Optimal Delivery Scheduling and Charging of EVs in the Navigation of a City Map

Fernando V. Cerna, Mahdi Pourakbari-Kasmaei, Member, IEEE, Rubén A. Romero, Senior Member, IEEE, and Marcos J. Rider, Senior Member, IEEE

Abstract—This paper presents a mixed integer linear programming model to optimize the costs of maintenance and extra hours for scheduling a fleet of battery electric vehicles (BEVs) so that the products are delivered to prespecified delivery points along a route. On this route, each BEV must have an efficient charging strategy at the prespecified charging points. The proposed model considers the average speed of the BEVs, the battery states of charge, and a set of deliveries allocated to each BEV. The charging points are located on urban roads and differ according to their charging rate (fast or ultra-fast). Constraints that guarantee the performance of the fleet’s batteries are also taken into consideration. Uncertainties in the navigation of urban roads are modeled using the probability of delay due to the presence of traffic signals, schools, and public works. The routes and the intersections of these routes are modeled as a predefined graph. The results and the evaluation of the model, with and without considering the extra hours, show the effectiveness of this type of transport technology. The models were implemented in AMPL and solved using the commercial solver CPLEX.

Index Terms—Battery electric vehicles, charging points, charging rates, mixed integer linear programming.

Notation
The notation used in this paper is presented as follows.

Sets

- $\Omega_i$: Set of intersections $i$.
- $\Omega_k$: Set of urban roads $k_i$.
- $\Omega_e$: Set of deliveries $e$.
- $\Omega_{vh}$: Set of BEVs $vh$.

Parameters

- $g_{km}$: Costs for maintenance ($R$/km).
- $g_{hx}$: Costs for extra hours ($R$/h).

Variables

- $t_{v_{h,e,i}}^i$: Type of intersection $i$ in delivery of each BEV $vh$ (-1: starting; 0: intermediate; 1: arrival).
- $d_{ki}$: Length of urban road $k_i$ (km).
- $v_{VE}$: Average speed of BEV $vh$ (km/h).
- $p_{max}$: Total operating time (h).
- $\tau_{PTS}$: Delay time due to PTS (h).
- $\tau_{PS}$: Delay time due to PS (h).
- $\tau_{PPW}$: Delay time due to PPW (h).
- $P_{PTS}$: Probability of delay due to PTS.
- $P_{PS}$: Probability of delay due to PS.
- $P_{PPW}$: Probability of delay due to PPW.
- $\Delta_{PTS}$: Binary value that characterizes the occurrence of a delay due to PTS in urban road $k_i$.
- $\Delta_{PS}$: Binary value that characterizes the occurrence of a delay due to PS in urban road $k_i$.
- $\Delta_{PPW}$: Binary value that characterizes the occurrence of a delay due to PPW in urban road $k_i$.
- $SOC_{vh}^o$: Initial state of charge of the battery of BEV $vh$ (kWh).
- $K^g$: Energy spent during the navigation of BEV $vh$ (kWh/km).
- $M$: Big value used in the linearization process.
- $K^s$: Storage capacity of the battery of BEV $vh$ (kWh).
- $Y$: Maximum number of discretizations of the variable $n_{v_{h,e,i}}^{i}$.
- $r_{ki}^n$: Numerical value that characterizes the type of urban road $k_i$ (1: main road; 0: secondary road).
- $\tau_{MIN,UR}^k$: Minimum time for ultra-fast charging in urban road $k_i$ (h).
- $\tau_{MAX,UR}^k$: Maximum time for ultra-fast charging in urban road $k_i$ (h).
- $\tau_{MIN,R}^k$: Minimum time for fast charging in urban road $k_i$ (h).
- $\tau_{MAX,R}^k$: Maximum time for fast charging in urban road $k_i$ (h).
- $n_{k_i}^p$: Unit charging rate in urban road $k_i$ (kW).
- $n_{v_{h,e,i}}^{i}$: Number of charging points in urban road $k_i$.

Manuscript received July 9, 2016; revised October 18, 2016 and January 12, 2017; accepted February 16, 2017. Date of publication February 22, 2017; date of current version August 21, 2018. This work was supported in part by the Brazilian Institution CNPq under Grant 2016/14319-7 and Grant 2016/14319-7, Paper no. TSG-00915-2016.

F. V. Cerna, M. Pourakbari-Kasmaei, and R. A. Romero are with the Departamento de Engenharia Elétrica, Faculdade de Engenharia de Ilha Solteira, Universidade Estadual Paulista, Ilha Solteira 15385-000, Brazil (e-mail: fvcerna83@gmail.com; mahdi.pourakbari@ieee.org; ruben@dee.feis.unesp.br).

M. J. Rider is with the Department of Systems and Energy, School of Electrical and Computer Engineering, University of Campinas, Campinas 13083-852, Brazil (e-mail: mj rider@dsee. fee.unicamp.br).

Digital Object Identifier 10.1109/TSG.2017.2672801

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I. INTRODUCTION

The development of less polluting technologies has become one of the most important targets for various sectors of industry. Among these, the transportation sector has sought to develop products that meet pollutant emission reduction targets and decrease the dependence on oil and its derivatives as energy sources [1]. In this context, electric vehicles (EVs) are presented as alternative transportation vehicles that are less polluting and more efficient. Each EV’s battery can be charged at a charging station connected to an electric network, which supplies propulsion for its mobility system [2]. The introduction of this technology and its several applications represents a great opportunity for synergy between the electricity and transport sectors. An important application of EVs is in the service sector, such as for product delivery, postal services, emergency services (police, ambulance), and so on. These services must ensure efficiency in deliveries by choosing the optimal route for each delivery while taking into account the battery state of charge, delay times in navigation, and charging at prespecified points located along the roads. Therefore, it is necessary to develop intelligent schemes to ensure the flexible navigation and efficient charging of EV fleets, considering time constraints, requirements, capacities, and so on [3].

In the literature, works related to the charging of BEVs in the navigation of a city map are scarce. In some works, the development of algorithms for routing with potential points of charging and optimal energy allocation has been considered [1], [2]. Another approach to optimal location of EV charging points was considered in [3]. This approach uses a fuzzy control to find the optimal location of charging points in an electrical distribution network, and takes into account the sensitivity indices obtained from the Newton-Raphson load flow assisting in the appropriate selection of charging point.

