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Applications of celestial mechanics in natural objects and spacecrafts

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Abstract Studies related to Celestial Mechanics started long ago, and it is one of the oldest fields in Astronomy. It started to try to explain the motions of the stars in the sky, in particular the irregular motion of some of those of then, which were really the planets of the Solar System. In the 20th century, with the arrival of the "Space Age", many applications related to the motion of artificial spacecrafts appeared. This new field was called "Astrodynamics", to designate the use of Celestial Mechanics in man-made objects. Several aspects, like orbit determination, maneuvers to change the orbit of the spacecraft, etc., are covered by this topic. The present Focus Issue in Celestial Mechanics publishes a list of papers in topics related to applications in Celestial Mechanics to both situations: natural and artificial satellites.

Keywords Celestial mechanics · Astrodynamics · Orbital dynamics

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The science of "Celestial Mechanics" was born to study the motion of the celestial bodies. It started with the observation of the motion of the stars in the night sky, which has been made probable, since the first form of humans was able to understand what they could see in the sky. The time and the evolution of technology improved our knowledge in Celestial Mechanics. A major series of steps were made in the years near 1600. Tycho Brahe (1546–1601) made accurate observations of the motion of the stars which moved against the fixed ones. Those data were than studied by Johannes Kepler (1571–1630), which made the important "Three Laws of Kepler" of the orbital motion:

1. The orbit of each planet is an ellipse with the Sun occupying one of the focus.

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2. for each planet, the radius vector of the planet covers equal areas in equal times, so its motion is faster at perihelion and slower at aphelion.
3. the cube of the semi-major axis of the planet's orbit is proportional to the square of its orbital period.

Based on those laws, Isaac Newton (1642–1727) formulated the famous Law of Universal Gravitation, showing that mass attracts mass according to the product of their masses divided by the square of the distances between them.

Many other advances came with time: the invention of the telescope, the construction of large arrays of observations using radio and optical telescopes, etc. Currently, not only the Solar System is well mapped, but also a large number of exoplanets are known (Domingos et al. 2006; Rodler et al. 2012), and this number increases every day. Planets similar to the Earth are already in the observation range.

On the technological side, since the pioneer works of K. E. Tsiolkovsky (1857–1935) and Robert Goddard (1882–1945), men started to build and launch spacecrafts in space, around the Earth and beyond. In that sense, the field of Astrodynamics appeared and has been developed. It studies the motion of spacecrafts in space. The gravity of the celestial bodies are the first force to be considered, but many others can also be important, like the radiation pressure, the atmospheric effects, the non-uniform distribution of mass in many bodies, propulsion, etc, according to the mission to be accomplished and the accuracy required for the desired trajectory. Actually, Astrodynamics is a recent field, compared to Celestial Mechanics. It was born with the remarkable work made by Walter Hohmann (1880–1945), which developed basic principles and tools that should be used for space navigation (Hohmann 1925). Its turning point considered by many as the start of the so-called "space age" was the launch of the "Sputnik"—first artificial satellite of the Earth—by the former Soviet Union, in 1957.

Other key historical accomplishments were the first man's travel in space (Yuri Gagarin in 1961), the first man in the Moon (Neil Armstrong in 1969), and the grand tour of the Voyager spacecraft by the giant planets of the Solar System (70 and 80 decades).

We can claim that live on Earth has been continuously improved by the space exploration and its spinoffs. Communications using satellites are available everywhere anytime. Remote sensing from space helps to find natural resources on Earth. It is almost impossible to think of a world with no weather forecast, which is strongly based on satellites. A more recent benefit, positioning on Earth, changed the way of finding specific points on Earth.

