Dual-pumped degenerate Kerr oscillator in a silicon nitride microresonator

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We demonstrate a degenerate parametric oscillator in a silicon-nitride microresonator. We use two frequency-detuned pump waves to perform parametric four-wave mixing and operate in the normal group-velocity-dispersion regime to produce signal and idler fields that are frequency degenerate. Our theoretical modeling shows that this regime enables generation of bimodal phase states, analogous to the $\chi^{(2)}$-based degenerate OPO. Our system offers potential for realization of CMOS-chip-based coherent optical computing and an all-optical quantum random number generator.

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Researchers recently proposed using degenerate optical parametric oscillators (OPO’s) to realize a novel form of coherent optical computing by simulating the quantum Ising model [1–3]. Such an approach offers the possibility of solving non-deterministic polynomial time (NP)-hard computation problems [4]. A first proof-of-principle demonstration based on a degenerate 2nd-order nonlinearity $\chi^{(2)}$ OPO using bulk optics offers the promise that this new paradigm could be realized [3, 5]. The approach is based on the non-equilibrium phase transition that occurs at threshold for optical parametric oscillation, in which the generated field with respect to the pump field locks to one of two possible states offset by $\pi$. Below threshold, the signal field is described by a squeezed vacuum state, where the strength of squeezing depends on the ratio of the intracavity power to the threshold power [6, 7]. The vacuum fluctuations of the out-of-phase quadrature are suppressed, while those of the in-phase quadrature are enhanced. It is along the in-phase quadrature that, at threshold, the system bifurcates into two possible states with opposite phases. By utilizing a network of coupled OPO’s, more complex phase-locked states can be achieved which encode the ground-state of the quantum Ising model [1], where the coupling strengths of its spins are equivalent to the coupling between the OPO’s. It has been suggested that such an OPO system offers potential for a simulation device that can outperform any other classical computation [8]. Recently, a 4-OPO system based on the $\chi^{(2)}$ nonlinearity has been demonstrated [2]. Alternatively, the bi-phase state output can be used to realize a quantum random number generator [3] that compared to other implementations requires only classical signal detection (no single photon detection), is intrinsically unbiased, and thus does not need any post-processing of the raw output, which has applications in cryptography [9], Monte Carlo simulations [10], and quantitative finance [11]. The scalability and robustness of these coupled OPO’s will be crucial for these applications.

An alternative approach suggested in [2] is to utilize a $\chi^{(3)}$ nonlinearity to realize an analogous degenerate OPO with dynamics that would mirror the $\chi^{(2)}$ system. The silicon-based photonics platform appears to be highly suited for this application since it allows for a robust, compact, integrated system that offers feasibility for scaling and stability. In recent years, there have been various nonlinear optics demonstrations in microresonators [12–27]. In particular, there has been significant development in frequency comb generation
through optical parametric oscillation in high-$Q$ microresonators [14, 16, 18, 20, 26] that utilizes parametric four-wave mixing (FWM) interactions based on the third-order nonlinearity $\chi^{(3)}$. The silicon nitride (Si$_3$N$_4$) platform offers potential as a platform for coupled degenerate OPOs due to its CMOS compatibility, low linear and nonlinear losses at telecommunication wavelengths, and high nonlinearity and dispersion engineering [28]. Recently, there have been demonstrations of inverse FWM using optical fibers in the normal group-velocity dispersion (GVD) regime [29], and Inagaki, et al. [30] has shown in initial experiments that a dual-pumped $\chi^{(3)}$ fiber OPO's multiplexed in the time domain can be used to simulate a 1-D Ising model.

In this paper, we theoretically and experimentally demonstrate that a dual-pumped, microresonator-based $\chi^{(3)}$ OPO can achieve degenerate oscillation. In our scheme, a Si$_3$N$_4$ microresonator is dual-pumped to generate a frequency-degenerate signal and idler pair through parametric FWM [Fig. 1(a)]. We perform theoretical analysis to determine the conditions under which degenerate oscillation and the associated random binary phase state is achieved and find that it requires normal GVD using a dual-pump. We show experimentally that frequency-degenerate oscillation is achieved by controlling the frequency offset between the two pumps.

