A natural cosmological constant from chameleons

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A R T I C L E   I N F O

A B S T R A C T

We present a simple model where the effective cosmological constant appears from chameleon scalar fields. For a Kachru–Kallosh–Linde–Trivedi (KKLT)-inspired form of the potential and a particular chameleon coupling to the local density, patches of approximately constant scalar field potential cluster around regions of matter with density above a certain value, generating the effect of a cosmological constant on large scales. This construction addresses both the cosmological constant problem (why Λ is so small, yet nonzero) and the coincidence problem (why Λ is comparable to the matter density now).

1. Introduction

The cosmological constant problem is one of the most challenging problems in theoretical physics today. Indeed, the problem is twofold. Firstly the observed cosmological constant today is about 123 orders of magnitude lower than the natural value implied by quantum loop corrections, the uppermost cut-off for effective field theory, namely the Planck scale. In the days before the observation of the cosmological constant, when it was thought to be zero, the problem was easier, since some kind of yet undiscovered symmetry could perhaps force it to be zero. Supersymmetry for instance alleviates the problem a bit, since in exact global supersymmetry $\Lambda = 0$, so broken supersymmetry requires that the cosmological constant be of the order of the supersymmetry breaking scale, instead of the Planck scale. Moreover unbroken supergravity (local supersymmetry) requires that any cosmological constant be negative.

The observation of a non-zero, positive cosmological constant dashed these hopes, and introduced yet a second more philosophical puzzle, the coincidence problem: why is the cosmological constant (which should be constant for all times) of the order of the matter density today, when cosmologically there is nothing particularly special about the moment in time we happen to live in. Experimentally, ρ_Λ is about twice the density of dark matter today. In Copernican terms, we appear to live at a special time in the history of the Universe when ρ_Λ, ρ_{DM} and ρ_{matter} are all comparable.

It is then perhaps natural to think of a dynamical scalar field which happens now to be in a region of the potential which is almost constant but has a very small value (though explaining that very small value is not easy), an idea known generically as quintessence. Yet such a scalar must be very light, and there are very strong constraints on light scalars from gravity, as these would generate an, as of yet unobserved, fifth force. Chameleon scalars [1,2] were introduced as a way to avoid those constraints: they are scalars whose mass depends on the local matter density, so on Earth the fields are very massive, avoiding laboratory gravity experimental constraints, as well as those from lunar laser ranging etc. (see [2] for further discussions). The coupling of chameleons with the local density takes the form

$$V_{eff}(\phi) = V(\phi) + \rho A(\phi) \tag{1.1}$$

and we will see that rather generally the coupling function $A(\phi)$ can be written as

$$A(\phi) = e^{g \frac{\phi}{M_{Pl}}} \tag{1.2}$$

where M_{Pl} is the reduced Planck mass and g is the coupling between the scalar ϕ and the matter type in the energy density ρ. This generic form makes contact with Brans–Dicke theories, and g is defined by the small ϕ limit as the coupling; it is the form usually assumed in chameleon theories. On the other hand, on planetary and Solar System scales, the scalar force is suppressed due to the fact that the scalar profile only varies within a thin shell inside large bodies, hence only the mass within this thin

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shell effectively interacts via the scalar fifth force (see [1–3] for more details). This leaves the possibility that the chameleon scalars have some interesting behaviour on lower density scales such as vacuum [4,5], space [2] (within the Solar System: outside the atmosphere, yet nonzero density) and on cosmological scales.

Generically, quintessence is strongly constrained by both the allowed variation of the masses of fundamental particles and the low mass required of a quintessence field to drive dark energy [6] (see [7] for an analysis within the context of the chameleon). Theoretically as well, one would still need to explain why the value of the potential at the minimum is so small, which is non-trivial.

In this paper we present a new approach to the cosmological constant problem, based on the chameleon scalar idea. The potential for the chameleon scalar that we will choose is phenomenologically motivated, based on the KKLT-inspired models used in [3,8,9].

