{\zeta}\right| < \frac{M}{v_{\text{SM}}} \times 10^{-1} = 2.6 \times 10^{-2}$ where we have been conservative choosing a ϵ^2 value of 1% (recall $\epsilon \equiv m_D M^{-1}$). However, this upper bound on $\left|\frac{y}{\zeta}\right|$ implies a lower bound on $\eta > \left|\frac{y}{\zeta}\right|^{-2} \frac{m_{\nu\text{light}}}{4.15} \approx \left|\frac{y}{\zeta}\right|^{-2} \frac{\sqrt{\Delta m^2_{\text{atm}}}}{4.15} \approx 17.17$ (with $\Delta m^2_{\text{atm}} = 2.32 \times 10^{-3}$ eV^2 being the atmospheric squared-mass difference) which is not a perturbative value for η. This happens because the values for v_σ corresponding for $D = 15$ is smaller than the values allowed in the range in eq. (2.16). Similar conclusions are found if we consider the case when $\Omega_{a,\text{DM}} h^2 < \Omega^{\text{Planck}}_{\text{DM}} h^2$. Therefore, the effective Lagrangian $\mathcal{L}_{Z_{15}}$ can not provide a natural framework for DM and the neutrino masses in $(2, 3)$ ISS case. For this reason we do not show the benchmark region for this model in Figure 1.

However, models explaining the DM relic density via ALPs and/or sterile neutrinos for the $(2, 3)$ ISS case can be found provided we slightly relax some constraints mentioned in Section 2. Actually, if an extra Z_N symmetry is allowed, we found that, for example, the solution $(p, |q|) = (2, 3)$ makes possible a model with $(p, q, r, s, t) = (2, -3, -3, 2, 2)$ in eqs. (2.1), (2.7) and (2.18). Nevertheless, the discrete gauge symmetry $Z_{10} \times Z_4$ with charges given in Table 3, respectively, must be considered. It is straightforward to check that for this model, ALPs provide 100% of the DM abundance provided $v_\sigma \cong 2.03 \times 10^{11}$ GeV with g of order one. In more details, for this benchmark point, we have that

$$m_a \simeq |g|^{\frac{1}{2}} \times 0.29\ \text{eV},\ g_{a\gamma} \simeq 1.14 \times 10^{-14}\ \text{GeV}^{-1}, \tag{3.8}$$

with the neutrino mass spectrum given by

$$M \simeq \zeta \times 8.4\ \text{TeV};\ \mu_S \simeq \eta \times 496.8\ \text{keV};\ m_{\nu\text{light}} \simeq \left[y^\intercal \zeta^{-1} \eta \left(\zeta^\intercal\right)^{-1} y\right] \times 210.8\ \text{eV}. \tag{3.9}$$

We note that for $\eta \leq 10^{-2}$ and the other coupling constants of order one, a suitable neutrino mass spectrum is achieved. For this model we have shown in Figure 1 a benchmark region denoted as M23a where ALPs provide 100% of DM abundance.

For the case that the DM abundance is made of ALPs and sterile neutrinos the scenario slightly changes. We have chosen the case when the DM is made of $\approx$ 43% of sterile neutrinos and $\approx$ 57% of ALPs as an illustrating example. However, these can take other

ISS Model	m_a $\times 10^{-11}$ eV	$g_{a\gamma}$ $\times 10^{-17}$ GeV^{-1}	M (TeV), μ_S (keV) $m_{\nu\text{light}}$ (eV)
M22a	(19.0−56.0)	(5.5 − 7.8)	(1.5 − 4.5), (0.1 − 0.7), (1.0 − 1.4)
M22b	(0.52−0.14)	(1.1 − 1.6)	(24.3 − 65.6), (11.3 − 58.9), (0.4 − 0.6)
M33a	(19.0−56.0)	(2.7 − 3.9)	(1.5 − 4.5), (0.1 − 0.7), (1.0 − 1.4)
M33b	(0.52−0.14)	(2.2 − 3.1)	(24.3 − 65.6), (11.3 − 58.9), (0.4 − 0.6)
M23a	$(0.06-0.30)\times 10^{11}$	(720−1200)	(7.5 − 21.0), (411.7 − 1940.7), (133.8 − 224.4)
M23b	$(0.03-0.2)\times 10^{11}$	(830−1400)	(5.6 − 15.8), (2.7 − 12.7), (1.4 − 2.5)

Table 4. Main features of the models discussed in the text. We have considered the constant $g \subset [10^{-3}, 2]$ in the mass term of the ALP, and $\Omega_{\rm DM}^{\rm Planck} h^2$ at 3σ.

values provided the DM abundance made of sterile neutrinos is less than $\lessapprox 50\%$, consistently with the constraints over its parameter space [56, 77, 92]. Doing so, we obtain

$$m_a \simeq |g|^{\frac{1}{2}} \times 0.46 \text{ eV}, \; g_{a\gamma} \simeq 1.02 \times 10^{-14} \text{ GeV}^{-1}, \tag{3.10}$$

and

$$M \simeq \zeta \times 10.6 \text{ TeV}; \; \mu_S \simeq \left[\frac{\eta}{10^{-2}}\right] \times 7.1 \text{ keV}; \; m_{\nu\text{light}} \simeq \left[y^{\intercal}\zeta^{-1}\left(\frac{\eta}{10^{-2}}\right)(\zeta^{\intercal})^{-1} y\right] \times 1.9 \text{ eV}, \tag{3.11}$$

where we have used $v_\sigma \cong 2.28\times 10^{11}$ GeV as an example. In this case for $\eta \approx 10^{-2}$ the sterile neutrino has $m_{\nu_S} \approx 7.1$ keV. In particular, this mass for the sterile neutrino may explain the recently indicated emission lines at 3.5 keV from galaxy clusters and the Andromeda galaxy [33, 34]. The benchmark region for this model is denoted as M23b in Figure 1.

