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Solving the Multiobjective Environmental/Economic Dispatch Problem using Weighted Sum and ε-Constraint Strategies and a Predictor-Corrector Primal-Dual Interior Point Method

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Abstract This paper proposes a technique for solving the multiobjective environmental/economic dispatch problem using the weighted sum and ε-constraint strategies, which transform the problem into a set of single-objective problems. In the first strategy, the objective function is a weighted sum of the environmental and economic objective functions. The second strategy considers one of the objective functions: in this case, the environmental function, as a problem constraint, bounded above by a constant. A specific predictor-corrector primal-dual interior point method which uses the modified log barrier is proposed for solving the set of single-objective problems generated by such strategies. The purpose of the modified barrier approach is to solve the problem with relaxation of its original feasible region, enabling the method to be initialized with unfeasible points. The tests involving the proposed solution technique indicate i) the efficiency of the proposed method with respect to the initialization with unfeasible points, and ii) its ability to find a set of efficient solutions for the multiobjective environmental/economic dispatch problem.

Keywords Predictor-corrector primal-dual interior point method · Modified barrier · Environmental/economic dispatch problems

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1 Introduction

The economic dispatch (EDP) and environmental dispatch (EnDP) problems are found in the area of electric power systems, which calculate power generation based on its economic aspects EDP, and on the concern to reduce pollutant emissions EnDP. EDP is a nonlinear optimization problem aimed at optimizing the allocation of power among the available generation units, minimizing the cost of fuels used in thermoelectric power generation, and respecting the operational constraints of the generation system. EnDP is a nonlinear optimization problem that seeks to minimize pollutant emissions from burning fossil fuels for the generation of thermoelectric power, while also respecting the constraints in meeting power demands and the operational constraints of generators. The economic/environmental dispatch problem (EEDP) investigated here is formulated as a multiobjective optimization problem, for which the aim is to optimize power generation costs while concomitantly reducing pollutant emissions, which are conflicting objectives.

There are two most used strategies in the literature to transform the EEDP multiobjective optimization problem into a set of single-objective optimization problems: i) the weighted sum strategy (WS), which considers a weighted sum of the environmental and economic functions as the objective function; and ii) the ε-constraint strategy (ε-C), which considers minimization of the economic problem subject to the constraints of maximum allowable pollutant emissions by each power generation unit. Both strategies are described in (Haimes et al. 1971) and (Miettinen 1999).

Various researchers have proposed solution methods for solving the multiobjective EEDP problem. El-Keib et al. (1994) and Hu and Wee (1994) used the WS strategy. El-Keib et al. (1994) solved the problem using Lagrangian relaxation, while Hu and Wee (1994) solved it by means of hierarchical systems. Ramanathan (1994) formulated the EEDP using both WS and ε-C strategies. The solution technique he used also involved Lagrangian relaxation. Ferial and EI-Hawary (1994) presented a summary of the main solution techniques applied to EEDP up to 1994. Heuristics-based approaches to solve the EEDP have also been researched. Wong et al. (1995) used the WS strategy and solved the EEDP problem using the simulated annealing method. Abido (2003) solved the EEDP directly using a strength Pareto evolutionary algorithm for solving multiobjective problems. Abido (2006) compared three heuristic algorithms for the direct solution of the EEDP: non-dominated sorting genetic algorithms, niched Pareto genetic algorithms, and strength Pareto evolutionary algorithms. Bayón et al. (2012), for the first time, presented the exact analytical solution for the EEDP. Jubril et al. (2013) solved the WS strategy using semidefinite programming. In the last paper, the authors also point out that the evolutionary approaches only estimate the Pareto front, while the nonlinear approaches guarantee the capture of the entire Pareto front for convex functions.

In Zhang et al. (2009), an interior point method (IPM) is proposed to solve the EEDP, using the ε-constraint strategy. The interior point methods are the state-of-the-art for solving the set of single-objective sub-problems generated by the ε-constraint strategy (when this strategy is applied to solving the EEDP). This method, however, works in the interior of the feasible region, and does not have the ability of handling with unfeasible initial points.

This paper proposes a predictor-corrector primal-dual modified log-barrier (PCPDMLB) method which is capable of handling with such unfeasible initial points. Obtaining feasible initial points for the EEDP is generally a difficult task, especially when it is formulated using the ε-C strategy. This feature further complicates the use of interior point methods for solving the problem, once that these methods work in the interior of the feasible region.

The PCPDMLB is a variant of the predictor-corrector primal-dual interior point methods presented by Wu et al. (1994) and Mehrotra and Sun (1992). However, our approach uses the modified log-barrier function defined by Polyak (1992), which allows for the initialization of the problem with unfeasible points. The iterative procedure of the method operates with points inside the relaxed region and outside the original region, seeking to achieve a sequence of feasible points such that, in optimality, the solution is feasible and inside the closure of the original region.

Differently from the method described in Mehrotra and Sun (1992), the PCPDMLB method considers a centering procedure that exploits the barrier parameter in the predictor and corrector steps to calculate the search directions, preventing the exterior points from being projected outside the relaxed region. The modified log-barrier procedure influences the determination of the search directions of the proposed method, differentiating it from those normally applied in classical predictor-corrector primal-dual IPM. The search directions of the predictor step are used immediately in a single iteration, in the corrector step, when the second-order approximants of the system of search directions pertaining to the complementarity conditions are considered.

The contributions of this paper are summarized as follows: i) the modified log-barrier approach has not been applied to the multiobjective EEDP; ii) the PCPDMLB method is capable of handling with unfeasible points, thus avoiding preprocessing or specific heuristics for finding a feasible initial point for the EEDP; iii) the PCPDMLB method proposes a new centering procedure that exploits the barrier parameter in the predictor and corrector steps to calculate the search directions; and iv) the sparse structure of the search direction system is exploited in the predictor and corrector steps of the method proposed. The proposed PCPDMLB method is described in the next section.

2 Predictor-Corrector Primal-Dual Modified Log-Barrier Method

The proposed predictor-corrector primal-dual interior point algorithm is based on the work of Monteiro et al. (1990), Kojima et al. (1989), Wu et al. (1994), and Mehrotra and Sun (1992), who used procedures based on the logarithmic barrier function presented by Frisch (1955) and further developed by Fiacco and McCormick (1990).

In Polyak (1992), the author applies the concept of a relaxed boundary region to the functional inequality constraints $h(x) \leq 0$, such that $h(x) \leq -\mu$ (where μ is the parameter associated with the modified barrier). The works presented in Sousa et al. (2009, 2012) also apply the concept of a relaxed boundary region; however, this concept is applied to the non-negativity region of the slack variables $z \geq 0$, such that $h(x) + z = 0$; $z \geq -\mu$. This new scheme seems to work much better for the optimal power flow (OPF) problem studied in Sousa et al. (2009, 2012).

The work here presented follows the approach described in Sousa et al. (2009, 2012), but here we introduce a new and efficient way of performing the predictor step which also incorporates a centering procedure. The search directions of the method are determined using a proposed predictor-corrector procedure which is a variant of the one presented by Mehrotra and Sun (1992). The proposed method considers a centering procedure that exploits the barrier parameter in the predictor and corrector steps to calculate the search directions, preventing the exterior points from being projected outside the relaxed region. Our method also differs from that proposed in Sousa et al. (2009, 2012) in the sense that sparse structure of the search direction system is exploited in the predictor and corrector steps. The modified log-barrier procedure influences the determination of the search directions of the proposed method, differentiating it from those normally applied in classical predictor-corrector primal-dual interior point methods. Also, our method differs from the methods described in Sousa et al. (2009, 2012) in the sense that those methods were proposed for solving the OPF problem, while our method is proposed for coping with unfeasible initial points associated with the set of single-objective subproblems generated by the ε-constraint strategy when applied to the EEDP, which are nonlinear programming problems.