However, unlike [3], when the coupling g between the chameleon and the local matter density was a fixed number of order one given by string theory, we will allow an arbitrary value for g, and find that we need a very large value for the coupling in our model, which is allowed in chameleon models, see [10]. The potential has a minimum which we set exactly at $V = 0$.

Within this model, we will find that the value for the scalar field potential is approximately constant near concentrations of matter (with density greater than a minimum density), and the fact that the value of this potential is so small and comparable with the density of matter arises simply from the condition of minimization of the effective potential (1.1).[1] In this way we translate both the issue of the smallness of the cosmological constant and the coincidence problem ($\rho_\Lambda \sim \rho_{DM}$) into just choosing the shape of the potential, which we argue is quite natural. Of course, we still have the old cosmological constant problem: why don't quantum corrections affect the $V = 0$ value of the minimum?

2. The model

The universal coupling to the matter density in (1.1) appears because the metric that couples universally to matter is not the Einstein frame metric $g_{\mu\nu}$, but the metric

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}. \tag{2.1}$$

Such a situation appears naturally in Kaluza–Klein (KK) compactifications, when the relation between the higher dimensional metric ds_D^2 and the lower dimensional metric ds_d^2 is of the type

$$ds_D^2 = R^2(\phi) ds_d^2 + g_{mn} dx^m dx^n + \dots$$
$$\equiv g_{MN} dx^M dx^N$$
$$ds_d^2 = g_{\mu\nu} dx^\mu dx^\nu$$
$$R = \Delta^{-\frac{1}{d-2}}; \quad \Delta = \sqrt{\det g_{mn}}, \tag{2.2}$$

so R is a modulus for the volume of compactification. If matter couples naturally to the D-dimensional metric ds_D^2, we obtain

$$A(\phi) = \frac{R(\phi)}{R_*}, \tag{2.3}$$

where R_* is a particular value for R, close to the average value of R in the Universe, to be defined shortly. The exact form of the kinetic term for R depends on the details of the compactification,

but in general it is such that it leads to (1.2), therefore the canonical scalar is

$$\phi = \frac{M_{\rm Pl}}{g} \ln \frac{R}{R_*}. \tag{2.4}$$

The mechanism described here depends only on having (1.2) and (2.3), not on the fact that R is obtained from KK compactification as the volume modulus, but we can use the KK compactification ansatz to motivate the form of the potential $V(R)$. Indeed the volume of the extra dimensions must be stabilized at some value, therefore we can approximate the potential by a quadratic

$$V(R) = M_{\rm Pl}^4 \left[-\alpha(R - R_*) + \beta(R - R_*)^2 + \frac{\alpha^2}{4\beta} \right], \tag{2.5}$$

where $\alpha, \beta > 0$ are both dimensionless, around the minimum at

$$R_{\rm min} = R_* + \frac{\alpha}{2\beta}. \tag{2.6}$$

The constant in the potential (2.5) was chosen such that the minimum is at $V(R_{\rm min}) = 0$, and the value R_* was introduced such that (2.5) is valid only for $R > R_*$. For $R < R_*$, we assume that the potential is well approximated by a very steep exponential,

$$V(R) = M_{\rm Pl}^4 v \left\{ \left[e^{\gamma(R^{-k} - R_*^{-k})} - 1 \right] + \frac{\alpha^2}{4v\beta} \right\}, \tag{2.7}$$

with $\gamma, k > 0$ and $v > 0$ all dimensionless. This form is for instance the leading exponential (at small R) arising from the KKLT-like potential generated by a superpotential $W = W_0 + A e^{-ia\varrho}$, with $a < 0$ instead of KKLT's $a > 0$, as explained in [3], so it is a rather natural possibility. Here $\varrho = i\sigma$ has an imaginary part σ related to the scalar R as $\sigma = R^{-2}\sqrt{\pi}$ for $n \leq 4$ large extra dimensions, and $\sigma = R^{-8/n}\sqrt{\pi}$ for $n \geq 4$. $a < 0$ can be achieved even in the context of KKLT by adding gluino condensation on a D9-brane with magnetic flux [12], as well as being needed in a more general context because of T-duality on a gaugino condensation potential [13]. The parameter v is included here to allow us to fix the value of $\frac{dV}{dR(R_*)}$ for $R < R_*$ independently of R_*.

