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COINCIDENCE OF MAPS ON TORUS FIBRE BUNDLES OVER THE CIRCLE

João Peres Vieira

Abstract. The main purpose of this work is to study coincidences of fibre-preserving self-maps over the circle S^1 for spaces which are fibre bundles over S^1 and the fibre is the torus T. We classify all pairs of self-maps over S^1 which can be deformed fibrewise to a pair of coincidence free maps.

1. Introduction

Given a fibration $M \xrightarrow{p} S^1$ and fibre-preserving maps $f, g\colon M \to M$ over S^1, the question is if the pair (f, g) can be deformed by fibrewise homotopy over S^1 to a coincidence free pair (f', g').

This problem was motivated by the case in that $f = \mathrm{Id}$, and in this case, the question is if the map g can be deformed by fibrewise homotopy over S^1 to a fixed point free map g', which has been considered by many authors, among them see [4], [6], [8] and [9].

Let us consider fibre-preserving maps $f, g\colon M \to M$, where M is a fibre bundle over the circle S^1 and the fibre is a closed surface S. These fibre bundles are obtained from the space $S \times [0, 1]$ by identifying the points $(x, 0)$ with the points $(\phi(x), 1)$, where ϕ is a homeomorphism of the surface S. The cases when $f = \mathrm{Id}$ and the fibre S is either the torus T or the Klein bottle K, were completely solved in [8] and [9], respectively.

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In this work, we consider the fibre $S = T$. We denote the total space by $M(\phi)$. We investigate when given fibre-preserving maps $f, g\colon M(\phi) \to M(\phi)$ over S^1, i.e. $p \circ f = p$ and $p \circ g = p$, the pair (f, g) can be deformed by fibrewise homotopy over S^1 to a coincidence free pair (f', g').

The set of homotopy classes of the pairs (f, g) such that $(f|_T, g|_T)$ can be deformed to a coincidence free pair is given by Theorem 3.6.

This paper is organized into four sections. In Section 2 we prove that our problem is equivalent to the existence of a section. This is given by Theorem 2.2. We show that to find this section it is equivalent to find a lifting in an algebraic diagram. This is the Proposition 2.10. We also present some results on the torus T and fibre bundles over S^1 and fibre T. These results include the Nielsen number of a pair of maps of the torus and the fundamental group of the spaces $M(\phi)$, $M(\phi) \times_{S^1} M(\phi)$ and $M(\phi) \times_{S^1} M(\phi) \setminus \Delta$ where Δ is the diagonal in $M(\phi) \times_{S^1} M(\phi)$, which is the pullback of $p\colon M(\phi) \to M(\phi)$ by $p\colon M(\phi) \to M(\phi)$.

In Section 3 we classify all T-bundles over S^1. This is the Proposition 3.4. We also obtain a presentation for the fundamental groups of $M(\phi)$, $M(\phi) \times_{S^1} M(\phi)$ and $M(\phi) \times_{S^1} M(\phi) \setminus \Delta$.

In Section 4, we present a necessary and sufficient condition for the existence of the lifting in the diagram

$$\begin{tikzcd}
& \pi_1(\mathcal{F}) \simeq \pi_2(T, T \setminus 1) \arrow[d] \\
& \pi_1(E_{S^1}(M(\phi))) \simeq \pi_1(M(\phi) \times_{S^1} M(\phi) \setminus \Delta) \arrow[d, "q_\#"] \\
\pi_1(M(\phi)) \arrow[ur, dashed, "\psi"] \arrow[r, "(f,g)_\#"'] & \pi_1(M(\phi) \times_{S^1} M(\phi))
\end{tikzcd}$$

with base points suitable. These conditions are related to existence of solutions of a system of equations involving the presentation of the groups above.

In Section 5, we classify all the pairs of maps (f, g), which can be deformed, by a fibrewise homotopy over S^1, to a pair of coincidence free maps (f', g'), which is Theorem 5.1.

2. Preliminary and general results

2.1. Coincidence theory. Let $f, g\colon X \to Y$ be maps between finite CW-complexes. Denote by $\mathrm{Coin}(f, g) = \{x \in X \mid f(x) = g(x)\}$.

Suppose that x_1, x_2 are in $\mathrm{Coin}(f, g)$. Then we say that x_1, x_2 are Nielsen equivalent according to f and g if there exists a path $\sigma\colon [0, 1] \to X$ such that $\sigma(0) = x_1$, $\sigma(1) = x_2$ and $f \circ \sigma$ is homotopic to $g \circ \sigma$ relative to end points.

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