Our configuration consists of two pump fields denoted by $A_1$ and $A_2$ that are offset by frequency $\pm \delta$ from the degeneracy point [see Fig. 1(b)]. There has been a number of experimental studies that have investigated parametric FWM with two pump waves [31, 34]. Energy conservation implies that the generated signal and idler fields with amplitudes denoted by $A_s$ and $A_i$, respectively, can grow at a frequency offset $\Delta$ from degeneracy for the signal/idler pair. In order for degenerate oscillation to occur, the parametric gain for the signal and idler must peak at $\Delta = 0$. We model the nonlinear interaction using the nonlinear Schrödinger equation,

$$\frac{dA}{dz} = -i \beta_2 \frac{\partial^2 A}{\partial \tau^2} + i \gamma |A|^2 A,$$

where $\beta_2$ is the GVD parameter and $\gamma$ is the nonlinear parameter proportional to $\chi^{(3)}$. We write the total field with respect to the center mode frequency as

$$A(z, \tau) = A_1(z)e^{i\delta \tau} + A_2(z)e^{-i\delta \tau} + A_s(z)e^{i\Delta \tau} + A_i(z)e^{-i\Delta \tau}.$$  

(2)

We assume that the pump waves are undepleted and have equal powers $P = |A_1|^2 = |A_2|^2$. The solution for the signal/idler pair has an exponential form $e^{\gamma z^2/2}$, where $g$ is the gain parameter,

$$g = \sqrt{[\delta^2 - \Delta^2]} \times [2\gamma P + \beta_2 (\delta^2 - \Delta^2)].$$

(3)

Our analysis indicates that with suitable control of the pump power and offset frequency, it is possible to find a condition for achieving parametric gain at modest normal GVD (i.e., $\beta_2 > 0$), which can be readily achieved by engineering the waveguide cross-section [40]. We investigate the parametric gain profile based on the parameters of a 690×1300 nm cross section Si$_3$N$_4$ waveguide pumped with two single-frequency lasers. Figure 2 shows the dependence of the gain on the dispersion length $L_D = 1/\delta^2|\beta_2|$ and the nonlinear length $L_{NL} = 1/2\gamma P$. We show three different regimes, each with a different ratio between $L_D$ and $L_{NL}$ for both normal and anomalous GVD and find that maximum parametric gain can

![Fig. 1. (a) Four-wave mixing energy level diagram in which two pump pumps are converted to two frequency-degenerate photons. (b) Scheme for our $\chi^{(3)}$ OPO in Si$_3$N$_4$ microresonator, where $A_1$ and $A_2$ are the pump fields, $A_s$ and $A_i$ are the generated signal and idler. $\delta$ is the frequency offset between the pumps and the center frequency, $\Delta$ is the frequency offset from degeneracy between the signal/idler and the center frequency.](image)

![Fig. 2. Parametric gain profile assuming two undepleted non-degenerate pumps. Dashed lines indicate the position of the pumps. Pump separation is 14 nm and the center wavelength between the pumps is 1550 nm. (a)-(c) show the gain profiles in normal GVD ($\beta_2 = 193$ ps$^2$/km) for the ratio between $L_D$ and $L_{NL}$ of 2, 0.5 and 0.2, respectively. The nonlinear coefficient ($\gamma = 0.94\ W^{-1}\ m^{-1}$) is based on the 690×1300 nm cross-section Si$_3$N$_4$ waveguide used in our experiment. (d)-(f) show the gain profiles in anomalous GVD ($\beta_2 = -193$ ps$^2$/km) for the same ratios. The ratio between $L_D$ and $L_{NL}$ is adjusted by changing the pump power. The effect of higher-order dispersion terms is negligible because of the small frequency separation between the pumps as well as the waveguide dispersion.](image)
be achieved at the degenerate signal/idler frequency only when $L_D > L_{NL}$ in the normal GVD regime [Fig. 2(a)]. Operating in this well-defined normal GVD regime is essential in order to implement the bi-phase state, as this ensures that the degenerate pairs have the lowest threshold for oscillation.