The method is defined for a general nonlinear programming problem whose general form is given in (1), which is equivalent to the problem (2) when the slack and excess variables z_1, z_2, and z_3 are added:

$$\begin{aligned} &\text{Min } f(x) \\ &subject\ to: \\ &\quad g(x) = 0; \\ &\quad h(x) \leq u; \\ &\quad l_1 \leq x \leq l_2; \end{aligned} \tag{1}$$

where $f(x): \mathbb{R}^n \to \mathbb{R}$, $g(x): \mathbb{R}^n \to \mathbb{R}^m$, $h(x): \mathbb{R}^n \to \mathbb{R}^r$, $x, l_1, l_2 \in \mathbb{R}^n$, $u \in \mathbb{R}^r$:

$$\begin{aligned} &\text{Min } f(x) \\ &subject\ to: \\ &\quad g(x) = 0; \\ &\quad h(x) + z_1 - u = 0; \ \ z_1 \geq 0; \\ &\quad x + z_2 - l_2 = 0; \ \ z_2 \geq 0; \\ &\quad l_1 - x + z_3 = 0; \ z_3 \geq 0. \end{aligned} \tag{2}$$

This problem can be addressed using the log-barrier strategy, as defined in (3):

$$\begin{aligned} &\text{Min } f(x) - \mu \sum_{j=1}^{r} \ln(z_1)_j - \mu \sum_{j=1}^{n} \ln(z_2)_j - \mu \sum_{j=1}^{n} \ln(z_3)_j \\ &subject\ to: \\ &\quad g(x) = 0; \\ &\quad h(x) + z_1 - u = 0; \\ &\quad x + z_2 - l_2 = 0; \\ &\quad l_1 - x + z_3 = 0; \end{aligned} \tag{3}$$

where $\mu > 0$ is the barrier parameter.

Based on Sousa et al. (2009, 2012), the modified barrier procedure incorporated into the method initially considers the problem (3) expressed in its standard form and adds the barrier parameter μ to the components of the variable $z_1 > 0$, relaxing and transforming them into $(z_1)_i > -\mu; i = 1, \ldots, r$, or, equivalently, into $z_1 > -\mu_1; e_1 = (1, \ldots, 1)^T \in R^r$. This procedure expands the original feasible set of the problem, enabling the method to start with unfeasible points, and to deal with the constraint $h(x) + z_1 - u = 0$.

The variable z_1 is rewritten in the problem as $z_1^{\diamond}$, according to (4):

$$(z_1^{\diamond})_i = 1 + \mu^{-1}(z_1)_i; \ i = 1, \ldots, r, \tag{4}$$

thus resulting in problem (5), which expands the original feasible set of problem (3):

$$\begin{aligned} &\text{Min } f(x) - \mu \sum_{j=1}^{r} \delta_j \ln(z_1^{\diamond})_j - \mu \sum_{j=1}^{n} \ln(z_2)_j - \mu \sum_{j=1}^{n} \ln(z_3)_j \\ &subject\ to: \ g(x) = 0; \\ &\qquad\qquad h(x) + z_1 - u = 0; \\ &\qquad\qquad x + z_2 - l_2 = 0; \\ &\qquad\qquad l_1 - x + z_3 = 0; \end{aligned} \tag{5}$$

where $\delta \in R^r$ is called the Lagrange multiplier estimate related to the new variable $z_1^{\diamond} > 0$, which is associated with constraint $h(x) + z_1 - u = 0$. Problem (5) is redefined by the unconstrained problem (6) by means of the modified barrier Lagrangian function L:

$$\begin{aligned} &\text{Min } L(x, z_1, z_2, z_3, \lambda_0, \lambda_1, \lambda_2, \lambda_3) \\ &\quad = f(x) - \mu \sum_{j=1}^{r} \delta_j \ln(z_1^{\diamond})_j - \mu \sum_{j=1}^{n} [\ln(z_2)_j + \ln(z_3)_j] \\ &\qquad + \sum_{j=1}^{m} (\lambda_0)_j [g_j(x)] + \sum_{j=1}^{r} (\lambda_1)_j [h_j(x) - (u)_j + (z_1)_j] \\ &\qquad + \sum_{j=1}^{n} (\lambda_2)_j [(x)_j - (l_2)_j + (z_2)_j] \\ &\qquad + \sum_{j=1}^{n} (\lambda_3)_j [-(x)_j + (l_1)_j + (z_3)_j] \end{aligned} \tag{6}$$

where $\lambda_0 \in \mathbb{R}^m$ is the vector of Lagrange multipliers associated with the equality constraints, and $\lambda_1 \in \mathbb{R}^r$ and $\lambda_2, \lambda_3 \in \mathbb{R}^n$ are the vectors of Lagrange multipliers associated with the inequality constraints of the problem (5).

The conditions required for first-order Karush-Kuhn-Tucker (KKT) optimality are applied to the unconstrained problem (6), according to (7), resulting in the nonlinear system shown in (8):

$$\nabla L(x, z_1, z_2, z_3, \lambda_0, \lambda_1, \lambda_2, \lambda_3) = 0. \tag{7}$$

$$\begin{aligned} &\nabla f(x) + \nabla g(x)^t \lambda_0 + \nabla h(x)^t \lambda_1 + \lambda_2 - \lambda_3 = 0; \\ &g(x) = 0; \\ &h(x) + z_1 - u = 0; \\ &x + z_2 - l_2 = 0; \\ &-x + z_3 + l_1 = 0; \\ &Z_1^{\diamond} \Lambda_1 e_1 - \delta\mu = 0; \\ &Z_2 \Lambda_2 e_2 - \mu e_2 = 0; \\ &Z_3 \Lambda_3 e_2 - \mu e_2 = 0, \end{aligned} \tag{8}$$

where $e_2 = (1, \ldots, 1)^T \in R^n$.

The PCPDMLB method and its algorithm will be developed in the following sections, considering the nonlinear system (8).

2.1 Updating the Lagrange Multiplier Estimate and Its Direction

The Lagrange multiplier estimate $\delta \in R^r$, which premultiplies the barrier parameter μ, is updated according to rule (9) proposed by Polyak (1992):

$$\delta_i^{k+1} = \frac{\mu_k \delta_i^k}{\mu_k + z_{1_i}^k}, \tag{9}$$

where $i = 1, \ldots, r$; with its respective direction d_δ^k determined by the following update of $\delta \in R^r$ in iteration $k+1$:

$$\delta_i^{k+1} = \delta_i^k + d_{\delta_i}^k = \frac{\mu_k \delta_i^k}{\mu_k + z_{1_i}^k} \Leftrightarrow d_{\delta_i}^k = \frac{\mu_k \delta_i^k}{\mu_k + z_{1_i}^k} - \delta_i^k$$
$$\Leftrightarrow d_{\delta_i^k} = \frac{\mu_k \delta_i^k - (\mu_k + z_{1_i}^k)\delta_i^k}{\mu_k + z_{1_i}^k} \Leftrightarrow d_{\delta_i}^k = -\frac{z_{1_i}^k \delta_i^k}{\mu_k + z_{1_i}^k},$$

where $i = 1, \ldots, r$.

Therefore, direction d_δ^k is calculated according to (10):

$$d_{\delta_i}^k = -\frac{z_{1_i}^k \delta_i^k}{\mu_k^+ z_{1_i}^k}, \tag{10}$$

where $i = 1, \ldots, r$.