A methodology that aims at minimizing the total investment in a charging infrastructure and the cost for charging EVs was developed in [4]. This methodology was used to forecast the number of EVs to be charged at charging rates of slow, regular, and urgent. A cost reduction was also presented in [5]. This reduction is done in operating costs (energy and degradation of the battery costs) for a control model in the charging and discharging of the EV. Similarly, a control strategy for optimal charging of EVs was developed in [6]. The strategy considers the constraints related to the demand of EVs, a calculation algorithm of the total power for charging, and charging management for each EV. Moreover, a charging method of the EVs based on fuzzy control was implemented in [7]. In this method, the charging process is done by considering the deviation of the frequency of power distribution grid of IEEE 39 nodes with the presence of solar sources. The method implemented in MATLAB/SIMULINK environment takes into account: battery capacity, initial state of charge, initial charging time, etc.

Works related to the optimization of deliveries in service chains address this problem. In [8], a mixed-integer programming model that maximizes the profits by different delivery times for all consumers was proposed. The optimal scheduling of deliveries of products was addressed in [9] via fuzzy logic. This approach considers a number of situations between the driver’s vehicle and each delivery to be done. Similarly, an integrated program delivery fleet that reduces carbon emissions by improving fleet efficiency and optimizing delivery times is developed in [10] and [11]. Some other works consider traffic information during the routing of vehicles. Thus, in [12] and [13], traffic information in real time was analyzed to determine optimal vehicle route. Based on this analysis, the decision-making processes that consider optimal routing policies, instant for the service, and simulation of vehicle flow using Markov processes are developed.

The problem of the shortest route was modeled in [14], which aimed to minimize charging costs along a route. This nonlinear model was modeled using dynamics programming for mono and multi vehicles. Another nonlinear model for minimizing navigation time and charging of an EV fleet was developed in [15]. An evolutionary genetic algorithm implemented in [16] aimed at determining the optimal routes for the navigation of fleets of EVs in order to minimize the costs associated with charging and operating time. Various models of the allocation of charging points in a city map have considered the density of traffic on urban roads throughout the day [17]–[20]. These works aimed to maximize the flow of EVs to be charged at the charging points. A heuristic algorithm that optimized the online service of charging points and considered time-dependent variable tariffs was proposed in [21] and extended in [22]. In [23], an online charging case was presented for managing a system of charging points. This online charging system was developed in Korea to optimize the charging of the mass transport system. Queuing theory was used in [24] to analyze the optimal management of charging stations, considering waiting rates and cost per service. Strategic scenarios were developed in [25]–[28], aiming to guarantee the optimal allocation of
charging points and to coordinate the management of EVs on the city map. The operation of the charging stations that maximized the operator’s gains and the intelligent control of the charging rate at the stations supplied by solar energy was developed in [29] and [30], respectively. In [31] and [32], smart scheduling schemes were developed for charging and discharging EVs during the day in order to optimize the power charging so as to minimize the impact on the power grid by minimizing the costs associated with charging and discharging.

Moreover, the traditional optimization methods are not robust for facing dynamic changes and usually a complete restart must be provided to obtain a feasible solution (e.g., dynamic programming). In contrast, evolutionary algorithms are used in such changing circumstances. However, none of them can guarantee finding the optimal global solution [33].

This paper proposes an MILP model that aims to optimize the maintenance cost and cost of extra hours in the route scheduling of a BEV fleet in the navigation of a city map. In this optimization problem, the average speed, battery state of charge, and set of deliveries allocated to each available BEV in the fleet are considered. Moreover, charging points located along the roads, which are differentiated by charging rate (fast and ultra-fast), are taken into account. To guarantee the performance of the fleet’s batteries, the corresponding constraints are applied. Uncertainties in the navigation of the city map are modeled using the probability of delays in the operation of the fleet due to PTS, PS, and PPW. To show the performance of the proposed approach, a city map containing roads and intersections was modeled as a graph of 71 nodes (intersections) and 131 edges (main and secondary roads). The analysis and evaluation of the proposed model, with and without considering the minimization of the extra hours of operation, showed promising results in the application of this type of transport technology. It should be noted that for both cases, the proposed model was implemented in AMPL language and the commercial solver CPLEX was used to find the optimal solution.

The main contributions of this paper are:
- An MILP model for the optimal route scheduling of EVs for charging during navigation considering prespecified delivery points that presents efficient computational behavior with conventional MILP solvers.
- A flexible model for analyzing the optimization of different scheduling strategies of EVs with consecutive deliveries so that they can be charged along their routes.
- The proposed model contributes a sustainable scenario in the use of energy resources, because toxic emissions in the city are minimized.

The remainder of this paper is structured as follows. In Section II, the hypotheses and uncertainties in the navigation are presented in detail. The mathematical formulation of an MINLP model and the linearization process are discussed in Section III. Section IV presents the case studies and results. Section V contains concluding remarks.

II. MATHEMATICAL MODELING

In this section, the main hypotheses for the problem related to the charging infrastructure, delivery points, and so on are considered in detail. Moreover, the constraints related to the city map and the uncertainties in the navigation of the BEVs are taken into consideration.

A. Hypotheses

The following hypotheses are taken into account to address the problem.

1- A charging infrastructure exists on the city map.
2- The charging points located on the urban roads are known.
3- The delivery points located at the intersections of roads on the city map are known.
4- The BEV fleet and the charging infrastructure belong to the same owner.
5- The start and end points of the fleet are at the same place, called the warehouse.
6- The operator pre-schedules the deliveries assigned to each BEV, and these deliveries are made consecutively.
7- The batteries of the fleet begin with the same state of charge (SOC).
8- The charging rates differ according to the type of road: main (ultra-fast charging) or secondary (fast charging).

B. Modeling of the City Map

The city map, characterized by roads and their intersections, is modeled as a multidirectional graph and adjacency matrix [34]. Fig. 1 shows the adjacency matrix of rows and columns equal to the number of nodes (roads intersections) of this graph. The unit values represent the links between nodes $k$ (starting node) and $i$ (arrival node) that determine the element $k_i$ in the graph. Note that the bidirectional arrows (red) represent the main roads, while the unidirectional arrows (black) represent the secondary roads.