Minimizing the effective potential (1.1), one finds

$$R_* \frac{dV}{dR}(R) = -\rho(R), \tag{2.8}$$

so for $R < R_*$, in the steep exponential side of the potential, using the fact that $R/R_* \simeq 1$, we find

$$\frac{\rho(R)}{\rho_*} \simeq e^{\gamma(R^{-k} - R_*^{-k})} = \frac{V(\rho) + vM_{\rm Pl}^4 - V_0}{vM_{\rm Pl}^4} \tag{2.9}$$

where we have used the implicit dependence $\rho(R)$ to denote by $V(\rho)$ the potential at the minimum value of the effective potential, and we have denoted by $\rho_* = \rho(R_*)$, i.e. the minimum density so that we are in the region $R \leq R_*$. We then have

$$V(\rho) \simeq vM_{\rm Pl}^4 \frac{\rho - \rho_*}{\rho_*} + V_0 \tag{2.10}$$

where $V_0 = V(R_*) = M_{\rm Pl}^4 \alpha^2/4\beta$.

At $R = R_*$, we can equate both the value of the potential, and of the derivative in (2.5) and in (2.7). Using the potential for $R > R_*$, we have

$$\frac{\rho_*}{R_*} = -\frac{dV}{dR}(R_*) = \alpha M_{\rm Pl}^4 \tag{2.11}$$

leading to

$$V_0 = M_{\rm Pl}^4 \frac{\alpha^2}{4\beta} = \frac{\rho_*}{4} \frac{\alpha}{\beta R_*}. \tag{2.12}$$

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[1] Note that within chameleon-type models it was argued that the chameleon, stabilized at the (*average*) ρ-dependent minimum of V_{eff}, acts as a quintessence field, see for instance [11] for an early example in supergravity. This is not what we propose here; here the dark energy is approximately *constant in time*, yet lumped around matter distributions.

Using the potential for $R < R_*$ on the other hand, we obtain

$$\frac{\rho_*}{R_*} = -\frac{dV}{dR}(R_*) = v M_{\text{Pl}}^4 \gamma k R_*^{-k-1} \Rightarrow v = \frac{\rho_*}{M_{\text{Pl}}^4 \gamma k R_*^{-k}} \qquad (2.13)$$

We can now input experimental constraints on the parameters. In [3], a constraint on $\gamma k R_*^{-k}$ was found from Earth laboratory experiments. The constraint was given for $g \sim \mathcal{O}(1)$, but we now write it for general g, as

$$\frac{R_*^k}{\gamma k} \left[\log_{10}\left(\frac{\gamma k}{R_*^k}\right) - 24 + \log_{10} g^2 \right] \lesssim g 10^{-29}. \qquad (2.14)$$

On the other hand, from the condition that the Milky Way Galaxy be screened (it has a thin shell),

$$\left(\frac{3\Delta\mathcal{R}}{\mathcal{R}}\right)_{\text{G}} = \frac{\phi_{\text{cosmo}} - \phi_{\text{solar system}}}{2g M_{\text{Pl}} \Phi_{\text{G}}} < 1, \qquad (2.15)$$

where $\Phi_{\text{G}} \sim 10^{-6}$ is the Newtonian potential of the galaxy, ϕ_{cosmo} and ϕ_{solar} are the values of the scalar field on cosmological and solar system scales respectively, related to R as in (2.4) and $\mathcal{R}$ is the radius of the galaxy. We find that

$$\ln \frac{R_{min}}{R_*} \lesssim 2g^2 10^{-6} \qquad (2.16)$$

Consider now the case $g \sim c \times 10^3$, with $c =$ a few, then (2.14) becomes