2.2 Search Directions—Predictor-Corrector Procedure

In the following sections, the search directions of the method are presented by the detailed description of the predictor and corrector steps of the method.

2.2.1 Predictor Step

The predictor step uses a Taylor first-order approximation to evaluate (8) on the point $r^{k+1} = r^k + dr^k$, thus obtaining the Newton's system (11):

$$J(r^{(k)})d^{(k)} = -\nabla L(r^{(k)}), \tag{11}$$

where

$$r^{k+1} = \left(x^{k+1}, z_1^{k+1}, z_2^{k+1}, z_3^{k+1}, \lambda_0^{k+1}, \lambda_1^{k+1}, \lambda_2^{k+1}, \lambda_3^{k+1}\right)^T,$$
$$r^k = \left(x^k, z_1^k, z_2^k, z_3^k, \lambda_0^k, \lambda_1^k, \lambda_2^k, \lambda_3^k\right)^T,$$
$$d^{(k)} = \left(d_x^k, d_{z_1}^k, d_{z_2}^k, d_{z_3}^k, d_{\lambda_0}^k, d_{\lambda_1}^k, d_{\lambda_2}^k, d_{\lambda_3}^k\right)^T,$$

and $J(r^{(k)})$ is the Jacobian matrix, whose element (i, j) is given by (12):

$$\left[\frac{\partial L_i(r)}{\partial r_j}\right]_{r=r^k}. \tag{12}$$

In (11), the second-order terms that occur in the complementarity equations are neglected in the predictor step and reused in the corrector step of the method.

The system (11) is rewritten as (13):

$$\begin{aligned}
&\nabla L_{xx}^2 d_x^k + \nabla g(x)^t d_{\lambda_0}^k + \nabla h(x)^t d_{\lambda_1}^k + d_{\lambda_2}^k - d_{\lambda_3}^k = m^k,\\
&\nabla g(x) d_x^k = t_0^k,\\
&\nabla h(x) d_x^k + d_{z_1}^k = t_1^k,\\
&d_x^k + d_{z_2}^k = t_2^k,\\
&-d_x^k + d_{z_3}^k = t_3^k,\\
&Z_1^\diamond d_{\lambda_1}^k + \Lambda_1 d_{z_1}^k = \pi_1^{\diamond k},\\
&Z_2 d_{\lambda_2}^k + \Lambda_2 d_{z_2}^k = \pi_2^k,\\
&Z_3 d_{\lambda_3}^k + \Lambda_3 d_{z_3}^k = \pi_3^k,
\end{aligned} \tag{13}$$

where

$$\nabla_{xx}^2 L = \nabla^2 f(x) + \lambda_0^t \nabla^2(x) + \lambda_1^t \nabla^2 h(x),$$

and

$$-\nabla L(r^{(k)}) = \left(m^k, t_0^k, t_1^k, t_2^k, t_3^k, \pi_1^k, \pi_2^k, \pi_3^k\right)^T,$$

are the first-order residuals of the approximation performed, and are defined by (14):

$$\begin{aligned}
m^k &= -\nabla f(x) - \nabla g(x)^t \lambda_0 - \nabla h(x)^t \lambda_1 - \lambda_2 + \lambda_3;\\
t_0^k &= -g(x);\\
t_1^k &= -h(x) - z_1 + u;\\
t_2^k &= -x - z_2 + l_2;\\
t_3^k &= x - z_3 - l_1;\\
\pi_1^k &= -Z_1^\diamond \Lambda_1 e_1 + \delta\mu;\\
\pi_2^k &= -Z_2 \Lambda_2 e_2 + \mu e_2;\\
\pi_3^k &= -Z_3 \Lambda_3 e_2 + \mu e_2.
\end{aligned} \tag{14}$$

Note that in our approach, the barrier parameter is also used in the predictor step (see the last 3 equations in (14)) to prevent the exterior points from being projected outside the relaxed region. The search directions $d^{(k)}$ of the predictor step are determined based on the solution of the linear system (14), using the sparse block structure of the Jacobian matrix. The directions of the predictor step $d^{(k)}$ are determined according to (15). Note that the insertion of the variable δ^k, whose update in each iteration k, given by $\delta^k + d_\delta^k$, influences the calculation of the directions d_x^k, $d_{\lambda_0}^k$, and $d_{\lambda_1}^k$. This procedure differentiates the IPM here proposed from classical log-barrier approaches:

$$d_{\delta_i}^k = -\frac{z_i^k \delta_i^k}{\mu_k + z_{1_i}^k};\ i = 1, \ldots, r;$$
$$d_{\lambda_0}^k = (\nabla g(x)\theta_k \nabla g(x)^t)^{-1}(-t_0^k + \nabla g(x)\theta_k(m^k + p^k + \mu \nabla h(x)(Z_1^\diamond)^{-1} d_\delta^k));$$

$$
\begin{aligned}
d_x^k &= \theta_k m^k + \theta_k p^k - \theta_k \nabla g(x)^t d_{\lambda_0}^k + \mu(\nabla h(x)(Z_1^{\diamond})^{-1} d_\delta^k)\\
d_{z_1}^k &= t_1^k - \nabla h(x) d_x^k;\\
d_{z_2}^k &= t_2^k - d_x^k;\\
d_{z_3}^k &= t_3^k + d_x^k;\\
d_{\lambda_1}^k &= (Z_1^{\diamond})^{-1}[\pi_1^k + \mu d_\delta^k - \Lambda_1(t_1^k - \nabla h(x) d_x^k)];\\
d_{\lambda_2}^k &= Z_2^{-1}[\pi_2^k - \Lambda_2(t_2^k - d_x^k)];\\
d_{\lambda_3}^k &= Z_3^{-1}[\pi_3^k - \Lambda_3(t_3^k + d_x^k)],
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
\theta_k^{-1} &= \nabla h(x)^t (Z_1^{\diamond})^{-1} \Lambda_1 \nabla h(x) + Z_2^{-1}\Lambda_2\\
&\quad + Z_3^{-1}\Lambda_3 + \nabla_{xx}^2 L,\\
p^k &= -\nabla h(x)^t (Z_1^{\diamond})^{-1}\pi_1^k + \nabla h(x)^t (Z_1^{\diamond})^{-1}\Lambda_1 t_1^k\\
&\quad - Z_2^{-1}\pi_2^k + Z_2^{-1}\Lambda_2 t_2^k + Z_3^{-1}\pi_3^k - Z_3^{-1}\Lambda_3 t_3^k,
\end{aligned}
\tag{16}
$$

and $(Z_1^{\diamond})^{-1}, Z_2^{-1}, Z_3^{-1}, \Lambda_1, \Lambda_2, \Lambda_3$ are the diagonal matrices, whose elements are given, respectively, by

$$((z_1^{\diamond})^{-1})_i, (z_2^{-1})_i, (z_3^{-1})_i, (\lambda_1)_i, (\lambda_2)_i, (\lambda_3)_i, \quad i = 1, \ldots, n,$$

2.2.2 Corrector Step

Using the directions determined in the predictor step $d^{(k)}$ in the current iteration k, the method must calculate the directions of the corrector step $\tilde{d}^{(k)}$based on second-order terms that were neglected in the predictor step for the complementarity equations. The search direction procedure of the corrector step is analogous to that performed in the predictor step and determines the search directions in this step by solving the linear system (17):

$$J(r^{(k)})\tilde{d}^{(k)} = -\nabla \tilde{L}(r^{(k)}), \tag{17}$$

where $\nabla \tilde{L}(r^{(k)})$ is obtained considering $\nabla L(r^{(k)})$ of the predictor step, using the terms $\pi_1^k, \pi_2^k,$ *and* π_3^k modified by the inclusion of the second-order terms neglected in the predictor step, which are expressed as (18):

$$
\begin{aligned}
\tilde{\pi}_1^k &= \pi_1^k - D_{z_1} D_{\lambda_1} e_1;\\
\tilde{\pi}_2^k &= \pi_2^k - D_{z_2} D_{\lambda_2} e_2;\\
\tilde{\pi}_3^k &= \pi_3^k - D_{z_3} D_{\lambda_3} e_2.
\end{aligned}
\tag{18}
$$

$D_{z_1}, D_{z_2}, D_{z_3}, D_{\lambda_1}, D_{\lambda_2}, D_{\lambda_3}$ are the diagonal matrices whose elements are $(d_{z_1})_i, i = 1, \ldots, r$, $(d_{z_2})_i, (d_{z_3})_i, i = 1, \ldots, n$, and $(d_{\lambda_1})_i, i = 1, \ldots, r$, $(d_{\lambda_2})_i, (d_{\lambda_3})_i, i = 1, \ldots, n$, respectively.