Also in Fig. 1 (right side), note that each main road is represented by two urban roads with opposite directions. The first main road (1, 4) is represented by two unidirectional roads 1→4, and 4→1, and second road (4, 5) by 4→5, and 5→4. Moreover, these four unidirectional roads are elements (in the
Characteristics of urban roads, such as the length, between nodes Fig. 2 is represented by two unidirectional roads, allocated and secondary roads according to the charging rate value. It is worth mentioning that, in the occurrence of these uncertainties in each urban road are limited may present some, all, or none of the influences PTS, PS, and PPW. The algorithm starts with establishing the distribution of the occurrence of each urban road. As a consequence of these influences, the delay times during navigation of the fleet are generated. These numerical values represent the occurrence probability of each of these influences on urban road ki. Therefore, the modeling of influences is important for determining the total delay time of each BEV during navigation on the city map.

To model the uncertainties PTS, PS, and PPW, a given horizon (a day) is taken into account. Therefore, the values of \( p_{PTS} \), \( p_{PS} \), and \( p_{PPW} \); in this paper these values are logically set to 0.5, 0.4, and 0.2, respectively [12]. Note that, PPW has the lowest probability of occurrence, because public works are not made for all urban roads in the city map; the delay time for PTS is more frequent to occur, especially in the city center, and nearby the crowded roads at the presence of more traffic lights, signs, etc. In the case of the presence of schools, basically, this probability represents the rush hours (opening and closing hours) existence of the building in the urban road ki. Therefore, in this paper each uncertainty (PTS, PS, and PPW) is modeled considering its probability of occurrence \( p_{1}, p_{2}, \) and \( p_{3} \) in each urban road ki in a specific day. Thus, this probability of occurrence determines the existence of a delay time during the navigation of the BEV through the urban road ki. These probabilities of occurrence are not determined (or calculated) for the specific periods in a day (e.g., each hour). It is worth mentioning that, considering the hourly operation of the BEV fleet would increase the model complexity and consequently results in decreasing the computational efficiency.

Aiming to determine the occurrence of delay times due to influences PTS, PS, and PPW on each urban road ki, an algorithm for modeling this stochastic phenomenon is proposed. Fig. 3 shows the flowchart of the simulating algorithm for the aforementioned uncertainties. The algorithm starts with established values \( \Delta_{ki}^{PTS} \), \( \Delta_{ki}^{PS} \), and \( \Delta_{ki}^{PPW} \) initialized to zero. It is worth emphasizing that \( \Delta_{ki}^{PTS} \), \( \Delta_{ki}^{PS} \), and \( \Delta_{ki}^{PPW} \) are the results of running the simulation algorithm, and they determine the occurrence of each influence PTS, PS, or PPW on urban road ki. Thus, these results are used as input data to the proposed model and complement the navigation and charging times in the constraint (15). Then, an iterative process is carried out for all urban roads ki. Thereafter, random values between zero and one are assigned to \( p_{1}, p_{2}, \) and \( p_{3} \). These random values are compared independently under the conditions \( p_{1} \leq p_{PTS} \), \( p_{2} \leq p_{PS} \), and \( p_{3} \leq p_{PPW} \). Note that depending on the values \( p_{1}, p_{2}, \) and \( p_{3} \), the conditions can be true or false, and each urban road ki may present some, all, or none of the influences PTS, PS, and PPW.
III. THE PROPOSED MODEL

The proposed model is formulated initially as an MINLP problem, as (1)–(17). It is worth mentioning that the “∑” in the equations stands for a “while”.

\[
\text{Min } F
\]

where \( F \) aims in the first case to minimize the costs for fleet maintenance (\( f_1 \)) and in the second case to minimize the maintenance cost and costs related to the extra hours of the fleet of BEVs (\( f_1 + f_2 \)). These functions are defined as follows.

\[
f_1 = \sum_{v \in \Omega_v} \sum_{e \in \Omega_e} \sum_{i \in \Omega_i} d_{ki} \omega_{v,h,e,ki}
\]

\[
f_2 = \sum_{v \in \Omega_v} \Delta t_{vh}
\]

The constraints of this problem are considered in detail as follows.

\[
\sum_{v \in \Omega_v} \omega_{v,h,e,ki} - \sum_{v \in \Omega_v} \omega_{v,h,e,ij} = -1 \quad \forall v \in \Omega_v, \forall e \in \Omega_e
\]

\[
\forall i \in \Omega_i/ t^i_{v,h,e,i} = -1
\]

\[
\sum_{v \in \Omega_v} \omega_{v,h,e,ki} - \sum_{v \in \Omega_v} \omega_{v,h,e,ij} = 0 \quad \forall v \in \Omega_v, \forall e \in \Omega_e
\]

\[
\forall i \in \Omega_i/ t^i_{v,h,e,i} = 0
\]

\[
\sum_{v \in \Omega_v} \omega_{v,h,e,ki} - \sum_{v \in \Omega_v} \omega_{v,h,e,ij} = 1 \quad \forall v \in \Omega_v, \forall e \in \Omega_e
\]

\[
\forall i \in \Omega_i/ t^i_{v,h,e,i} = 1
\]

Constraints (2), (3), and (4) guarantee the determination of the shortest route for the navigation of BEV \( vh \) in delivery \( e \). Constraint (2) is applied to obtain the shortest urban road \( d_{ij} \) to be traversed, starting from intersection \( i \) (\( t^i_{v,h,e,i} = -1 \)) for all possible intersections \( j \). Fig. 4 illustrates an example related to constraint (2). Note that the intersection \( i \) (\( t^i_{v,h,e,i} = -1 \)) represents only the starting node to the intermediate nodes \( 1 \) (\( t^1_{v,h,e,1} = 0 \)), \( 2 \) (\( t^2_{v,h,e,2} = 0 \)), \( 3 \) (\( t^3_{v,h,e,3} = 0 \)), and \( 4 \) (\( t^4_{v,h,e,4} = 0 \)). Therefore, there is no BEV \( vh \) coming to the node \( i \). Moreover, the red line (dashed) represents the shortest urban road \( d_{i2} \) to be traversed; this urban road \( i2 \) indicates the unit value of the variable \( \omega_{v,h,e,i2} \). Thus, the sum of the variables \( \omega_{v,h,e,ij} \) related to all possible urban roads \( ij \) results in 1.