$$\gamma k R_*^{-k} \gtrsim g^{-1} 10^{30} = c^{-1} 10^{27} \qquad (2.17)$$

and (2.16) becomes

$$\frac{\alpha}{\beta R_*} \lesssim 4g^2 10^{-6} \sim 4c^2 \qquad (2.18)$$

We finally get

$$V_0 \lesssim c^2 \rho_*; \quad v \lesssim \frac{c \times 10^{-27} \rho_*}{M_{\text{Pl}}^4} \qquad (2.19)$$

leading to a potential in the $R < R_*$ – i.e., $\rho > \rho_*$ – region (in the case we are close to saturating the bounds)

$$V \simeq b \times 10^{-27}(\rho - \rho_*) + d \rho_*, \qquad (2.20)$$

with $b \sim$ a few, and $d \lesssim c^2 \sim$ a few, to be constrained better from experiments shortly. Note that the value of b is irrelevant, all that matters is that changing ρ has almost no effect on the value of V for $\rho > \rho_*$, which stays close to $d\rho_*$. Finally, note that in order to have a consistent picture, there must be regions in the Universe which are on the quadratic piece of the potential, and that requires $\rho_* > \rho_\Lambda$. Eq. (2.11) (and (2.13), (2.20) in the last inequality) then implies that

$$\alpha > \frac{\rho_\Lambda}{M_{\text{Pl}}^4 R_*} \sim \frac{10^{-122}}{R_*} \gtrsim \frac{1}{(c\gamma k)^{1/k}} 10^{-122+27/k} \qquad (2.21)$$

We now look to understand the consequences of the potential (2.20). Consider the case where $\rho_{\text{galaxy cluster}} \sim$ a few ρ_* (where galaxy clusters were chosen as the largest matter structure, and their density means the density of the dark and normal matter inside them), which means that the chameleon inside galaxy clusters is in the $R < R_*$ region, but a bit away from R_*, and moreover that

$$V(R \lesssim R_*) \simeq V(R_*) = d \rho_* \sim \rho_{\text{galaxy cluster}}. \qquad (2.22)$$

More precisely, consider that

$$\frac{d \rho_*}{\rho_{\text{galaxy cluster}}} = \frac{\Omega_\Lambda}{\Omega_{matter}} \simeq \frac{72\%}{28\%} \simeq 2.5 \qquad (2.23)$$

where the right hand side is the experimental value (and matter includes dark and normal matter).[2] Then outside the galaxy clusters, where the density falls to almost zero, the chameleon drops down to $R = R_{\text{min}}$, where $V(R_{min}) = 0$. In this way, the chameleon creates a potential V that is almost constant in regions with matter (independent of the distribution of matter inside the patch), and almost zero outside. If the ratio to the matter density is taken as in (2.23), this creates an extra energy contribution that accounts for the energy of the observed cosmological constant.

Then, as the Universe expands, $\rho_{\text{galaxy cluster}}$ drops due to the Hubble expansion (the volume of the galaxy cluster expands[3]), but as long as we are still in the $R < R_*$ region, we still have

$$V(\rho) \simeq d \rho_* \qquad (2.24)$$

therefore the value of this extra energy contribution is constant in time, i.e. it is *effectively a cosmological constant*.[4] One can ask, is the effect of light propagation through this set of patches of cosmological constant the same as through an average cosmological constant, when considered at very large scales? It is a nontrivial question, but one can ask exactly the same one in the case of the $\Lambda = 0$ FRW model, since the matter distribution is also in reality not constant, but sharply peaked. The usual answer is that *on the average* we have a constant density, on exactly the same scales as for our patchwise Λ, so one can consider the average density. It is definitely something to check, but it is very difficult, and since it is exactly the same situation in the case of the matter density which almost everyone takes for granted, this analysis goes beyond the scope of this paper. If the effect on light propagation on very large scales is the same as the effect of a cosmological constant, the acceleration of the Universe observed in type IA supernovas is the same.