Note that the directions $\tilde{d}_{z_1}^k, \tilde{d}_{z_2}^k, \tilde{d}_{z_3}^k, \tilde{d}_{\lambda_1}^k, \tilde{d}_{\lambda_2}^k, \tilde{d}_{\lambda_3}^k$ defined in the predictor step are used for the redefinition of the residuals $\tilde{\pi}_1^k, \tilde{\pi}_2^k, \tilde{\pi}_3^k$ of the corrector step.

The directions $(\tilde{d})^k = (\tilde{d}_\delta^k, \tilde{d}_x^k, \tilde{d}_{\lambda_0}^k, \tilde{d}_{z_1}^k, \tilde{d}_{z_2}^k, \tilde{d}_{z_3}^k, \tilde{d}_{\lambda_1}^k, \tilde{d}_{\lambda_2}^k, \tilde{d}_{\lambda_3}^k)$ of the corrector step and the residual $\tilde{p}^k$are determined in the same way as in the predictor step, considering the residuals of the corrector step $\tilde{\pi}_1^k, \tilde{\pi}_2^k, \tilde{\pi}_3^k$ determined in (18). Directions $\tilde{d}_{\lambda_0}^k$, $\tilde{d}_x^k$, and $\tilde{d}_{\lambda_1}^k$ are updated explicitly by:

$$
\begin{aligned}
\tilde{d}_{\lambda_0}^k &= (\nabla g(x)\theta_k \nabla g(x)^t)^{-1}(-t_0^k + \nabla g(x)\theta_k(m^k + \tilde{p}^k + \ldots\\
&\quad \ldots + \mu \nabla h(x)(Z_1^{\diamond})^{-1} d_\delta^k));\\
\tilde{d}_x^k &= \theta_k m^k + \theta_k \tilde{p}^k - \theta_k \nabla g(x)^t \tilde{d}_{\lambda_0}^k + \mu(\nabla h(x)(Z_1^{\diamond})^{-1} d_\delta^k);\\
\tilde{d}_{\lambda_1}^k &= Z_1^{-1}[\tilde{\pi}_1^k + \mu d_\delta^k - \Lambda_1(t_1^k - \nabla h(x) d_x^k)];
\end{aligned}
$$

2.3 Step Length

Once the directions have been determined, the step length in this direction should be calculated to obtain new points, which are expressed by (19):

$$
\begin{aligned}
\delta_i^{k+1} &= \frac{\mu_k \delta_i^k}{\mu_k^+ z_{1_i}^k}; \quad i = 1, \ldots, r;\\
x^{k+1} &= x^k + \alpha_P \tilde{d}_x^k;\\
z_1^{k+1} &= z_1^k + \alpha_P \tilde{d}_{z_1}^k;\\
z_2^{k+1} &= z_2^k + \alpha_P \tilde{d}_{z_2}^k;\\
z_3^{k+1} &= z_3^k + \alpha_P \tilde{d}_{z_3}^k;\\
\lambda_0^{k+1} &= \lambda_0^k + \alpha_D \tilde{d}_{\lambda_0}^k;\\
\lambda_1^{k+1} &= \lambda_1^k + \alpha_D \tilde{d}_{\lambda_1}^k;\\
\lambda_2^{k+1} &= \lambda_2^k + \alpha_D \tilde{d}_{\lambda_2}^k;\\
\lambda_3^{k+1} &= \lambda_3^k + \alpha_D \tilde{d}_{\lambda_3}^k.
\end{aligned}
\tag{19}
$$

In (19), the points should be determined so as to ensure the non-negativity of the following variables: $x^{k+1}, z_1^{k+1}, z_2^{k+1}, z_3^{k+1}, \lambda_1^{k+1}, \lambda_2^{k+1}, \lambda_3^{k+1}$.

Using the strategy presented by Granville (1994), the primal (α_P) and dual (α_D) steps are calculated according to (20) and (21):

$$
\alpha_P = \min\left\{
\begin{array}{l}
\min\limits_{(z_1)_i>0 \text{ e } (d_{z_1}^k)_i<0}\left\{-\frac{(z_1^k)_i}{(d_{z_1}^k)_i}\right\},\\
\min\limits_{(z_2)_j>0 \text{ e } (d_{z_2}^k)_j<0}\left\{-\frac{(z_2^k)_j}{(d_{z_2}^k)_j}\right\},\\
\min\limits_{(z_3)_j>0 \text{ e } (d_{z_3}^k)_j<0}\left\{-\frac{(z_3^k)_j}{(d_{z_3}^k)_j}\right\}, 1
\end{array}
\right\}
\tag{20}
$$

$$
\alpha_D = \min\left\{
\begin{array}{l}
\min\limits_{(\lambda_1)_i>0 \text{ e } (d_{\lambda_1}^k)_i<0}\left\{-\frac{(\lambda_1^k)_i}{(d_{\lambda_1}^k)_i}\right\},\\
\min\limits_{(\lambda_2)_j>0 \text{ e } (d_{\lambda_2}^k)_j<0}\left\{-\frac{(\lambda_2^k)_j}{(d_{\lambda_2}^k)_j}\right\},\\
\min\limits_{(\lambda_3)_j>0 \text{ e } (d_{\lambda_3}^k)_j<0}\left\{-\frac{(\lambda_3^k)_j}{(d_{\lambda_3}^k)_j}\right\}, 1
\end{array}
\right\}.
\tag{21}
$$

2.4 Updating the Barrier Parameter

The barrier parameter is updated according to the rule proposed by Polyak (1992), which ensures the reduction of L. This update is defined by (22):

$$\mu_{k+1} = \mu_k (1 - \sigma_k)/\phi; \;\; \phi \geq 1, \tag{22}$$

where

$$\sigma_k = \max_i \left\{ \frac{\mu_k}{(z_1)_i + \mu_k} \right\}, \quad i = 1, \ldots, r.$$

The initial value of ϕ should not be too high, thus avoiding oscillatory behavior, or too low, thus preventing the method from stopping prematurely. Therefore, $\phi = 1.618$ (golden ratio) was fitted to the PCPDMLB method to calculate the barrier parameter, since this value enabled a better performance of the proposed method.

2.5 Stop Criterion

In this work, the stop criterion is determined based on Wright (1997) .Typical tests to ensure that a solution $(x, z_1, z_2, z_3, \lambda_0, \lambda_1, \lambda_2, \lambda_3)$ is a locally optimal solution for the predictor step are determined by

i) Primal feasibility: $\|t_i^k\| \leq \varepsilon_1; \quad i = 0, 1, 2, 3;$
ii) Dual feasibility: $\|m^k\| \leq \varepsilon_2;$
iii) Complementary slackness: $\|\tilde{\pi}_i^k\| \leq \varepsilon_3; \quad i = 1, 2, 3,$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are predefined tolerances. Of course, other criteria can be adopted according to their application to a specific problem.