The illustrative example of Fig. 5 is related to the constraint (3) and shows the navigation of BEV \( vh \) on urban roads formed with intermediate intersections, where the navigation from intersection \( k \) to \( i \) and from intersection \( i \) to \( j \) constitutes a minimum route \( ki \) and \( ij \). In this case, the intermediate intersection \( i \) represents the arrival node for the BEV \( vh \) coming from the possible nodes \( 2 \) (\( t^2_{v,h,e,2} = 0 \)), \( 4 \) (\( t^4_{v,h,e,4} = 0 \)), and \( 10 \) (\( t^1_{v,h,e,10} = 0 \)) and at the same time it represents the starting node for the BEV \( vh \) going to the possible nodes \( 6 \) (\( t^6_{v,h,e,6} = 0 \)), \( 8 \) (\( t^8_{v,h,e,8} = 0 \)), and \( 12 \) (\( t^{12}_{v,h,e,12} = 0 \)). Thus, the sum of the variables \( \omega_{v,h,e,ki} \) related to the possible urban roads \( ki \) (intersections \( k \) with values 2, 4, and 10) results in 1 due to the selection of the shortest urban road \( 2i \). Similarly, the sum of the variables \( \omega_{v,h,e,ij} \) results in 1, (intersections \( j \) with values 6, 8, and 12) by selecting the shortest urban road \( i8 \).

In (4), the urban road \( ki \) is selected such that the length that must be traversed, starting from intermediate intersection \( k \) (\( r^{1}_{v,h,e,k} = 0 \)) to the possible arrival intersection \( i \) (\( r^{i}_{v,h,e,i} = 1 \)), is the shortest \( d_{ki} \). Fig. 6 is used to illustrate constraint (4). Note that the intersection \( i \) represents only the arrival node and the intersections \( k \) are represented by intermediate nodes \( 2 \) (\( r^{2}_{v,h,e,2} = 0 \)), \( 6 \) (\( r^{6}_{v,h,e,6} = 0 \)), \( 7 \) (\( r^{7}_{v,h,e,7} = 0 \)), and \( 8 \) (\( r^{8}_{v,h,e,8} = 0 \)). Therefore, there exists no BEV \( vh \) starting from \( i \) to the other intermediate nodes. Here the red line (dashed) represents the
shortest urban road $\delta_i$ to be traversed, starting from all possible intersections $k$ to the arrival intersection $i$ ($v_{vh,e,i} = 1$). Thus, the sum of the variables $\omega_{vh,e,ki}$ related to the urban roads $ki$ (intersection $k$ with values 2, 6, 7, and 8) results in 1.

In order to initialize the state of charge $SOC^0_{vh}$ of the BEV $vh$ at intersection $i$ ($v_{vh,e,i} = -1$) for the first delivery $e$, constraint (5) is considered. Note that the SOC is the state of energy available in the battery at a given moment.

$$SOC_{vh,e,i} = SOC^0_{vh}, \forall vh \in \Omega_v, \forall e \in \Omega_e/e = 1,$$
$$\forall i \in \Omega_n/v_{vh,e,i} = -1$$

The state of charge, $SOC_{vh,e-1,v}$, for the battery of BEV $vh$ in delivery $e-1$ at intersection $v$ ($v_{vh,e,v} = 1$) is equal to its state of charge in delivery $e$ at intersection $u$ ($v_{vh,e,u} = -1$), (6).

$$SOC_{vh,e,u} = SOC_{vh,e-1,v}, \forall vh \in \Omega_v, \forall e \in \Omega_e, \forall u \in \Omega_u,$$
$$\forall v \in \Omega_n/v_{vh,e,u} = -1 \land v_{vh,e-1,v} = 1$$

Fig. 7 shows an illustrative example related to constraints (5) and (6). Note that the arrows in red and blue represent the shortest routes to deliveries, $e = 1$ (intersection 4) and $e = 2$ (intersection 8) in their corresponding points. Constraint (5) is related to the initial state of charge of the BEV battery, $SOC_{vh,1,1}$, in the warehouse (starting at intersection 1 with $v_{vh,1,1} = -1$). This state of charge is represented by $SOC^0_{vh}$, and is assigned to each BEV $vh$ starting at intersection 1, towards the first delivery $e = 1$ (for this case, in arrival intersection 4 with $v_{vh,1,4} = 1$).

In order to explain constraint (6), in Fig. 7, node 4 is taken into consideration. Note that this node represents, for the shortest route in red ($e = 1$), the arrival intersection with $v_{vh,1,4} = 1$, and, for the shortest route in blue ($e = 2$), the starting intersection with $v_{vh,2,4} = -1$. Therefore, it is necessary to consider that for BEV $vh$, which existed in node 4, the state of charge of the first delivery ($SOC_{vh,1,4}$) should be equal to the state of charge of the second delivery ($SOC_{vh,2,4}$). In order to determine the state of charge of the battery for BEV $vh$, constraint (7), which contains two terms, including the available state of charge $SOC^a_{vh,e,ki}$ and the charged energy $e_{vh,e,ki}$ in urban road $ki$, is considered. Each SOC has its own limitations on energy storage in the fleet’s batteries, represented in (8).

$$SOC_{vh,e,i} = SOC^a_{vh,e,i} + \sum_{ki \in \Omega_u} e_{vh,e,ki}, \forall vh \in \Omega_v, \forall e \in \Omega_e$$
$$\forall i \in \Omega_n/v^a_{vh,e,i} \geq 0$$

(7)

$$0.10 \times K^x \leq SOC_{vh,e,i} \leq 0.98 \times K^x$$
$$\forall vh \in \Omega_v, \forall e \in \Omega_e$$
$$\forall i \in \Omega_n$$

(8)

$$SOC^a_{vh,e,i} = \sum_{ki \in \Omega_u} \omega_{vh,e,ki}(SOC_{vh,e,k} - K^x d_{ki})$$
$$\forall vh \in \Omega_v,$$
$$\forall e \in \Omega_e, \forall i \in \Omega_n/v^a_{vh,e,i} \geq 0$$

(9)

$$0 \leq SOC^a_{vh,e,i}$$. \forall vh \in \Omega_v, \forall e \in \Omega_e,$$
$$\forall i \in \Omega_n/v^a_{vh,e,i} \geq 0$$

(10)