In the past of the current era, when we had no galaxies, but rather a constant density distribution, we had effectively a cosmological constant, since the potential $V(\rho) \simeq d\rho_*$ everywhere, as $\rho > \rho_*$.[5] In particular, that would be true of the time of the last scattering, so for the CMB, we effectively had a cosmological constant, and nothing is changed. The same would be true for the baryon acoustic oscillations. For the light propagation during the current epoch, with galaxies, the same comment as for the supernovae case above would apply. Going back in time beyond the Big Bang Nucleosynthesis (BBN) would depend on particular cosmological models and models for the chameleon potential away from the

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[2] Since $\rho_{\text{galaxy cluster}} \sim 10^2 \rho_\Lambda$, we obtain that $d \rho_* \sim 2 \times 10^2 \rho_\Lambda$.

[3] Of course, the region in a cluster where the density exceeds ρ_Λ is decoupled from the expansion, and with an even larger density it is virialized and actually contracts. But the region (shell) where ρ drops to about ρ_Λ should expand with the Hubble expansion, and inside it, the density is constant, and equal to $d\rho_*$ (with $d \sim c^2$). We can then choose this value of ρ that expands with the Hubble expansion as being ρ_Λ. It is left for future work to check that ρ_* is of order ρ_Λ or a bit larger numerically. We have a self-consistent solution: We have a cosmological constant due to the averaging over patches of constant energy density, expanding with the Hubble expansion. In turn, the patches expand with the Hubble expansion because the shell where $\rho = \rho_*$ expands due to the presence of ρ_Λ.

[4] Note that we have then clusters of energy density constant in space and time, which should average into a cosmological constant in the same way as patches of matter, increasingly localized on smaller scales (clusters of galaxies, galaxies, stars) average to the FRW solution. We hope to check this by numerical simulations in the future.

[5] Since $\rho_{\text{galaxy cluster}} \sim 10^2 \rho_\Lambda \sim$ a few ρ_*, it follows that $\rho_* \sim$ a few tens of $\rho_{\text{total,now}} \sim \rho_\Lambda$, so going back to the time when the density was almost uniform (or in fact to any z larger than about (a few tens)$^{1/3} \sim 3$–4, when $\rho_{\text{total}} \simeq \rho_*$), that total evolving density would have increased above ρ_*.

minimum. The chameleon could be for instance also the inflaton,[6] but in any case the observational constraint is that the chameleon must reach the minimum of the potential, and stabilize going up the steep exponential side, due to the chameleon coupling $A(\phi)\rho$ in the effective potential, *before* the time of the BBN. Then, after BBN we will already be at $\rho > \rho_*$, with $V(\rho) \simeq d\rho_*$.

We have simulated the effect of the cosmological constant with this chameleon field. Considering our original motivation, we can ask: is this construction *natural*, that is, was it natural to obtain a very small value for the observed cosmological constant, and was it natural to have a value for ρ_Λ so close to ρ_{matter} today? The smallness of the cosmological constant is related to the smallness of $\rho_* = R_* \alpha M_{\text{Pl}}^4$. In [3,8,9] it was shown that we can obtain a potential like the desired phenomenological potential from the KKLT construction with large extra dimensions, and the required values for R_* and α, β, v are all obtained due to the $e^{ia\varrho}$ exponentials in the superpotential and the large extra dimensions. So in that particular case, the naturalness of the small cosmological constant would be reduced to the naturalness of large extra dimensions. As an example, consider eq. 6.13 in [3], which says that $\rho_* = \alpha R_* M_{\text{Pl}}^4 \sim A^2 |a|^2 / M_{\text{Pl}}^2 e^{|a|(\sigma_* + \sigma_{\min})} \sim M_{\text{Pl}}^4 e^{|a|(\sigma_* + \sigma_{\min} - 2\sigma_0)}$, with $|a|\sigma = R^{-k}$ and A being the constant in the superpotential below eq. (2.7). We see that the smallness of ρ_* (or α) is reduced to the fact that the large extra dimensional volume $|a|\sigma$ appears in an exponent, and so its very small variations (between σ_*, $\sigma_{\min}$ and the constant σ_0) are much amplified. Of course, the model there was a string theoretic model, with $g \sim \mathcal{O}(1)$, whereas we want $g \sim c \times 10^3$, so we can't use it directly, but we just presented it as an example of the mechanism. The naturalness of the large extra dimensions itself is debated, but mechanisms like the warping used by Randall–Sundrum argue to make it natural.