2.6 Algorithm

STEP 1 – (Initialization of the Algorithm):
Adjust $k = 0$. Select initial values for: $x^0, \; \lambda_0^0, \lambda_1^0, \lambda_2^0, \lambda_3^0$; select the barrier parameter μ^0, and the relative errors ε_1,ε_2, ε_3 , and sufficiently small positive numbers. Calculate z_1^0, z_2^0, z_3^0 as follows:

$$z_1^0 - \mu e_1 = u - h(x^0),$$
$$z_2^0 = l_2 - x^0,$$
$$z_3^0 = x^0 - l_3.$$

STEP 2 – (Residuals—Predictor Step):
Calculate the residuals $(m^k, t_0^k, t_1^k, t_2^k, t_3^k, \pi_1^k, \pi_2^k, \pi_3^k)$ of the predictor step defined in (14), the residual p^k and the matrix θ_k^{-1} defined in (16).

STEP 3 – (Movement Directions—Predictor step):
Calculate the movement directions of the predictor step: $(d_\delta^k, d_x^k, d_{z_1}^k, d_{z_2}^k, d_{z_3}^k, d_{\lambda_0}^k, d_{\lambda_1}^k, d_{\lambda_2}^k, d_{\lambda_3}^k)$ defined in (15).

STEP 4 – (Residuals – Corrector step):
Calculate the residuals of the corrector step $\tilde{\pi}_1^k, \tilde{\pi}_2^k, \tilde{\pi}_3^k$ defined in (18).

STEP 5 – (Optimality Test):
Stop Criteria:

i) Primal feasibility: $\|t_i^k\| \leq \varepsilon_1; \quad i = 0, 1, 2, \; 3;$
ii) Dual feasibility: $\|m^k\| \leq \varepsilon_2;$
iii) Complementary slackness: $\|\tilde{\pi}_i^k\| \leq \varepsilon_3; \quad i = 1, 2, 3.$

If the stop criteria are satisfied, then **STOP**. The solution obtained is optimal. Otherwise, go on to step 6.

STEP 6 – (Movement Directions—Corrector Step):
Calculate the residual $\tilde{p}^k$and the movement directions of the corrector step $(\tilde{d}_\delta^k, \tilde{d}_x^k, \tilde{d}_{\lambda_0}^k, \tilde{d}_{z_1}^k, \tilde{d}_{z_2}^k, \tilde{d}_{z_3}^k, \tilde{d}_{\lambda_1}^k, \tilde{d}_{\lambda_2}^k, \tilde{d}_{\lambda_3}^k)$.

STEP 7 – (Unlimitedness Test):
If $(\tilde{d}_{z_1})_i \geq 0$ or $(\tilde{d}_{z_2})_j \geq 0$ ou $(\tilde{d}_{z_3})_j \geq 0, \; \begin{matrix} \forall i = 1, \ldots, r \\ \forall j = 1, \ldots, n \end{matrix}$, and z_1^k, z_2^k, z_3^k are feasible, then **STOP**, the solution is unlimited. Otherwise, go on to step 8.

STEP 8 – (Step Length):
Considering the primal and dual feasibility conditions, the lengths of steps α_P and α_D are determined by (20) and (21), respectively.

STEP 9 – (New Point):
Update the barrier parameter μ_{k+1} according to (22), the Lagrange multiplier estimates δ_i^{k+1} and the variables $x^{k+1}, z_1^{k+1}, z_2^{k+1}, z_3^{k+1}, \lambda_0^{k+1}, \lambda_1^{k+1}, \; \lambda_2^{k+1}, \lambda_3^{k+1}$ according to (19).
Perform $k \leftarrow k + 1$ and return to step 2.

2.7 PCPDLB Method

In this section, problem (1) is changed according to (23). The purpose of this alteration is to describe a simpler problem where functional inequality constraints $h(x) \leq 0$ are not present. Such problem appears in Sect. 3.5, when the WS strategy is applied to solving the multiobjective EEDP:

$$\begin{aligned} &\text{Min } f(x) \\ &subject\ to: \\ &\quad g(x) = 0; \\ &\quad l_1 \leq x \leq l_2. \end{aligned} \tag{23}$$

Due to this simplification in (23), the modified barrier function is no longer necessary in this case, since the inequality constraints are no longer present in (23). Therefore, problem (23) can be solved by a predictor-corrector primal-dual interior point algorithm (PCPDLB) that takes into account a classical barrier function. This method can be obtained by assuming some simplifications on the proposed PCPDMLB method, as described as follows:

i) Primal and dual variables z_1 e λ_1 associated with $h(x) \leq 0$ are not considered in the solution technique described in section 2;
ii) The directions and residuals of the predictor and corrector steps related to z_1 e λ_1 are not considered, as well as all the calculations associated with such variables.

Taking in account all the modifications previously mentioned, the algorithm of the PCPDLB method can be obtained by assuming some simplifications in the algorithm described in section 2.6 for the PCBDMLB method, such as

- The residuals m^k and p^k of the predictor step and the matrix θ_k^{-1} are calculated according to (24):

$$\begin{aligned} m^k &= -\nabla f(x) - \nabla g(x)^t \lambda_0 - \lambda_2 + \lambda_3; \\ p^k &= -Z_2^{-1}\pi_2^k + Z_2^{-1}\Lambda_2 t_2^k + Z_3^{-1}\pi_3^k - Z_3^{-1}\Lambda_3 t_3^k; \\ \theta_k^{-1} &= Z_2^{-1}\Lambda_2 + Z_3^{-1}\Lambda_3 + \nabla_{xx}^2 L. \end{aligned} \tag{24}$$

- The directions $d_{\lambda_0}^k$ and d_x^k of the predictor step are calculated by (25):

$$\begin{aligned} d_{\lambda_0}^k &= (\nabla g(x)\theta\nabla g(x)^t)^{-1}(-t_0^k + \nabla g(x)\theta m^k \\ &\quad + \nabla g(x)\theta p^k); \\ d_x^k &= \theta m^k + \theta p^k - \theta\nabla g(x)^t d_{\lambda_0}^k. \end{aligned} \tag{25}$$

- The directions $d_{\lambda_0}^k$ and d_x^k of the corrector step are calculated by (26):

$$\begin{aligned} \tilde{d}_{\lambda_0}^k &= (\nabla g(x)\theta\nabla g(x)^t)^{-1}(-t_0^k + \nabla g(x)\theta m^k \\ &\quad + \nabla g(x)\theta \tilde{p}^k) \\ \tilde{d}_x^k &= \theta m^k + \theta \tilde{p}^k - \theta\nabla g(x)^t \tilde{d}_{\lambda_0}^k, \end{aligned} \tag{26}$$

where

$$\tilde{p}^k = -Z_2^{-1}\tilde{\pi}_2^k + Z_2^{-1}\Lambda_2 t_2^k + Z_3^{-1}\tilde{\pi}_3^k - Z_3^{-1}\Lambda_3 t_3^k. \tag{27}$$

The EDP and EnDP are described in the next section, as well as the multiobjective EEDP investigated in this paper.

3 Dispatch Problems

The EDP calculates thermoelectric power generation based on the optimization of its economic aspects, while the EnDP calculates power generation based on the reduction in the emission of pollutants produced by the fuel consumed to generate power. The formulation of the two problems is described in detail below.