In (9), the available energy in the battery $SOC^a_{vh,e,i}$ is calculated as the difference between the $SOC^0_{vh,e,k}$ at intersection $k$ and $K^x d_{ki}$, which is the energy used during travel on urban road $ki$ ($\omega_{vh,e,ki} = 1$). Finally, the non-negativity of $SOC^a_{vh,e,i}$ is guaranteed by (10) at intersection $i$ with $v^a_{vh,e,i} \geq 0$. Fig. 8 illustrates in detail the concepts of constraints (7) and (9). Note that the shortest urban roads $ki$ to be traversed are represented in red (dashed). Moreover, the variables $SOC_{vh,e,k}$ and $SOC^a_{vh,e,i}$ represent the state of charge of the BEV battery at the intermediate intersections $k$ and $i$, respectively. Moreover, the number of charges $n^R_{vh,e,ki}$ on urban road $ki$ and the obtained energy, $e_{vh,e,ki}$, by charging from these supply points.
are shown. Constraint (7) considers the calculation of $SOC_{vh,e,i}$ of the battery BEV at the intermediate intersection $i$. This calculation takes into account the available energy, $SOC_{vh,e,i}^{a}$, and the energy to be recharged by BEV at charging points on the urban road $ki$, $\varepsilon_{vh,e,ki}$, is added to it. Moreover, the available energy in the intermediate intersection $i$, $SOC_{vh,e,i}^{a}$, is calculated via constraint (9) and considers the reduction of energy $K^{*}d_{ki}$ in addition to the state of charge $SOC_{vh,e,k}$ (at intermediate intersection $k$), as shown in Fig. 8.

$$v_{vh,e,ki} = \omega_{vh,e,ki}^{R} + \frac{RU_{vh,e,ki}}{RU_{vh,e,ki}} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

$$\tau_{MIN,UR}^{k_{i}} \leq \tau_{vh,e,ki}^{RU} \leq \tau_{MAX,UR}^{k_{i}} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}/\Omega_{1} = 1$$

$$\tau_{MIN,R}^{k_{i}} \leq \tau_{vh,e,ki}^{RU} \leq \tau_{MAX,R}^{k_{i}} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}/\Omega_{1} = 0$$

$$0 \leq n_{vh,e,ki}^{P} \leq n_{vh,e,ki}^{R} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

The energy $\varepsilon_{vh,e,ki}$ is calculated by constraint (11), where the charging depends on $n_{vh,e,ki}^{R}$ and $\tau_{vh,e,ki}^{RU}$ at each point along urban road $ki$. By considering (12) and (13), the minimum and maximum $\tau_{vh,e,ki}^{RU}$ for urban road $ki$ with $\Omega_{1} = 1$ and $\Omega_{1} = 0$ are guaranteed. In (14), the maximum number of charging points to be visited on urban road $ki$ is guaranteed.

Finally, with constraint (15), the total time of operation of BEV $vh$ considering the value $t_{max}$ and extra hours $\Delta t_{vt,h}$ to be penalized for the second case is determined in (1). Note that the total time of operation of BEV $vh$ is composed of $\tau_{PS}^{a} \Delta k_{i}$, $\tau_{PTS}^{a} \Delta k_{i}$, and $\Delta_{PPW}^{a} \Delta k_{i}$, which are the navigation times and delays due to PTS, PS, and PPW, respectively, and also the charging times $n_{vh,e,ki}^{R} \tau_{vh,e,ki}^{RU}$ for urban road $ki$ chosen for the navigation of BEV $vh$. Constraints (16) and (17), respectively, stand for the main binary and integer decision variables $\omega_{vh,e,ki}$ and $n_{vh,e,ki}^{R}$.

$$\sum_{v_{h} \in \Omega_{v}} \sum_{v_{h} \in \Omega_{v}} \omega_{vh,e,ki} \left( d_{ki} / t_{vh,e,ki}^{RU} \right) + \sum_{v_{h} \in \Omega_{v}} \sum_{v_{h} \in \Omega_{v}} \omega_{vh,e,ki} \left( \tau_{PTS}^{a} \Delta k_{i} \right) + \sum_{v_{h} \in \Omega_{v}} \sum_{v_{h} \in \Omega_{v}} \omega_{vh,e,ki} \left( \tau_{PS}^{a} \Delta k_{i} \right) + \sum_{v_{h} \in \Omega_{v}} \sum_{v_{h} \in \Omega_{v}} \omega_{vh,e,ki} \left( \tau_{PPW}^{a} \Delta k_{i} \right) + \sum_{v_{h} \in \Omega_{v}} \sum_{v_{h} \in \Omega_{v}} \omega_{vh,e,ki} \left( n_{vh,e,ki}^{R} \tau_{vh,e,ki}^{RU} \right) \leq \tau_{max}^{a} + \Delta t_{vh} \forall v_{h} \in \Omega_{v}$$

$$A. Linearization$$

In the proposed model, a set of linearization procedures is applied to the non-linear constraints (9), (11), and (15). This set is inspired by the Big-M method [35]–[37], and is presented as linear constraints shown in Fig. 9. Note that these non-linear constraints in the proposed model are related to the product of two variables; these variables are shown in Fig. 9 as $\varepsilon$ (continuous variable) and $\omega$ (binary variable). When the product, $\sigma$, is evaluated for values of $\omega$ (0 or 1), $\sigma$ results in 0 or $\varepsilon$, respectively. Therefore, a collection of linear constraints is necessary to obtain the same values of $\sigma$. Thus, the linear constraints shown in Fig. 9 are evaluated for the values of $\omega$. If $\omega = 0$, then the resulting inequalities produce the set point $\sigma = 0$ (green line on the left side). Similarly, if $\omega = 1$, then the inequalities form the set of points $\sigma = \varepsilon$ (green line on the right side). Note that, defining a proper $M$ is vital in the Big-M method and a large $M$ may result in serious numerical difficulties on a computer, while a small $M$ may not guarantee the optimal solution [38].

Using the linearization technique in [35], the following is the linear formulation for (9).

$$SOC_{vh,e,i}^{a} = \sum_{v_{h} \in \Omega_{v}} \sum_{v_{h} \in \Omega_{v}} \omega_{vh,e,ki} \left( n_{vh,e,ki}^{R} \tau_{vh,e,ki}^{RU} \right) \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

$$0 \leq -\Delta_{vh,e,ki}^{a} + SOC_{vh,e,k} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

$$0 \leq \Delta_{vh,e,ki}^{a} + SOC_{vh,e,k} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

$$\Delta_{vh,e,ki}^{a} \leq M \ast \omega_{vh,e,ki} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

To linearize constraint (11), $n_{vh,e,ki}^{R}$ is discretized with $Y$ binary variables $W_{vh,e,ki,y}$, as in (19.b) and replaced in (19.a).

$$\varepsilon_{vh,e,ki} = \frac{p_{vh}}{\sum_{k_{i}=1}^{Y} \Delta Z_{vh,e,ki,y} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}}$$

$$n_{vh,e,ki}^{R} = \sum_{k_{i}=1}^{Y} W_{vh,e,ki,y} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}$$

$$0 \leq -\Delta W_{vh,e,ki,y} + \tau_{vh,e,ki}^{RU} \forall v_{h} \in \Omega_{v}, \forall e \in \Omega_{e}, \forall k_{i} \in \Omega_{a}, \forall y = 1 \ldots Y$$
The charging rates guarantee that the optimal global solution would be found. In addition, the BEVs’ batteries had a capacity of 30 kWh. Table I shows the deliveries allocated to each BEV vh by the fleet operator.