As for the coincidence problem, an explanation for the fact that ρ_Λ is close to the matter density ρ_{matter} today is more nuanced. The potential is correlated to the matter energy density, $V = V(\rho)$, but is approximately constant, at the value $d\rho_*$, and the fact that this is close to ρ_{matter} today is still somewhat coincidental.

However, we also have another feature that is different from other constructions of an effective cosmological constant. Though it was well approximated by a cosmological constant until now, in the near future, this potential contribution will drop to zero. Indeed, once $\rho_{\text{galaxy cluster}}$ drops below ρ_*, (2.20) will not be valid anymore. In fact, asymptotically, as $\rho_{\text{galaxy cluster}}$ becomes very small, eventually R will settle at $R_{\min}$, with $V(R_{\min}) = 0$, i.e. with no dark energy at all. So from this point of view, the coincidence is less drastic: as soon as ρ_Λ becomes comparable to ρ_{matter} we can observe it, but it also means that we are close to the point where it will start disappearing. After that, we will still have some dark energy, though it will take the form of a decreasing potential energy contribution. Note that there should be other ways to distinguish between our V and a cosmological constant, in particular by focusing on the edge region of the patches of constant V, where V drops to zero, so one should see a difference in the motion of matter and the propagation of light. However, this is a complicated issue, that could be analyzed by numerical simulations, and as such falls outside the scope of this paper. We hope to come back to it in the future.

To close the discussion of the model, we will review some experimental constraints on the model which were derived in [3] for the case $g \sim \mathcal{O}(1)$, to apply in our case of $g \sim 10^3$. We already discussed (2.14) and (2.16). Putting together (2.18) and (2.21), we find

$$\sqrt{\beta} \gtrsim \frac{10^3}{2g} \frac{\sqrt{\rho_\Lambda}}{M_{\text{Pl}}^2 R_*} \tag{2.25}$$

and then from [3] the mass of the chameleon on the largest scales is (since $\rho_\Lambda = H_0^2 M_{\text{Pl}}^2$)

$$m_{\text{cosmo}} = \sqrt{2} g M_{\text{Pl}} \sqrt{\beta} R_* \gtrsim \frac{10^3}{\sqrt{2}} H_0, \tag{2.26}$$

independent of g. One thing which does change with respect to [3] is the range of the chameleon in various environments for $\rho > \rho_*$,

$$m \gtrsim 10^{15} g \frac{\sqrt{\rho}}{M_{\text{Pl}}} \tag{2.27}$$

leading to

$$m^{-1} \lesssim \frac{0.2 \text{ mm}}{g\sqrt{\rho[\text{g/cm}^3]}} = \frac{0.2 \text{ μm}}{c\sqrt{\rho[\text{g/cm}^3]}}, \tag{2.28}$$

i.e., a factor of g smaller range.

3. Conclusions

In this paper we have given an alternative to a simple cosmological constant based on chameleon scalars. Because of the chameleon coupling, a value for the chameleon potential energy as a function of the matter density was approximately constant above a certain value ρ_*, namely (2.20), but zero for sufficiently small ρ (close to vacuum). That means that for certain choices of parameters, we can have patches of approximately constant energy density around the largest matter structures, and zero outside them. The patches have constant energy in time, thus effectively simulating a cosmological constant, as long as the matter density of the largest matter structures stays above a certain value. Eventually, as matter will get diluted, the potential energy will start to drop, leaving no cosmological constant in the far future.

This mechanism generates a small cosmological constant despite the vacuum being at $V = 0$, thus alleviating the cosmological constant problem. The small value of the effective cosmological constant can be in principle obtained rather naturally, like in a large extra dimensions scenario. It would be interesting to see whether this scenario can be embedded in a consistent fundamental theory. We took the coupling g to be $\sim$ a few $\times 10^3$, though in string theory g is usually a fixed number of order 1.

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