3.1 Economic Dispatch Problem

The purpose of the EDP is the optimal allocation of electricity demand among the available generation units, while satisfying operational constraints and minimizing generation costs. The EDP was proposed by Steinberg and Smith (1943) and Gent and Lamont (1971). Mathematically, the EDP consists of minimizing the sum of the costs of generation $C_i(p_i); i \in I$ of thermoelectric power units, which are usually described by quadratic functions of output power $p_i; i \in I$, respecting the constraints of meeting the system's total demand and the operational constraints of the generation units, as described in (28):

$$\begin{aligned} &\text{Min } CT = \sum_{i \in I} C_i(p_i) \\ &\textit{subject to}: \\ &\quad \sum_{i \in I} p_i = D \\ &\quad p_i^{\min} \leq p_i \leq p_i^{\max}, \quad i \in I, \end{aligned} \tag{28}$$

where

$$C_i(p_i) = a_i^c p_i^2 + b_i^c p_i + c_i^c; \; i \in I. \tag{29}$$

CT	Total electricity generation cost;
$C_i(p_i)$	Generation cost of unit i;
a_i^c, b_i^c, c_i^c	Coefficients of the cost function (29);
p_i	Power output of unit i;
I	Set of thermoelectric generation units of the system;
$p_i^{\min}, p_i^{\max}$	Lower and upper operational constraints, respectively, of unit i;
D	System demand.

3.2 Environmental Dispatch Problem

According to Gent and Lamont (1971), for a long time, the optimal operation of thermoelectric power generation considered only economic criteria while disregarding any environmental criteria. In countries that use predominantly fossil fuel, the most serious impacts caused by electricity generation are gas emissions into the atmosphere as a byproduct of combustion. The dispatch strategy aimed at minimizing these emissions rather than at achieving the goal of reducing thermoelectric generation costs is known as environmental dispatch.

The optimization model for environmental dispatch proposed by EL-Hawary et al. (1992), which aims to minimize the sum of the costs of pollutant emissions $E_i(p_i); i \in I$ of the system, is given in (30):

$$\begin{aligned}&\text{Min } ET = \sum_{i\in I} E_i(p_i)\\&subject\ to:\\&\quad \sum_{i\in I} p_i = D\\&\quad p_i^{\text{Min}} \le p_i \le p_i^{Max}, \quad i \in I,\end{aligned} \tag{30}$$

where

$$E_i(p_i) = a_i^e p_i^2 + b_i^e p_i + c_i^e;\ i \in I. \tag{31}$$

ET Total cost of pollutant emissions;
$E_i(p_i)$ Cost of pollutant emissions of unit i;
$a_i^e,\ b_i^e, c_i^e$ Coefficients of the cost function (29).

A more general economic/environmental dispatch problem, which involves the conflicting goals of EDP and EnDP, is described as a multiobjective problem in the next section.

3.3 Economic/Environmental Dispatch Problem

The EEDP is formulated as a multiobjective optimization problem which involves the minimization of two conflicting objectives: the total cost of generation *CT* and the total cost of pollutant emissions *ET*, subject to the operational constraints and the constraints of meeting demand, as described in (32):

$$\begin{aligned}&\text{Min } \left\{\sum_{i\in I} C_i(p_i), \sum_{i\in I} E_i(p_i)\right\}\\&subject\ to:\\&\quad \sum_{i\in I} p_i = D\\&\quad p_i^{\min} \le p_i \le p_i^{\max}, i \in I.\end{aligned} \tag{32}$$

Since the minimization of power generation costs and emissions costs are conflicting objectives, which cannot be minimized simultaneously, strategies have been proposed for finding efficient solutions for the problem, such as the weighted sum and ε-constraint strategies, which are classically used for solving multiobjective problems. The use of these problem-solving strategies (29) enables it to be redefined as a set of single-objective problems. In this study, the two approaches are investigated in order to find effective solutions for the EEDP. These approaches are described below.

3.4 Weighted Sum Strategy

The weighted sum strategy defined by Miettinen (1999) considers as the objective function of the EEDP a weighted sum of the economic and environmental objective functions. Using this strategy, one obtains the set of single-objective economic/environmental dispatch problems given in (33):

$$\begin{aligned}&\text{Min } \beta \sum_{i\in I} C_i(p_i) + (1-\beta) \sum_{i\in I} E_i(p_i)\\&subject\ to:\\&\quad \sum_{i\in I} p_i = D\\&\quad p_i^{\text{Min}} \le p_i \le p_i^{\text{Max}}, \quad i \in I.\\&\quad \beta \in [0, 1]\end{aligned} \tag{33}$$

The EEDP (33) modeled by means of the weighted sum strategy of economic/environmental dispatch (30) is solved for successive values of $\beta \in [0, 1]$. For each value of β, a single-objective subproblem is solved, determining an optimal solution of this subproblem. A sequence of solutions obtained for each value of β determines the solutions, efficient or not, dominated in the EEDP (33), through which it is possible to trace the Pareto-optimal curve of the problem.

3.5 The ε-Constraint Strategy

The ε-constraint strategy was suggested by Haimes et al. (1971) and defined in Miettinen (1999). The basic idea is to keep one of the objective functions, environmental, or economic, as an additional constraint of the dispatch problem, economic, or environmental, bounded above by a value that represents the maximum pollutant emissions or the maximum allowable cost. In this article, we will consider an economic dispatch problem with a maximum emission constraint, modeled as described in (34):

$$\begin{aligned}&\text{Min } \sum_{i\in I} C_i(p_i)\\&subject\ to:\\&\quad \sum_{i\in I} p_i = D\\&\quad p_i^{\min} \le p_i \le p_i^{\max}, \quad i \in I\\&\quad E_i(p_i) \le E_i^{\max}, \quad i \in I,\end{aligned} \tag{34}$$

where

$E_i^{\max}$ Maximum emission cost permissible for each power generation unit i.

The method developed in Sect. 2 is applied to the problem (34) using the modified barrier procedure, which will allow it to be initialized with unfeasible solutions. The next section describes in detail the method's application and its results.

4 Application and Results

4.1 System Data

The PCPDMLB method presented in Sect. 2 is applied to a multiobjective environmental/economic dispatch problem with 40 generation units, using the weighted sum and ε-constraint problem-solving strategies. The algorithm pre-

Table 1 Coefficients of the generation and emission cost functions and operational constraints of the system