The literature is very reach in topics related to this field, starting from basic impulsive maneuvers (Roth 1967; Prussing 1970, 1969; Eckel 1963; Hohmann 1925; Smith 1959; Bender 1962; Jin and Melton 1991; Jezewski and Mittleman 1982; Hoelker and Silber Hoelker and Silber; Shternfeld 1959; Gross and Prussing 1974; Eckel 1982; Prussing and Chiu 1986; Ting 1960; Walton et al. 1975). Then, it goes to orbital maneuvers based in low thrust (Casalino and Colasurdo 2002; Casalino et al. 1999; Brophy and Noca 1998; Zee 1963; Lion and Handelsman 1968; Sukhanov and Prado 2007, 2008; Macau 2000; Macau and Grebogi 2006), swing-by techniques (Flandro 1966; Marsh 1988; Dunham and Davis 1985; Farquhard and Dunham 1981; Dunham and Davis 1985; Prado and Broucke 1995; Prado 2007; Gomes et al. 2013; DeMelo et al. 2009), and even gravitational capture (Belbruno and Miller 1993; Pierson and Kluever 1994; Neto and Prado 1998). It covers also some more applied concepts, like searches for specific orbits (Chiaradia et al. 2003; Carvalho et al. 2010; Araujo et al. 2012; Domingos et al. 2008; D'Amario et al. 1982; Farquhar et al. 1985; Gomes and Domingos 2015; Salazar et al. 2015a, b, 2014, 2012; Gomes et al. 2007).

The present special issue of the "Computer and Applied Mathematics" journal, called "Focus on Orbital Dynamics", is the second special issue covering activities related to Celestial Mechanics. It has 17 papers, which are summarized below.

The paper "Atmospheric close approach with the earth considering drag and lift forces" by Gomes, V. M.; Prado, A. F. B. A.; Pineros, J. M.; and Golebiewska, J. studies the so-called "Aero-Gravity Assisted", which is a maneuver that increases the energy gains of the standard pure gravity maneuver using the atmosphere of the planet. The paper considers close approaches between a spacecraft and the Earth, in cases where the passage is close enough to the Earth and the spacecraft passes by its atmosphere. Drag and lift are considered. Earth and Sun are assumed to be in circular orbits. The main goal is to show the variations of energy of the orbits of the spacecraft due to this close approach, in particular the effects of the atmosphere in the trajectory of the spacecraft.

The paper "Optimal round trip lunar missions based on the patched-conic approximation" by Gagg Filho, L. A. and Fernandes, S. S. approaches the problem of optimal bi-impulsive trajectories to make a round trip to the Moon. The optimization criterion, to be minimized, is the total velocity increment. The dynamical model used is the patched-conic approximation. Each phase of the motion is considered separately to solve the optimization problem. The parameters to be optimized are the phase angle of the point where the space vehicle reaches the sphere of influence of the Moon and the initial velocity at departure. The Sequential Gradient Restoration Algorithm (SGRA) is used to search for the optimal solutions. The results show a good agreement with the similar results available in the literature.

The paper "Optimal low-thrust transfers between coplanar orbits with small eccentricities" by Fernandes, S. S.; Carvalho, F. C.; and Moraes, R. V. obtains a first-order analytical solution for the problem of optimal time-fixed low-thrust limited power transfers in an inverse-square force field, between coplanar orbits with small eccentricities. To obtain the results, canonical transformation theory is used. The short periodic terms are eliminated from the maximum Hamiltonian, expressed in nonsingular orbital elements, through an infinitesimal canonical transformation based in the method of Hori. Closed-form analytical solutions are obtained. The existence of conjugate points is investigated using the Jacobi condition, for long-duration maneuvers.

The paper "Laser altimeter for the deep space mission aster. Modeling and simulation of the instrument operation above a surface with crater" by Brum, A. G. V.; Cruz, F. C.; and Hetem Jr., A. studies the first Brazilian deep space mission, ASTER. This mission is planned to carry onboard a laser altimeter to investigate the triple asteroid system $2001SN_{263}$. The studies involved the modeling of the instrument and its operation and the result was the creation of a package of computer programs to simulate the operation of a pulsed laser altimeter with the operating principle based on the measurement of the time of flight of the traveling pulse. The work main goal is to study the special case involving the modeling of a surface with crater. The approach was the comparison of the return signal obtained from the crater with the expected signal in the case of a flat and homogeneous surface.