By considering the linearization of (11), constraint (15) changes to the following form.

\[
\sum_{\forall e \in \Omega_{v}} \sum_{\forall k \in \Omega_{\delta}} \omega_{h,e,\delta,\epsilon} \left( \frac{d_{k}}{v_{h}} \right) \\
+ \sum_{\forall e \in \Omega_{v}} \sum_{\forall k \in \Omega_{\delta}} \omega_{h,e,\delta} \left( t_{PS} \Delta_{k} \right) \\
+ \sum_{\forall e \in \Omega_{v}} \sum_{\forall k \in \Omega_{\delta}} \omega_{h,e,\delta} \left( t_{PS} \Delta_{k} \right) \\
+ \sum_{\forall e \in \Omega_{v}} \sum_{\forall k \in \Omega_{\delta}} Y \\
\leq \tau_{vh}^{max} + \Delta t_{vh} \\
\forall v_{h} \in \Omega_{v}. \tag{20}
\]

B. The MILP Model

The optimal route scheduling of the EVs for charging during navigation considering prespecified delivery points is modeled as an MILP problem as follows:

\[
\min (1) \\
\text{subject to: (1) - (7), (8), (10), (12) - (14), (16) - (20).
\]

IV. TEST AND RESULTS

To evaluate the proposed MILP model, the system presented in Fig. 2 was used. In addition, the BEVs’ batteries had a SOC of 8.20 kWh, while their maximum capacity, K, was set to 30 kWh. Table I shows the deliveries e allocated to each BEV vh by the fleet operator.

In this paper, to implement and solve the proposed models, a modeling language for mathematical programming AMPL [39] and the commercial solver CPLEX [40] were used on a 2.67-GHz computer with 3 GB of RAM in order to guarantee that the optimal global solution would be found. The charging rates \( p_{k}^{h} \) were 6 kW (fast) and 10 kW (ultra-fast) for the main and secondary urban roads \( k \), respectively.

The energy used during the navigation, \( K^{e} \), was 0.16 kWh/km, with an average speed of 40 km/h for each BEV [30]. Two cases were carried out, the first case to minimize the costs for fleet maintenance and the second case to minimize the maintenance cost and costs related to the extra hours of the fleet of BEVs.

The cost coefficients \( g^{km} \) and \( g^{h} \) in (1) were in a ratio of 1 to 100, respectively, considering a greater weight for extra hours in the second case. In the operation of BEV vh, the time \( \tau_{vh}^{max} \) was 8 hours (habitual working hours). In addition, the delay times for uncertainties \( t_{PS} \), \( t_{PS} \), and \( t_{PPW} \) were 1, 2, and 5 minutes, respectively [31].

Table II, III, and IV show the shortest routes for BEVs 1, 2, and 3, respectively, in both cases. These tables represent the unit values of the binary variable, \( \omega_{h,e,\delta,\epsilon} \) related to constraints (2), (3), and (4), as explained. An analysis done for BEV 1 is used as a descriptive example below. Thus, in delivery 1 of BEV 1, the proposed model determines the unit values of \( \omega_{1,1,34,26} \) and \( \omega_{1,1,26,25} \). These values indicate the BEV navigation by urban roads 34 → 26, and 26 → 25 or by shortest route 34 → 26 → 25 for both cases. Note that, in the first urban road 34 → 26, node 34 represents the warehouse, and

<p>| TABLE I |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| <strong>TABLE I</strong>    | SCHEDULING OF DELIVERIES FROM STARTING POINT TO ARRIVAL POINT (STARTING → ARRIVAL) FOR BEVs |</p>
<table>
<thead>
<tr>
<th>BEV</th>
<th>Delivery1</th>
<th>Delivery2</th>
<th>Delivery3</th>
<th>Delivery4</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34 → 25</td>
<td>25 → 10</td>
<td>10 → 7</td>
<td>7 → 5</td>
<td>5 → 34</td>
</tr>
<tr>
<td>2</td>
<td>34 → 22</td>
<td>22 → 2</td>
<td>2 → 19</td>
<td>19 → 41</td>
<td>41 → 34</td>
</tr>
<tr>
<td>3</td>
<td>34 → 38</td>
<td>38 → 50</td>
<td>50 → 61</td>
<td>61 → 46</td>
<td>46 → 34</td>
</tr>
<tr>
<td>4</td>
<td>34 → 29</td>
<td>29 → 51</td>
<td>51 → 68</td>
<td>68 → 60</td>
<td>60 → 34</td>
</tr>
<tr>
<td>5</td>
<td>34 → 33</td>
<td>33 → 57</td>
<td>57 → 65</td>
<td>65 → 70</td>
<td>70 → 34</td>
</tr>
</tbody>
</table>