i	Cost of generation			Cost of pollutant emissions			$p_i^{\min}$	$p_i^{\max}$
	a_i^c	b_i^c	c_i^c	a_i^e	b_i^e	c_i^e		
1	0,0069	6,73	94,705	0,0057	0,033	7,248	36	114
2	0,0069	6,73	94,705	0,0046	0,0458	19,834	36	114
3	0,02028	7,07	309,54	0,0025	0,0469	18,317	60	120
4	0,00942	8,18	369,03	0,0028	−0,0446	19,22	60	190
5	0,0114	5,35	148,89	0,0058	0,0008	10,18	47	97
6	0,01142	8,05	222,33	0,0053	0,0481	14,774	68	140
7	0,00357	6,99	278,71	0,0052	0,0167	6,007	110	300
8	0,00492	6,6	391,98	0,0056	0,0478	17,934	135	300
9	0,00573	6,6	455,76	0,0057	0,0499	14,468	135	300
10	0,00605	12,9	722,82	0,0052	0,0411	17,984	130	300
11	0,00515	12,9	635,2	0,0033	−0,0553	11,002	94	375
12	0,00569	12,8	654,69	0,0059	0,0281	21,727	94	375
13	0,00421	12,5	913,4	0,0047	0,01	16,742	125	500
14	0,00752	8,84	1760,4	0,0047	−0,0319	5,492	125	500
15	0,00708	9,15	1728,3	0,004	0,0498	17,754	125	500
16	0,00708	9,15	1728,3	0,0056	0,046	19,684	125	500
17	0,00313	7,97	647,85	0,0059	−0,0208	13,608	220	500
18	0,00313	7,97	649,69	0,0043	−0,0417	6,374	220	500
19	0,00313	7,97	647,83	0,0051	−0,0034	17,277	242	550
20	0,00313	7,97	647,81	0,0049	0,0463	6,81	242	550
21	0,00298	6,63	785,96	0,0024	0,0092	20,634	254	550
22	0,00298	6,63	785,96	0,004	0,0387	11,574	254	550
23	0,00284	6,66	794,53	0,005	0,0479	9,36	254	550
24	0,00284	6,66	794,53	0,0036	0,0462	19,848	254	550
25	0,00277	7,1	801,32	0,0027	0,0497	12,101	254	550
26	0,00277	7,1	801,32	0,0038	0,0356	18,162	254	550
27	0,52124	3,33	1055,1	0,0056	0,0054	21,305	10	150
28	0,52124	3,33	1055,1	0,006	0,0088	18,734	10	150
29	0,52124	3,33	1055,1	0,0025	0,0472	19,399	10	150
30	0,0114	5,35	148,89	0,0024	−0,0435	14,765	47	97
31	0,0016	6,43	222,92	0,0029	0,0491	5,914	60	190
32	0,0016	6,43	222,92	0,0049	−0,0328	7,28	60	190
33	0,0016	6,43	222,92	0,0051	0,0311	7,546	60	190
34	0,0001	8,95	107,87	0,0042	−0,0313	20,767	90	200
35	0,0001	8,62	116,58	0,005	0,0069	22	90	200
36	0,0001	8,62	116,58	0,006	−0,0009	9,143	90	200
37	0,0161	5,88	307,45	0,0058	0,03	7,102	25	110
38	0,0161	5,88	307,45	0,0022	0,0423	11,21	25	110
39	0,0161	5,88	307,45	0,0056	0,0327	11,206	25	110
40	0,00313	7,97	647,83	0,0026	−0,0408	6,195	242	550

sented in Sect. 2.6 was implemented in C++ language and executed using the data of the problem listed in Table 1.

The coefficients a_i^c, b_i^c, c_i^c of the economic function (generation costs) and the operational constraints of the system under study were taken from Coelho and Mariani (2006) and Arantes et al. (2006). The coefficients of the pollutant emissions cost function a_i^e, b_i^e, c_i^e, which are not given in these references, were generated randomly based on the relation between the environmental and economic coefficients for a system involving six generation units described by Souza

(2010). The operational constraints and coefficients of the economic and environmental functions used in this application are also given in Table 1.

The energy demand D of the system considered here is 10,500 MW. The values applied to initialize the algorithm described in Sect. 2.6 are presented below. The initial point p^0, given below and used in all the computational tests of this study, was extracted from Coelho and Mariani (2006), and corresponds to a feasible point of the economic dispatch problem, without considering the environmental dispatch function.

$p^{(0)}$=(113,997453; 113,626347; 97,399937; 179,733101; 90,494299; 105,400153; 259,599877; 299,9; 284,601078; 204,799816; 94,1; 94,2; 214,759791; 394,279373; 394,279398; 394,279381; 489,279397; 489,27939; 511,279371; 511,279371; 523,27939; 523,279437; 523,279474; 523,279398; 523,279375; 523,27937; 10,1; 10,1; 10,1; 88,297938; 189,9; 189,9; 189,9; 164,88839; 164,812509; 199,9; 91,371556; 93,306261; 109,9; 511,279371).

For the stop criterion of the method, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.01$ was chosen. The following initial values were adopted for the dual variables: $\lambda_0^0 = 0$; $(\lambda_1^0)_i = 10^{-6}$; $i = (1, 2, 3, \ldots, 40)$; $(\lambda_2^0)_i = 1$; $i = (1, 2, 3, \ldots, 40)$; $(\lambda_3^0)_i = 10^{-6}$, $i = (1, 2, 3, \ldots, 40)$.

4.2 Results—Weighted Sum Strategy

The PCPDLB method presented in Sect. 2.7 was applied to the solution of the EEDP (33) with 40 generation units. The system data are described in Sect. 4.1.

In the WS problem (33), for each discrete value adopted for the interval [0,1], a point is obtained on the Pareto-optimal curve, which relates the values of CT and ET. $\beta = 0$ implies that the emission is zero, while $\beta = 1$ implies that the generation cost is nil. For these extreme values of β, the environmental function ET varies from 14.506,046 ($\beta = 0$) to 18.012,24 ($\beta = 1$). Seven values were chosen for the construction of the Pareto-optimal curve: $\beta_{(j)}$, $j = 1, \ldots, 7, :$ $\beta_{(1)} = 0$, $\beta_{(2)} = 0, 02$ $\beta_{(3)} = 0, 06$, $\beta_{(4)} = 0, 16$, $\beta_{(5)} = 0, 24$, $\beta_{(7)} = 0, 33e$, and $\beta_{(7)} = 1$. The values obtained for $ET^{(j)}$ and $CT^{(j)}$, respectively, are listed in Table 1 in [\$] (standard monetary unit). Figure 1 illustrates the curve of the efficient solutions obtained.

4.3 Results–ε-Constraint Strategy

To implement the ε-constraint strategy to the EEDP described in (34), it is necessary to establish the maximum allowable values of the cost of pollutant emissions $E_i^{\max}$, $i \in I$. To obtain the Pareto-optimal curve using the ε-constraint strategy, problem (34) was solved for 7 sets of values of $E_i^{\max}$, $i \in I$. These values were chosen by applying the

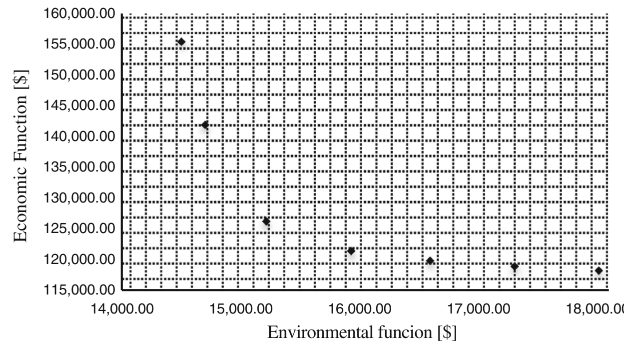

Fig. 1 Pareto-optimal curve obtained by using the weighted sum strategy

PCPDLB method to the solution of the problems (7 situations) of the weighted sum studied in Section 4.2. To calculate these values, the powers p_i, i ε I obtained by applying the PCPDLB methods to the 7 situations under study were substituted in the emission cost function (31), thus calculating the value of E_i, i ε I for each situation. The $E_i^{\max}$ boundaries for each of the seven cases were calculated considering the integer value closest to the values of E_i, i ε I obtained in each of the situations.

Table 4 lists the values obtained in [\$] (standard monetary unit) for each set of $E_i^{\max}$; $i \in I$, considering the β values used in each of the 7 situations in question.

For the initial point $p^{(0)}$, the respective emission cost values can be calculated by substituting $p^{(0)}$ in the emission cost function (31), and obtaining the vector $E^{(0)}$, shown below:

$E^{(0)}$ = (85,05; 84,033; 46,601; 101,65; 57,749; 78,722; 360,78; 535,93; 490,35; 244,5; 35,01; 76,728; 235,66; 723,55; 659,21; 908,37; 1415,8; 1015,3; 1348,7; 1311,3; 682,6; 1127,1; 1403,5; 1029,7; 777,42; 1077,3; 21,93; 19,434; 20,13; 29,635; 119,8; 177,75; 197,36; 129,79; 159,07; 248,72; 58,265; 34,31; 82,436; 664,99).