The paper "Aerodynamic resistance in upper atmosphere—case of the last stage delta rocket fall in Argentina", by Moreschi, L. D. and Schulz, W. studies the fall of the third stage of a Delta-II launcher in the province of Corrientes, Argentina. Using known orbital data before entry and the location of the impact point, it was possible to refine a trajectory simulation 6D code to propagate the solutions of the dynamic equations until the crash point. It was possible to follow the trajectory until the impact point. The paper compared the effects of two atmospheric models: the static model USSA-76 versus the dynamic model NRLMSISE-00.

The paper "Design of a linear time-invariant control system based on a multi-objective optimization approach" by Santos, W. G.; Rocco, E. M.; and Boge, T. explores the optimal design of a linear time-invariant control system composed of three different types of linear parallel actuators. A solution is proposed to find the best selection of actuators' gains to improve the performance parameters. A discrete multi-objective optimization problem is formulated with the objective functions: overshoot and settling time of the closed-loop response. The results showed that a better performance can be obtained with a systematic multi-objective optimization methodology.

The paper "Studying the lifetime of orbits around moons in elliptic motion", by Domingos, R. C. and Gomes, V. M. studies the problem of finding lifetime of orbits around moons in elliptic motion around their mother planet. The lifetime ends when a collision with the surface of the moon occurs. The mathematical model is the second order expansion of the potential of the disturbing planet, assumed in an elliptical orbit. The results are expressed by graphs showing the lifetime of the orbit as a function of its initial inclination and eccentricity. The inclinations considered are above the critical value of the third-body perturbation. The influence of the eccentricity of the primaries is investigated.

The paper "Numerical and analytical approach for the spin stabilized satellite attitude propagation" by Zanardi, M. C.; Orlando, V.; Motta, G. B.; Pelosi, T.; and Silva, W. R. makes a comparison between numerical and analytical results of the attitude propagation of a spin stabilized spacecraft. External torques are introduced in the equations of motion and the comparisons are made using the following torques: gravity gradient, aerodynamic, solar radiation, magnetic residual, and eddy current. The analytical approach describes the equations of motion in terms of the spin velocity, spin axis, right ascension, and declination angles. The applications are made for the Brazilian satellites SCD1 and SCD2.

The paper "Permanent magnet hall thrusters development for future Brazilian space missions", by Martins, A. A.; Ferreira, J. L.; Miranda, R.; Schellin, A.; Souza, L. A.; Costa, E. G.; Coelho, H. O.; Serra, A. C. B.; and Nathan, F. S. considers the development of a Permanent Magnet Hall Thruster (PHALL) for the UNIESPAÇO program, which is part of the Brazilian Space Activities Program (PNAE) since 2004. The project consists on plasma source design, construction, and characterization of the Hall type propulsion engine using several plasma diagnostics sensors. The paper shows a brief description of the engine, its diagnostics instrumentation, and measured plasma parameters for possible applications of PHALL thrust in orbit transfer maneuvers of the future Brazilian geostationary satellites. The plasma density and temperature are shown. It is also shown the specific impulse, total thrust, propellant flow rate, and necessary power consumption for the orbit raising maneuvers. A numerical simulation of raising the satellite from an altitude of 700 km to 36,000 km using a PHALL thrust operating in the 100–500 mN thrust range is made.

The paper "Unscented Kalman filter for spacecraft attitude estimation using modified Rodrigues parameters and real data", by Garcia, R. V.; Kuga, H. K.; and Zanardi, M. C. formulates a Sigma-Point Kalman filter for attitude estimation using the Modified Rodrigues Parameters and real data of attitude sensors. The filter is used for attitude estimation. The gyro-based model is considered for attitude propagation. The attitude of the satellite is estimated using real data supplied by gyros, Earth sensors, and Sun sensors that are on board the CBERS-2 (China Brazil Earth Resources Satellite) spacecraft. The UKF with MRP shows the results which are similar to those obtained when using the Euler angles.