Finally, for clearness, we show in Table 4 an overview of the main results of all considered models. Specifically, we show energy scales for the neutrino masses and the ALP parameter space for each ISS case.

4 Discussion and summary

We have connected two interesting motivations for going beyond the standard model: neutrino masses and ALPs as dark matter. A natural scenario for achieving that is the ISS mechanism. In particular, we have considered the minimal versions of the ISS mechanism in agreement with all the neutrino constraints. Nevertheless, in the considered framework, the mass scales for the ISS mechanism are generated by gravity induced non-renormalizable operators when the scalar field containing the ALP gets a vacuum expectation value, v_σ. Naturalness of these scales imposes strong constraints on these operators and, when combining these with the ALP acceptable range for v_σ, only two solutions are possible: $(p, |q|) = (2, 3)$ for $6 \times 10^{10} \lesssim v_\sigma \lesssim 1 \times 10^{11}$ GeV and $(p, |q|) = (3, 5)$ for $2 \times 10^{13} \lesssim v_\sigma \lesssim 8 \times 10^{13}$ GeV. This implies that operators given M and μ_S scales can only belong to these two categories. Then, a simultaneous application of constraints coming from the texture of ISS mass matrix, the violation of the unitarity, the mass of exotic charged leptons, the stability of the effective Lagrangian against gravitational effects and the suitable ALP parameter space (m_a and $g_{a\gamma}$) to provide the total DM density almost set the rest of terms in the Lagrangian, only leaving a few of possibilities for all of ISS cases.

Among the minimal ISS mechanisms, the $(2, 2)$ and $(3, 3)$ ISS cases are quite similar. It is due to the fact that in both of them $n_{N_R} = n_{S_R}$ implying that neutrino mass spectrum is characterized by only two mass scales, M and $m_{\nu\text{light}}$. Thus, the results obtained are almost identical. Although, there is a slightly difference in the value of $g_{a\gamma}$ due to the presence of more fermions in the $(3, 3)$ ISS case. In both cases, we find two effective models denoted as M22a,b and M33a,b in Table 4. Since there is not sterile neutrino in these cases, the total DM density is made of ALPs. We also remark that, although, the ALPs in these models can decay to two photons and, in the $(2, 2)$ ISS mechanism, to two massless active neutrinos, these are cosmologically stable because those decays are strongly suppressed by factors of $1/M_{\text{Pl}}^2$ and/or $1/v_\sigma^2$.

On the other hand, the $(2, 3)$ ISS case is phenomenologically more interesting due to the presence of a sterile neutrino in the mass spectrum. It implies that the DM density can be made of ALPs and ν_S. We have found a model satisfying all of previously mentioned constraints and, at the same time, offering the total DM. Because sterile neutrinos in the $(2, 3)$ ISS mechanism can give, roughly speaking, at most $\approx 43\%$ of the DM density, it is necessary that the remaining $\approx 57\%$ of DM be made of ALPs. It is also possible that ALPs give the total DM density. It occurs when the mixing angle between active and sterile neutrinos is very suppressed in order to make the the Dodelson-Widrow mechanism inefficient. Both cases were studied in detail and denoted as M23a and M23b, respectively.

Regarding the search for ALPs, the benchmark regions in Figure 1 are out of reach of the current and future experimental searches for axion/ALP such as ALPS II, IAXO, CAST [47], since these currently have not enough sensitivity to probe the ALP/axion-photon couplings and masses that are motivated in models with scales $v_\sigma \gtrsim 10^{13}$ GeV. Nevertheless, for the (2,2) and (3,3) ISS cases the benchmark regions are remarkably within the target regions in proposed experiments based on LC circuits [90, 93], which are designed to search for QCD axions and ALPs and cover many orders of magnitude in the parameter space of these particles, beyond the current astrophysical and laboratory limits [47–49]. Specifically, the ABRACADABRA experiment [90] may explore ALP masses as low as $\sim 10^{-10}$ eV for a coupling to photons of the order of $\sim 10^{-18}$ GeV^{-1}, which are well below our benchmark regions (Figure 1).

Finally, despite the fact that neutrino mass spectrum is not completely predicted in the models found, the matrix scales in the ISS mechanism are estimated to be in agreement with the neutrino constraints. Moreover, we have numerically checked, in all models, that there are solutions with coupling constants of order one that also satisfy LFV processes and the unitary condition. These processes can easily avoid without fine tuning in the models discussed in this paper. Specifically, we have found that the BR($\mu \to e\gamma$) in all cases are as small as $\sim 10^{-20} - 10^{-15}$ which are consistent with the current experimental value BR$(\mu \to e\gamma) < 5.7 \times 10^{-13}$ [94] and with future sensitivities around 6×10^{-14} [95].

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