<p>| TABLE II |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| <strong>TABLE II</strong>    | SHORTEST ROUTE FOR BEV 1 DURING THE NAVIGATION |</p>
<table>
<thead>
<tr>
<th>BEV</th>
<th>Delivery1</th>
<th>Delivery2</th>
<th>Delivery3</th>
<th>Delivery4</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34 → 26</td>
<td>26 → 25</td>
<td>24 → 23</td>
<td>23 → 22</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>22 → 21</td>
<td>21 → 19</td>
<td>19 → 18</td>
<td>18 → 1 2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>23 → 4</td>
<td>4 → 15</td>
<td>22 → 21</td>
<td>21 → 20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>19 → 40</td>
<td>40 → 41</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| TABLE III |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| <strong>TABLE III</strong>   | SHORTEST ROUTE FOR BEV 2 DURING THE NAVIGATION |</p>
<table>
<thead>
<tr>
<th>BEV</th>
<th>Delivery1</th>
<th>Delivery2</th>
<th>Delivery3</th>
<th>Delivery4</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34 → 26</td>
<td>26 → 25</td>
<td>24 → 23</td>
<td>23 → 22</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>22 → 21</td>
<td>21 → 19</td>
<td>19 → 18</td>
<td>18 → 1 2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>23 → 4</td>
<td>4 → 15</td>
<td>22 → 21</td>
<td>21 → 20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>19 → 40</td>
<td>40 → 41</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| TABLE IV |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| <strong>TABLE IV</strong>    | SHORTEST ROUTE FOR BEV 3 DURING THE NAVIGATION |</p>
<table>
<thead>
<tr>
<th>BEV</th>
<th>Delivery1</th>
<th>Delivery2</th>
<th>Delivery3</th>
<th>Delivery4</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34 → 26</td>
<td>26 → 25</td>
<td>24 → 23</td>
<td>23 → 22</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>22 → 21</td>
<td>21 → 19</td>
<td>19 → 18</td>
<td>18 → 1 2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>23 → 4</td>
<td>4 → 15</td>
<td>22 → 21</td>
<td>21 → 20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>19 → 40</td>
<td>40 → 41</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
in urban road 26→25, node 25 represents the delivery point 1 (BEV 1 arriving). In delivery 2, the variables with unit values \( \omega \), \( \omega_{1.2,12} \), \( \omega_{1.2,11} \), and \( \omega_{1.2,11,10} \), determine three urban roads \( k_i \) to be traversed, such as: 25→12; 12→11; and 11→10 or the shortest route 25→12→11→10 for both cases. Note that, in this shortest route, nodes 25 and 10 represent delivery points 1 (BEV 1 starting) and 2 (BEV 1 arriving), respectively. In the case of delivery 3, the variables with unit value, \( \omega_{1.3,10,27} \); \( \omega_{1.3,27,26} \); \( \omega_{1.3,26,25} \); \( \omega_{1.3,25,12} \); and \( \omega_{1.3,12,7} \), determine five urban roads, 10→27; 27→26; 26→25; 25→12; and 12→7 to be traversed for both cases. Moreover, the nodes 10 and 7 represent delivery points 2 (BEV 1 starting) and 3 (BEV 1 arriving), respectively. For the last delivery, the variables with unit value are \( \omega_{1.4,7,8} \); \( \omega_{1.4,8,11} \); \( \omega_{1.4,11,26} \); \( \omega_{1.4,26,25} \); \( \omega_{1.4,25,24} \); \( \omega_{1.4,24,23} \); \( \omega_{1.4,23,14} \); and \( \omega_{1.4,14,5} \), which determine the urban roads 7→8; 8→11; 11→26; 26→25; 25→24; 24→23; and 23→14, and 14→5 to be traversed in both cases. In this delivery, the nodes 7 and 5 represent delivery points 3 (BEV 1 starting) and 4 (BEV 1 arriving), respectively. Finally, in the return to the warehouse, the variables with unit values are \( \omega_{1.5,5,6} \); \( \omega_{1.5,6,13} \); \( \omega_{1.5,13,24} \); and \( \omega_{1.5,24,34} \), which determine the urban roads 5→6; 6→13; 13→24; and 24→34, to be traversed in both cases. Note that, the nodes 5 and 34 represent the delivery point 4 (BEV 1 starting) and the warehouse (BEV 1 arriving), respectively.

Thus, Table II shows the route of BEV 1 for each case. These routes coincided, resulting in a 153 km distance from the warehouse to delivery 4 and 39.3 km returning to the warehouse. A total operation time of 8h 37m 48s for the first case and 7h 58m 48s for the second case showed a reduction of 22.3%. Note that the reduction was related to the total charging time of BEV 1. The shortest routes for BEV 2 in both cases are shown in Table III. Note that for both cases the routes coincided, presenting a total distance of 121 km from the warehouse to delivery 4 and 37.5 km returning to the warehouse. In addition, the total operating time for the first case was 7h 57m 36s, while for the second case it was 7h 15m 0s, which shows a total reduction of 8.9%. With coincident routes and an average speed of 40 km/h for the fleet, it can be concluded that this reduction was directly related to the total charging time for BEV 2.

Table IV shows the shortest route for BEV 3 for both cases. Note that the routes coincided minimally in both cases, because a change in the route of delivery 3 affected the navigation time. For the second case, this resulted in a reduction of the total navigation time of approximately 1.12% compared to the first case. Although for both cases the total distance of the shortest route was approximately 238 km, the total charging time of the second case showed a reduction of 36.9% compared to the first case, while the operating times of the first and second cases were 11h 56m 24s and 10h 6m 36s, respectively. Figs. 10 and 11 illustrate the shortest routes of BEVs 4 and 5, respectively, the first case in red and the second case in blue. In Fig. 10, for both cases, the shortest routes from the warehouse to delivery 4 and returning to the warehouse were 129 km and 60 km, respectively. The total operation time for the second case was 8h 3m 36s, which was about 27.08% less than the first case, which had a 9h 5m 24s operating time. This reduction is the result of different charging times during the navigation of BEV 4.

In Fig. 11, the change in the route (when returning to the warehouse) resulted in additional navigation time for the first case, about 0.78% more than the second case. Moreover, the charging time for the second case was reduced about 14.19%. Also, the total operating time for the first and second cases were 10h 51m 36s and 10h 16m 12s, respectively. For both cases, the shortest route was about 220 km. Finally, the shortest routes obtained for each BEV in the fleet, for both cases, demonstrate the effectiveness of the proposed model regarding operation times. An example of the effectiveness model in the reduction of operation times is demonstrated in the operation of BEVs 4 and 5. Note that the operating time of the BEVs and their components are analyzed in detail in Table V.
Table V

Navigation, Charging, and Delay Times in the Operation of the Fleet

<table>
<thead>
<tr>
<th>BEV</th>
<th>Operation Time (h)</th>
<th>Navigation Time (h)</th>
<th>Charging Time (h)</th>
<th>Delay Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Case</td>
<td>Second Case</td>
<td>First Case</td>
<td>Second Case</td>
</tr>
<tr>
<td>1</td>
<td>9.55</td>
<td>8.90</td>
<td>5.72</td>
<td>5.72</td>
</tr>
<tr>
<td>2</td>
<td>9.05</td>
<td>8.34</td>
<td>5.09</td>
<td>5.09</td>
</tr>
<tr>
<td>3</td>
<td>13.08</td>
<td>11.25</td>
<td>7.20</td>
<td>7.12</td>
</tr>
<tr>
<td>4</td>
<td>9.76</td>
<td>8.73</td>
<td>5.41</td>
<td>5.41</td>
</tr>
<tr>
<td>5</td>
<td>11.72</td>
<td>11.13</td>
<td>6.35</td>
<td>6.40</td>
</tr>
</tbody>
</table>

Fig. 12. Navigation times and delays due to the presence of an uncertainty.