The paper "Mathematical modeling of spacecraft guidance and control system in 3D space orbit transfer mission", by Shirazi, A. and Mazinan, A. H. studies the performance of a spacecraft during an orbital maneuver where the thrust direction is used as the control. The maneuver is not planar, and there are initial and terminal constraints on the orbits. It is used polynomial functions for the guidance. A genetic algorithm is used to solve the problem. The attitude control system is also modeled to get the spacecraft response. Gas thrusters are used to control the attitude. The results showed that the approach used in the paper satisfies the mission requirement.

The paper "Rigorous treatment of the averaging process for co-orbital motions in the planetary problem", by Robutel, P. developed an analytical Hamiltonian formalism to study the motion of two planets in co-orbital resonance. By constructing a complex domain of holomorphy for the planetary Hamiltonian, the paper estimates the size of the transformation that maps this Hamiltonian to its first order averaged over one of the fast angles. This procedure allows proving rigorous theorems on the behavior of co-orbital motions over a large timescale.

The paper "Orbital maneuvers to reach and explore a triple asteroid", by Formiga, J. K. S. and Santos, D. P. S. studies orbital maneuvers related to a mission to a triple asteroid. A genetic algorithm is used to find multi-impulsive maneuvers to go from the Earth to the asteroid, searching for maneuvers with minimum fuel consumption. Besides that, Swing-By maneuvers with the two smaller bodies are calculated to show the possible gains of energy. The "restricted three-body problem" is used as the mathematical model, with the goal of determining the accuracy of the approximated model.

The paper "Exploring the Moon gravity to escape from the Earth-Moon system", by Santana, S. H. S.; De Melo, C. F.; Macau, E. E. N.; and Winter, O. C. studies escape trajectories from the Earth–Moon system using a gravity assist maneuver in the Moon. It uses a semi-analytical approach. Several studies were performed to find a set of initial conditions (eccentricity and semi-major axes) for the maneuvers. It was possible to identify which orbits generate escape trajectories.

The paper "Close approach of a cloud of particles around an oblate planet", by Gomes, V. M.; Oliveira, G. M. C.; Prado, A. F. B. A.; and Sanchez, D. M. studies close approaches between a cloud of particles and an oblate planet. This cloud is created during the passage of a spacecraft by the periapsis of its orbit. The system is formed by the Sun and the planet, assumed to be in circular orbits around the center of mass of the system, and the cloud of particles. The particles make a close approach, and then, they are dispersed by the gravitational force of the planet. The model uses the planar restricted circular three-body problem plus the effects of the oblateness of the planet. The results show the differences between the behavior of the cloud after the passage, considering or not the effects of the oblateness of the planet.

The paper "Analysis of the orbital evolution of exoplanets", by Carvalho, J. P. S.; Moraes, R. V.; Prado, A. F. B. A.; and Winter, O. C. considers the motion of an exoplanet. A study with respect to possible collisions of the planet with the central star is developed in this paper. An expanded model up to the fifth order is used to analyze the effect of this potential in the orbital elements of the extra solar planet. Numerical simulations were performed using the software Mercury, to compare the results. The analysis showed that the planet collided with the central star in the moment of the first inversion for orbits with high inclinations in various situations. The results of the simulations of the equations developed in this study are consistent with the N-body numerical simulations. The flip of the inclination was also studied.

The paper "On retrograde orbits, resonances and stability", by Morais, M. H. M. presents new results with respect to the stability of retrograde configurations with respect to prograde configurations, in the low-mass ratio regime of the planar circular restricted 3-bodyproblem. Then, it explores the case with mass ratio 0.001 and shows new stability maps in a grid of semi-major axis versus eccentricity for the 2/1 and 1/2 retrograde resonances. The paper also explains how the stability borders of the 2/1 and 1/2 retrograde resonances are related to the geometry of the resonant orbits.

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