The components i in which $E_i^{(0)} > E_i^{\max}$ characterize infeasibility at the initial point of the problem, and are highlighted in bold in Table 4 . For each case in which there was infeasibility, an initial value was stipulated for the modified barrier parameter μ of the PCPDMLB method, which relaxes and expands the feasible region of the problem, considering the violated constraints. The μ values adopted in each of the 7 situations are given in Table 3.

The PCPDMLB method initially seeks to find feasible solutions starting from the expansions in the feasible regions, and then to determine efficient solutions aimed at optimizing the generation costs for a set of maximum emission constraints considered, respecting the demand constraints and operational constraints.

Table 4 shows the results obtained from the application of the PCPDMLB method to the EEDP given in (34). The values of ET and CT are given in [\$].

Table 2 Results of the weighted sum strategy

j	$\beta_{(j)}$	$ET^{(j)}$	$CT^{(j)}$	No. of iter.
0	0	14.506,05	155.409,77	21
1	0,02	14.702,97	141.917,20	29
2	0,06	15.220,54	126.240,83	25
3	0,16	15.932,26	121.389,82	33
4	0,24	16.597,23	119.860,60	31
5	0,33	17.307,61	118.950,55	25
6	1	18.012,24	118.280,141	26

Table 3 Results of the ε-constraint strategy

j	μ	ET	CT	No. of iter.
1	1000	14.511,47	154.938,77	44
2	950	14.711,18	141.496,30	40
3	880	15.230,38	126.070,44	36
4	750	15.939,42	121.338,66	46
5	580	16.619,47	119.788,53	41
6	460	17.312,92	118.917,44	36
7	520	18.012,24	118.280,14	26

The number of iterations in each application is influenced by the initial solution, which is the same in all the cases. The initial solution may be closer to one solution than to another. By comparing Tables 2 and 4, we observe that the method PCPDMLB (used to solve the EEDP through the ε-C strategy) takes more iteration steps than the PCPDLB (used to solve the EEDP through the WS strategy). This is explained by the fact that the initial point, which is the same used for both methods, is infeasible for PCPDMLB (as shown in Table 3), but is feasible for PCPDLB. Therefore, PCPDMLB has to work on the expanded feasible region, implying in a larger number of iterations for finding the optimal solution. Figure 2 illustrates the curve of efficient solutions calculated by the PCPDMLB.

The PCPDMLB method was able to find efficient solutions, even with initially much higher environmental constraints than the environmental limit (the largest difference reached a value close to 1,000).

The modified barrier strategy was necessary and important for the resolution of the problem (27), considering the 7 cases of the present application, when it was necessary to give the initial modified barrier parameter μ^0 values that varied from 520 to 1,000. Even so, the PCPDMLB method determined efficient solutions for this problem, with a computational time in the order of milliseconds.

Table 4 Values adopted for $E_i^{\max}$; $i \in I$ in each of the 7 situations studied here

i	Values of β						
	0	0.02	0.06	0.16	0.24	0.33	1
1	86	86	86	86	86	86	86
2	85	85	85	85	85	85	85
3	60	60	60	60	60	60	60
4	112	112	112	112	112	112	112
5	65	65	65	65	65	65	65
6	126	126	126	126	126	126	126
7	479	479	479	479	479	479	480
8	**472**	**512**	537	537	537	537	537
9	**460**	497	543	543	543	543	543
10	498	469	397	**214**	**123**	**112**	**112**
11	455	455	455	300	157	63	35
12	458	427	369	213	131	77	77
13	571	537	473	288	**187**	**103**	**92**
14	**574**	**573**	**573**	**522**	**508**	**500**	**369**
15	652	644	631	**547**	**514**	**488**	**291**
16	**474**	**477**	**485**	**459**	**459**	**465**	**396**
17	**463**	**493**	**557**	**688**	**847**	**1,076**	1,479
18	**631**	**666**	**742**	**879**	1,041	1,061	1,061
19	**532**	**564**	**634**	**771**	**934**	**1,166**	1,559
20	**526**	**559**	**630**	**769**	**934**	**1,167**	1,515
21	752	752	752	752	752	752	752
22	**651**	**710**	**841**	1,140	1,243	1,243	1,243
23	**518**	**569**	**683**	**960**	**1,281**	1,548	1,549
24	**726**	**791**	**931**	1,134	1,135	1,135	1,135
25	856	857	857	857	857	857	857
26	**692**	**747**	**867**	1,127	1,188	1,188	1,188
27	149	89	39	27	25	24	23
28	156	88	37	25	23	22	21
29	83	70	31	23	22	22	21
30	34	34	34	34	34	34	34
31	120	120	120	120	120	120	120
32	178	178	178	178	178	178	178
33	198	198	198	198	198	198	198
34	183	183	183	183	183	183	183
35	224	224	224	224	224	224	224
36	249	249	249	249	249	249	249
37	81	81	81	81	81	81	81
38	43	43	43	43	43	43	43
39	83	83	83	83	83	83	83
40	771	771	771	771	771	771	771

5 Conclusions

The search of efficient solutions for the multiobjective EEDP has been investigated in the literature by means of IPM applied to sets of single-objective problems which are generated by the WS strategy or by the ε-Constraint (ε-C) strategy. However, in all such studies presented in the literature, some preprocessing or specific heuristics are necessary in order to

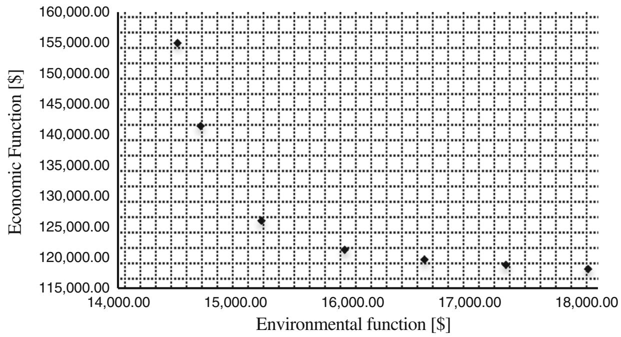

Fig. 2 Pareto-optimal curve obtained by using the ε-constraint strategy

find a feasible initial point for the IPMs, since those methods generally work in the interior of the feasible region.

This paper investigates WS and ε-C strategies applied for finding efficient solutions for the multiobjective EEDP, via PCPDMLB method. The main contribution of PCPDMLB method in this context is that it is capable of handling with unfeasible points, thus avoiding preprocessing or specific heuristics for finding a feasible initial point. This is accomplished by the modified log-barrier function approach, defined by Polyak (1992). This approach allows one to initialize the problem with unfeasible points by defining a relaxed region. The PCPDMLB here proposed also considers a centering procedure that exploits the barrier parameter in the predictor and corrector steps to calculate the search directions, preventing the exterior points from being projected outside the relaxed region. The sparse structure of the search direction system is also exploited in the predictor and corrector steps of the method proposed. The modified log-barrier procedure influences the determination of the search directions of the proposed method, differentiating it from those normally applied in classical predictor-corrector primal-dual interior point methods.

The results obtained demonstrate the efficiency of the PCPDMLB method when applied to a test case of 40 power generation units. Without the modified log-barrier procedure, it would be necessary, for solving the set of problems generated by the ε-constraint strategy, to find a solution that would satisfy 40 initial quadratic constraints associated with the pollutant emission.

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