Table VI

Charging Times, Number of Charges and Unit Charging Rate for the BEV 1 During Navigation

<table>
<thead>
<tr>
<th>Deliveries</th>
<th>BEV 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Delivery 1</td>
<td>34</td>
<td>26</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Delivery 2</td>
<td>25</td>
<td>12</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Delivery 3</td>
<td>10</td>
<td>27</td>
<td>0.28</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Delivery 4</td>
<td>7</td>
<td>8</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Delivery 5</td>
<td>5</td>
<td>6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Return</td>
<td>6</td>
<td>13</td>
<td>0.63</td>
<td>1.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note that for BEV 2 and 3, the charging time component in the second case presents a significant reduction from the first case due to the presence of uncertainties during navigation. Therefore, optimization of the operation time results in reducing the charging time when the shortest route has higher delay times from the presence of uncertainties.
navigated. It is observed that in both cases BEV 1 presents the same total number of recharges after operation. Moreover, in the first case, only three recharges were ultra-fast (10 kW), and in the second case, all five recharges were ultra-fast. Note that in both cases, most of the recharges were made on urban roads $ki$ belonging to deliveries 3 and 4. Total charging time after the operation of BEV 1 resulted in 2.91 and 2.26 hours for the first and second cases, respectively. Also, note that the lowest recharge time for the first case resulted in 0.28 hours (delivery 3), and 0.26 hours (delivery 1) in the second case. Finally, the results show a reduction of time of 0.65 hours in total recharge time for BEV 1. This reduction results in significant monetary terms when considering a large fleet.

Figs. 13 (a)–(e) show the profile of $SOC_{vh,i}$ for the BEVs for both cases, and Fig. 13 (f) shows the total number of charges, $\sum_{V \in \Omega} \sum_{i \in \Omega} n^R_{vh,(e,ki)}$ for each BEV. In Fig. 13 (a), for the first case, BEV 1 shows five states of charge of 9.6% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), 24.3% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), 20.9% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 23.6% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), and 12.7% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$) of $K$. For the second case, it shows three states of charge with values of 31.2% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), 40.3% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 2$), and 38.3% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 2$). Note that, for both cases, more than 50% of the charging was ultra-fast. Thus, the shape of profile $SOC_{vh,i}$ for the second case shows the peak values of the stored energy in the battery of BEV 1 obtained with ultra-fast charging rates and reducing the charging time during operation. Note that, in the first case has more energy peaks (achieved with fast charging rates) than the second case, and these energy peaks are achieved with fast charging rates. For BEV 2 in the first case, Fig. 13 (b) shows three states of charge of 23.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 34.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and 42.7% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$) at the fast charging rate. In the second case, there are five states of charge of 28.3% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), 21.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 17.0% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 2$), 19.0% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), and 25.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$) at the ultra-fast rate for most of the time. Therefore, the second case shows an energy storage regime smaller than in the first case for most urban roads $ki$ to be traversed. The energy stored values in the battery for the first case result of recharges with charging times near 1 hour.

In Fig. 13 (c), BEV 3 shows five states of charge, 15.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 21.6% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), 25.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 36.7% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 2$), and 29.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$) along with the route in the first case. The second case shows three states of charge with values of 31.3% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$), 81.6% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 4$), and 92.9% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 1$) at the ultra-fast rate. Moreover, the profile $SOC_{vh,i}$ for the second case presents an energy peak close to the battery capacity. This peak presents the largest amount of charging in ultra-fast mode, with the charging times at approximately 30 minutes. After this peak, a new recharge is done at the same rate, providing the required energy to conclude the operation.

For BEV 4 in Fig. 13 (d), the obtained states of charge are, for the first case, 14.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 27.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 36.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and 32.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and for the second case, 55.8% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 2$), 21.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and 21.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 2$). Note that, the profile $SOC_{vh,i}$ in the second case presents the peak energy near about 50% of the battery capacity, and also in the first case the peaks are smaller and consecutive. In Fig. 13 (e) the states of charge for BEV 5 are, for the first case, 19.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 31.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 32.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), 12.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and 21.3% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and for the second case 62.9% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 2$), 72.0% ($p^R_{ki} = 6$; $n^R_{vh,e,ki} = 1$), and 32.0% ($p^R_{ki} = 10$; $n^R_{vh,e,ki} = 2$). Thus, in the first case the BEV 5 begins without recharges and during navigation makes partial recharges less than 30% of battery capacity. In the second case the profile $SOC_{vh,i}$ of the BEV 5 after beginning operation stores energy with a peak value about 60% of the battery capacity. This stored energy peak value allows navigation of BEV 5 for more than half the shortest route determined by the proposed model.

Note that, the stored energy profile for each BEV shows the energy regime necessary for the attendance of all deliveries with the return to the warehouse in both cases. Fig. 13 (f) shows the total number of charges for the BEVs during navigation. Note that BEVs 1, 3, and 5 had an equal number of charges for both cases. Also, for BEVs 2 and 4, the number of charges in the second case was more than in the first case. Finally, the cost for the first case was approximately 4.46% higher than for the second case. Thus, it is evident that an optimization strategy for this problem should minimize the extra hours after working hours.

V. CONCLUSION

In this paper, an MILP model for optimizing the maintenance costs and extra hours in the route scheduling of a BEV fleet during the delivery of products at prespecified delivery points was proposed. The BEV on each route must have an efficient charging strategy. The proposed model considers a number of operational constraints related to the number of deliveries, average speed, location of delivery points, and charging, as well as the performance of the fleet’s batteries.
The uncertainties are modeled using the probability of delays due to the PTS, PS, and PPW. A city map with 71 interections and 131 urban roads, main and secondary, was used. Results showed that considering the extra hours due to the presence of PTS, PS, and PPW. The Table VII contains the data related to the city map.

### References