Furthermore, in order to ensure degenerate oscillation in a microresonator cavity, the position of the two pumps and the signal/idler within the cavity resonance is critical. Figure 3 shows the resonator conditions in the normal GVD regime. We define the resonator mode frequencies near the frequencies of pump 1, pump 2, and the degenerate signal/idler frequencies as $f_1$, $f_2$, and $f_r$, respectively (indicated in red in Fig. 3). The center frequency between the two pump resonance modes $f_c = \frac{f_1 + f_2}{2}$ is offset from the signal/idler resonance mode by $\Delta f = f_r - f_c$. This frequency offset is dependent on the dispersion and, for a cold cavity with normal GVD, it is given by $\Delta f_{\text{cold}} = v_g/4\pi L_D$, where $v_g$ is the group velocity. In a hot cavity, the frequency offset is due to both dispersion and the Kerr nonlinearity and is given by $\Delta f_{\text{hot}} = \frac{v_g}{4\pi} (\frac{L_E}{L_D} - \frac{1}{L_{NL}})$.

For pump frequencies of $f_{p,1}$ and $f_{p,2}$ (indicated in blue in Fig. 3), we define detuning of the pump frequency from the peak of the resonance as $\delta f = f_{p,1} - f_1 = f_{p,2} - f_2$. The signal/idler position with respect to the resonance for the zero detuning case, where both pumps are tuned to the peak of the respective resonances, is shown in [Fig. 3(a)]. As discussed earlier, for the degenerate signal/idler to oscillate, $L_D > L_{NL}$ must be satisfied, which corresponds to a negative $\Delta f_{\text{hot}}$ that could jeopardize the oscillation of the degenerate pair. If $|\Delta f_{\text{hot}}| > \delta
\nu/2$, where $\delta
\nu$ is the cavity linewidth, the degenerate pair is suppressed. However, we can take advantage of pump detuning to achieve the optimum oscillation conditions in the resonator. First, since $L_{NL}$ is dependent on the total power built-up inside the resonator, pump detuning affects the parametric gain profile required for oscillation. In addition, the pump frequency tuning can be used to control the optimal signal/idler frequency position provided that $|\Delta f_{\text{hot}} - \delta f| < \delta
\nu/2$ [Fig. 3(b)]. This allows for all fields to lie within their respective cavity resonances allowing for degenerate oscillation to occur. In practice, the ultimate detuning range that can be achieved is limited by the thermal effect in the resonator from pumping two different resonances.

We numerically simulate the degenerate OPO using the modified Lugliato-Lefever equation [22,28], which we previously developed for modeling parametric frequency comb generation in microresonators [39]. The model includes effects of higher-order dispersion and self-steepening, which enables simulations over a wide range of conditions. The dispersion of the microresonator mode is simulated using a finite-element mode solver [Fig. 3(a)]. The simulated spectra of the OPO is shown in Fig. 3(b) for a Si$_3$N$_4$ microresonator with a free spectral range (FSR) of 200 GHz and a cross section of 690×1300 nm, which corresponds to normal GVD at the pump wavelengths of 1535 and 1565 nm for the fundamental TM mode. The two pumps are each offset from the frequency degeneracy point by $\delta = 4\times\text{FSR}$. The simulated spectra show that parametric oscillation of the degenerate signal/idler pair occurs only under pump detuning conditions consistent with our analysis above. Multiple simulations with different initial noise conditions indicate that the degenerate signal field settles into one of two phase states which are offset by $\pi$ [Fig. 3(c)], which is precisely the required behavior for
Power (dB) 
-40  -30  -20  -10  0
-40  -30  -20  -10  0
-40  -30  -20  -10  0

Fig. 5. OPO spectra from two pumps for pump offsets $\delta$ of (a) 6.5$\times$FSR, (b) 5$\times$FSR, and (c) 4$\times$FSR. The FSR is 200 GHz. The pump offset of 4$\times$FSR (c) enables the generation of a degenerate signal/idler pair between the pumps, which results in the generation of the bi-phase state.

In conclusion, we theoretically and experimentally demonstrate frequency-degenerate parametric oscillation via dual-pump FWM in $\text{Si}_3\text{N}_4$ microresonators operating in the normal GVD regime. We show that the $\text{Si}_3\text{N}_4$ platform offers the potential of developing a scalable CMOS-compatible platform that utilizes the unique properties of light to solve challenging computational problems and for use as a system for random number generation.

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**References**